

# MATHCOUNTS® COMPETITION SERIES

EST. 1983

2017-2018  
**SCHOOL HANDBOOK**



Check out  
this year's math  
problems on

**PG. 11!**

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# 2017-2018

# MATHCOUNTS®

# SCHOOL HANDBOOK

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*The National Association of Secondary School Principals has placed all three MATHCOUNTS programs on the NASSP Advisory List of National Contests and Activities for 2017-2018.*

# HOW TO USE THIS SCHOOL HANDBOOK

## If You're a New Coach



Welcome! We're so glad you're a coach this year.  
Check out the **Guide for New Coaches**  
starting on the next page.

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## If You're a Returning Coach



Welcome back! Thank you for coaching again.  
Get the **2018-2019 Handbook Materials**  
starting on page 8.

# GUIDE FOR NEW COACHES

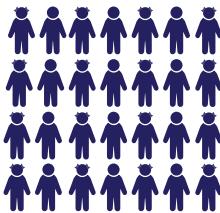
**Welcome to the MATHCOUNTS® Competition Series!** Thank you so much for serving as a coach this year. Your work truly does make a difference in the lives of the students you mentor. We've created this Guide for New Coaches to help you get acquainted with the Competition Series and understand your role as a coach in this program.

If you have questions at any point during the program year, please feel free to contact the MATHCOUNTS national office at (703) 299-9006 or [info@mathcounts.org](mailto:info@mathcounts.org).

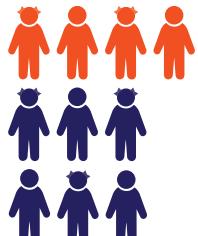
## THE MATHCOUNTS COMPETITION SERIES IN A NUTSHELL

The **MATHCOUNTS Competition Series** is a national program that provides students the opportunity to compete in live, in-person math contests against and alongside their peers. Created in 1983, it is the longest-running MATHCOUNTS program and is open to all sixth-, seventh- and eighth-grade students.

**HOW DOES IT WORK?** The Competition Series has 4 levels of competition—school, chapter, state and national. Here's what a typical program year looks like.



**Schools register in the fall and work with students during the year. Coaches administer the School Competition, usually in January.** Any number of students from your school can participate in your team meetings and compete in the School Competition. MATHCOUNTS provides the School Competition to coaches in November. Many coaches use this to determine which student(s) will advance to the Chapter Competition.



**Between 1 and 10 students from each school advance to the local Chapter Competition, which takes place in February.** Each school can send a team of 4 students plus up to 6 individual competitors. All chapter competitors—whether they are team members or individuals—participate in the individual rounds of the competition; then just the 4 team members participate in the team round. Schools also can opt to send just a few individual competitors, rather than forming a full team. Over 500 Chapter Competitions take place across the country.



**Top students from each Chapter Competition advance to their State Competition, which takes place in March.** Your school's registration fees cover your students as far as they get in the Competition Series. If your students make it to one of the 56 State Competitions, no additional fees are required.



**Top 4 individual competitors from each State Competition receive an all-expenses-paid trip to the National Competition, which takes place in May.** These 224 students combine to form 4-person state teams, while also competing individually for the title of National Champion.

**WHAT DOES THE TEST LOOK LIKE?** Every MATHCOUNTS competition consists of 4 rounds—Sprint, Target, Team and Countdown Round. Altogether the rounds are designed to take about 3 hours to complete. Here's what each round looks like.



### Sprint Round

40 minutes  
30 problems total  
no calculators used  
focus on speed and accuracy



### Target Round

Approx. 30 minutes  
8 problems total  
calculators used  
focus on problem-solving and mathematical reasoning

*The problems are given to students in 4 pairs. Students have 6 minutes to complete each pair.*



### Team Round

20 minutes  
10 problems total  
calculators used  
focus on problem-solving and collaboration

*Only the 4 students on a school's team can take this round officially.*



### Countdown Round

Maximum of 45 seconds per problem  
no calculators used  
focus on speed and accuracy

*Students with highest scores on Sprint and Target Rounds compete head-to-head. This round is optional at the school, chapter and state level.*

**HOW DO I GET MY STUDENTS READY FOR THESE COMPETITIONS?** What specifically you do to prepare your students will depend on your schedule as well as your students' schedules and needs. But in general, working through lots of different MATHCOUNTS problems and completing practice competitions is the best way to prepare to compete. Each year MATHCOUNTS provides the *School Handbook* to all coaches, plus lots of additional free resources online.

The next sections of this Guide for New Coaches will explain the layout of the *MATHCOUNTS School Handbook* and other resources, plus give you tips on structuring your team meetings and preparation schedule.

## THE ROLE OF THE COMPETITION COACH

Your role as the coach is such an important one, but that doesn't mean you need to know everything, be a math expert or treat coaching like a full-time job. Every MATHCOUNTS coach has a different coaching style and you'll find the style that works best for you and your students. But in general **every good MATHCOUNTS coach must do the following.**

- Schedule and run an adequate number of practices for participating students.
- Help motivate and encourage students throughout the program year.
- Select the 1-10 student(s) who will represent the school at the Chapter Competition in February.
- Take students to the Chapter Competition or make arrangements with parents and volunteers to get them there.



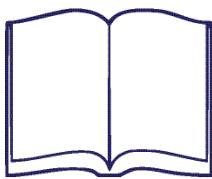
**LOOKING FOR TOOLS TO HELP YOU BECOME A TOP-NOTCH COACH? CHECK OUT OUR VIDEOS AT THE COACH SECTION OF THE MATHCOUNTS WEBSITE!**

You don't need to know how to solve every MATHCOUNTS problem to be an effective coach. In fact, many coaches have told us that they themselves improved in mathematics through coaching. Chances are, you'll learn with and alongside your students throughout the program year.

You don't need to spend your own money to be an effective coach. You can prepare your students using solely the free resources and this handbook. We give coaches numerous detailed resources and recognition materials so you can guide your Mathletes® to success even if you're new to teaching, coaching or competition math, and even if you use only the free resources MATHCOUNTS provides all competition coaches.

## MAKING THE MOST OF YOUR RESOURCES

As the coach of a registered competition school, you already have received what we at MATHCOUNTS call the **School Competition Kit**. Your kit includes the following materials for coaches.



### 2017-2018 MATHCOUNTS School Handbook

The most important resource included in the School Competition Kit. Includes 250 problems.



### Student Recognition Ribbons and Certificates

10 participation certificates and 1 ribbon for each registered chapter competitor.

You'll also get access to electronic resources. The following resources are available to coaches online at [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches). This section of the MATHCOUNTS website is restricted to coaches and you already should have received an email with login instructions. *If you have not received this email, please contact us at info@mathcounts.org to make sure we have your correct email address.*

#### Official 2018 MATHCOUNTS School Competition

Released in November 2017  
Includes all 4 test rounds and the answer key

#### 2017 MATHCOUNTS School, Chapter + State Competitions

Released by mid-April 2018  
Each level includes all 4 test rounds and the answer key

#### MATHCOUNTS Problem of the Week

Released each Monday  
Each multi-step problem relates to a timely event

You can use the **2018 MATHCOUNTS School Competition** to choose the students who will represent your school at the Chapter Competition. Sometimes coaches already know which students will attend the Chapter Competition. If you do not need the School Competition to determine your chapter competitors, then we recommend using it as an additional practice resource for your students.

