Solving SuDoKu Using SET Operations

By

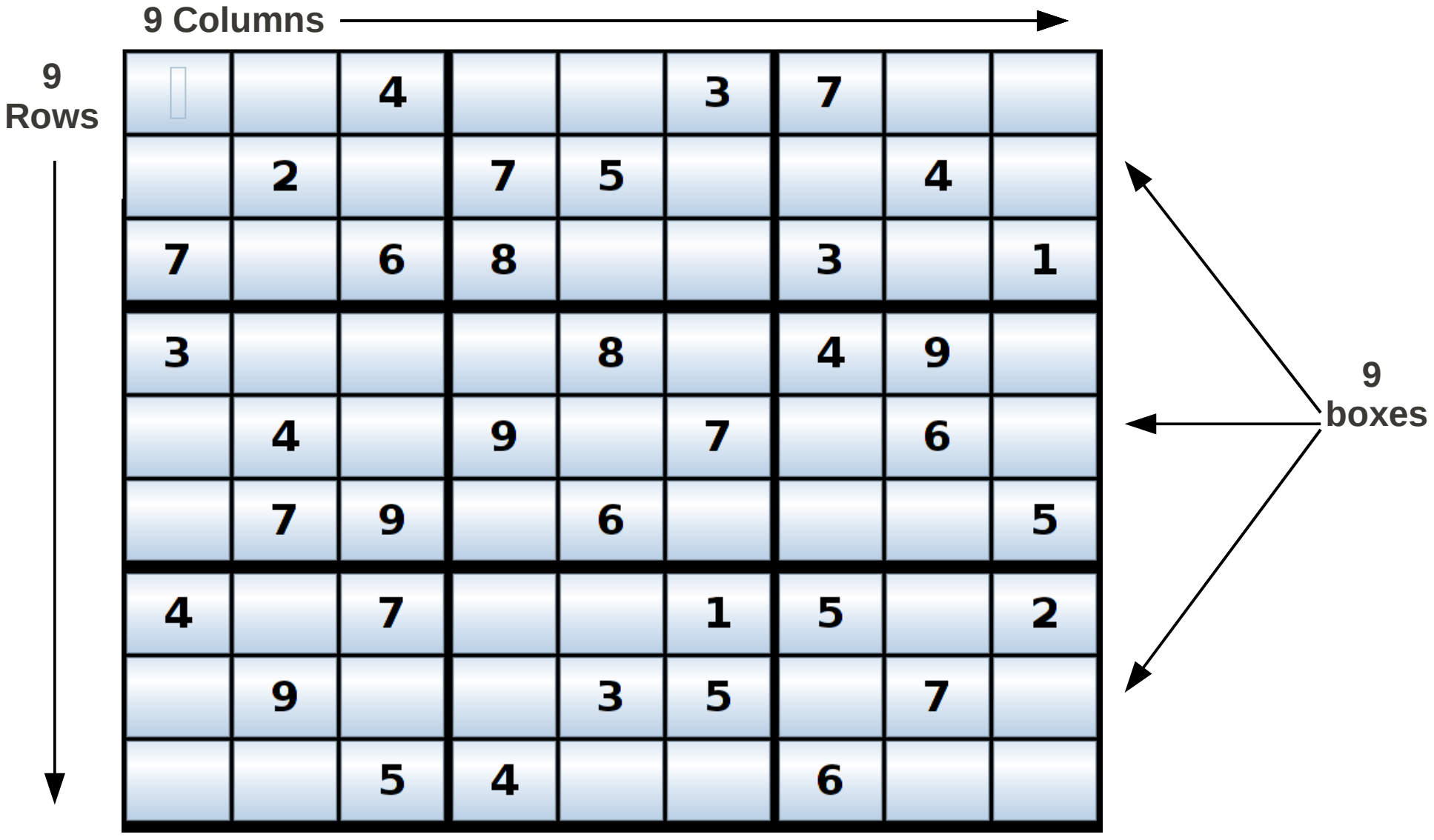
Pradeep Kashyap R.
8th sem CS 'A'
USN: 4JC10CS403

Introduction

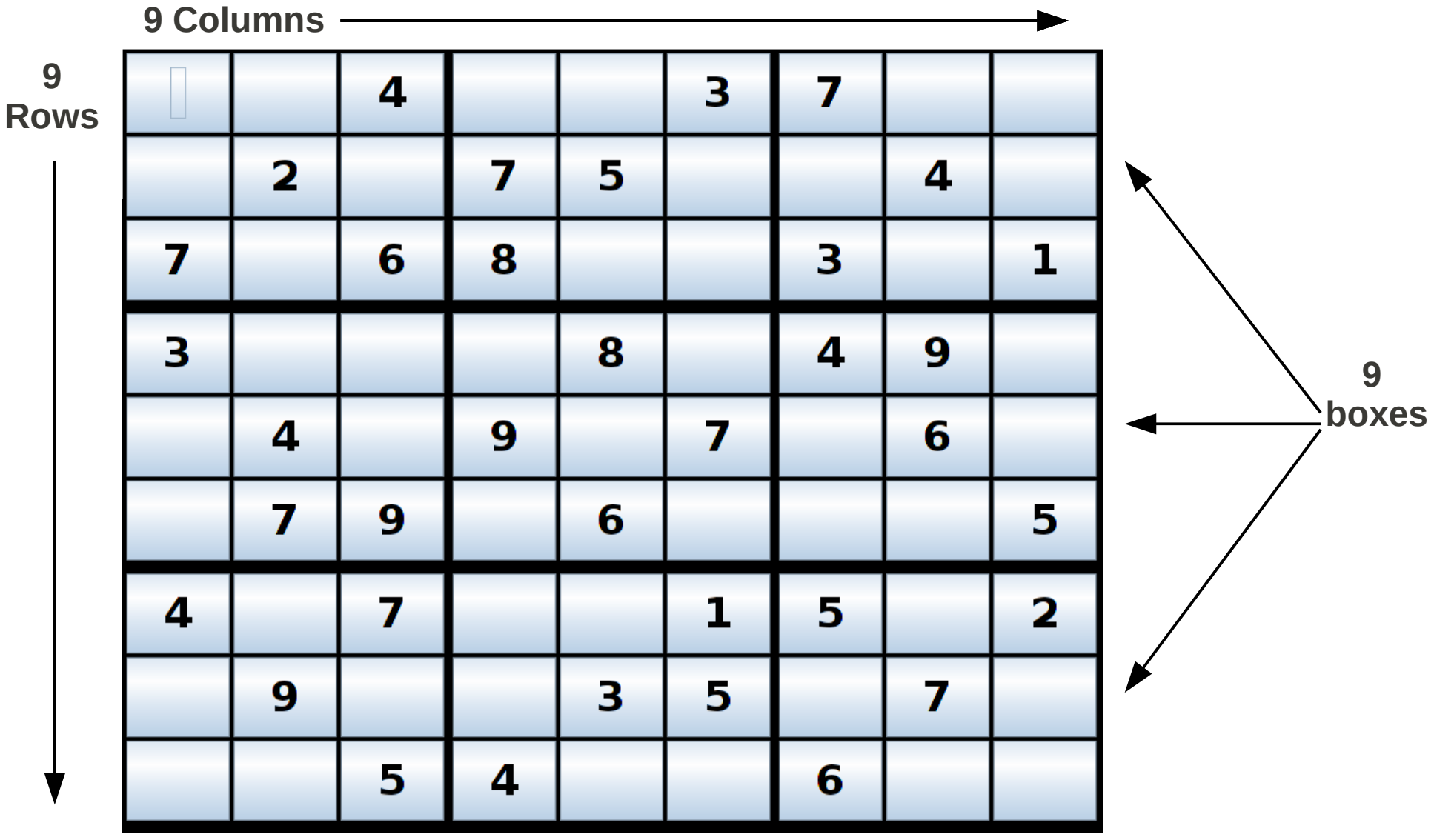
Sudoku is a logical, number placement Puzzle.

The objective is to fill 9x9 grid such that each row, each coloumn, and each 3x3 sub-grid to get values from 1-9 without repetition.

