Q1. Solution

Eigenvalues and eigenvectors are concepts in linear algebra that play a crucial role in understanding linear transformations and diagonalization of matrices.

- Eigenvalues (λ) are scalar values that represent how a linear transformation stretches or compresses a vector. They indicate the factor by which the eigenvectors are scaled when the transformation is applied.
- Eigenvectors (v) are non-zero vectors that remain in the same direction after the transformation, although they may be scaled by the corresponding eigenvalue.

Eigen-Decomposition is an approach in linear algebra that decomposes a square matrix into a set of eigenvectors and eigenvalues.

Mathematically, for a square matrix A, the eigen-decomposition can be expressed as: A = QΛQ^(-1), where Q is a matrix whose columns are the eigenvectors of A, and Λ is a diagonal matrix containing the corresponding eigenvalues.

Example: Consider a 2x2 matrix A: �=(3113)*A*=(3113)

To find the eigenvalues, we solve the characteristic equation det(A - λI) = 0: |3−�113−�|=(3−�)2−1=0||3−*λ*113−*λ*||=(3−*λ*)2−1=0 Solving this equation, we get eigenvalues λ = 4, 2.

Next, for each eigenvalue, we find the corresponding eigenvector by solving the equation (A - λI)v = 0: For λ = 4, we get the eigenvector v1 = [1, 1]. For λ = 2, we get the eigenvector v2 = [-1, 1].

Therefore, the eigen-decomposition of A is:

�=(1−111)(4002)(1−111)−1*A*=(11−11)(4002)(11−11)−1

Q2.

Eigen decomposition is a method used to decompose a square matrix into a set of eigenvectors and eigenvalues. It is significant in linear algebra because it allows for the diagonalization of matrices, simplifying computations and providing insights into the behavior of linear transformations.

Q3.

A square matrix A is diagonalizable using the Eigen-Decomposition approach if it has a complete set of linearly independent eigenvectors. This condition implies that A must have n linearly independent

eigenvectors, where n is the dimension of the matrix. A square matrix is diagonalizable if and only if it has n linearly independent eigenvectors.
Proof: Let A be an n × n square matrix with n linearly independent eigenvectors. Then, A can be diagonalized as $A = Q\Lambda Q^{(-1)}$, where Q is the matrix of eigenvectors and Λ is the diagonal matrix of eigenvalues. Since there are n linearly independent eigenvectors, the matrix Q has full rank and is invertible. Therefore, A is diagonalizable.

Q4.
The spectral theorem states that for a symmetric matrix, the eigenvalues are real, and the eigenvectors corresponding to distinct eigenvalues are orthogonal. In the context of the Eigen-Decomposition approach, the spectral theorem ensures that the diagonalization of a symmetric matrix is possible, leading to the representation of the matrix in terms of its eigenvalues and eigenvectors.
Example: Consider a symmetric matrix A: �=(2−1−12)*A*=(2−1−12)
The eigenvalues are λ = 3, 1, and the corresponding eigenvectors are [1, 1] and [1, -1]. These eigenvectors are orthogonal, satisfying the conditions of the spectral theorem.

Q5.
Eigenvalues of a matrix are found by solving the characteristic equation $\det(A - \lambda I) = 0$, where A is the matrix, λ is the eigenvalue, and I is the identity matrix. Eigenvalues represent the scaling factors by which the corresponding eigenvectors are stretched or compressed when the matrix transformation is applied.

Q6.
Eigenvectors are non-zero vectors that remain in the same direction (up to scaling) after the matrix transformation is applied. They are associated with eigenvalues and represent the directions of maximum variance or stretching in the data.

Q7.
Geometrically, eigenvectors represent the directions along which a linear transformation has a simple behavior, such as stretching or compressing. Eigenvalues represent the scaling factors or magnitudes of these transformations along the corresponding eigenvectors.

Q8.
Some real-world applications of eigen decomposition include:

- Principal Component Analysis (PCA) for dimensionality reduction in data analysis
- Eigenfaces for facial recognition and image processing
- Eigendecomposition of correlation or covariance matrices in finance for risk management and portfolio optimization
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Q9. Yes, a matrix can have more than one set of eigenvectors and eigenvalues. However, each set corresponds to different transformations or characteristics of the matrix.

Q10. The Eigen-Decomposition approach is useful in data analysis and machine learning in several ways:

- Principal Component Analysis (PCA): PCA uses eigen decomposition to reduce the dimensionality of data while preserving the most significant information.
- Spectral Clustering: Eigen decomposition is used to compute the eigenvectors of similarity matrices for clustering high-dimensional data.
- Eigenfaces: In facial recognition systems, eigen decomposition is used to represent faces as linear combinations of eigenvectors for efficient classification.