The **2017-2018 MATHCOUNTS School Handbook** will be your primary resource for the Competition Series this year. It is designed to help your students prepare for each of the 4 rounds of the test, plus build critical thinking and problem-solving skills. This section of the Guide for New Coaches will focus on how to use this resource effectively for your team.

**WHAT'S IN THE HANDBOOK?** There is a lot included in the *School Handbook*, and you can find a full table of contents on pg. 8 of this book, but below are the sections that you'll use the most when coaching your students.

- **Handbook Problems:** 250 math problems divided into Warm-Ups, Workouts and Stretches. These problems in-



**CHECK OUT OUR  
ONLINE COACH RESOURCE  
VIDEOS: MAKING THE  
MOST OF YOUR  
COACHING RESOURCES  
HOW TO USE THE  
HANDBOOK**

- crease in difficulty as the students progress through the book. (pg. 11)
- **Solutions to Handbook Problems:** complete step-by-step explanations for how each problem can be solved. These detailed explanations are only available to registered coaches. (pg. 56)
  - **Answers to Handbook Problems:** key available to the general public. Your students can access this key, but not the full solutions to the problems. (pg. 49)
  - **Problem Index + Common Core State Standards Mapping:** catalog of all handbook problems organized by topic, difficulty rating and mapping to Common Core State Standards. (pg. 53)

There are 3 types of handbook problems to prepare students for each of the rounds of the competition. You'll want to have your students practice all of these types of problems.

<b>Warm-Ups</b>	<b>Workouts</b>	<b>Stretches</b>
14 Warm-Ups in handbook 10 questions per Warm-Up no calculators used  	8 Workouts in handbook 10 questions per Workout calculators used  	3 Stretches in handbook Number of questions and use of calculators vary by Stretch  <i>Each Stretch covers a particular math topic that could be covered in any round. These help prepare students for all 4 rounds.</i>
<i>Warm-Ups prepare students particularly for the Sprint and Countdown Rounds.</i>   	<i>Workouts prepare students particularly for the Target and Team Rounds.</i>   	   

**IS THERE A SCHEDULE I SHOULD FOLLOW FOR THE YEAR?** On average coaches meet with their students for an hour once a week at the beginning of the year, and more often as the competitions approach. Practice sessions may be held before school, during lunch, after school, on weekends or at other times, co-ordinating with your school's schedule and avoiding conflicts with other activities.

Designing a schedule for your practices will help ensure you're able to cover more problems and prepare your students for competitions. We've designed the *School Handbook* with this in mind. Below is a suggested schedule for the program year that mixes in Warm-Ups, Workouts and Stretches from the *School Handbook*, plus free practice competitions from last year. This schedule allows your students to tackle more difficult problems as the School and Chapter Competition approach.

<b>Mid-August – September 2017</b> Warm-Ups 1, 2 + 3 Workouts 1 + 2	<b>October 2017</b> Warm-Ups 4, 5 + 6 Workout 3 Probability Stretch	<b>November 2017</b> Warm-Ups 7 + 8 Workouts 4 + 5 Patterns Stretch	<b>December 2017</b> Warm-Ups 9, 10 + 11 Workout 6 Travel Stretch
<b>January 2018</b> Warm-Ups 12, 13 + 14 Workouts 7 + 8 <i>2018 MATHCOUNTS School Competition</i> <i>Select chapter competitors (optional at this time)</i>		<b>February 2018</b> Practice Competition: 2017 School Competition Practice Competition: 2017 Chapter Competition <i>Select chapter competitors (required by this time)</i> <i>2018 MATHCOUNTS Chapter Competition</i>	

You'll notice that in January or February you'll need to select the 1-10 student(s) who will represent your school at the Chapter Competition. This must be done before the start of your local Chapter Competition. You'll submit the names of your chapter competitors either online at [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches) or directly to your local Chapter Coordinator.

It's possible you and your students will meet more frequently than once a week and need additional resources. If that happens, don't worry! You and your Mathletes can work together using the **Interactive MATHCOUNTS Platform**, powered by NextThought. This free online platform contains numerous *MATHCOUNTS School Handbooks* and past competitions, not to mention lots of features that make it easy for students to collaborate with each other and track their progress. You and your Mathletes can sign up for free at [mathcounts.nextthought.com](http://mathcounts.nextthought.com).

And remember, just because you and your students will meet once a week doesn't mean your students can only prepare for MATHCOUNTS one day per week. Many coaches assign "homework" during the week so they can keep their students engaged in problem solving outside of team practices. Here's one example of what a 2-week span of practices in the middle of the program year could look like.



**CHECK OUT THE  
INTERACTIVE  
MATHCOUNTS PLAT-  
FORM TO GET EVEN  
MORE HANDBOOK  
PROBLEMS + PAST  
COMPETITIONS!**

Monday	Tuesday	Wednesday (Weekly Team Practice)	Thursday	Friday
-Students continue to work individually on Workout 4, due Wednesday	-Students continue to work on Workout 4 -Coach emails team to assign new Problem of the Week, due Wednesday	-Coach reviews solutions to Workout 4 -Coach gives Warm-Up 7 to students as timed practice and then reviews solutions -Students discuss solutions to Problem of the Week in groups	-Coach emails math team to assign Workout 5 as individual work, due Wednesday	-Students continue to work individually on Workout 5
-Students continue to work individually on Workout 5, due Wednesday	-Students continue to work on Workout 5 -Coach emails team to assign new Problem of the Week, due Wednesday	-Coach reviews solutions to Workout 5 -Coach gives Warm-Up 8 to students as timed practice and then reviews solutions -Students discuss solutions to Problem of the Week in groups	-Coach emails math team to assign Workout 6 as group work, due Wednesday	-Students work together on Workout 6 using online Interactive Platform

**WHAT SHOULD MY TEAM PRACTICES LOOK LIKE?** Obviously every school, coach and group of students is different, and after a few practices you'll likely find out what works and what doesn't for your students. Here are some suggestions from veteran coaches about what makes for a productive practice.

- Encourage discussion of the problems so that students learn from each other
- Encourage a variety of methods for solving problems
- Have students write math problems for each other to solve
- Use the **Problem of the Week** (posted online every Monday)
- Practice working in groups to develop teamwork (and to prepare for the Team Round)
- Practice oral presentations to reinforce understanding

On the following page is a sample agenda for a 1-hour practice session. There are many ways you can structure math team meetings and you will likely come up with an agenda that works better for you and your group. It also is probably a good idea to vary the structure of your meetings as the program year progresses.

## MATHCOUNTS Team Practice Sample Agenda – 1 Hour

### *Review Problem of the Week (20 minutes)*

- Have 1 student come to the board to show how s/he solved the first part of the problem.
- Discuss as a group other strategies to solve the problem (and help if student answers incorrectly).
- Have students divide into groups of 4 to discuss the solutions to the remaining parts of the problem.
- Have 2 groups share answers and explain their solutions.

