EECS 398 W25 Midterm Review

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Announcements

- The Final Exam is on **Monday, April 28th from 10AM-12PM**.
- Watch recording of Suraj Final Review.
- Study Tips
  - Go through lecture notebooks & homeworks to help make cheat sheet (one page, double-sided, **handwritten**).
  - Do discussion problems.
  - Take F24 Final.

Agenda

- We'll be working through https://study.practicaldsc.org/fi-review-saturday/index.html.
- We'll post these annotated slides and the recording after, along with enabling solutions on the study site for this worksheet.

Linear Regression - Angela

Problem 1.1

We want to use multiple regression to fit a prediction rule of the form

$$H(x_i^{(1)}, x_i^{(2)}, x_i^{(3)}) = w_0 + w_1 x_i^{(1)} x_i^{(3)} + w_2 (x_i^{(2)} - x_i^{(3)})^2.$$

Write down the design matrix X and observation vector $\vec{y}$ for this scenario. No justification needed.

Problem 1

Consider the dataset shown below.

$x^{(1)}$	$x^{(2)}$	$x^{(3)}$	y
0	6	8	-5
3	4	5	7
5	-1	-3	4
0	2	1	2

Linear Regression - Angela

Problem 1

Consider the dataset shown below.

$x^{(1)}$	$x^{(2)}$	$x^{(3)}$	y
0	6	8	-5
3	4	5	7
5	-1	-3	4
0	2	1	2

Problem 1.2

For the X and $\vec{y}$ that you have written down, let $\vec{w}$ be the optimal parameter vector, which comes from solving the normal equations $X^T X\vec{w} = X^T\vec{y}$. Let $\vec{e} = \vec{y} - X\vec{w}$ be the error vector, and let e_i be the ith component of this error vector. Show that

$$4e_1 + e_2 + 4e_3 + e_4 = 0.$$

Feature Engineering - Angela

Problem 2.1

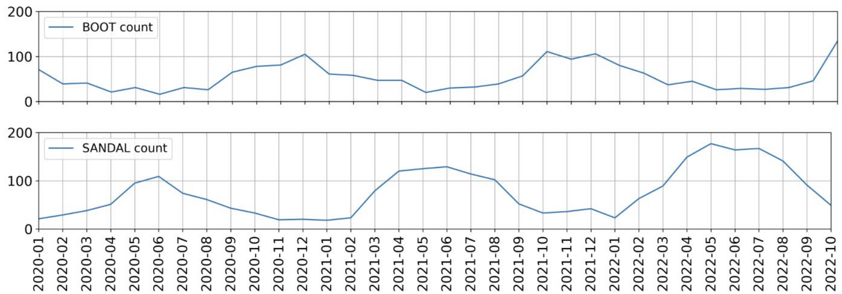
$$\text{predicted boot}_i = w_0$$

w_0:

- 0
- 50
- 100
- Not enough info

Problem 2

The two plots below show the total number of boots (top) and sandals (bottom) purchased per month in the `df` table. Assume that there is one data point per month.



For each of the following regression models, use the visualizations shown above to select the value that is **closest** to the fitted model weights. If it is not possible to determine the model weight, select "Not enough info". For the models below:

- The notation boot refers to the number of boots sold.
- The notation sandal refers to the number of sandals sold.
- $\text{summer} = 1$ is a column with value 1 if the month is between March (03) and August (08), inclusive.
- $\text{winter} = 1$ is a column with value 1 if the month is between September (09) and February (02), inclusive.

Feature Engineering - Angela

Problem 2.2

$$\text{predicted boot}_i = w_0 + w_1 \cdot \text{sandals}_i$$

w_0:

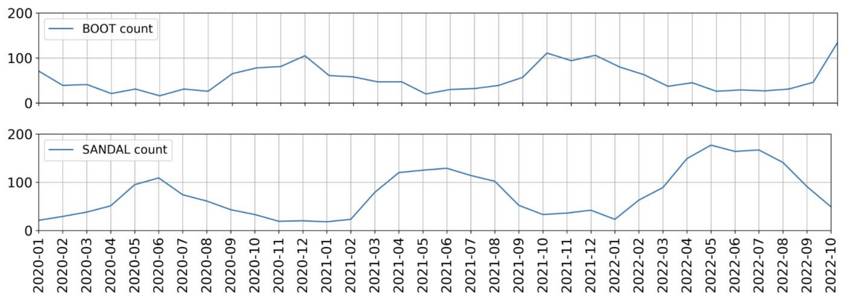
- -100
- -1
- 0
- 1
- 100
- Not enough info

w_1:

- -100
- -1
- 1
- 100
- Not enough info

Problem 2

The two plots below show the total number of boots (top) and sandals (bottom) purchased per month in the `df` table. Assume that there is one data point per month.