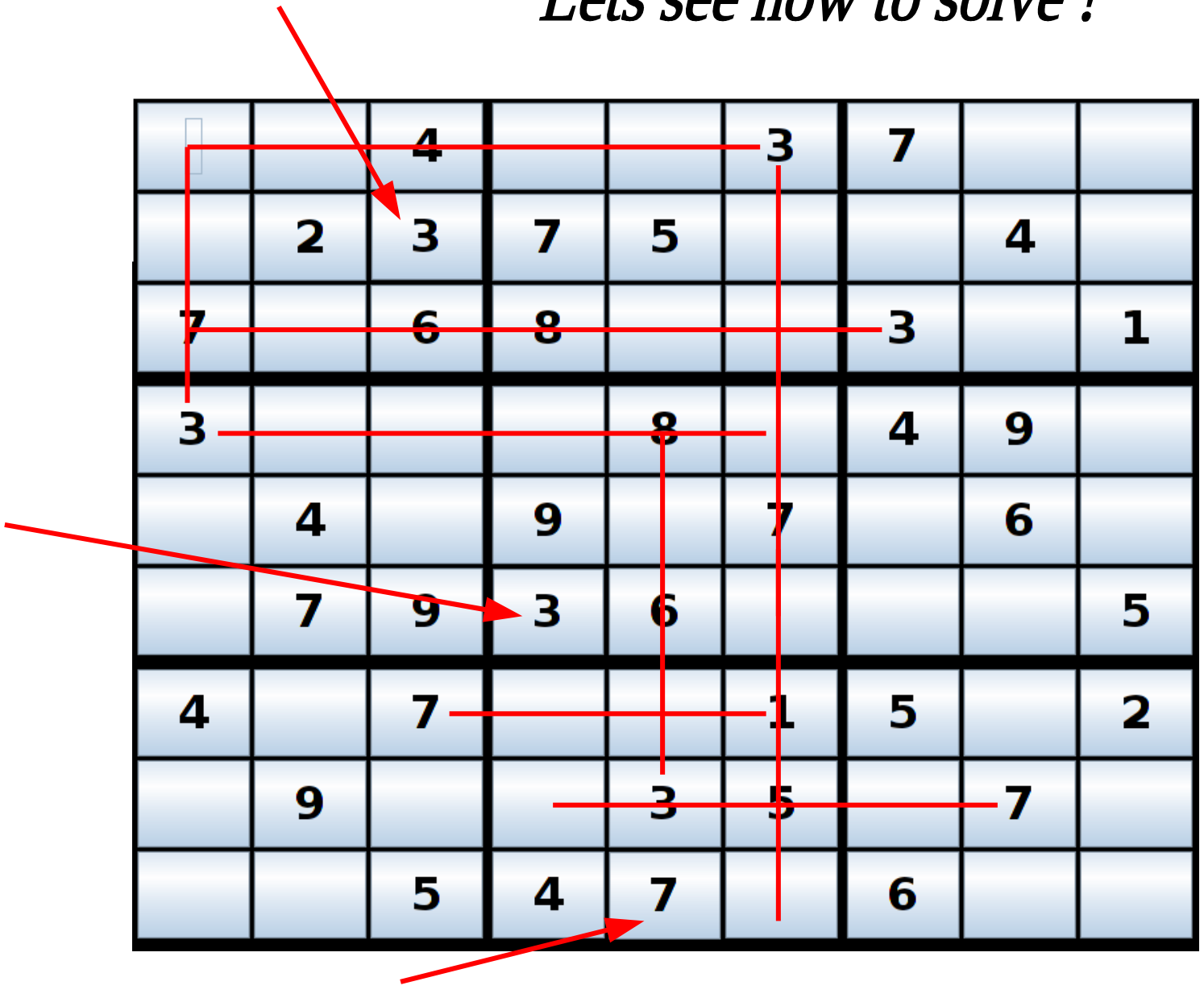
It Looks like this !!



Lets see how to solve !



Lets see how to solve !



Lets see how to solve !

9	8	**4**	1	2	**3**	**7**	5	6
1	**2**	3	**7**	**5**	6	9	**4**	8
7	5	**6**	**8**	4	9	**3**	2	**1**
3	6	1	5	**8**	2	**4**	**9**	7
5	**4**	2	**9**	1	**7**	8	**6**	3
8	**7**	**9**	3	**6**	4	2	1	**5**
4	3	**7**	6	9	**1**	**5**	8	**2**
6	**9**	8	2	**3**	**5**	1	**7**	4
2	1	**5**	**4**	7	8	**6**	3	9

Solving Using SETs

Why sets?

Properties of sudoku such as no "Repetition" of values in same row, same column and in the same Box (3x3 grid) and

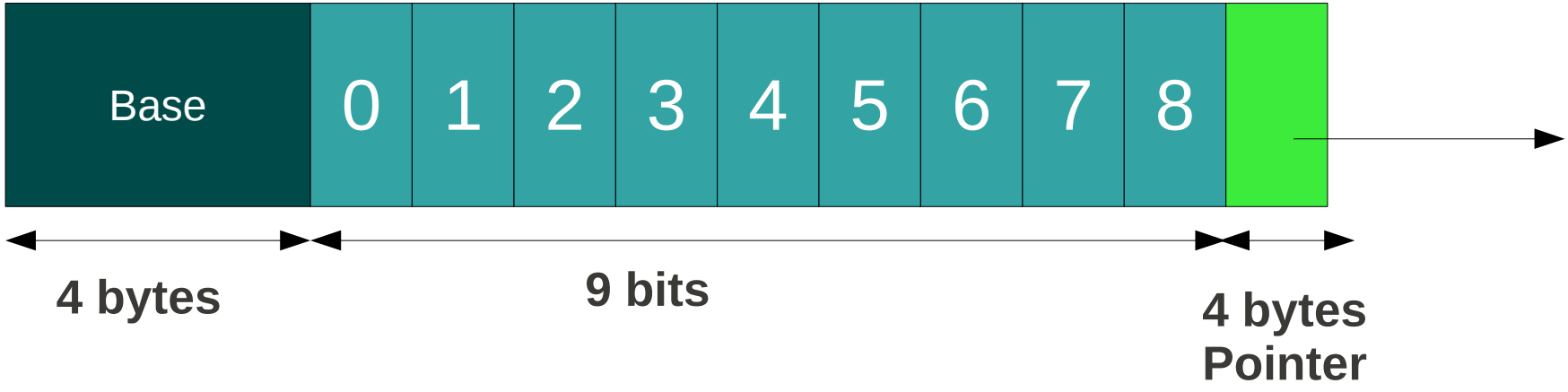
Each cell can have a "Set" of Possible Values which can occupy it; makes SET suitable Data Structure.

UNION and DIFFERENCE and the most frequently used operations to solve Sudoku.

This technique determines whether a given sudoku is solvable without guess or not.

