PHYS 673 Project Report

Pradeep Kumar - 30222080

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1 Warm up

1.1 Hamiltonian

a. The Hamiltonian for the system consisting of a two-level atom and a quantized single-mode cavity, including the driving laser, is given by:

$$H = H_{\text{atom}} + H_{\text{cavity}} + H_{\text{int}} + H_{\text{drive}}$$

Where:

$$\begin{aligned} H_{\text{atom}} &= \hbar\omega_a \sigma_+ \sigma_-, \\ H_{\text{cavity}} &= \hbar\omega_c a^\dagger a, \\ H_{\text{int}} &= \hbar g(\sigma_+ + \sigma_-)(a + a^\dagger), \\ H_{\text{drive}} &= \hbar E(a e^{i\omega_L t} + a^\dagger e^{-i\omega_L t}). \end{aligned}$$

b.. To move to the frame rotating with the input/drive laser frequency ω_L, we apply a unitary transformation

$$U = e^{i\omega_L t(\sigma_+\sigma_- + a^\dagger a)}$$

The transformed Hamiltonian becomes [1]:

$$H' = UHU^\dagger + i\hbar\frac{dU}{dt}U^\dagger$$

$$\begin{aligned} UHU^\dagger = & \hbar\omega_a\sigma_+\sigma_- + \hbar\omega_c a^\dagger a + \hbar g(a\sigma_+ + a^\dagger\sigma_-) + \hbar E(a + a^\dagger) \\ & + \hbar g(a^\dagger\sigma_+ e^{-2i\omega_L t} + a\sigma_- e^{2i\omega_L t}) \end{aligned}$$

$$i\hbar\frac{dU}{dt}U^\dagger = -\hbar\omega_L(\sigma_+\sigma_- + a^\dagger a)$$

Therefore,

$$\begin{aligned} H' = & \hbar\Delta_a\sigma_+\sigma_- + \hbar\Delta_c a^\dagger a + \hbar g(a\sigma_+ + a^\dagger\sigma_-) + \hbar E(a + a^\dagger) \\ & + \hbar g(a^\dagger\sigma_+ e^{-2i\omega_L t} + a\sigma_- e^{2i\omega_L t}) \end{aligned}$$

Where $\Delta_a = \omega_a - \omega_L$ and $\Delta_c = \omega_c - \omega_L$ are the detunings of the atom and cavity from the driving laser frequency, respectively.

c. After making the rotating wave approximation (RWA), we neglect fast oscillating terms($e^{\pm 2i\omega_L t}$). Applying the RWA gives:

$$H_{\text{RWA}} = \hbar\Delta_a\sigma_+\sigma_- + \hbar\Delta_c a^\dagger a + \hbar g(a\sigma_+ + a^\dagger\sigma_-) + \hbar E(a + a^\dagger)$$

1.2 Liouvillian

a. The contribution to the system Liouvillian from the cavity field, considering its energy decay rate κ, is given by the Lindblad term:

$$\mathcal{L}_{\text{cavity}}(\rho) = \frac{\kappa}{2}\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$$

Where:

- ρ is the density matrix of the system.
- $a^\dagger$ and a are the creation and annihilation operators for the cavity field.

b. The contribution to the system Liouvillian from the atom's spontaneous emission, considering its energy decay rate γ_s, is also given by a Lindblad term:

$$\mathcal{L}_{\text{spontaneous}}(\rho) = \frac{\gamma_s}{2}\left(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-\right)$$

Where:

- σ_+ and σ_- are the raising and lowering operators for the atom.

c. The contribution to the system Liouvillian from the atom's pure dephasing, considering its dephasing rate γ_p, is given by another Lindblad term:

$$\mathcal{L}_{\text{dephasing}}(\rho) = \frac{\gamma_p}{2}\left(2\sigma_z\rho\sigma_z - \sigma_z^2\rho - \rho\sigma_z^2\right)$$
$$\mathcal{L}_{\text{dephasing}}(\rho) = \gamma_p\left(\sigma_z\rho\sigma_z - \rho\right)$$

Where:

- σ_z is the Pauli-Z operator representing the atomic coherence.