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- $\text{winter} = 1$ is a column with value 1 if the month is between September (09) and February (02), inclusive.

Feature Engineering - Angela

Problem 2.3

$\text{predicted boot}_i = w_0 + w_1 \cdot (\text{summer}{=}1)_i$

w_0:

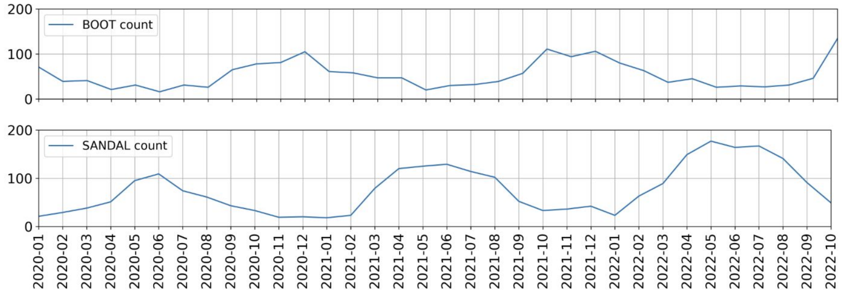
- ◯ -100
- ◯ -1
- ◯ 0
- ◯ 1
- ◯ 100
- ◯ Not enough info

w_1:

- ◯ -80
- ◯ -1
- ◯ 0
- ◯ 1
- ◯ 80
- ◯ Not enough info

Problem 2

The two plots below show the total number of boots (top) and sandals (bottom) purchased per month in the `df` table. Assume that there is one data point per month.



For each of the following regression models, use the visualizations shown above to select the value that is **closest** to the fitted model weights. If it is not possible to determine the model weight, select "Not enough info". For the models below:

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- The notation sandal refers to the number of sandals sold.
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- $\text{winter} = 1$ is a column with value 1 if the month is between September (09) and February (02), inclusive.

Feature Engineering - Angela

Problem 2.4

$\text{predicted sandal}_i = w_0 + w_1 \cdot (\text{summer=1})_i$

w_0:

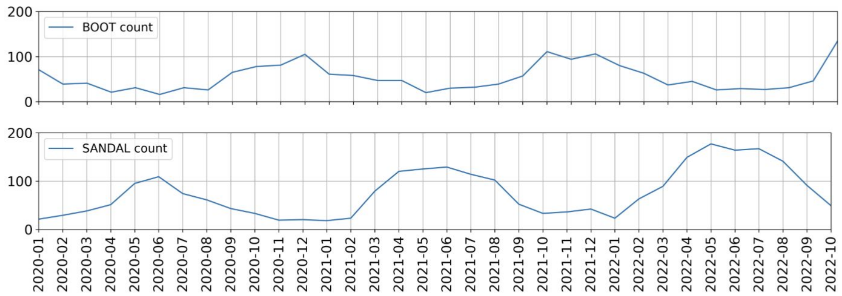
- -20
- -1
- 0
- 1
- 20
- Not enough info

w_1:

- -80
- -1
- 0
- 1
- 80
- Not enough info

Problem 2

The two plots below show the total number of boots (top) and sandals (bottom) purchased per month in the `df` table. Assume that there is one data point per month.



For each of the following regression models, use the visualizations shown above to select the value that is **closest** to the fitted model weights. If it is not possible to determine the model weight, select "Not enough info". For the models below:

- The notation boot refers to the number of boots sold.
- The notation sandal refers to the number of sandals sold.
- $\text{summer} = 1$ is a column with value 1 if the month is between March (03) and August (08), inclusive.
- $\text{winter} = 1$ is a column with value 1 if the month is between September (09) and February (02), inclusive.