Implementation Details

Implementation of SET using BitVector

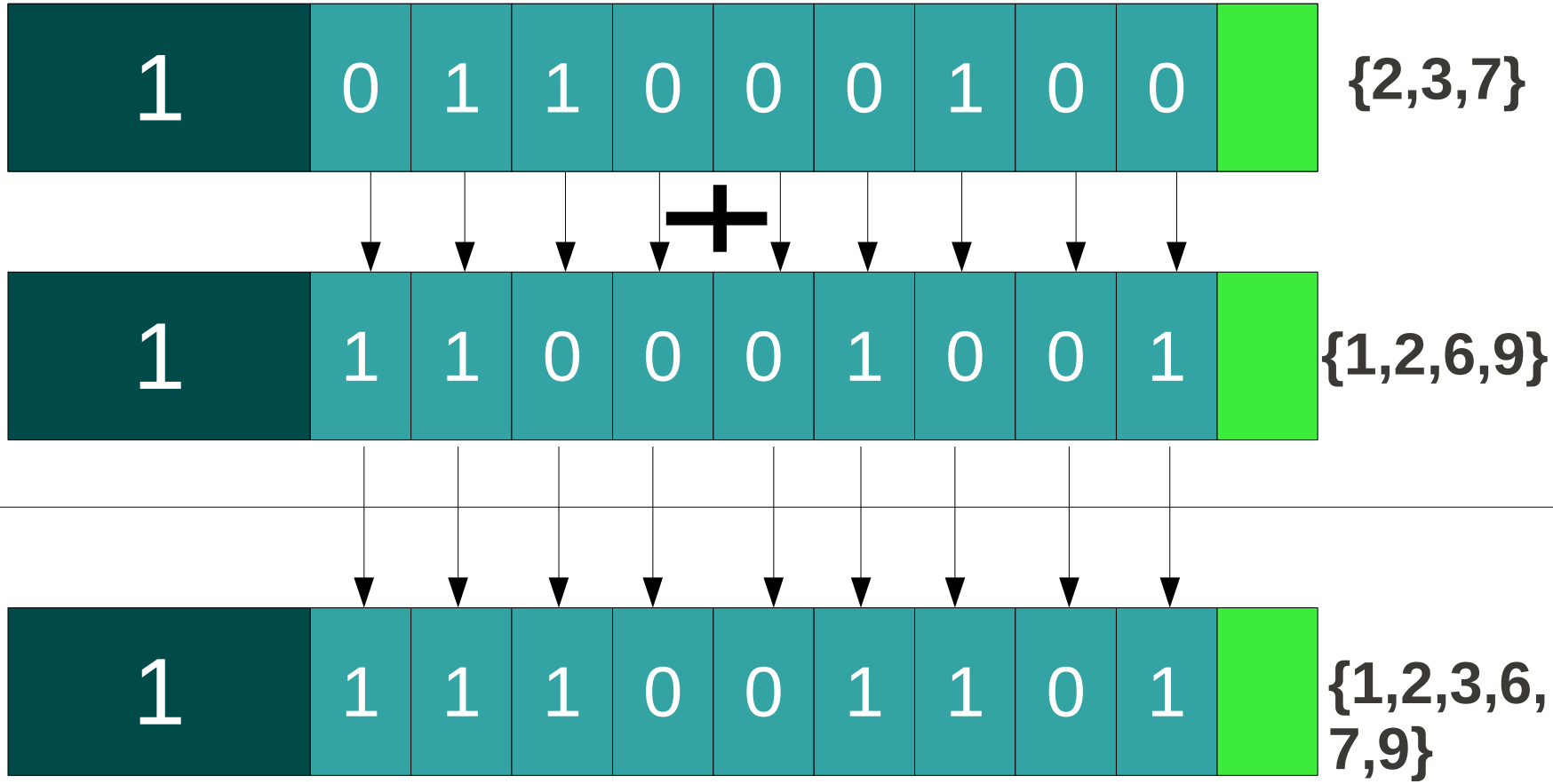


Union, *Intersection,* *difference are* ***O(1)***

OR operation *AND opeartion* ***A-B = A*** $\cap \overline{B}$

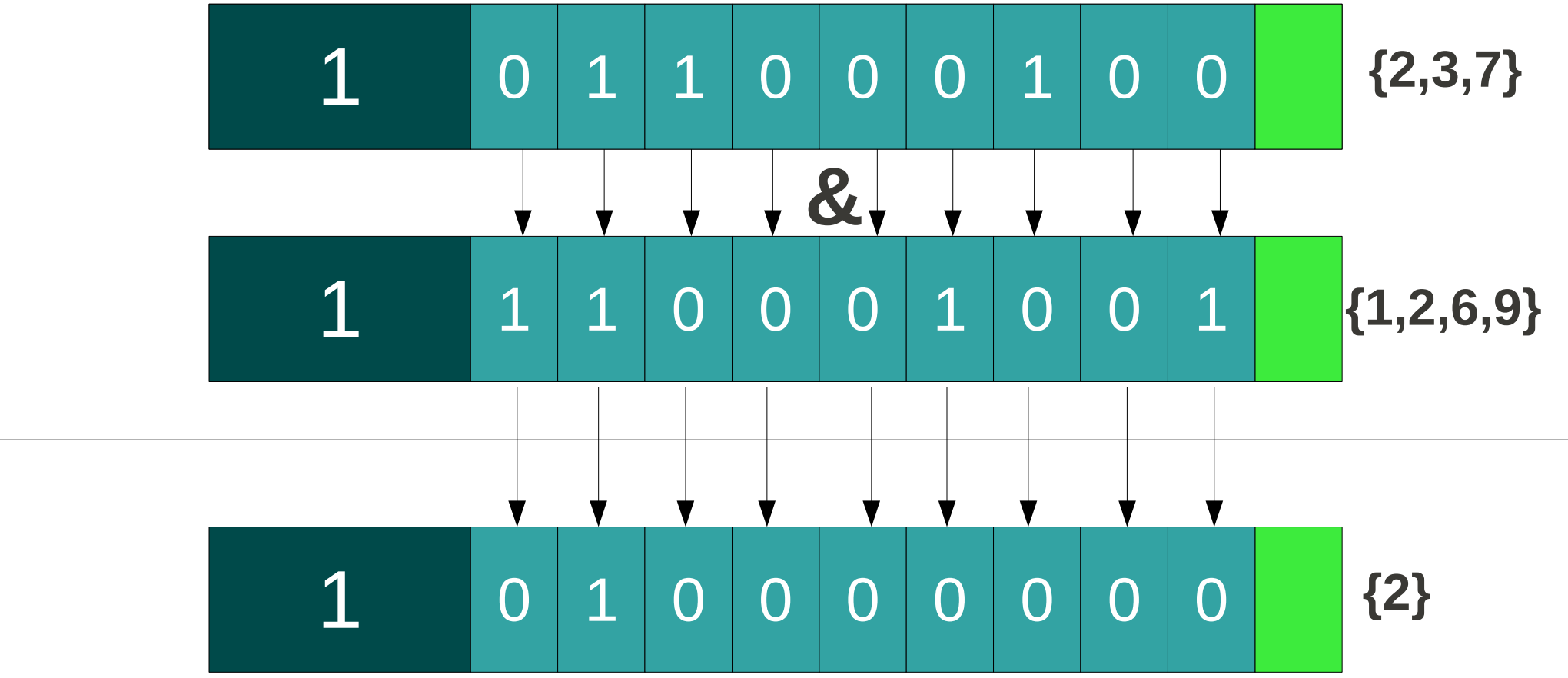
Implementation Details

Union Using OR-operation



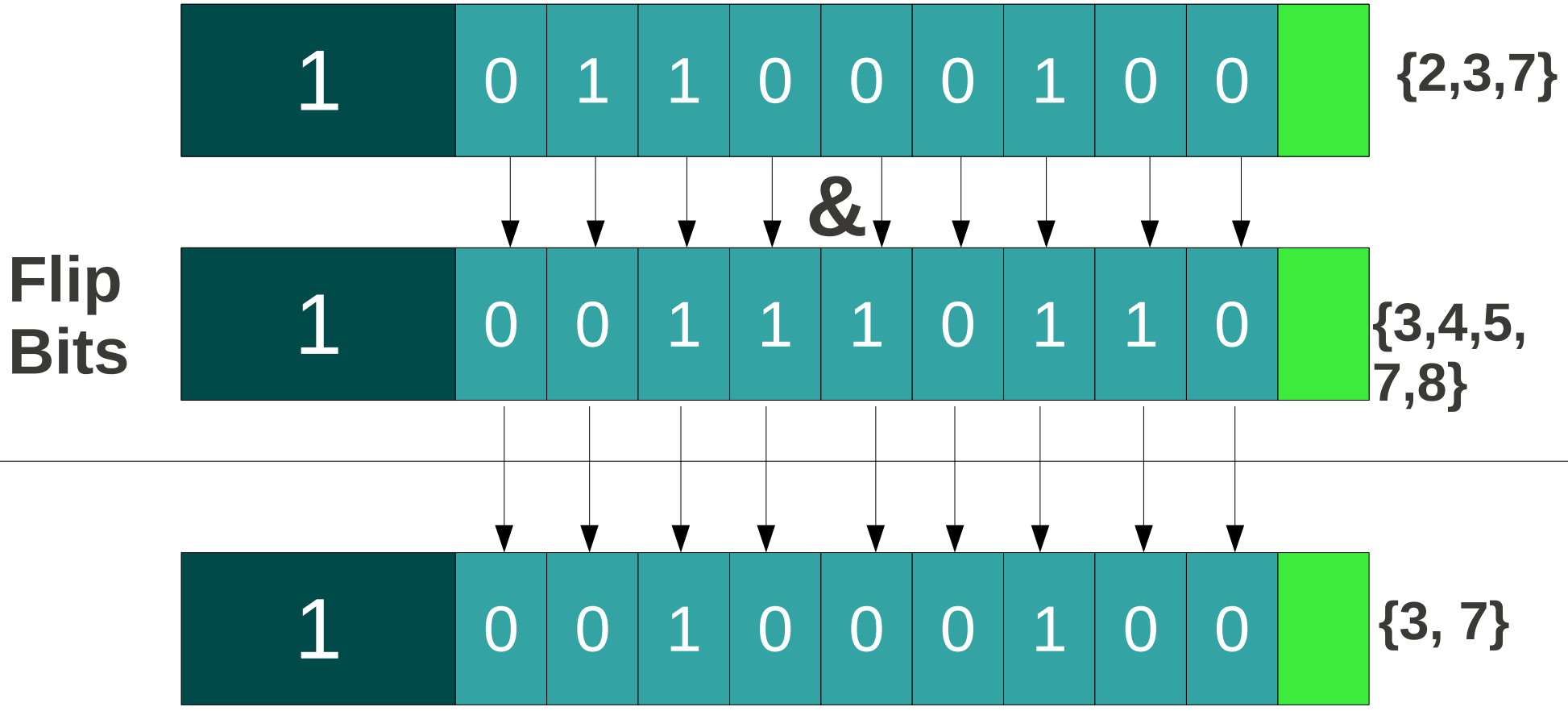
Implementation Details

Intersection Using AND-operation

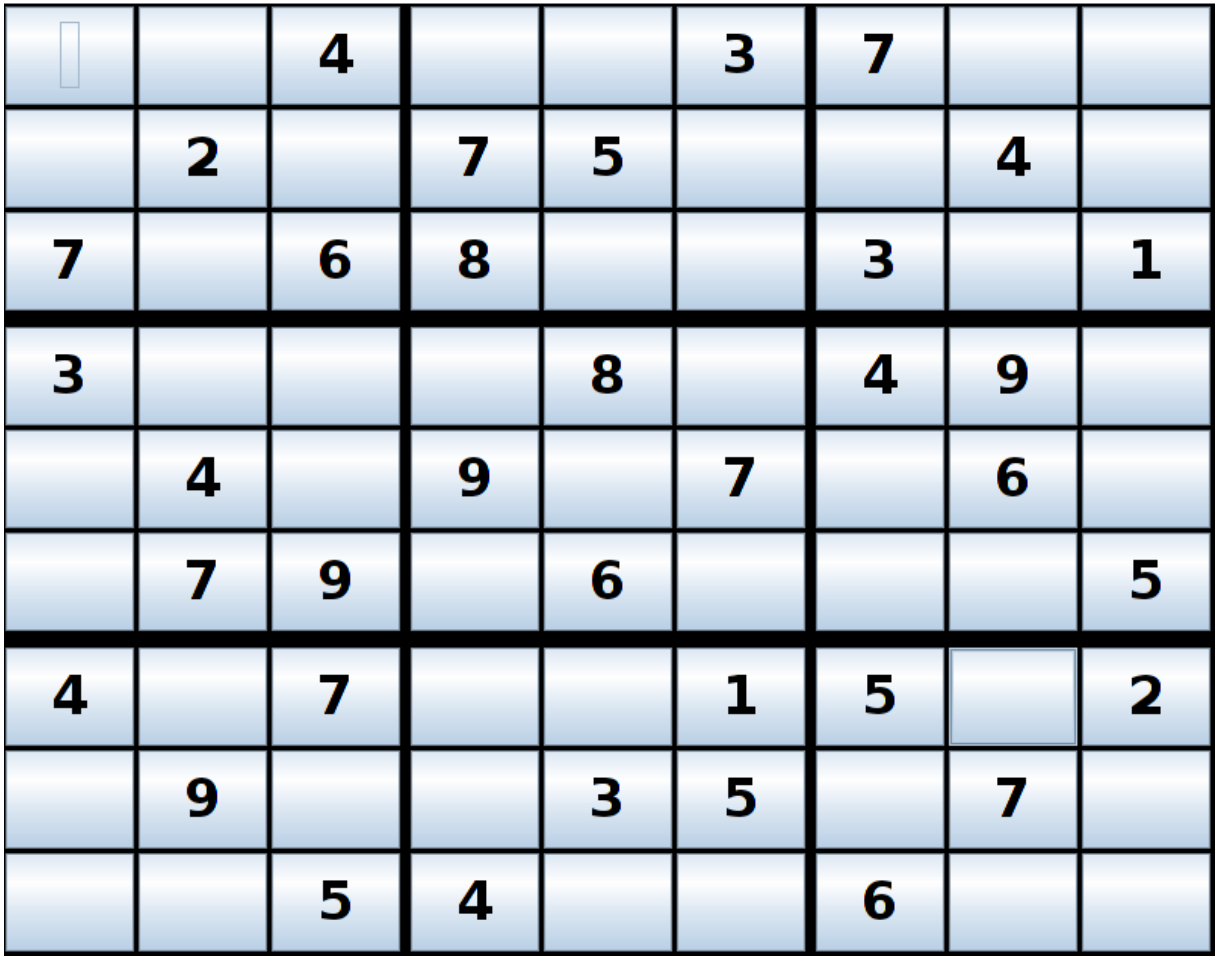


Implementation Details

Difference Using AND and Complement



Solving Using SETs



Solving Using SETs

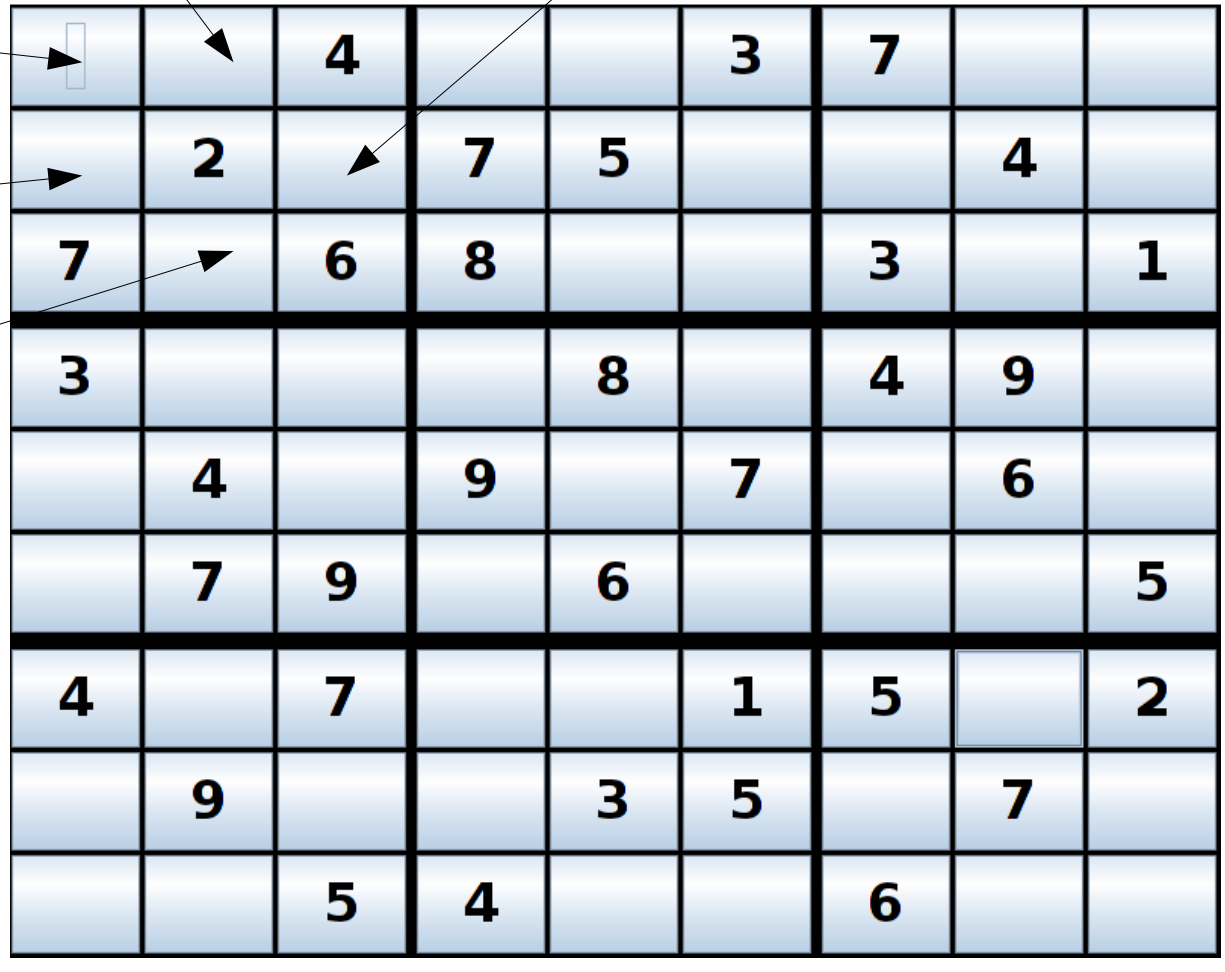
PossibleValues are
{ 1, 5, 8 }

PossibleValues are
{ 1, 3, 8 }

PossibleValues are
{ 1, 5, 8, 9 }

PossibleValues are
{ 1, 8, 9 }

PossibleValues is/are
{ 5 }



Defining PossibleValues Mathematically

$$C = \{ 1,2, \ldots ,9 \}$$

$$\forall\, 0 \leq i , j \leq 8$$

Cell[i][j].PossibleValues=

$$C - (row[i] \cup column[j] \cup Box(\, boxVal(i,j)\,))$$

Where,

$$boxVal(i,j) = \left(\left\lfloor i / 3 \right\rfloor \right) * 3 + \left(\left\lfloor j / 3 \right\rfloor \right)$$

Methods of solving sudoku
Using SET operations

- ***Basic Filling.***

- ***Filling by Elimination.***

- ***Filling By***
  - ***Row Analysis.***
  - ***Column Analysis.***

Methods of solving sudoku
Using SET operations : Basic Filling

Basic Filling:

In this technique cells whose cardinality of PossibleValues is one are filled. i.e

If |PossibleValues| = 1 then fill the cell with the only value in the Set PossibleValues.