1.3 Cavity response via QuTip

1.3.1 Weak Coupling

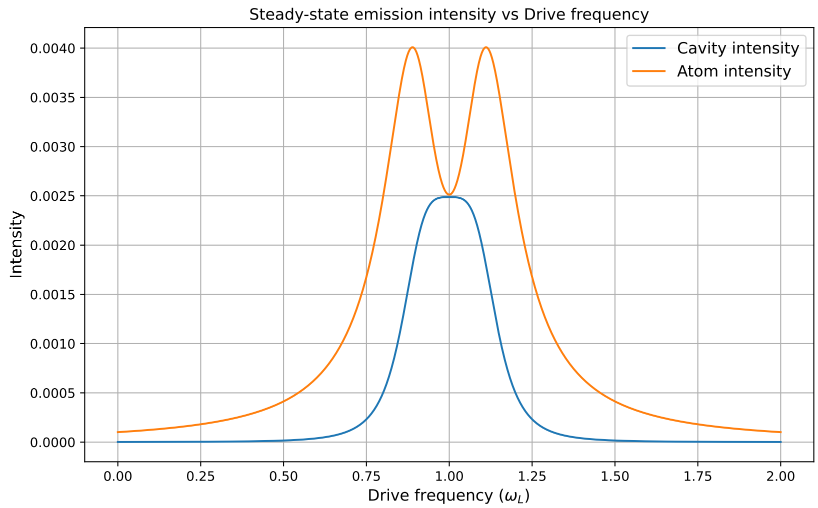

Figure 1: Intensity versus ω_L with parameters : $\omega_a = \omega_c = 1,$ g $= 0.1$ and $\kappa = 0.02$

Comments : In the weak coupling regime we can observe the Lorentzian curve with the width proportional to κ.

1.3.2 Strong Coupling

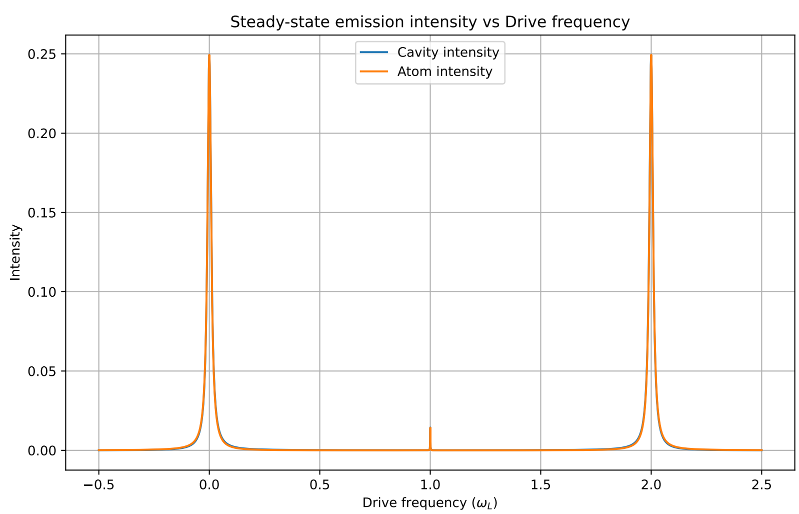

Figure 2: Intensity versus ω_L with parameters : $\omega_a = \omega_c = 1,$ g $= 1$ and $\kappa = 0.001$

Comments : In the strong coupling regime we observe that width of the peaks is very thin as κ is very small also the separation between the peaks is large because of larger g.

