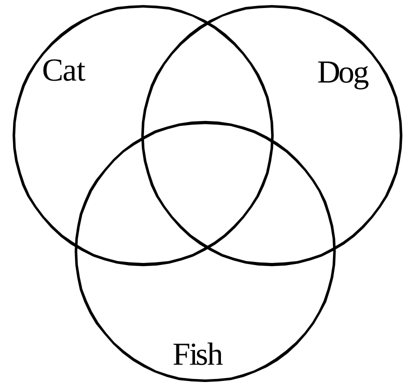
# Venn Diagrams



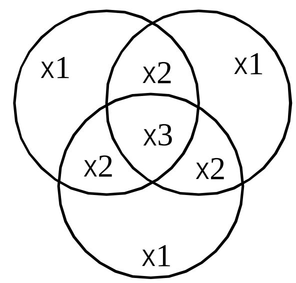
Using more than two variables can be more difficult in a Venn diagram.

**Example:**

Every student who appled for admission to a veterinary school has at least one pet: 30 have a cat, 28 have a dog and 26 have fish. If 13 students have fish and a cat, 15 students have fish and a dog, 11 students have both a cat and a dog, and 4 students have a cat, a dog, and fish. How many students applied to veterinary school? Begin at the center of the diagram below and work your way out.




Using a Venn Diagram is easy enough, but you can also use the following logic: 30 have cats + 28 have a dog + 26 have fish = 84 owners. People who own two (or more) different animals were counted multiple times, so we subtract them: 84 - 13 - 15 - 11 = 45. This seems right, but it is different from the answer we got with the diagram. Notice that when we subtracted people with two pets, we subtracted the people who have all three pets three times (we actually wanted to subtract them twice). Add 4 back to get 49.




**Example:**

7 out of 8 dentists recommend brushing your teeth after every meal, and 6 out of 8 dentists recommend flossing after every meal, and 5 out of 8 dentists recommend chewing sugarless gum after every meal, and every dentist recommends you do at least one of these three things. What is the fewest possible number of dentists who recommend doing all three?

**Hard Practice:**

**1.** How many integers from 1 to 60 are multiples of 3, 4, or 5?

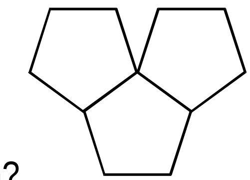
**2.** A small auto dealership sells expensive foreign automobiles. You are looking for a black convertible Porsche. The lot has 50 cars that meet at least one of your criteria. They have 18 Porsches, 25 black cars, and 16 convertibles. There are 3 black convertibles, 4 black Porsches, and 5 convertible Porsches. How many black convertible Porsches are there?

# Permutations



**Consider the following:**
Five students are running a race. In how many ways can the five students place 1st, 2nd, and 3rd?

An organization is choosing colors for the three pentagons in its logo. They have narrowed their choice to 7 colors, and want to have three diffierent colors in their logo. How many possible logos can be created by choosing three of the seven colors and using one for each pentagon?



There is a formula for **permutations** and some notation which we must consider. **P(n,r)** means the number of permutations of n things taken r at a time.

For example, five people in a race taking three places is **P(5,3).**

$$P(5,3) = 5 \cdot 4 \cdot 3$$

The formula for **P(n,r).**

$$P(n,r) = \frac{n!}{(n-r)!}$$

**Got that?** Now, you can really forget this... use common sense, don't memorize a formula. You should, however, remember what **P(7,2)** means.

**Practice:**
**1.** Find **P(10,3)**.
**2.** Find **P(8,6)**.
**3.** For the pick 3 lottery, six balls numbered 1 through 6 are placed in a hopper and randomly selected one at a time without replacement to create a three-digit number. How many different three-digit numbers can be created?
**4.** How many ways can five books be ordered on a shelf from left to right?
**5.** Eight people are asked to select a leadership team: president, vice-president, and secretary among themselves. How many different leadership teams are possible?

# Permutations



**Letter Arrangements:**

Perhaps the most commonly asked questions involving permutations involve arrangements of letters (probably because these problems are easy to write).

**Ex.**

How many different four-letter 'words' can be formed by rearranging the letters in the word MATH?

**Solution:** There are four letters to choose from for the first spot, leaving three for the second, two for the third, and one remaining for the end of the word:

$$P(4,4) = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

**Ex.**

How many different four-letter 'words' can be formed by rearranging the letters in the word COUNTS?