Methods of solving sudoku
Using SET operations : Basic Filling

PossibleValues =

{ 1,2,3,4,5,6,7,8,9} -

({ 1,3,6,7,8 }

∪

{ 2,4,7,9}

∪

{ 2,4,6,7})

		4			3	7		
	2		7	5			4	
7		6	8			3		1
3				8		4	9	
	4		9		7		6	
	7	9		6				5
4		7			1	5		2
	9			3	5		7	
		5	4			6		

Methods of solving sudoku
Using SET operations : Basic Filling

PossibleValues =

{ 1,2,3,4,5,6,7,8,9} -

({ 1,3,6,7,8 }

∪

{ 2,4,7,9}

∪

{ 2,4,6,7})

		4			3	7		
	2		7	5			4	
7		6	8			3		1
3				8		4	9	
	4		9		7		6	
	7	9		6				5
4		7			1	5		2
	9			3	5		7	
		5	4			6		

Methods of solving sudoku
Using SET operations : Basic Filling

PossibleValues =

{ 1,2,3,4,5,6,7,8,9} -

(

{ 1,2,3,4,6,7,8,9 }

∪

{ 2,4,6,7})

		4			3	7		
	2		7	5			4	
7		6	8			3		1
3				8		4	9	
	4		9		7		6	
	7	9		6				5
4		7			1	5		2
	9			3	5		7	
		5	4			6		

Methods of solving sudoku
Using SET operations : Basic Filling

PossibleValues =

{ 1,2,3,4,5,6,7,8,9} -

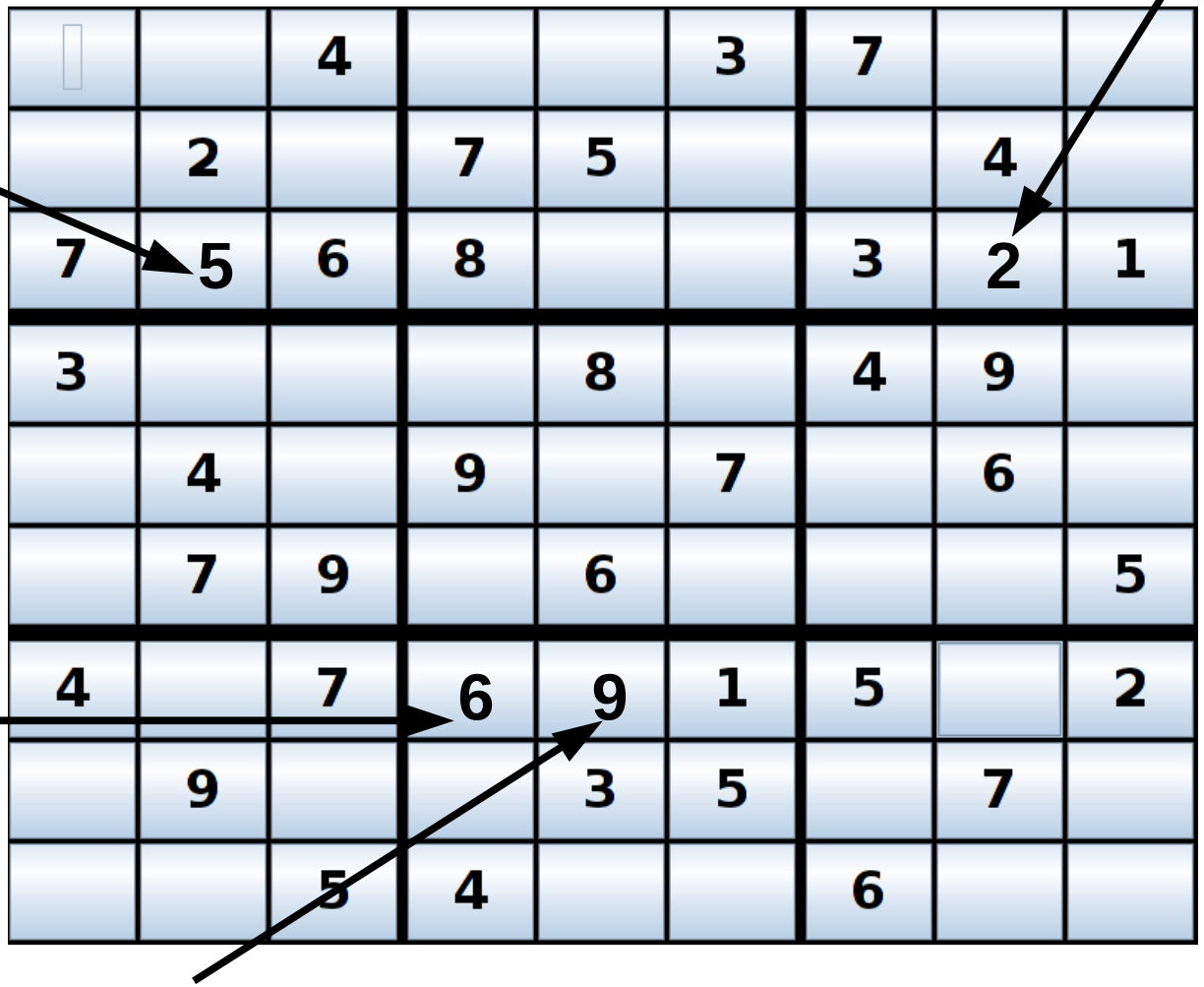
({ 1,2,3,4,6,7,8,9 })

		4			3	7		
	2		7	5			4	
7		6	8			3		1
3				8		4	9	
	4		9		7		6	
	7	9		6				5
4		7			1	5		2
	9			3	5		7	
		5	4			6		

Methods of solving sudoku
Using SET operations : Basic Filling

PossibleValues =

{ 5 }



Methods of solving sudoku
Using SET operations : Basic Filling

Using this method, all Sudokus of 'very easy' and 'easy' level are solvable.

Very few Sudoks of 'medium' level can also be solved.

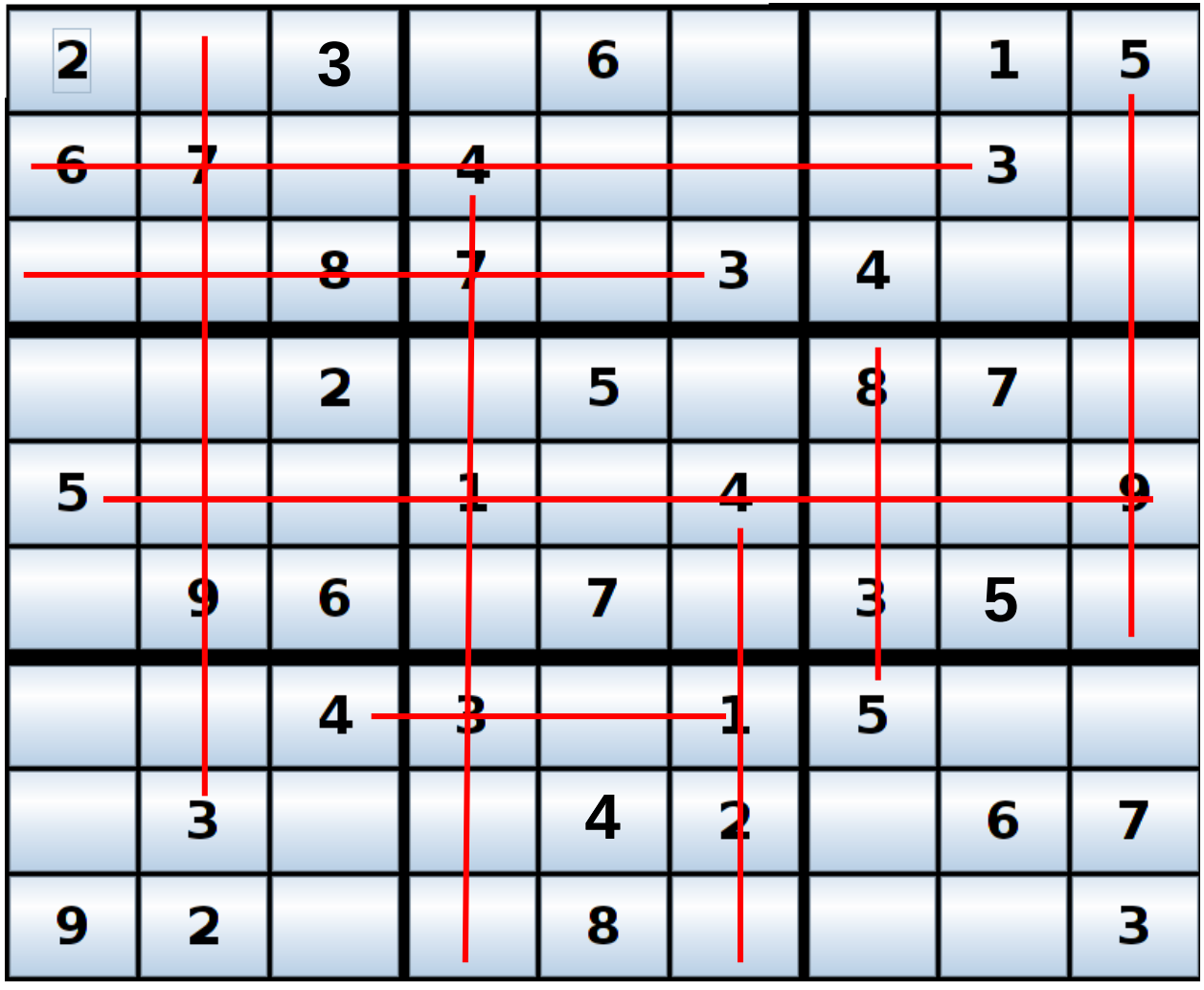
Methods of solving sudoku
Using SET operations :Filling by Elimination

Filling By elimination:

In this technique, a cell 'c' in a box 'b' is filled with a value 'x' if 'x' is not a PossibleValue in other cells in the same box 'b'. i.e.

