Q Find Dual of the below LPP.

Maximize $Z = 10x_1 + 13x_2 + 19x_3$

subject to constraints :

$$6x_1 + 5x_2 + 3x_3 \leq 26$$
$$4x_1 + 2x_2 + 5x_3 \leq 7$$

$$x_1, x_2, x_3 \geq 0$$

1) Since objective function is Maximization Type, the dual LPP should be Minimization type.

2) In Primal Problem there are 3 variables, therefore, Dual will have 3 constraints.

3) The Primal Problem has 2 constraints, dual LPP will have 2 variables, Let us call them y_1, and y_2.

Let us now write the dual Problem

$$W = 26y_1 + 7y_2$$

S.T.
$$6y_1 + 4y_2 \geq 10$$
$$5y_1 + 2y_2 \geq 13$$
$$3y_1 + 5y_2 \geq 19$$

$$y_1, y_2 \geq 0$$

Q   Find Dual of the below LPP.

$$\text{Minimize} \quad Z = 3x_1 + 5x_2 + 7x_3$$

S.T.
$$x_1 + x_2 + 3x_3 \leq 10$$
$$4x_1 - x_2 + 2x_3 \geqslant 15$$
$$x_1, x_2, x_3 \geqslant 0$$

This is a ~~minim~~ minimization type problem, ~~it can be~~ its dual can be obtained using various methods (different Books suggest different methods). We will standardize the constraints by converting "$\leq$" type constraints into "$\geqslant$" type. For this, we will multiply by (-1) on both the sides.

$$\text{Minimize} \quad Z = 3x_1 + 5x_2 + 7x_3$$

S.T.
$$-x_1 - x_2 - 3x_3 \geqslant -10$$
$$4x_1 - x_2 + 2x_3 \geqslant 15$$
$$x_1, x_2, x_3 \geqslant 0$$

Now we will write the dual :

~~Maximize~~

Primal Obj F^n is Minimize $\rightarrow$ Dual O.F. should be maximize type

Primal has 3 variables $\rightarrow$ Dual should have 3 constraints

Primal has 2 constraints $\rightarrow$ Dual should have 2 variables

Dual LPP

Objective F^n :

$$\text{Maximize}: \quad W = -10y_1 + 15y_2$$

S.T.

$$(-1)y_1 + 4y_2 \leq 3$$

$$(-1)y_1 + (-1)y_2 \leq 5$$

$$(-3)y_1 + 2y_2 \leq 7$$

$$y_1, y_2 \geq 0$$

This is the final answer

Some authors or books may do the working in different manner, so the end result may appear differently

For Example, if we define $y_1' = -y_1$

Then,

Obj F^n :

$$\text{Maximize}: \quad W = 10y_1' + 15y_2$$

S.T.

$$y_1' + 4y_2 \leq 3$$

$$y_1' - y_2 \leq 5$$

$$3y_1' + 2y_2 \leq 7$$

$$y_1 \geq 0, \quad y_2 \leq$$

$$y_2 \geq 0 ; \quad y_1' \leq 0$$