1.3.3 Detuned limit

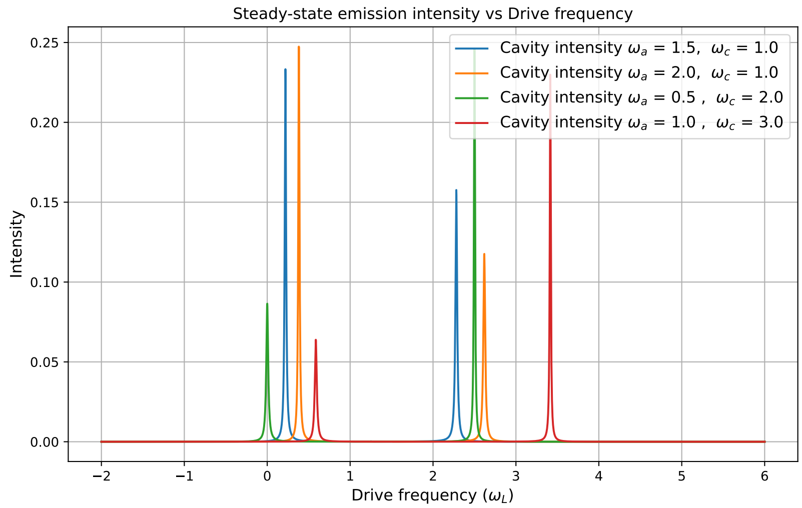

Figure 3: Cavity Intensity vs ω_L with parameters : g = 1 and $\kappa = 0.01$

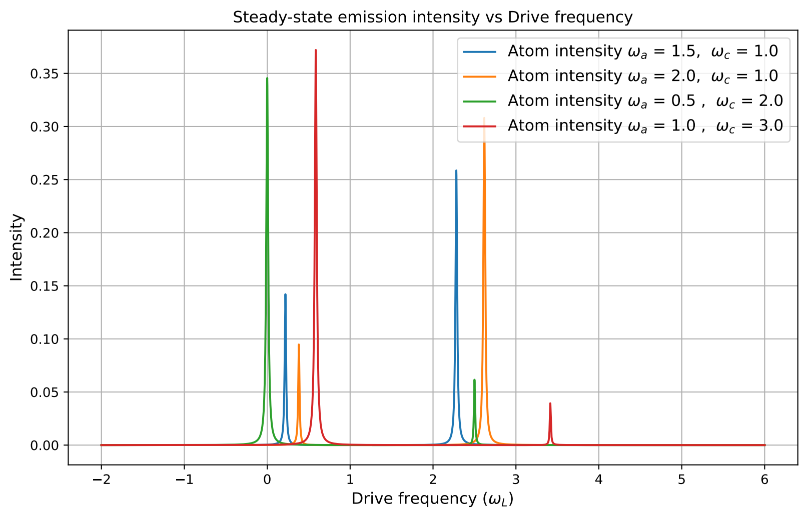

Figure 4: Atom Intensity vs ω_L with parameters : g = 1 and $\kappa = 0.01$

Comments : We observe the asymmetrical sprectra depending upon the values for ω_c and ω_a, with higher intensity when the drive frequency is in resonance with ω_c or ω_a.

1.3.4 Nonlinear Saturation

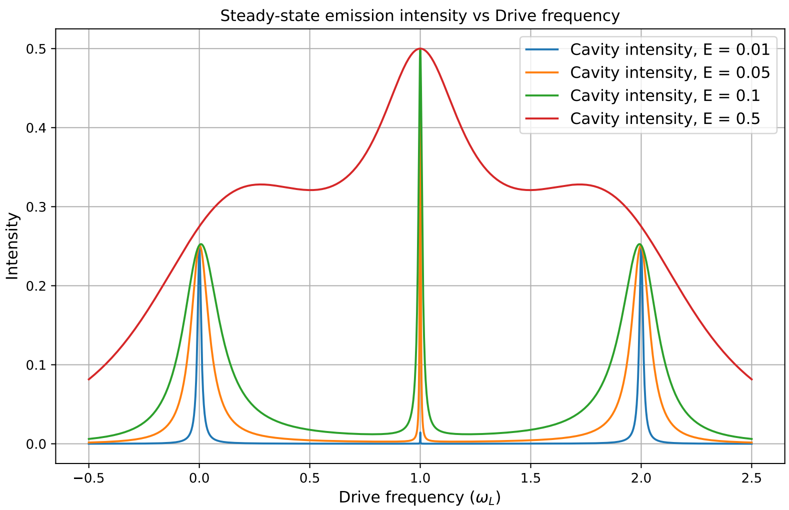

Figure 5: Cavity Intensity vs ω_L with parameters : $\omega_a = \omega_c = 1$, g = 1, $\kappa = 0.001$ and Fock space dimension d=2