Feature Engineering - Angela

Problem 2.5

$$\text{predicted sandal}_i = w_0 + w_1 \cdot (\text{summer=1})_i + w_2 \cdot (\text{winter=1})_i$$

w_0:

- -20
- -1
- 0
- 1
- 20
- Not enough info

w_1:

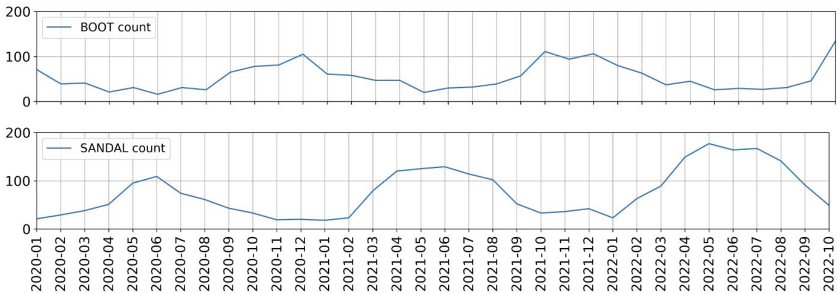
- -80
- -1
- 0
- 1
- 80
- Not enough info

w_2:

- -80
- -1
- 0
- 1
- 80
- Not enough info

Problem 2

The two plots below show the total number of boots (top) and sandals (bottom) purchased per month in the `df` table. Assume that there is one data point per month.



For each of the following regression models, use the visualizations shown above to select the value that is **closest** to the fitted model weights. If it is not possible to determine the model weight, select "Not enough info". For the models below:

- The notation boot refers to the number of boots sold.
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Cross-Validation - Abhi

Problem 3

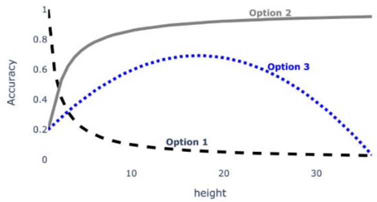
We will aim to build a classifier that takes in demographic information about a state from a particular year and predicts whether or not the state's mean math score is higher than its mean verbal score that year.

In honor of the rotisserie chicken event on UCSD's campus in March of 2023, `sklearn` released a new classifier class called `ChickenClassifier`.

Problem 3.1

`ChickenClassifier`s have many hyperparameters, one of which is `height`. As we increase the value of `height`, the model variance of the resulting `ChickenClassifier` also increases.

First, we consider the training and testing accuracy of a `ChickenClassifier` trained using various values of `height`. Consider the plot below.



- Note that **accuracy** is a metric that measures how well a classifier performs by comparing the number of correct predictions to the total number of predictions.

Which of the following depicts **training accuracy vs. `height`**?

- Option 1
- Option 2
- Option 3

Which of the following depicts **testing accuracy vs. `height`**?

- Option 1
- Option 2
- Option 3

Cross-Validation - Abhi

Problem 3

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In honor of the rotisserie chicken event on UCSD's campus in March of 2023, `sklearn` released a new classifier class called `ChickenClassifier`.

`ChickenClassifier`s have another hyperparameter, `color`, for which there are four possible values: `"yellow"`, `"brown"`, `"red"`, and `"orange"`. To find the optimal value of `color`, we perform k-fold cross-validation with $k = 4$. The results are given in the table below.

	Fold 1	Fold 2	Fold 3	Fold 4	row mean
yellow	0.56	0.59	0.39	0.76	0.575
brown	0.42	0.52	0.65	0.48	0.5175
red	0.49	0.51	0.66	0.83	0.6225
orange	0.6	0.49	0.65	0.54	0.57
column mean	0.5175	0.5275	0.5875	0.6525	

Problem 3.2

Which value of `color` has the best average validation accuracy?

- ○ `"yellow"`
- ○ `"brown"`
- ○ `"red"`
- ○ `"orange"`

Cross-Validation - Abhi

Problem 3

We will aim to build a classifier that takes in demographic information about a state from a particular year and predicts whether or not the state's mean math score is higher than its mean verbal score that year.

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Problem 3.3

True or False: It is possible for a hyperparameter value to have the best average validation accuracy across all folds, but not have the best validation accuracy in any one particular fold.

- ○ True
- ○ False

Cross-Validation - Abhi

Problem 3

We will aim to build a classifier that takes in demographic information about a state from a particular year and predicts whether or not the state's mean math score is higher than its mean verbal score that year.

In honor of the rotisserie chicken event on UCSD's campus in March of 2023, `sklearn` released a new classifier class called `ChickenClassifier`.