'x' can appear only in the cell 'c' of the box 'b'

Filling by elimination



Defining Mathematically : Filling by Elimination

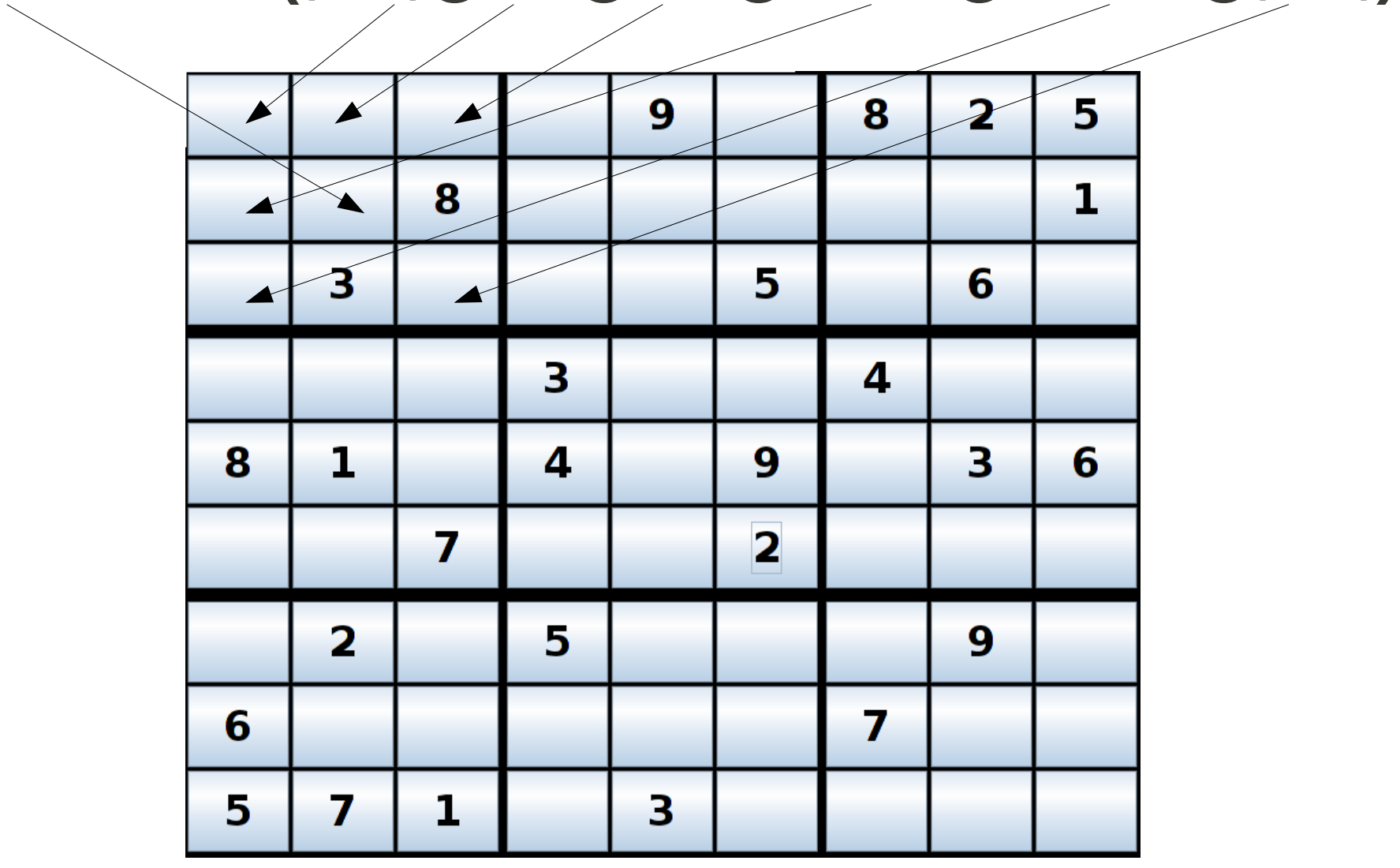
If a cell 'j' is under consideration in the BOX 'i' then

$$\text{Result} = \text{BOX}[i].\text{cell}[j].\text{PossibleValues} - \bigcup_{q=0}^{8} \text{BOX}[i].\text{cell}[q].\text{PossibleValues}, \quad \text{Where, } q \neq j$$

If |Result| = 1 then cell[j] = Result

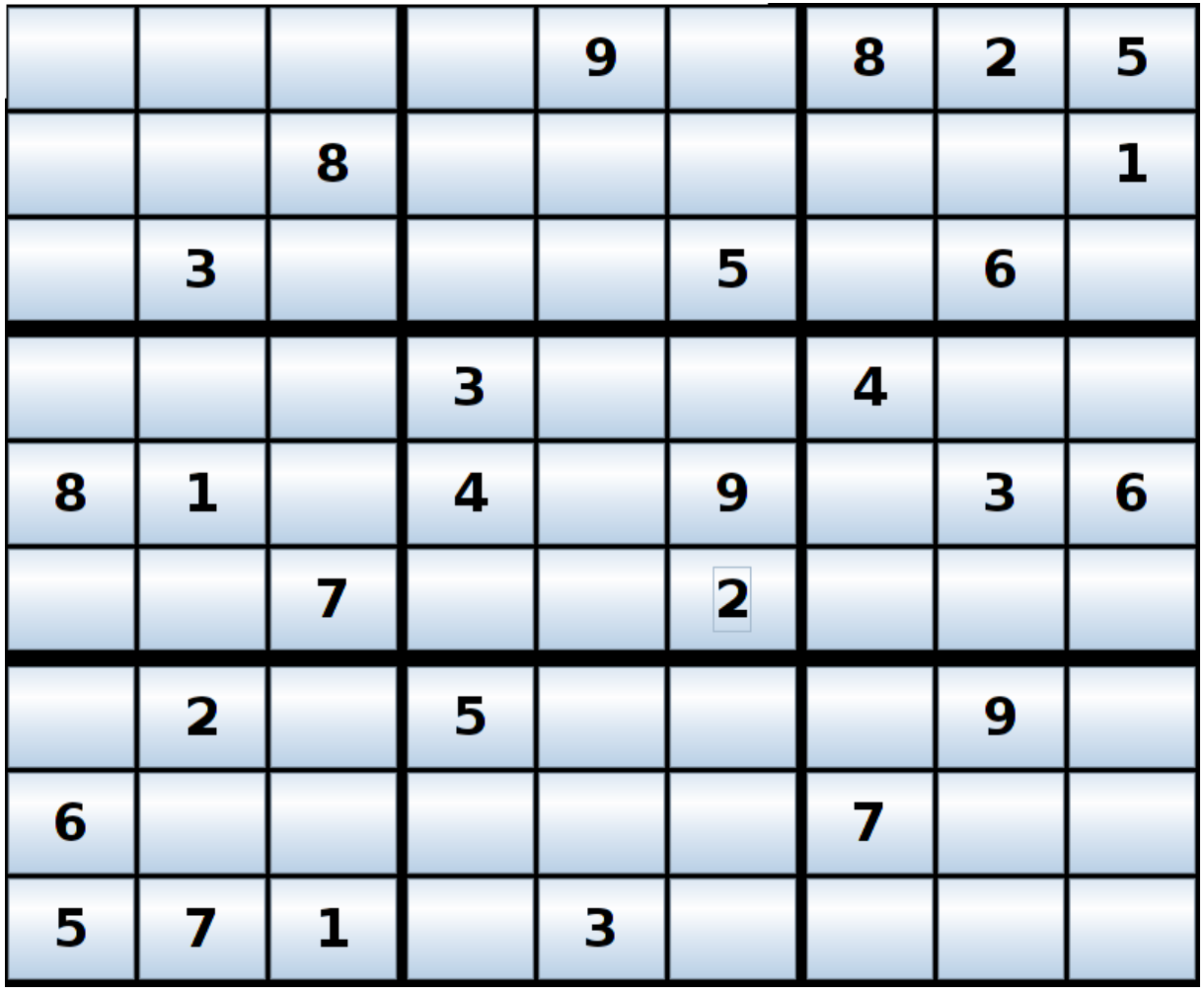
Filling by elimination

Result= {4,5,6,9 } — ({1,4,7}∪{4,6}∪{4,6}∪{2,4,7,9}∪{1,2,4,7,9}∪{2,4,9})