### *Timed Practice with Warm-Up (15 minutes)*

- Have students put away all calculators and have one student pass out Warm-Ups (face-down).
- Give students 12 minutes to complete as much of the Warm-Up as they can.
- After 12 minutes is up, have students hold up pencils and stop working.

### *Play Game to Review Warm-Up Answers (25 minutes)*

- Have students divide into 5 groups (size will depend on number of students in meeting).
- Choose a group at random to start and then rotate clockwise to give each group a turn to answer a question. When it is a group's turn, ask the group one question from the Warm-Up.
- Have the group members consult their completed Warm-Ups and work with each other for a maximum of 45 seconds to choose the group's official answer.
- Award 2 points for a correct answer on questions 1-3, 3 points for questions 4-7 and 5 points for questions 8-10. The group gets 0 points if they answer incorrectly or do not answer in 45 seconds.
- Have all students check their Warm-Up answers as they play.
- Go over solutions to select Warm-Up problems that many students on the team got wrong.



**OK I'M READY TO START. HOW DO I GET STUDENTS TO JOIN?** Here are some tips given to us from successful competition coaches and club leaders for getting students involved in the program at the beginning of the year.

- Ask Mathletes who have participated in the past to talk to other students about participating.
- Ask teachers, parent volunteers and counselors to help you recruit.
- Reach parents through school newsletters, PTA meetings or Back-to-School-Night presentations.
- Advertise around your school by:
  1. posting intriguing math questions (specific to your school) and referring students to the first meeting for answers.
  2. designing a bulletin board or display case with your MATHCOUNTS poster (included in your School Competition Kit) and/or photos and awards from past years.
  3. attending meetings of other extracurricular clubs (such as honor society) so you can invite their members to participate.
  4. adding information about the MATHCOUNTS team to your school's website.
  5. making a presentation at the first pep rally or student assembly.

**Good luck in the competition! If you have any questions during the year, please contact the MATHCOUNTS national office at (703) 299-9006 or [info@mathcounts.org](mailto:info@mathcounts.org).**

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**COACH RESOURCES:**  
**WWW.MATHCOUNTS.ORG/COACHES**

# 2017-2018 HANDBOOK MATERIALS

**Thank you for being a coach in the MATHCOUNTS Competition Series this year!**

We hope participating in the program is meaningful and enriching for you and your Mathletes.

Don't forget to log in at [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches) for additional resources!

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# HIGHLIGHTED RESOURCES

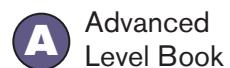
Also access resources at  
[www.mathcounts.org/coaches!](http://www.mathcounts.org/coaches)



Great for  
Coaches



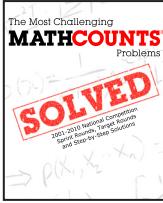
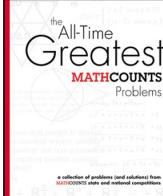
Great for  
Mathletes



Advanced  
Level Book



Free  
Resource

<p><b>OPLET</b>            Online database of over 13,000 problems and over 5,000 step-by-step solutions. Create personalized quizzes, flash cards, worksheets and more!</p> <p><b>Save \$25 when you buy your subscription by Oct. 13, 2017</b>  <i>Renewers:</i> use code RENEW18  <i>First-Time Subscribers:</i> use code NEW18</p>	<p><b>Practice Competitions for MATHCOUNTS, Vol. I &amp; II</b></p>  <p>Practice books written by repeat national-level coach Josh Frost. Each volume includes 4 complete mock-competitions plus solutions.</p>	<p><b>Most Challenging MATHCOUNTS Problems Solved</b></p>  <p>Advanced level practice book with 10 years of national-level Sprint Rounds, plus detailed step-by-step solutions to each problem.</p>
<p><b>C</b>  <a href="http://www.mathcounts.org/myoplet">www.mathcounts.org/myoplet</a></p> <p><b>All Time Greatest MATHCOUNTS Problems</b></p>  <p>A collection of some of the most creative, interesting and challenging MATHCOUNTS competition problems.</p> <p><b>C M A</b>  <a href="http://www.mathcounts.org/store">www.mathcounts.org/store</a></p>	<p><b>Interactive MATHCOUNTS Platform</b></p>  <p>Online platform of past and current handbook and competition problems. Interactive features make collaboration easy and fun!</p> <p><b>C M X</b>  <a href="http://mathcounts.nexthought.com">mathcounts.nexthought.com</a></p>	<p><b>MATHCOUNTS Trainer App</b></p>  <p>Train your Mathletes with this fun app, featuring real-time leaderboards and lots of past MATHCOUNTS problems.</p> <p><b>M X</b>  <a href="http://aops.com/mathcounts_trainer">aops.com/mathcounts_trainer</a> or download at the App Store</p>
<p><b>Past Competitions</b></p>  <p>Last year's School, Chapter and State competitions are free online! Other years' competitions can be purchased.</p> <p><b>C M X</b>  <a href="http://www.mathcounts.org/pastcompetitions">www.mathcounts.org/pastcompetitions</a>  <a href="http://www.mathcounts.org/store">www.mathcounts.org/store</a></p>	<p><b>Problem of the Week</b></p> <p>A new, multi-step problem every week! Each problem focuses on a particular set of math skills and coincides with a timely event, holiday or season. Get the problem at the beginning of the week and the step-by-step solution the following week.</p> <p><b>C M X</b>  <a href="http://www.mathcounts.org/potw">www.mathcounts.org/potw</a></p>	<p><b>MATHCOUNTS Minis</b></p> <p>A fun monthly video series featuring Richard Rusczyk from Art of Problem Solving. Each video looks at a particular math skill and walks through how to solve different MATHCOUNTS problems using creative problem-solving strategies.</p> <p><b>C M X</b>  <a href="http://www.mathcounts.org/minis">www.mathcounts.org/minis</a></p>

# CRITICAL 2017-2018 DATES

## 2017



Aug. 15 –  
Dec. 15

Submit your school's registration to participate in the Competition Series and receive this year's School Competition Kit, which includes a hard copy of the 2017-2018 *MATHCOUNTS School Handbook*. Kits are shipped on an ongoing basis between mid-August and December 31.

The fastest way to register is online at [www.mathcounts.org/compreg](http://www.mathcounts.org/compreg). You also can download the MATHCOUNTS Competition Series Registration form and mail or email it with payment to:

MATHCOUNTS Foundation – Competition Series Registrations  
1420 King Street, Alexandria, VA 22314  
*Email:* reg@mathcounts.org

To add students to your school's registration, log in at [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches) to access the Dashboard. **Questions?** Call the MATHCOUNTS national office at (703) 299-9006 or email us at info@mathcounts.org.



Nov. 1

The 2018 School Competition will be available online. All registered coaches can log in at [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches) to download the competition.



Nov. 3  
(postmark)

**Deadline to register for the Competition Series at reduced registration rates** (\$30 per student, \$300 for full registration of 10 students). After November 3, registration rates will be \$35 per student, \$350 for full registration.