Problem 3.4

Now, instead of finding the best `height` and best `color` individually, we decide to perform a grid search that uses k-fold cross-validation to find the combination of `height` and `color` with the best average validation accuracy.

For the purposes of this question, assume that:

- We are performing k-fold cross validation.
- Our training set contains n rows, where n is greater than 5 and is a multiple of k.
- There are h_1 possible values of `height` and h_2 possible values of `color`.

Consider the following three subparts:

- A. What is the size of each fold?
- B. How many times is row 5 in the training set used for training?
- C. How many times is row 5 in the training set used for validation?

Choose from the following options.

- ○ k
- ○ $\frac{k}{n}$
- ○ $\frac{n}{k}$
- ○ $\frac{n}{k} \cdot (k-1)$
- ○ $h_1 h_2 k$
- ○ $h_1 h_2 (k-1)$
- ○ $\frac{n h_1 h_2}{k}$
- ○ None of the above

Regularization & Classification - Abhi

Problem 4

Consider the least squares regression model, $\vec{h} = X\vec{w}$. Assume that X and $\vec{h}$ refer to the design matrix and hypothesis vector for our training data, and $\vec{y}$ is the true observation vector.

Let $\vec{w}^*_{\text{OLS}}$ be the parameter vector that minimizes mean squared error without regularization. Specifically:

$\vec{w}^*_{\text{OLS}} = \underset{\vec{w}}{\arg\min} \frac{1}{n} \|\vec{y} - X\vec{w}\|_2^2$

Let $\vec{w}^*_{\text{ridge}}$ be the parameter vector that minimizes mean squared error with L_2 regularization, using a non-negative regularization hyperparameter λ (i.e. ridge regression). Specifically:

$\vec{w}^*_{\text{ridge}} = \underset{\vec{w}}{\arg\min} \frac{1}{n} \|\vec{y} - X\vec{w}\|_2^2 + \lambda \sum_{j=1}^{p} w_j^2$

For each of the following problems, fill in the blank.

Problem 4.1

If we set $\lambda = 0$, then $\|\vec{w}^*_{\text{OLS}}\|_2^2$ is ____ $\|\vec{w}^*_{\text{ridge}}\|_2^2$

- less than
- equal to
- greater than
- impossible to tell

Regularization & Classification - Abhi

Problem 4

Consider the least squares regression model, $\vec{h} = X\vec{w}$. Assume that X and $\vec{h}$ refer to the design matrix and hypothesis vector for our training data, and $\vec{y}$ is the true observation vector.

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For each of the following problems, fill in the blank.

Problem 4.2

For each of the remaining parts, you can assume that λ is set such that the predicted response vectors for our two models ($\vec{h} = X\vec{w}^*_{\text{OLS}}$ and $\vec{h} = X\vec{w}^*_{\text{ridge}}$) is different.

The **training** MSE of the model $\vec{h} = X\vec{w}^*_{\text{OLS}}$ is ____ than the model $\vec{h} = X\vec{w}^*_{\text{ridge}}$.

- ○ less than
- ○ equal to
- ○ greater than
- ○ impossible to tell

Regularization & Classification - Abhi

Problem 4

Consider the least squares regression model, $\vec{h} = X\vec{w}$. Assume that X and $\vec{h}$ refer to the design matrix and hypothesis vector for our training data, and $\vec{y}$ is the true observation vector.

Let $\vec{w}^*_{\text{OLS}}$ be the parameter vector that minimizes mean squared error without regularization. Specifically:

$\vec{w}^*_{\text{OLS}} = \arg\min_{\vec{w}} \frac{1}{n} \|\vec{y} - X\vec{w}\|_2^2$

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$\vec{w}^*_{\text{ridge}} = \arg\min_{\vec{w}} \frac{1}{n} \|\vec{y} - X\vec{w}\|_2^2 + \lambda \sum_{j=1}^{p} w_j^2$

For each of the following problems, fill in the blank.

Problem 4.3

Now, assume we've fit both models using our training data, and evaluate both models on some unseen testing data.

The **test** MSE of the model $\vec{h} = X\vec{w}^*_{\text{OLS}}$ is ____ than the model $\vec{h} = X\vec{w}^*_{\text{ridge}}$.