**Solution:** This is still just a basic permutation. There are:

$$P(6,4) = 6 \cdot 5 \cdot 4 \cdot 3 = 360$$ 'words' this time.

**Ex.**

How many different seven-letter 'words' can be formed by rearranging the letters in the word ALGEBRA?

**Solution:** This is more difficult. Notice that there are two A's. No need to panic, just pretend for a moment that the A's are different. We'll call one of the them $A_1$ and the other $A_2$.

$$P(7,7) = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5{,}040$$ 'words'.

However, we overcounted. $A_1LGEBRA_2$ is the same as $A_2LGEBRA_1$, just like $A_1A_2LEGBR$ is the same as $A_2A_1LEGBR$. To eliminate the extra cases, we need to divide by the ways that the As can be arranged, which in this case is just 2.

$$\frac{P(7,7)}{2} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2} = 2{,}520$$ 'words'.

**Practice:**

Find the number of arrangements of the letters of the word COOKBOOK.

# Permutations



**Restrictions:**
Often there are restrictions placed on permutation problems:

**Examples:**

**1.** How many *even* five-digit numbers contain each of the digits 1 through 5?

**2.** How many of the arrangements of the letters in the word COUNTING contain a double-N?

**3.** In how many of the arrangements of the letters in the word EXAMPLE are the letters A and M adjacent to each other?

**Practice:**

**1.** How many arrangements of the letters in the word START begin with a T?

**2.** How many arrangements of the letters in the word BEGIN start with a vowel?

**3.** How many arrangements of the letters in the word BEGINNING have an N at the beginning?

**4.** How many of the arrangements of the digits 1 through 6 have the 1 to the left of the 2?

**Practice:**

**1.** Seven students line up on stage. If Molly insists on standing next to Katie, how many different ways are there to arrange the students on stage from left to right?

**2.** How many arrangements of the letters in the word ORDERED include the word RED?

**3.** How many arrangements of the letters in the word ORDERED begin and end with the same letter?

**4.** There are nine parking spots in front of the building for six teachers and the principal. If the principal always gets one of the three shady spots, how many ways can all seven cars be parked in the lot?

# Combinations



**Consider the following problems:**

**1.** There are six scrabble letters left in the bag at the end of the game (FHJSU and Y). If you reach in and grab two letters, how many different pairs of letters are possible?

**2.** Kelly wants to offer three sodas at her snack stand. She has a list of 8 sodas to choose from. How many combinations of sodas are possible?

**3.** Roger has won a contest at the fair, and gets to choose four different prizes from a set of nine. How many **combinations** of four prizes can he **choose** from a set of nine?

The primary difference is that ORDER DOES NOT MATTER.
There is a formula for **combinations** and some notation which we must consider. **C(n,r)** means the number of combinations of n things taken r at a time.

For example, choosing four prizes from a set of nine is **C(9,4).**

$$C(9,4) = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 126$$

The formula for **C(n,r).**

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

Don't memorize it, understand it!

$C(n,r)$ is also notated as $\binom{n}{r}$ or $nCr$ and is often called 'choose'.

**Practice:**

**1.** Find $_6C_2$.

**2.** Find $C(6,4)$.

**3.** Explain why the answers above are equal.

**4.** How many ways can three books be chosen from a shelf of 20?

**5.** Eight people are asked to select a leadership team of three members. How many different leadership teams are possible?

Name________________________ Period _____

# Combinations and Permutations

Counting

**Solve each:**

**1.** How many ways can six songs be placed in order on a CD?

1.________

**2.** How many different arrangements of the letters in the word COUNTING are possible?

2.________

**3.** Of the 13 players on a soccer team, 11 are starters. How many different teams of 11 starters are possible?

3.________

**4.** Ten students rush into the cafeteria and take six seats at a table. How many possible combinations of students are left standing?

4.________

**5.** A phone number has seven digits and cannot start with a 0 or a 1.
How many phone numbers can be created if digits cannot be repeated?
(hint: How many choices are there for the first digit? the second? etc.)

5.________

**6.** Jacob has two red pegs to place in the game board below. How many different placements are possible?

o o o o o o
1 2 3 4 5 6

6.________

**7.** Jacob has three red pegs and three blue pegs to fill six holes in a game board. How many different color combinations are possible?

o o o o o o
1 2 3 4 5 6

7.________

**8.** Jacob has two red pegs, two green pegs, and two blue pegs to fill six holes in a game board. How many different color combinations are possible?

o o o o o o
1 2 3 4 5 6

8.________