Dec. 15  
(postmark)

### Competition Series Registration Deadline

In some circumstances, late registrations might be accepted at the discretion of MATHCOUNTS and the local coordinator. *Late fees will apply. Register on-time to ensure your students' participation.*

## 2018



Early Jan.

If you have not been contacted with details about your upcoming competition, call your local or state coordinator. Coordinator contact information is available at [www.mathcounts.org/findmycoordinator](http://www.mathcounts.org/findmycoordinator).



Late Jan.

If you have not received your School Competition Kit, contact the MATHCOUNTS national office at (703) 299-9006 or [info@mathcounts.org](mailto:info@mathcounts.org).



Feb. 1-28

### Chapter Competitions



March 1 –  
Apr. 1

### State Competitions



May 13-14

### 2018 Raytheon MATHCOUNTS National Competition in Washington, DC

# THIS YEAR'S HANDBOOK PROBLEMS



**You and your students might notice something special  
about some of the problems in this year's  
Warm-Ups and Workouts...**

Throughout this handbook are names—first and/or last—of people who donated to the MATHCOUNTS Foundation's Giving Tuesday campaign last year to help us cover half the cost of registering for the Competition Series for Mathletes from low-income schools. These donors help make this program possible for students across the country, so we decided it was fitting to include them in the primary preparation resource for participants in this program.

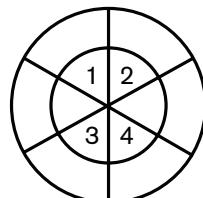
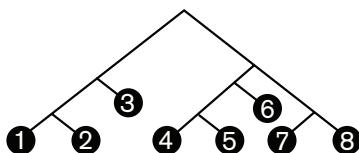
**To all of our 2016 Giving Tuesday Donors  
(whether or not you chose to be featured in this handbook)**

# THANK YOU!



# Probability Stretch

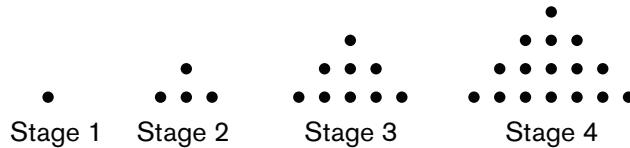
1. \_\_\_\_\_ % Petra randomly selects a card from a standard deck of 52 playing cards. What is the percent probability that the card shows a red number greater than 6? Express your answer to the nearest hundredth.
2. \_\_\_\_\_ Max has eight identical cups. Each cup contains a different combination of nickels, dimes and quarters, each totaling 45 cents. Max randomly selects a cup. What is the probability that the cup he selects contains at least three dimes? Express your answer as a common fraction.
3. \_\_\_\_\_ A bag contains five chips numbered 2 through 6. Danya draws chips from the bag one at a time and sets them aside. After each draw, she totals the numbers on all the chips she has already drawn. What is the probability that at any point in this process her total will equal 10? Express your answer as a decimal to the nearest tenth.
4. \_\_\_\_\_ A drawer contains five socks: two green and three blue. What is the probability that two socks pulled out of the drawer at random will match? Express your answer as a common fraction.
5. \_\_\_\_\_ A penny, a nickel and a dime are flipped. What is the probability that at least two coins land heads up and one of them is the nickel? Express your answer as a common fraction.
6. \_\_\_\_\_ % When the circuit containing blinking lights A and B is turned on, lights A and B blink together. Then A blinks once every 5 seconds and B blinks once every 11 seconds. Lindsey looks at the two lights just in time to see A blink alone. What is the percent probability that the next light to blink will be A blinking alone?
7. \_\_\_\_\_ % What is the percent probability that a randomly selected multiple of 3 less than or equal to 3000 is also a multiple of 5?
8. \_\_\_\_\_ Starting at the top and selecting paths randomly as you move downward, what is the probability of ending at an odd number? Express your answer as a common fraction.
9. \_\_\_\_\_ A five-digit number is made by randomly ordering the digits 1, 2, 3, 4 and 5. What is the probability that this number is divisible by 4? Express your answer as a common fraction.
10. \_\_\_\_\_ Pierre throws darts that land randomly in the dartboard shown here. The dartboard is a circle of radius 2 units, with an inner circle of radius 1 unit. Both circles are divided into six congruent sectors. What is the probability that a dart Pierre throws will land in one of the four inner numbered sectors? Express your answer as a decimal to the nearest hundredth.





# Patterns Stretch

11. \_\_\_\_\_ dots The first four stages of a dot pattern are shown. How many more dots are in the figure at Stage 47 than in the figure at Stage 27?



12. \_\_\_\_\_ The first three terms of a sequence are 1, 2 and 3. Each subsequent term is the sum of the three previous terms. What is the 11th term of this sequence?

13. \_\_\_\_\_ What is the sum of the terms in the arithmetic series  $2 + 5 + 8 + 11 + 14 + \dots + 89 + 92$ ?

14. \_\_\_\_\_ Three consecutive terms in an arithmetic sequence are  $x$ ,  $2x + 11$  and  $4x - 3$ . What is the constant difference between consecutive terms in this sequence?

15. \_\_\_\_\_ What is the sum of the terms in the geometric series  $1 + 4 + 16 + \dots + 1024$ ?

16. \_\_\_\_\_ What is the sum of the first 51 consecutive odd positive integers?

17. \_\_\_\_\_ What is the sum of the terms in the infinite series  $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$ ?

18. \_\_\_\_\_ What is the sum of the terms in the infinite series  $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots$ ? Express your answer as a common fraction.

19. \_\_\_\_\_ Let  $f(x) = 2x + 3$  and  $f^2(x) = f(f(x)) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 9$ . If  $f^5(x) = ax + b$ , what is the value of  $a + b$ ?

20. \_\_\_\_\_ degrees The degree measures of the interior angles of a quadrilateral form a geometric sequence whose terms have integer values and are all integer multiples of the first term. What is the largest possible degree measure of an angle in this quadrilateral?



# Travel Stretch

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

21. \_\_\_\_\_ mi/h Jack and Jill travel up a hill at a speed of 2 mi/h. They travel back down the hill at a speed of 4 mi/h. What is their average speed for the entire trip? Express your answer as a mixed number.



22. \_\_\_\_\_ : \_\_\_\_\_ p.m. At 2:20 p.m., Jack is at the top of the hill and starts walking down at the exact same time that Jill, who is at the bottom of the hill, starts walking up. If they maintain the same uphill and downhill speeds from the previous problem, and the distance from the bottom to the top of the hill is 1.5 miles, at what time will Jack and Jill meet?

23. \_\_\_\_\_ yards When Jack and Jill meet, as described in the previous problem, how many yards will they be from the bottom of the hill?

24. \_\_\_\_\_ minutes Alysha's average speed when walking from home to the market is 5 mi/h, and it takes her 21 minutes longer than when she drives to the market. If Alysha drives to the market, along the same route, at an average speed that is eight times her average walking speed, how many minutes does it take her to drive from home to the market?



25. \_\_\_\_\_ miles Based on problem 24, how many miles does Alysha travel to get from home to the market?