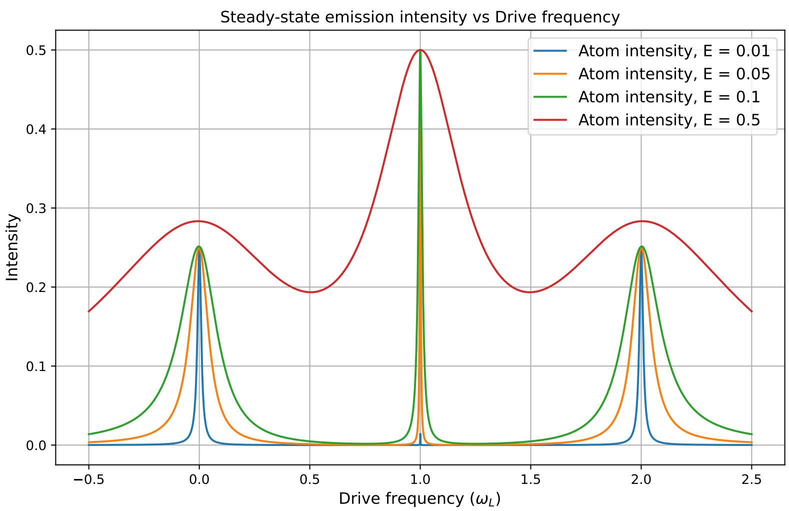

Figure 6: Atom Intensity vs ω_L with parameters : $\omega_a = \omega_c = 1$, g = 1, $\kappa = 0.001$ and Fock space dimension d=2

Comments : If we restrict to two dimensional fock space then we will only observe single photon excitation.

Fock space dimension = 8

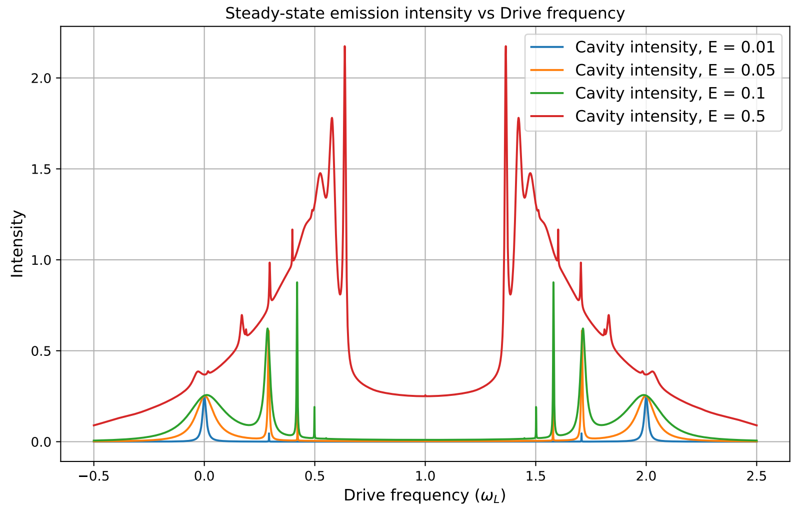

Figure 7: Cavity Intensity vs ω_L with parameters : $\omega_a = \omega_c = 1$, g = 1, $\kappa = 0.001$ and Fock space dimension d=8

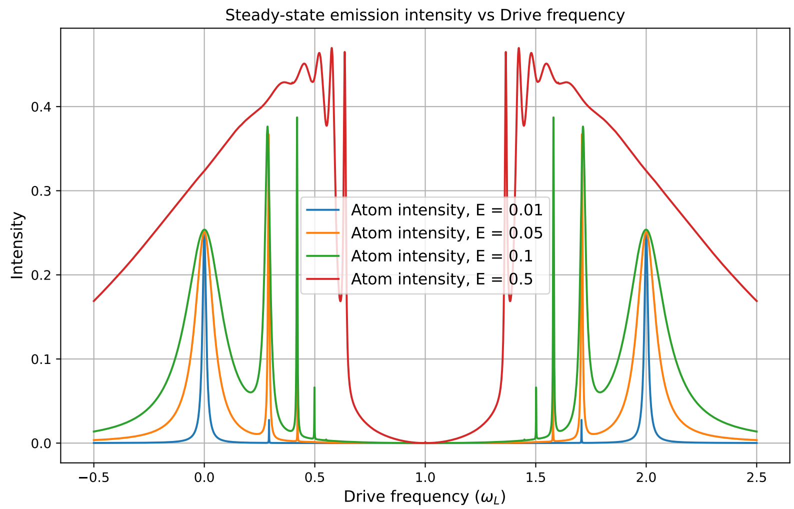

Figure 8: Atom Intensity vs ω_L with parameters : $\omega_a = \omega_c = 1$, g = 1, $\kappa = 0.001$ and Fock space dimension d=8