- less than
- equal to
- greater than
- impossible to tell

Regularization & Classification - Abhi

Problem 4

Consider the least squares regression model, $\vec{h} = X\vec{w}$. Assume that X and $\vec{h}$ refer to the design matrix and hypothesis vector for our training data, and $\vec{y}$ is the true observation vector.

Let $\vec{w}^*_{\text{OLS}}$ be the parameter vector that minimizes mean squared error without regularization. Specifically:

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$\vec{w}^*_{\text{ridge}} = \arg\min_{\vec{w}} \frac{1}{n} \|\vec{y} - X\vec{w}\|_2^2 + \lambda \sum_{j=1}^{p} w_j^2$

For each of the following problems, fill in the blank.

Problem 4.4

Assume that our design matrix X contains a column of all ones. The sum of the residuals of our model $\vec{h} = X\vec{w}^*_{\text{ridge}}$ ____.

- equal to 0
- not necessarily equal to 0

Regularization & Classification - Abhi

Problem 4

Consider the least squares regression model, $\vec{h} = X\vec{w}$. Assume that X and $\vec{h}$ refer to the design matrix and hypothesis vector for our training data, and $\vec{y}$ is the true observation vector.

Let $\vec{w}^*_{\text{OLS}}$ be the parameter vector that minimizes mean squared error without regularization. Specifically:

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$$\vec{w}^*_{\text{ridge}} = \arg\min_{\vec{w}} \frac{1}{n} \|\vec{y} - X\vec{w}\|_2^2 + \lambda \sum_{j=1}^{p} w_j^2$$

For each of the following problems, fill in the blank.

Problem 4.5

As we increase λ, the bias of the model $\vec{h} = X\vec{w}^*_{\text{ridge}}$ tends to ____.

- increase
- stay the same
- decrease

Regularization & Classification - Abhi

Problem 4

Consider the least squares regression model, $\vec{h} = X\vec{w}$. Assume that X and $\vec{h}$ refer to the design matrix and hypothesis vector for our training data, and $\vec{y}$ is the true observation vector.

Let $\vec{w}^*_{\text{OLS}}$ be the parameter vector that minimizes mean squared error without regularization. Specifically:

$\vec{w}^*_{\text{OLS}} = \arg\min_{\vec{w}} \frac{1}{n} \|\vec{y} - X\vec{w}\|_2^2$

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For each of the following problems, fill in the blank.

Problem 4.6

As we increase λ, the model variance of the model $\vec{h} = X\vec{w}^*_{\text{ridge}}$ tends to ____.

- ○ increase
- ○ stay the same
- ○ decrease

Regularization & Classification - Abhi

Problem 4

Consider the least squares regression model, $\vec{h} = X\vec{w}$. Assume that X and $\vec{h}$ refer to the design matrix and hypothesis vector for our training data, and $\vec{y}$ is the true observation vector.

Let $\vec{w}^*_{\text{OLS}}$ be the parameter vector that minimizes mean squared error without regularization. Specifically:

$$\vec{w}^*_{\text{OLS}} = \underset{\vec{w}}{\arg\min} \frac{1}{n} \|\vec{y} - X\vec{w}\|_2^2$$

Let $\vec{w}^*_{\text{ridge}}$ be the parameter vector that minimizes mean squared error with L_2 regularization, using a non-negative regularization hyperparameter λ (i.e. ridge regression). Specifically:

$$\vec{w}^*_{\text{ridge}} = \underset{\vec{w}}{\arg\min} \frac{1}{n} \|\vec{y} - X\vec{w}\|_2^2 + \lambda \sum_{j=1}^{p} w_j^2$$

For each of the following problems, fill in the blank.

Problem 4.7

As we increase λ, the observation variance of the model $\vec{h} = X\vec{w}^*_{\text{ridge}}$ tends to _____.

- ○ increase
- ○ stay the same
- ○ decrease

Gradient Descent - Abhi

Problem 5

Suppose we'd like to use gradient descent to minimize the function $f(x) = x^3 + x^2$. Suppose we choose a learning rate of $\alpha = \frac{1}{4}$.

Problem 5.1

Suppose $x^{(t)}$ is our guess of the minimizing input x^* at timestep t, i.e. $x^{(t)}$ is the result of performing t iterations of gradient descent, given some initial guess. Write an expression for $x^{(t+1)}$. Your answer should be an expression involving $x^{(t)}$ and some constants.