26. \_\_\_\_\_ minutes  Jana begins jogging along a path and, 5 minutes later, Zhao begins riding his bicycle along the same path, which has a length of 2 miles. Zhao rides his bicycle at a speed of 10 mi/h, and Jana's jogging speed is 6 mi/h. If they both begin at one end of the path and end at the other, how many minutes after Zhao reaches the end of the path will Jana reach the end of the path?

27. \_\_\_\_\_ minutes Based on problem 26, how many minutes after Zhao begins riding will he catch up with Jana? Express your answer as a mixed number.

28. \_\_\_\_\_ miles Again, based on problem 26, how many miles will Jana have traveled when Zhao catches up with her? Express your answer as a mixed number.

29. \_\_\_\_\_ mi/h  Ansel left the dock in his motorboat, traveled 10 miles, and then returned to the dock along the same route. On the return trip, Ansel was traveling against the current of the river, and his average speed relative to the water was 20 mi/h. If the round-trip took Ansel 64 minutes, what is the speed of the river's current?

30. \_\_\_\_\_ Based on problem 29, what fraction of Ansel's total travel time was spent traveling upstream? Express your answer as a common fraction.



# Warm-Up 1

31. \_\_\_\_\_ What is the value of  $5 - 5 \times 5 + 5 \div 5$ ?

32. \_\_\_\_\_ diagonals How many diagonals are in a convex heptagon?

33. \_\_\_\_\_ What is the first year after 2018 that is a palindrome?

34. \_\_\_\_\_ A standard 52-card deck of playing cards includes four aces. What is the probability that two cards selected randomly, without replacement, will both be aces? Express your answer as a common fraction.

35. \_\_\_\_\_ What is the value of  $\sqrt{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 10}$ ? Express your answer in simplest radical form.

36. \_\_\_\_\_ °F The temperature dropped from 13 °F to –5 °F. How many degrees Fahrenheit is the absolute value of the change in temperature?

37. \_\_\_\_\_ What is the value of  $1 \times 2 + 3 \div 6 \times 5 - 4$ ? Express your answer as a common fraction.

38. \_\_\_\_\_ If  $x \circledast y$  is defined as  $x^2 - y^2$ , what is the value of  $3 \circledast (2 \circledast 1)$ ?

39. \_\_\_\_\_ If the digits 7, 8, 2, 3 and 0 are used, each exactly once, to form a three-digit positive integer and a two-digit positive integer that differ by exactly 288, what is the sum of the three-digit integer and the two-digit integer?

40. \_\_\_\_\_ degrees In rectangle ABCD, point P lies on side BC and point Q lies in the interior of the rectangle so that triangle APQ is equilateral. If the measure of angle PAB is 17 degrees, what is the measure of angle QPC?



## Warm-Up 2

41. \_\_\_\_\_ balls Kim is knitting a baby blanket that requires 750 meters of yarn. There are 180 meters of yarn in each ball. How many balls of yarn must Kim buy to ensure she has enough yarn to complete her blanket?
42. \_\_\_\_\_ years old On Chris' birthday in 1992, he was half the age of his brother Joseph. On Chris' birthday in 1998, he was two-thirds the age of Joseph. How old will Chris be on his birthday in 2018?
43. \_\_\_\_\_ degrees On a standard 12-hour clock, the minute hand moves continuously, at a constant rate, making one full revolution every hour, and the hour hand moves similarly, making one full revolution every 12 hours. What is the measure of the smaller of the two angles between the minute hand and the hour hand, in degrees, when the clock reads 5:42?
44. \_\_\_\_\_ What is the value of the expression  $12 \times 37 + 12 \times 7 + 12 \times 6$ ?
45. \_\_\_\_\_ factors How many distinct positive factors does 2018 have?
46. \_\_\_\_\_ Two fair six-sided dice, with sides numbered 1 through 6, are rolled. What is the probability that the values on the two top faces add to at least 9? Express your answer as a common fraction.
47. \_\_\_\_\_ If the graph of the equation  $y = mx + b$  is a line passing through the points (6, 13) and (10, 31), what is the value of  $m$ ? Express your answer as a common fraction.
48. \_\_\_\_\_ Dewey buys soda in 12-ounce cans that cost \$1.00 each. Peppar buys soda in 20-ounce bottles that cost \$1.25 each. If Dewey and Peppar buy the same volume of soda in one week, then Peppar pays  $P\%$  less than Dewey. What is the value of  $P$ ?
49. \_\_\_\_\_ logs Gerald Scheetz is building a log cabin. If each log is 9 inches in diameter, how many logs must be stacked on top of one another to create a wall that has a height of 12 feet?
50. \_\_\_\_\_ units<sup>2</sup> A square with area 8 units<sup>2</sup> is inscribed in a circle. What is the area of the circle? Express your answer in terms of  $\pi$ .



# Warm-Up 3

51. \_\_\_\_\_ If  $y$  is a number such that  $y^2 = (y + 2018)^2$ , what is the value of  $y$ ?

52. \_\_\_\_\_ years old Maura is 5 years younger than her sister Cara. Seven years ago, Maura was half as old as her sister. How old is Maura now?

53. \_\_\_\_\_ A dartboard consists of three concentric circles with radii 10, 5 and 1, respectively, measured in inches. The area between the largest and middle circles is colored green, the area between the middle and smallest circles is colored yellow, and the area within the smallest circle, the bull's-eye, is colored red. If a thrown dart is guaranteed to hit the board, but its position on the board is uniformly random, what is the probability that it lands in the yellow portion of the board? Express your answer as a common fraction.

54. \_\_\_\_\_ days The 1990 and 2018 calendars are identical in the number of days in each month and the day of the week on which each day of each month occurs. In fact, the calendar repeats in these ways every 28 years until the year 2100. How many days are there in the 28 years preceding 2018?

55. \_\_\_\_\_ cm A right triangle has legs with lengths of 5 cm and 10 cm. What is the length of the altitude drawn to the hypotenuse of this triangle? Express your answer in simplest radical form.

56. \_\_\_\_\_ Min Zhang wrote down all of the two-digit multiples of 5. What is the probability that one of these numbers, chosen at random, has exactly two distinct primes that are factors? Express your answer as a common fraction.

57. \_\_\_\_\_ Given a set of numbers with median  $m$ , the median of all the numbers less than  $m$  is called the *lower quartile*. The median of all the numbers greater than  $m$  is called the *upper quartile*. The absolute difference between the lower and upper quartiles is called the *interquartile range*. What is the interquartile range for the numbers in the stem-and-leaf plot shown?

0	2	4	6	8	9
1	1	1	3	8	9
2	0	4	6	8	
3	0	3	5	7	7
4	1	5	7	7	

58. \_\_\_\_\_

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

Positive integers 1 to 36 are written in rows in a six-by-six array as shown. Each prime number is crossed off, as well as all the numbers in the diagonal extending up and to the right from that prime. For example, 11 is prime and is crossed off along with the 6 above and to the right. What is the sum of the remaining values after all the primes and associated diagonals have been eliminated?