Filling by elimination

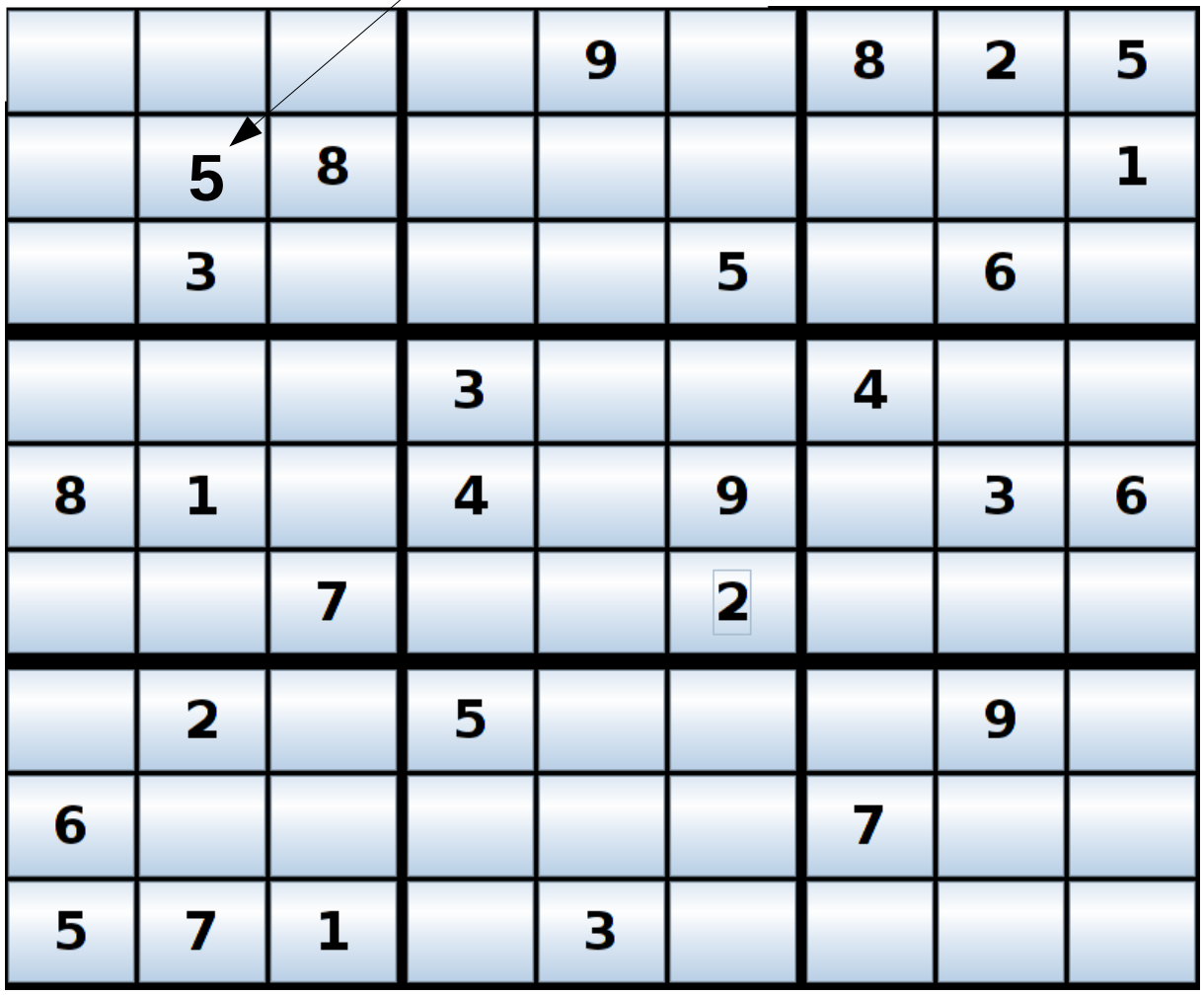
Result= {4,5,6,9 }— { 1, 2, 4, 6, 7, 9}



				9		8	2	5
		8						1
	3				5		6	
			3			4		
8	1		4		9		3	6
		7			2			
	2		5				9	
6						7		
5	7	1		3				

Filling by elimination

Result= {5}



Methods of solving sudoku
Using SET operations :Filling by Elimination

Using this method, all Sudokus of 'medium' are solvable.

Methods of solving sudoku
Using SET operations :Filling by Row / Column Analysis

Filling By Row Analysis:

In this technique, a cell 'c' of a row 'r' is filled with a value 'x' if 'x' is not a Possible Value in any of the other cell in the same row. i.e.

A cell in a row is filled with a value if it cannot appear in any other cell in the same row

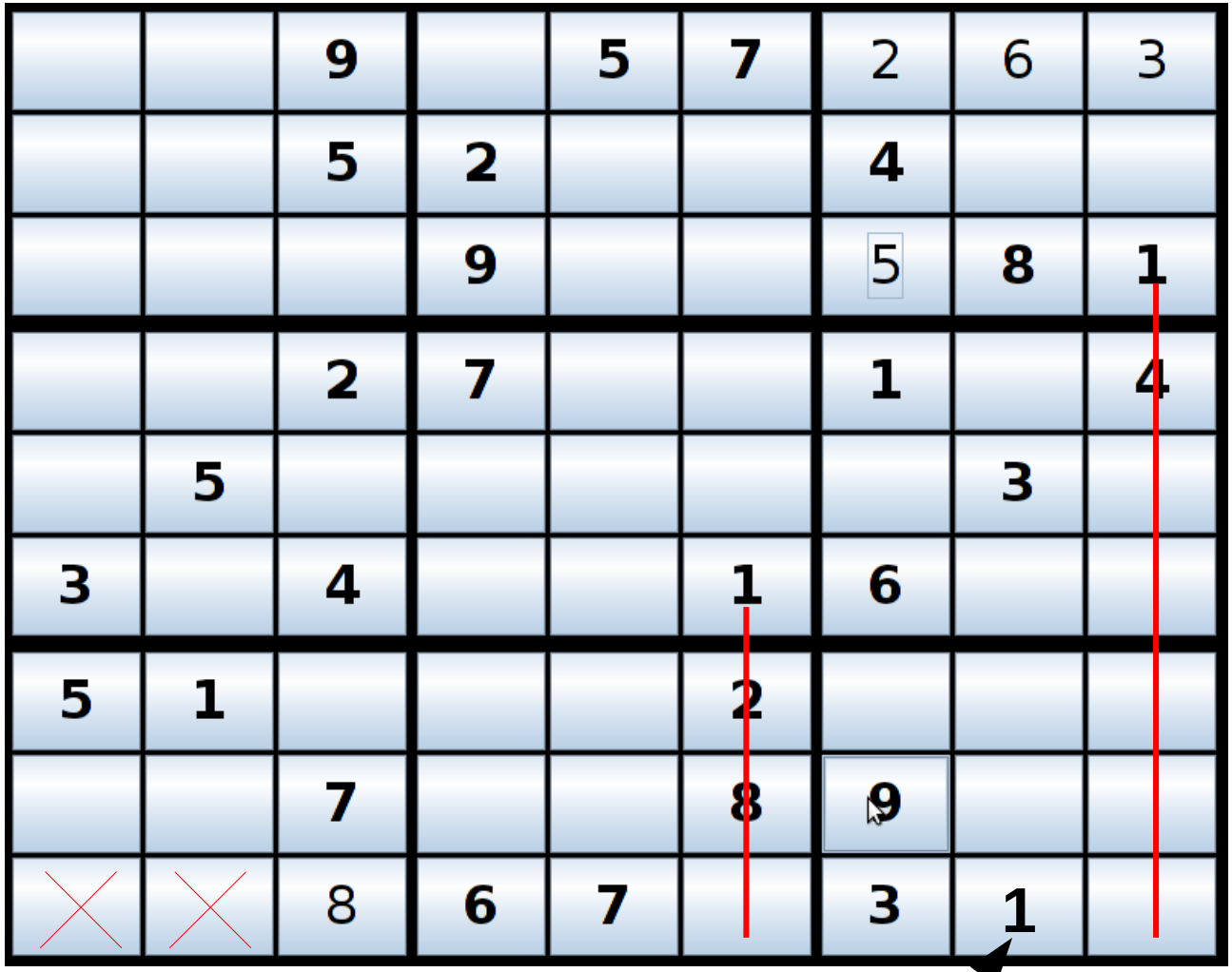
Methods of solving sudoku
Using SET operations :Filling by Row / Column Analysis

Filling By Column Analysis:

In this technique, a cell 'c' of a column 'L' is filled with a value 'x' if 'x' is not a Possible Value in any of the other cell in the same column. i.e.

