Project Proposal: Comparative Control Analysis of a 6-DOF Planar Quadrotor System

ECE515 - Control Systems Theory
Fall 2025

Motivation

The planar quadrotor represents a fundamental **underactuated system** where horizontal motion is achieved indirectly through attitude changes. This coupling between lateral position and pitch angle presents significant control challenges, making it an ideal platform to compare linear and nonlinear control techniques. Understanding the **limitations of linearization-based method**s versus exact feedback linearization provides crucial insights for real-world autonomous flight applications.

System Description

We consider a 6-DOF planar quadrotor with state vector $\mathbf{x} = [x, z, \theta, \dot{x}, \dot{z}, \dot{\theta}]^T$ and control inputs $\mathbf{u} = [F, M]^T$ (thrust and torque). The system dynamics are:

$$m\ddot{x} = -F\sin\theta, \quad m\ddot{z} = F\cos\theta - mg, \quad J\ddot{\theta} = M \tag{1}$$

In control-affine form: $\dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})\mathbf{u}$, where

$$f(\mathbf{x}) = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ 0 \\ -g \\ 0 \end{bmatrix}, \quad g(\mathbf{x}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{\sin x_3}{m} & 0 \\ \frac{\cos x_3}{m} & 0 \\ 0 & \frac{1}{J} \end{bmatrix}$$

Project Objectives

1. **Stability Analysis:** Use Lyapunov theory to prove that the unforced system with $\mathbf{u} = [mg, 0]^T$ is unstable around any hover equilibrium $\mathbf{x}_e = [d, h, 0, 0, 0, 0]^T$.

2. **LQR Control:** Linearize the system around hover equilibrium and design an LQR controller. Demonstrate stabilization performance and identify the limitations arising from linearization approximations (e.g., restricted region of attraction, performance degradation for large angles).

3. **Feedback Linearization:** Apply exact feedback linearization to transform the nonlinear system into a linear form without approximations. Design controllers for the linearized subsystems and demonstrate trajectory tracking capabilities.

4. **Comparative Analysis:** Compare LQR and feedback linearization approaches in terms of region of attraction, tracking performance, and robustness. Provide interactive MATLAB simulations for reproducibility.

Expected Outcomes

This project will provide a comprehensive comparison between linearization-based (LQR) and exact nonlinear (feedback linearization) control techniques. I expect to demonstrate that while LQR is effective near equilibrium, feedback linearization offers superior performance for large maneuvers and trajectory tracking. All results will be validated through MATLAB simulations with clear visualizations, making the project easily reproducible for evaluation purposes.