59. \_\_\_\_\_ What is the value of  $1,000,000! \div 999,999!?$

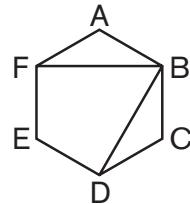
60. \_\_\_\_\_ inches A cubic yard of topsoil is to be spread evenly in the garden at Prove It! Math Academy. The garden measures 10 feet by 8 feet. How many inches deep will the topsoil be? Express your answer as a mixed number.



# Warm-Up 4

61. \_\_\_\_\_

Diagonals FB and BD are drawn in regular hexagon ABCDEF. What is the ratio of the sum of the areas of triangles ABF and BCD to the area of quadrilateral BDEF? Express your answer as a common fraction.



62. \_\_\_\_\_

What is the value of  $\frac{11! - (9+1)(9!)}{8(7!)}$ ?

63. \_\_\_\_\_ times

David's optometrist sold him a bottle of eyeglass cleaner containing 30 mL of glass-cleaning solution. Assuming there are 20 drops per milliliter, and assuming proper cleaning requires 3 drops of glass cleaner on each side of each lens, what is the maximum number of times David can properly clean his glasses before he must buy a new bottle of eyeglass cleaner?

64. \_\_\_\_\_ combinations

The lunch-ordering app for Pete's Pizza Parlor requires you to choose two distinct meats from among pepperoni, Canadian bacon and sausage; or choose two distinct vegetables from among mushrooms, onions, green peppers and black olives; or choose one meat and one vegetable from among the same choices. How many different pizza combinations are possible using the lunch-ordering app?

65. \_\_\_\_\_ pounds

Kathy Beckhardt weighs four of her sheep at the fair. She can weigh two of them at a time on the big scale. Sheep A and sheep B have an average weight of 150 pounds, sheep B and sheep C have an average weight of 127 pounds, and sheep C and sheep D have an average weight of 168 pounds. What is the average weight of sheep A and sheep D?

66. \_\_\_\_\_ meters

In circle O, the lengths of chords AB and BC are equal and  $m\angle ABC = 90$  degrees. Given that circle O has a radius of 3 meters, what is the length of arc ABC? Express your answer in terms of  $\pi$ .

67. \_\_\_\_\_ tiles

How many 4-inch square tiles are needed to cover a wall that measures 6 feet by 8 feet?

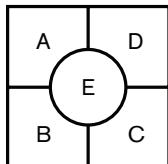
68. \_\_\_\_\_

What is the units digit of  $2^{2017} \times 7^{2017}$ ?

69. \_\_\_\_\_ integers

How many integers between 100 and 1000 contain no digits other than 3, 4 or 5?

70. \_\_\_\_\_ paths



The square shown is divided into five cells. How many paths can be drawn that start at any cell, move only to adjacent cells and visit each of the five cells exactly once?



# Warm-Up 5

71. \_\_\_\_\_ paper clips A pencil and 5 paper clips weigh the same as 2 erasers. A pencil weighs the same as 29 paper clips. How many paper clips weigh the same as an eraser?

72. \_\_\_\_\_ points What is the maximum number of points of intersection of a right triangle with a square, assuming no side of the triangle is collinear with any side of the square?

73. \_\_\_\_\_ If  $p(x) = ax^2 + bx + c$  is a quadratic polynomial satisfying  $p(0) = 4$ ,  $p(1) = 15$ ,  $p(2) = 36$ , what is the value of the product  $abc$ ?

74. \_\_\_\_\_ units A certain sphere has a volume that is numerically equal to three times its surface area. What is the radius of this sphere?

75. \_\_\_\_\_ candles A layered candle is made with 5 colors, shown here as candle A. How many different candles can be made using the same 5 colors, with BLUE as the middle layer, shown as candle B, and with no color next to a color that it touched in candle A?

A	B
BLUE	
GREEN	
RED	
ORANGE	
YELLOW	

76. \_\_\_\_\_ Suppose Luke spins the pointer on a fair 3-color spinner twice. What is the probability that the pointer lands on the same color twice? Express your answer as a common fraction.

77. \_\_\_\_\_ shots Kevin is playing basketball and up to now made  $\frac{1}{3}$  of his attempted shots. If he makes his next 5 shots, he will improve his shooting percent to 50%. How many shots has Kevin attempted up to now, when he has a  $\frac{1}{3}$  success rate?

78. \_\_\_\_\_ base eight What is  $110011_2$  when rewritten in base eight?

79. \_\_\_\_\_ points If the point  $(8, 9)$  is the center of a circle of radius 10 units, at how many points does the circle intersect the coordinate axes?

80. \_\_\_\_\_ If  $x + \frac{1}{y} = \frac{1}{5}$  and  $y + \frac{1}{x} = 20$ , what is the value of the product  $xy$ ?



## Warm-Up 6

81. \_\_\_\_\_ If  $3x + 5 = 13$ , what is the value of the expression  $(3x + 2)(3x + 3)(3x + 4)$ ?

82. \_\_\_\_\_ units<sup>2</sup> What is the maximum area of a rectangle with a diagonal of length 16 units?

83. \_\_\_\_\_ pairs How many pairs of numbers  $(a, b)$  satisfy rules I and II shown here?

- I.  $a = 0$  or  $b = -1$  or  $b = 1$
- II.  $a = -1$  or  $a = 1$  or  $b = 0$

84. \_\_\_\_\_ If each letter in the sum A.BC + D.EF represents a different nonzero digit, what is the least possible value of the sum? Express your answer as a decimal to the nearest hundredth.

85. \_\_\_\_\_ ways In the hardware store, Matt goes to the fastener aisle, which has wood screws, sheet metal screws, hex bolts, carriage bolts and lag bolts. How many ways can he choose 10 fasteners if he needs at least one of each kind?

86. \_\_\_\_\_ A family farm is equally divided among three heirs: Jim, Jan and John. John's share of the farm is then equally divided among his three heirs: Peter, Paul and Patricia. Paul decides to sell his share of the farm, and then later the family decides to sell the remainder of the farm all at once. What portion of the proceeds from the most recent sale should Jim receive? Express your answer as a common fraction.

87. \_\_\_\_\_ Olivia Justynski earned scores of 82, 86 and 92 on her first three tests. What score does she need on her fourth test to achieve an average score of 90 on the four tests?

88. \_\_\_\_\_ hours For each child, Kiddie Day Care charges \$330 per month for preschool and \$5.50 per hour for each hour of after-school care. If Cody's cost was \$770 for his son's child care last month, how many hours did his son spend in after-school care?

89. \_\_\_\_\_ When three consecutive positive integers are multiplied, the product is 16 times the sum of the three integers. What is the difference of the product minus the sum?

90. \_\_\_\_\_ The lift force on an airplane during flight is directly proportional to the surface area of the wing. Orville builds a model airplane and goes outside to play. Orville's little brother, Wilbur, builds a mini replica of Orville's plane that is half as long in every linear dimension. What is the ratio of the lift force on Wilbur's plane to that on Orville's plane? Express your answer as a common fraction.