Gradient Descent - Abhi

Problem 5

Suppose we'd like to use gradient descent to minimize the function $f(x) = x^3 + x^2$. Suppose we choose a learning rate of $\alpha = \frac{1}{4}$.

Problem 5.2

Suppose $x^{(0)} = -1$.

- What is the value of $x^{(1)}$?
- Will gradient descent eventually converge, given the initial guess $x^{(0)} = -1$ and step size $\alpha = \frac{1}{4}$?

Gradient Descent - Abhi

Problem 5

Suppose we'd like to use gradient descent to minimize the function $f(x) = x^3 + x^2$. Suppose we choose a learning rate of $\alpha = \frac{1}{4}$.

Problem 5.3

Suppose $x^{(0)} = 1$.

- What is the value of $x^{(1)}$?
- Will gradient descent eventually converge, given the initial guess $x^{(0)} = 1$ and step size $\alpha = \frac{1}{4}$?

Precision/Recall, Logistic Regression, Clustering - Caleb

Problem 6

For a given classifier, suppose the first 10 predictions of our classifier and 10 true observations are as follows:

Predictions	1	1	1	1	1	0	1	1	1	1
True Label	0	1	1	1	0	0	0	1	1	1

1. What is the accuracy of our classifier on these 10 predictions?

2. What is the precision on these 10 predictions?

3. What is the recall on these 10 predictions?

Problem 7

Suppose we want to use logistic regression to classify whether a person survived the sinking of the Titanic. The first 5 rows of our dataset are given below.

	Age	Survived	Female
0	22.0	0	0
1	38.0	1	1
2	26.0	1	1
3	35.0	1	1
4	35.0	0	0

Suppose after training our logistic regression model we get $\vec{w}^* = \begin{bmatrix} -1.2 \\ -0.005 \\ 2.5 \end{bmatrix}$, where -1.2 is an intercept term, -0.005 is the optimal parameter corresponding to passenger's age, and 2.5 is the optimal parameter corresponding to sex (1 if female, 0 otherwise).

Suppose after training our logistic regression model we get $\vec{w}^* = \begin{bmatrix} -1.2 \\ -0.005 \\ 2.5 \end{bmatrix}$, where -1.2 is an intercept term, -0.005 is the optimal parameter corresponding to passenger's age, and 2.5 is the optimal parameter corresponding to sex (1 if female, 0 otherwise).

Problem 7.1

Consider Sīlānah Iskandar Nāsīf Abī Dāghir Yazbak, a 20 year old female. What chance did she have to survive the sinking of the Titanic according to our model? Give your answer as a probability in terms of σ. If there is not enough information, write "not enough information."

	Age	Survived	Female
0	22.0	0	0
1	38.0	1	1
2	26.0	1	1
3	35.0	1	1
4	35.0	0	0

Suppose after training our logistic regression model we get $\vec{w}^* = \begin{bmatrix} -1.2 \\ -0.005 \\ 2.5 \end{bmatrix}$, where -1.2 is an intercept term, -0.005 is the optimal parameter corresponding to passenger's age, and 2.5 is the optimal parameter corresponding to sex (1 if female, 0 otherwise).

Problem 7.2

Sīlānah Iskandar Nāsīf Abī Dāghir Yazbak actually survived. What is the cross-entropy loss for our prediction in the previous part?

	Age	Survived	Female
0	22.0	0	0
1	38.0	1	1
2	26.0	1	1
3	35.0	1	1
4	35.0	0	0

Suppose after training our logistic regression model we get $\vec{w}^* = \begin{bmatrix} -1.2 \\ -0.005 \\ 2.5 \end{bmatrix}$, where -1.2 is an intercept term, -0.005 is the optimal parameter corresponding to passenger's age, and 2.5 is the optimal parameter corresponding to sex (1 if female, 0 otherwise).

Problem 7.3

At what age would we predict that a female passenger is more likely to have survived the Titanic than not? In other words, at what age is the probability of survival for a female passenger greater than 0.5?

Hint: Since $\sigma(0) = 0.5$*, we have that* $\sigma\left(\vec{w}^* \cdot \text{Aug}(\vec{x}_i)\right) = 0.5 \implies \vec{w}^* \cdot \text{Aug}(\vec{x}_i) = 0$.

