MATHCOUNTS®

2010
■ School Competition ■
Sprint Round
Problems 1–30

Name _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

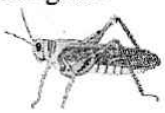
Total Correct	Scorer's Initials

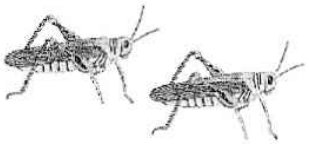
1. The table shown is partially filled in with the results of a survey done by the radio station KMAT. How many of the females surveyed listen to this station?

	listen	don't listen
males	62	
females		102
total	130	150

1. _____ females

2. In a recent month, a variety of insectivores at the zoo consumed 46,000 grasshoppers. The zoo paid \$12 per 1000 grasshoppers. If grasshoppers are available only in units of 1000, how many dollars were spent on grasshoppers consumed during that month?





2. \$ _____

3. Addison's age is three times Brenda's age. Janet is six years older than Brenda. Addison and Janet are twins. How old is Brenda?

3. _____ years

4. If a drip of water is equivalent to $\frac{1}{4}$ of a milliliter, how many drips are in a liter of water? Note: 1 liter = 1000 milliliters.

4. _____ drips

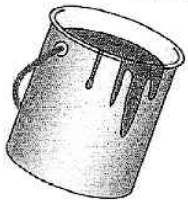
5. Given that $\aleph = 6$ and $\Diamond = 3$, what is the value of $\aleph^2 + \Diamond$?

5. _____

6. Tiffany scored 29 points in her school's basketball playoff game. She made a combination of 2-point baskets and 3-point baskets during the game. If she made a total of 11 baskets, how many 3-point baskets did she make?

6. _____ 3-point baskets

7. It would take John six hours to paint a particular room by himself. It would take Tom 12 hours to paint the same room by himself. If John and Tom work together, each at his individual rate, how many hours will it take them to paint the room?

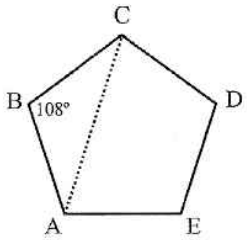


7. _____ hours

8. The mean of seven positive integers is 16. When the smallest of these seven integers is removed, the sum of the remaining six integers is 108. What is the value of the integer that was removed?

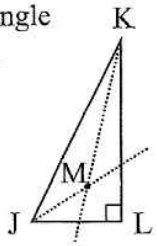
8. _____

9. In regular pentagon ABCDE, diagonal AC is drawn, as shown. Given that each interior angle of a regular pentagon measures 108 degrees, what is the measure of angle CAB?



9. _____ degrees

10. In right triangle JKL, angle J measures 60 degrees and angle K measures 30 degrees. When drawn, the angle bisectors of angles J and K intersect at a point M. What is the measure of obtuse angle JMK?

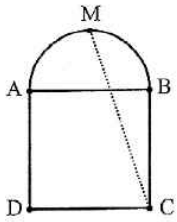


10. _____ degrees

11. When Sarah rowed down Black River with the current, she took one hour to go four miles. When she rowed back the same distance, at the same rowing speed, but against the current, her trip required two hours. What is the speed, in miles per hour, of the current in Black River?

11. _____ mph

12. Square ABCD is constructed along diameter AB of a semicircle, as shown. The semicircle and square ABCD are coplanar. Line segment AB has a length of 6 centimeters. If point M is the midpoint of arc AB, what is the length of segment MC? Express your answer in simplest radical form.



12. _____ cm

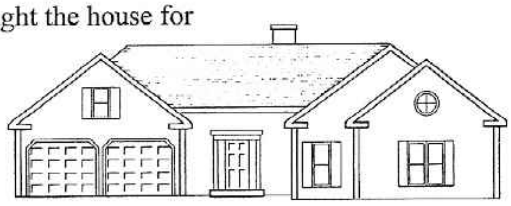
13. How many positive integers between 200 and 500 are divisible by each of the integers 4, 6, 10 and 12?

13. _____ positive integers

14. How many three-letter arrangements can be made if the first and third letters each must be one of the 21 consonants, and the middle (second) letter must be one of the five vowels? Two such arrangements to include are KOM and XAX.

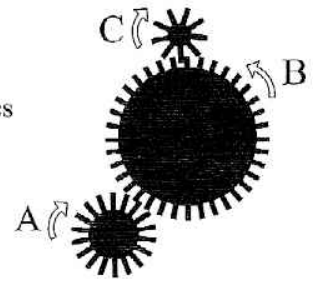
14. _____ arrangements

15. It cost Mr. Andrews $200,000 to build a house. He sold it to Ms. Bond at a 10% profit. Later Ms. Bond sold it to Mr. Cash at a 10% loss. What is the positive difference between the amount Ms. Bond bought the house for and the amount Ms. Bond sold the house for?