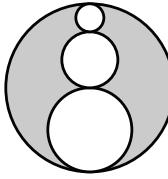
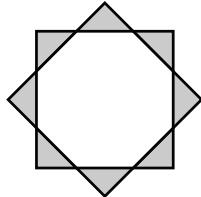


# Warm-Up 7

91. \_\_\_\_\_ What whole number  $n$  makes  $\frac{6}{78} < \frac{1}{n} < \frac{5}{55}$  true?
92. \_\_\_\_\_ In 2016 the Flying Turtles finished their baseball season with a record of 95 wins and 67 losses. The Dolphins finished the season with 84 wins and 78 losses. The Flying Turtles and Dolphins played each other 19 times during the season. If the Flying Turtles had  $F$  wins against teams other than the Dolphins, and the Dolphins had  $D$  wins against teams other than the Flying Turtles, what is the value of  $F + D$ ?
93. \_\_\_\_\_ units A point D is placed on the segment with endpoints  $(0, 8)$  and  $(8, 0)$ , and a point E is placed on the segment with endpoints  $(-3, 0)$  and  $(0, -2)$ . What is the shortest possible distance between D and E? Express your answer in simplest radical form.
94. \_\_\_\_\_ terms In the arithmetic sequence  $1, 3, 5, 7, 9, 11, \dots$ , how many terms appear after the term 315 but before the term 639?
95. \_\_\_\_\_ Allen Zhang rolls two fair 6-sided dice with faces numbered 1 through 6. What is the probability that the sum of his two rolls has an odd number of positive integer divisors? Express your answer as a common fraction.
96. \_\_\_\_\_ Six semicircles, each of radius  $r$ , are constructed inside a regular hexagon of side length  $s$ , one on each side, so that each semicircle is tangent to two others. What is the ratio of  $r$  to  $s$ ? Express your answer in simplest radical form.
97. \_\_\_\_\_ Gaylon starts writing down dates from January 1, 2018 onward as follows: 01012018, 01022018, 01032018, etc. What is the 2018th digit Gaylon writes down?
98. \_\_\_\_\_ lightning bolts Zeus threw, on average, 12 lightning bolts per day in the month of March. During the first week in April, he averaged 15 lightning bolts per day. How many lightning bolts does Zeus need to throw per day on average for the rest of April to maintain a 12-bolt-per-day average over March and April? Express your answer to the nearest integer.
99. \_\_\_\_\_ For what positive value of  $x$  is the equation  $9^{2x^2 - 6} = 27^{x^2 - 1}$  true?
100. \_\_\_\_\_ times The decibel is a unit used to describe the loudness of a sound. For every 20-decibel increase, a sound gets 10 times as loud. Normal conversation is about 60 decibels, and a loud rock concert is about 120 decibels. How many times as loud is a rock concert compared to normal conversation?



# Warm-Up 8

101. \_\_\_\_\_ Pamela Wickham writes a sequence of four consecutive integers on a sheet of paper. The sum of three of these integers is 206. What is the other integer?
102. \_\_\_\_\_ seconds Benjamin starts walking up on an escalator that moves down one flight of stairs every 20 seconds. Benjamin takes 10 seconds to walk up a single flight of stairs on the adjacent stationary staircase. Assuming Benjamin walks at the same speed on the escalator and stairs, how many seconds does it take him to walk up two flights on this escalator?
103. \_\_\_\_\_ divisors Let  $K = 168 \times 900 = 151,200$ . How many positive integer divisors does  $K$  have?
104. \_\_\_\_\_ Emma Kerwin creates a custom six-sided die by randomly choosing six different integers between 1 and 7, inclusive, to paint on the sides of a blank cube. What is the probability that the faces of her die sum to 24? Express your answer as a common fraction.
105. \_\_\_\_\_ \$ The owners of two food carts calculate their weekly profits for three weeks. The medians and the highest weekly profit values are the same for the two carts. The mean weekly profit of Cart A is \$27 more than that of Cart B. What is the absolute difference between the lowest weekly profit values of Cart A and Cart B?
106. \_\_\_\_\_  Each of the circles in the figure is tangent to exactly two others. The centers of all four lie on a line. If the diameters of the three inner circles are in a ratio of 1:2:3, what fraction of the largest circle is shaded? Express your answer as a common fraction.
107. \_\_\_\_\_ Two congruent squares overlap to form a regular octagon as shown. What is the ratio of the shaded area to the area of the regular octagon? Express your answer in simplest radical form. 
108. \_\_\_\_\_ hours It takes Avi one half-hour longer to make a basket than it takes Markus. After 28 hours, Markus has made one more basket than Avi has made. How many hours does it take Avi to make one basket?
109. \_\_\_\_\_ Suppose  $N$  is a positive integer such that  $N - 1$  is even,  $N - 2$  is divisible by 3,  $N - 3$  is divisible by 5, and  $N - 5$  is divisible by 7. What is the least possible value of  $N$ ?
110. \_\_\_\_\_ What fraction of the positive integer factors of  $1000^3$  are perfect squares? Express your answer as a common fraction.



# Warm-Up 9

111. \_\_\_\_\_

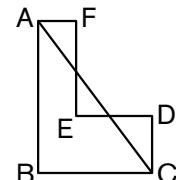
Sola's lucky numbers are 7 and 11. So he decides his lucky common fraction,  $f$ , will be formed by the repeating decimal  $f = \underline{0.711}$ . What is the value of  $f$  as a common fraction?

112. \_\_\_\_\_

Suppose  $m$  is the line given by the equation  $6x - 3y = 7$ , and suppose  $n$  is the line perpendicular to  $m$  and passing through the point  $(6, 2)$ . If  $k$  is the line of slope 5 and  $y$ -intercept 1, what is the  $x$ -coordinate of the intersection of  $n$  and  $k$ ? Express your answer as a common fraction.

113. \_\_\_\_\_

In hexagon ABCDEF, shown here, adjacent sides are perpendicular. If  $AB = 8$ ,  $BC = 6$ ,  $CD = 3$  and  $DE = 4$ , what fraction of the segment AC lies inside of the hexagon? Express your answer as a common fraction.



114. \_\_\_\_\_ times stronger

The Richter scale is used to describe the strength of an earthquake. An increase of 1 point on the Richter scale represents a tenfold increase in the strength of an earthquake. How many times stronger is an earthquake rated 7.5 on the Richter scale compared to an earthquake rated 5? Express your answer in simplest radical form.

115. \_\_\_\_\_

The third term of a geometric sequence of integers is 45. The seventh term of the sequence is 3645. What is the least possible sum of the first five terms of the sequence?

116. \_\_\_\_\_ words

In a new version of Scrabble, a sequence of letters is considered a word if the first and last letters are consonants and every letter in between is a vowel. In this game, how many four-letter words can be formed using each of the letters M, A, T, H, R, U, L, E and S no more than once?

117. \_\_\_\_\_ mi/h

Rebecca and Susan live at opposite ends of a 2-mile-long street. At 8:00 a.m., Rebecca starts jogging from her house toward Susan's end of the street. At 8:06 a.m., Susan starts jogging from her apartment toward Rebecca's end of the street. They pass each other at exactly 8:13 a.m. If Rebecca and Susan jog at the same constant speed, what is this speed, in miles per hour?