Comments : The outermost peaks corresponds to single photon excitation whereas the inner peaks corresponds to multi photon excitation. This is most prominent for the strong field E = 0.5

2 Modelling a real system

2.1 Downloaded the paper

2.2 Read the paper

2.3 Reproduction of Fig. 7a

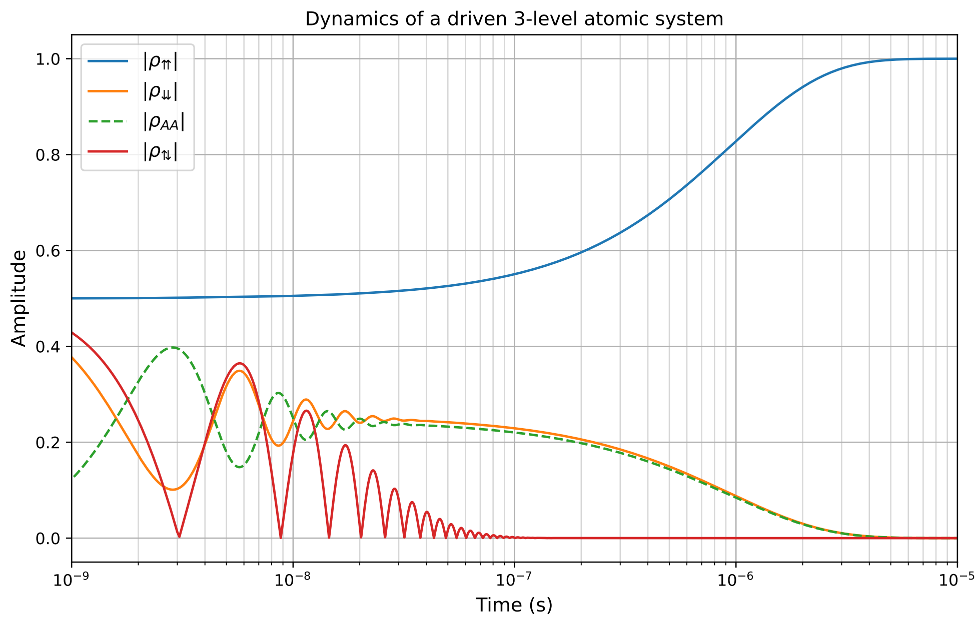

Figure 9: Three level model, with parameters : $\chi = 100$, $\frac{\gamma}{2\pi} = 35\text{MHz}$, $\Omega_{A1} = 5\gamma$ and $\delta = 0$ [2]

2.4 Comments on the relation of the data in the figure to photon generation

The initial state of the system is the equal superposition of $|\downarrow\rangle$ state and $|\uparrow\rangle$ state. The $|\downarrow\rangle$ is driven to $|A\rangle$ state and its population initially drops as seen in the drop of $|\rho_{\downarrow\downarrow}|$ the Figure 9, and the population of $|A\rangle$ state increases and then the population of $|A\rangle$ state starts to decrease as seen in the drop of $|\rho_{AA}|$. This drop is directly related to photon generation as the system emits radiation via relaxation to the $|\downarrow\rangle$ state at rate γ_{A1}, or the $|\uparrow\rangle$ state at rate γ_{A2}. Every time the population of $|A\rangle$ state drops as seen by the drop in $|\rho_{AA}|$, we will observe photons.

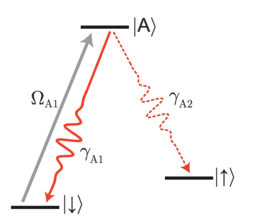

Figure 10: A three-level atomic system is driven to the $|A\rangle$ state at rate Ω_{A1}. From $|A\rangle$, the system emits radiation via relaxation to the $|\downarrow\rangle$ state at rate γ_{A1}, or the $|\uparrow\rangle$ state at rate γ_{A2}.

References

[1] D. A. Steck. "Quantum and atom optics." Revision 0.15, 21 February 2024. (2024), [Online]. Available: `http://steck.us/teaching`.

[2] E. I. Rosenthal, S. Biswas, G. Scuri, *et al.*, *Single-shot readout and weak measurement of a tin-vacancy qubit in diamond*, 2024. arXiv: `2403.13110 [quant-ph]`.