	Age	Survived	Female
0	22.0	0	0
1	38.0	1	1
2	26.0	1	1
3	35.0	1	1
4	35.0	0	0

Suppose after training our logistic regression model we get $\vec{w}^* = \begin{bmatrix} -1.2 \\ -0.005 \\ 2.5 \end{bmatrix}$, where -1.2 is an intercept term, -0.005 is the optimal parameter corresponding to passenger's age, and 2.5 is the optimal parameter corresponding to sex (1 if female, 0 otherwise).

Problem 7.4

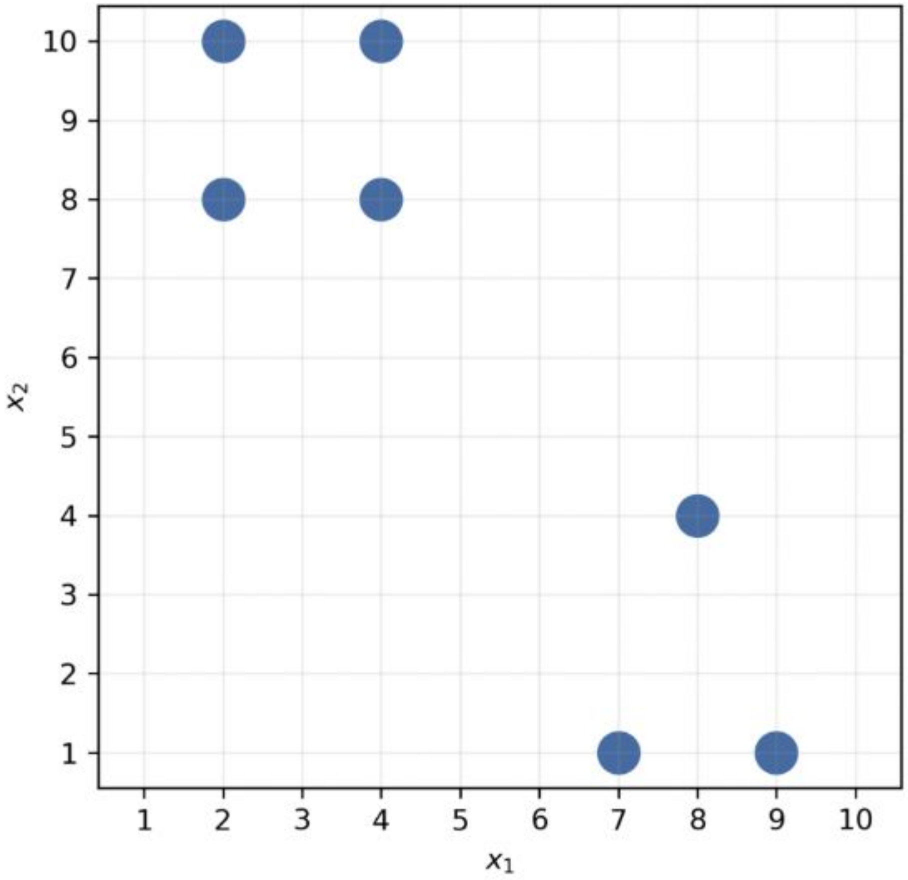
Let m be the **odds** of a given non-female passenger's survival according to our logistic regression model, i.e., if the passenger had an 80% chance of survival, m would be 4, since their odds of survival are $\frac{0.8}{0.2} = 4$.

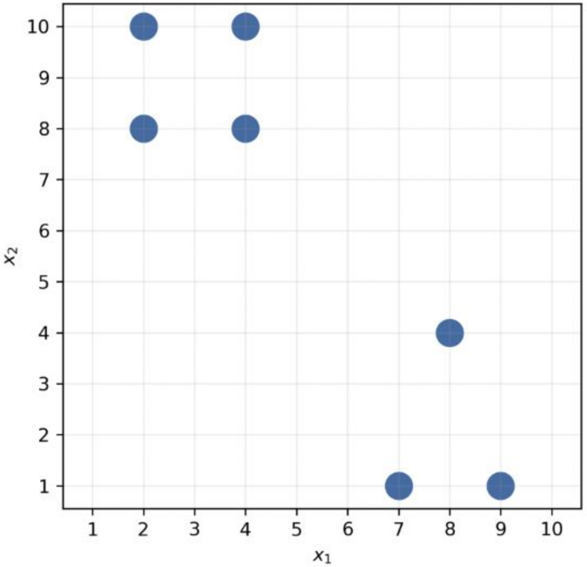
It turns out we can compute f, the odds of survival for a female passenger of the same age, in terms of m. Give an expression for f in terms of m.