15. $ _____

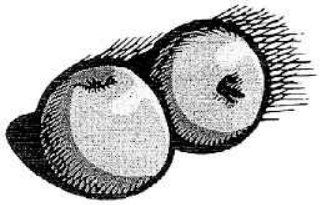
16. A machine has three interlocking gears. Gear A has 18 teeth and turns clockwise at 30 revolutions per minute. Gear B turns counter-clockwise and has 36 teeth. Gear C turns clockwise and has 9 teeth. How many revolutions does gear C turn in one minute?



16. _____ revolutions

17. When the length of a rectangle is increased by 20% and the width is increased by 10%, by what percent is the area increased?

17. _____ %



18. In a bag containing only red apples and green apples, the number of red apples is $\frac{3}{4}$ of the number of green apples. What fraction of the apples in the bag is red? Express your answer as a common fraction.

18. _____

19. The point (0, 0) is reflected across the vertical line $x = 1$. Its image is then reflected across the line $y = 2$. What are the coordinates of the resulting point?

19. (,)

20. In trapezoid ABCD, the parallel sides AB and CD have lengths of 8 and 20 units, respectively, and the altitude is 12 units. Points E and F are the midpoints of sides AD and BC, respectively. What is the area of quadrilateral EFCD in square units?

20. _____ sq units

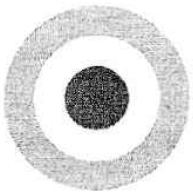
21. Ms. Osborne's class wants to decorate its classroom, and the students want to collect the money to fund the decorating. If every student gives \$1.50, then the class will be 95¢ short. If every student gives \$1.60, then the class will have \$1.15 left over. How many students are in Ms. Osborne's class?

21. _____ students

22. A bag contains exactly three red marbles, five yellow marbles and two blue marbles. If three marbles are to be drawn from the bag at the same time, what is the probability that all three will be the same color? Express your answer as a common fraction.

22. _____

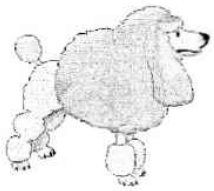
23. A target consists of concentric circles of radii 1 cm, 2 cm and 3 cm. The innermost circle is colored red, the middle ring is colored white, and the outer ring is colored blue. If a point is chosen at random on the target, what is the probability that it lies in the blue region? Express your answer as a common fraction.



23. _____

24. Fido can chew a strip of rawhide at a rate of five inches per minute. Fluffy can chew a strip of rawhide at a rate of two inches per minute. Fluffy starts chewing on a long strip of rawhide, and six minutes later, Fido starts chewing on the other end. Each of them chews until all of the rawhide is gone, and in the end they have each consumed half of the original piece of rawhide. How long was the original strip of rawhide?

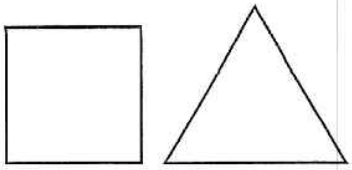
24. _____ inches



25. A student correctly answers 15 of the first 20 questions on an examination. He then answers $\frac{1}{3}$ of the remaining questions correctly. All of the questions are worth the same amount. If the student's final score is 50%, how many questions are on the exam?

25. _____ questions

26. A square and an equilateral triangle have equal perimeters. The area of the triangle is $16\sqrt{3}$ square centimeters. How long, in centimeters, is a diagonal of the square? Express your answer in simplest radical form.



26. _____ cm

27. Six students (four juniors and two seniors) must be split into three pairs. If the pairs are chosen randomly, what is the probability that the two seniors form one pair? Express your answer as a common fraction.

27. _____

28. Bus A is 150 miles due east of Bus B. Both busses start driving due west at constant speeds at the same time. It takes Bus A 10 hours to overtake Bus B. If they had started out at the same time, had driven at the same constant speeds, but had driven toward one another, they would have met in 2 hours. What is the speed, in miles per hour, of Bus A?

28. _____ mph

29. Let the function $a \,@\, b$ be defined as $3a + 2b$ for all real numbers a and b. If u and v are real numbers for which $u \,@\, v = v \,@\, u = 20$, what is the value of $2u \,@\, 3v$?

29. _____

30. If a positive two-digit integer is divided by the sum of its digits, the quotient is 2 with a remainder of 2. If the same two-digit integer is multiplied by the sum of its digits, the product is 112. What is the two-digit integer?

30. _____