A cell in a column is filled with a value if it cannot appear in any other cell in the same column

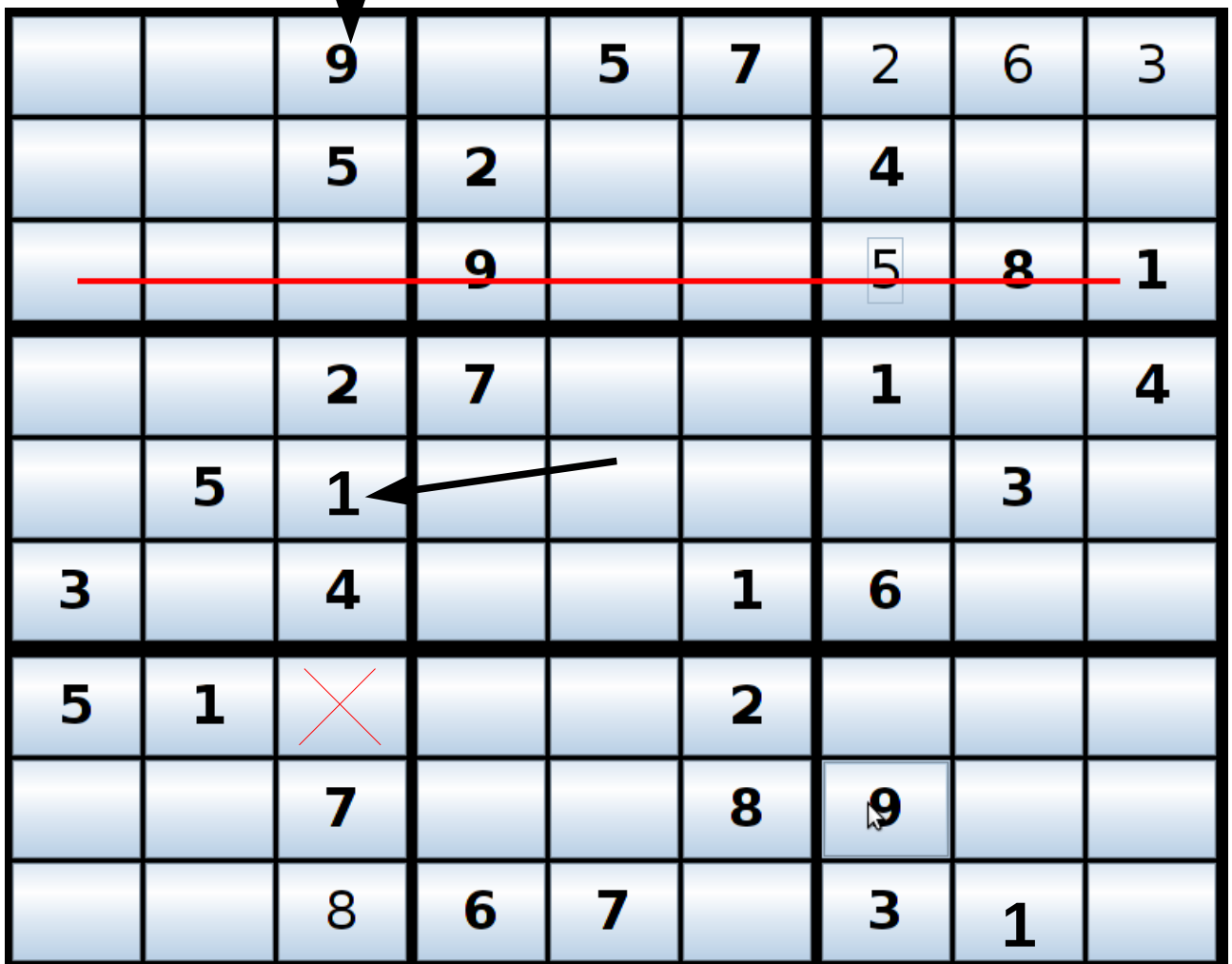
Row/ Column Analysis



Analyzing This Row for 1

Row/ Column Analysis

Analyzing This Column for 1



Row/ Column Analysis

Is Filling by Row / Column Analysis necessary to <u>solve and determine</u> whether a sudoku is solvable without guess ??

YES

Row/ Column Analysis

Analyzing This Row for 2

1	5	8	**2**	**6**	4	9	7	**3**
6	**4**	7	9	3	1	**2**	**8**	5
9	**3**	2	5	**7**	**8**			**6**
2		3		5	**9**			**7**
	7	**9**			6	**5**	3	
5		6	**3**		7		9	
7	9	4	**6**	**1**	2	3	**5**	8
3	**6**	**1**	8	9	5	7	**2**	4
8	2	5	7	**4**	**3**			**9**

Defining Mathematically: Row / column Analysis

If a cell 'j' is under consideration in the ROW 'r' then

$$\text{Result} = \text{cell}[r][j].\text{PossibleValues} - \bigcup_{q=0}^{8} \text{Cell}[r][q].\text{PossibleValues}, \quad \text{Where, } q \neq j$$

If |Result| = 1 then cell [r] [j] = Result

Defining Mathematically: Row / column Analysis

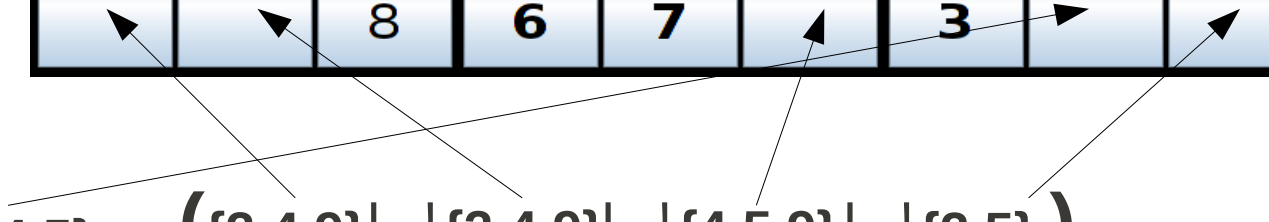
If a cell 'j' is under consideration in the Coloumn 'L' then

$$\text{Result} = \text{cell}[\,j\,][\,L\,].\text{PossibleValues} - \bigcup_{q=0}^{8} \text{Cell}\,[\,q\,]\,[L].\,\text{PossibleValues},\quad \text{Where, } q \neq j$$

If |Result| = 1 then cell [j] [L] = Result

Row/ Column Analysis

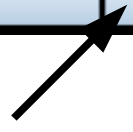
		9		5	7	2	6	3
		5	2			4		
			9			5	8	1
		2	7			1		4
	5						3	
3		4			1	6		
5	1				2			
		7			8	9		
		8	6	7		3		



Result= {1,2,4,5} — ({2,4,9} ∪ {2,4,9} ∪ {4,5,9} ∪ {2,5})

Row/ Column Analysis

		9		5	7	2	6	3
		5	2			4		
			9			5	8	1
		2	7			1		4
	5						3	
3		4			1	6		
5	1				2			
		7			8	9		
		8	6	7		3	1	



{ 1 }

Result= {1,2,4,5} — {2, 4, 5, 9}

Row/ Column Analysis

Using this method, almost all Sudokus of 'Hard' level can be solved.

Not all Sudokus are solvable Without guess

2	6	8	7	9	1	5	**3**	**4**
5	7	**1**	4	3	**6**	9	8	2
9	3	**4**	**5**	8	**2**	1	6	7
7	**8**		2	**1**	3	**4**		**6**
1		6	**8**	4	**7**	3	2	
3	4	**2**	6	**5**	9	7	**1**	**8**
6			**3**	2	**4**	**8**		**1**
4	1		**9**	6	8	**2**		
8	**2**		1	7	5	6	4	

Implementation Details

Tools and languages Used.

Languages Used:

- ***C++ (Backend)***
- ***Java (Frontend) swings***

Tools:

- ***LaTex (Reporting).***
- ***OpenOffice (presentation).***

Implementation Details

Lets execute the program !

Skip implementation details

Comparision and conclusion

- The technique discussed can solve and decide whether a sudoku is solvable without guess or not.

- Set Data structre and associated operation in O(1) helped to solve sudoku in O(n^3)

- Set Data structre was efficient interms of both memory and time.

- we were able to give a mathematical solution to SuDoKu

Comparision and conclusion

- Lot of algorithms exist for generating Sudokus and not for solving them.

General Brute force techniques take

$O(n^n)$ ***for backtracking technique***

Without Set Data Structure.

$O(n^2 \times (4n^2 (3n))) = O(n^2 \times 12n^3) = O(n^5)$

Comparision and conclusion

Using Set Data strucature it reduces to

$$n^2 \times (4(1) (3n)) = O (n^3)$$

Thank you :)

Questions ??