	Age	Survived	Female
0	22.0	0	0
1	38.0	1	1
2	26.0	1	1
3	35.0	1	1
4	35.0	0	0

Problem 8

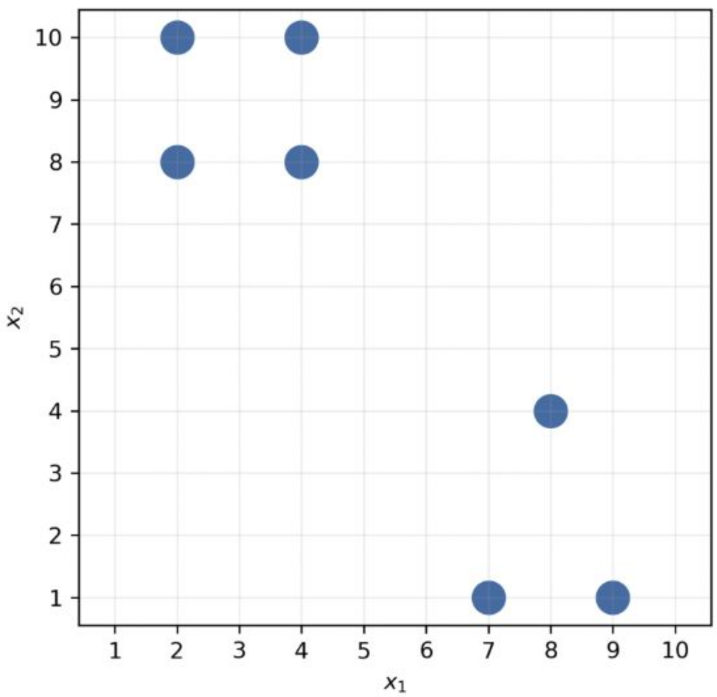
Consider the following dataset of $n = 7$ points in $d = 2$ dimensions.





Problem 8.1

Suppose we decide to use agglomerative clustering to cluster the data. How many possible pairs of clusters could we combine in the first iteration?

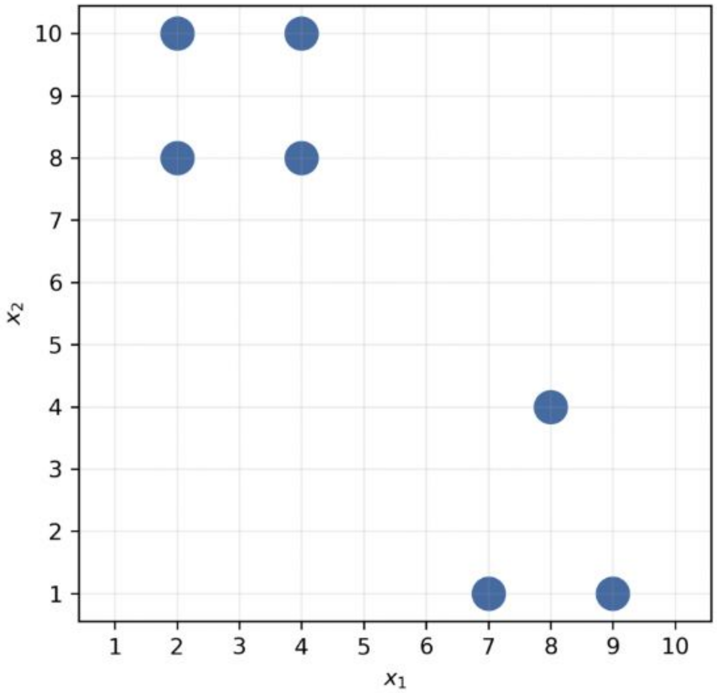


Problem 8.2

Suppose we want to identify $k = 2$ clusters in this dataset using k-means clustering.

Determine the centroids $\vec{\mu}_1$ and $\vec{\mu}_2$ that minimize inertia. (Let $\vec{\mu}_1$ be the centroid with a smaller x_1 coordinate.) Justify your answers.

Note: You don't have to run the k-Means Clustering algorithm to answer this question.



Problem 8.3

What is the total inertia for the centroids you chose in the previous part? Show your work.