118. \_\_\_\_\_ units<sup>2</sup>

We define a *Heronian triangle* to be a triangle with three integer side lengths and integer area. What is the least possible positive area of a Heronian triangle whose longest side has a length of 17 units?

119. \_\_\_\_\_

For each of the first eight prime numbers, Brian Edwards writes down all the number's positive factors. What is the sum of all the numbers Brian writes down?

120. \_\_\_\_\_

If  $\sqrt{x} - \sqrt{y} = 10$  and  $\sqrt{x} + \sqrt{y} = 14$ , what is the value of  $x + y$ ?



# Warm-Up 10

121. \_\_\_\_\_ What is the greatest prime factor of  $(1!)! \times (2!)! \times (3!)! \times (4!)!$ ?

122. \_\_\_\_\_ minutes The table shows how long it takes Anita's fully discharged cell phone battery to fully charge using three methods. When her phone battery fully discharged, Anita charged the phone for half an hour using the wall charger, and now she will continue charging it for 1 hour using her computer. How many minutes are required to fully charge the phone battery using the portable charger, if the phone is not used during or between chargings?

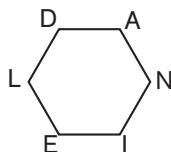
Method	Hours (to fully charge)
Wall Charger	1.5
Computer	3
Portable Charger	5

123. \_\_\_\_\_ In equilateral triangle ABC, M is the midpoint of side AB. If CMN is also an equilateral triangle, what fraction of the area of triangle  $\triangle ABC$  lies inside of  $\triangle CMN$ ? Express your answer as a common fraction.

124. \_\_\_\_\_ What is the greatest prime factor of  $3^7 - 27$ ?

125. \_\_\_\_\_ ways In how many ways can eight differently colored balls, including one red, one green and one yellow, be ordered left to right so that the green ball is to the right of the red ball (not necessarily adjacent) and the yellow ball is to the right of the green ball (not necessarily adjacent)?

126. \_\_\_\_\_ units Sides DL and AN in a regular hexagon DANIEL, shown here, are extended until they intersect at a point F. If the sides of the hexagon have length 6 units, what is the length of segment FE? Express your answer as a radical in simplest form.



127. \_\_\_\_\_ baskets Annette, Mary and Lynn team up to pick apples. Annette can pick 4 baskets of apples per hour, and Mary can pick 5 baskets of apples per hour. Annette, Mary and Lynn work together to pick 6 baskets of apples in half an hour. How many baskets of apples can Lynn pick by herself in 3 hours?

128. \_\_\_\_\_ Kayla Straub starts with a pile of 15 stones. She divides the pile into two new piles and finds the product of the numbers of stones in the two new piles. Kayla then divides one of the existing piles into two new piles. She finds the product of the numbers of stones in the two new piles and adds it to the previous product. Kayla continues this process, each time adding the product of the numbers of stones in the two new piles to the previous total, until she has 15 piles with one stone each. What is the greatest possible ending total?

129. \_\_\_\_\_ A sphere is inscribed in a cube. What is the ratio of the volume of the cube to that of the sphere? Express your answer as a common fraction in terms of  $\pi$ .

130. \_\_\_\_\_ Let  $\#x$  represent the greatest even integer less than  $x$ . If  $20 < x < 30$ , what is the maximum possible value of  $\#(5x) - \#(4x)$ ?



# Warm-Up 11

131. \_\_\_\_\_ points If two distinct ellipses and a square are drawn, what is the maximum possible number of points at which at least two of the three planar figures intersect?

132. \_\_\_\_\_ Isosceles triangles ABC and DEF have six interior angles altogether, but these six angles have only three different measures among them. If the sum of these three different measures is 156 degrees, and both triangles have at least one angle of measure  $m$  degrees, what is the value of  $m$ ?

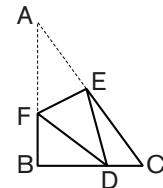
133. \_\_\_\_\_

Regular hexagon PQRSTU lies inside of trapezoid ABCD, as shown, so that vertices P and Q trisect the base AB, S and T lie on the base CD, and sides PU and QR are parallel to sides AD and BC, respectively. The shaded area is what fraction of the area of trapezoid ABCD? Express your answer as a common fraction.

134. \_\_\_\_\_ integers How many positive integers in the set of numbers from 1 to 1000, inclusive, are multiples of 2, 3 and 5 but not 8?

135. \_\_\_\_\_ In the sum ABCD + EFGH, each letter represents a digit selected independently at random from the set {1, 2, 3, 4}. What is the probability that the sum of the two four-digit numbers contains the digit 5 at least once? Express your answer as a common fraction.

136. \_\_\_\_\_  $\text{cm}^2$  In right triangle ABC, with  $AB = 44 \text{ cm}$  and  $BC = 33 \text{ cm}$ , point D lies on side BC so that  $BD:DC = 2:1$ . If vertex A is folded onto point D to create quadrilateral BCEF, as shown, what is the area of triangle CDE?



137. \_\_\_\_\_ passes After the first eight games of the football season, Jason Doan had completed 70% of his passes. During the ninth game, he completed 49 of his 50 passes, raising his season pass completion rate to 74%. How many total passes did he throw during the first nine games?

138. \_\_\_\_\_ The mean of seven distinct positive integers is 20. What is the difference between the greatest and least possible medians of the seven integers?

139. \_\_\_\_\_ integers How many two-digit positive integers have a units digit that is equal to the product of its two digits?

140. \_\_\_\_\_ Colleen Kipfstuhl rolls a standard fair six-sided die. If she rolls a number with an odd number of positive integer divisors, she steps 1 meter to her right. Otherwise, she steps 1 meter to her left. After four rolls of the die, what is the probability Colleen ends up right where she started? Express your answer as a common fraction.



# Warm-Up 12

141. \_\_\_\_\_ What is the absolute difference between the sum of the multiples of 2, from 1 to 100, inclusive, and the sum of the multiples of 3, from 1 to 100, inclusive?

142. \_\_\_\_\_ If  $p(x)$  is a cubic polynomial with  $p(0) = 4$ ,  $p(1) = 10$ ,  $p(-1) = 2$  and  $p(2) = 26$ , what is the value of  $p(3)$ ?

143. \_\_\_\_\_ inches What is the greatest possible perimeter of an obtuse triangle, each of whose side lengths is a whole number of inches less than or equal to 100?

144. \_\_\_\_\_ fist bumps After playing a math game, each member of the MATHCOUNTS national office staff gives a fist bump to every coworker. If 25 members of the national office staff participate as described, how many total fist bumps occur?

145. \_\_\_\_\_ students Several students were trying out for a class play. When asked which roles they were willing to play, 12 of them were willing to play the knight, 15 were willing to play the princess and 6 were willing to play the sorcerer. Of these students, 8 were willing to play either the knight or the princess, 5 were willing to play the knight or the sorcerer, and 4 were willing to play the princess or the sorcerer. Exactly 3 of these students were willing to play any of the roles. How many students were willing to play the sorcerer but no other role?

146. \_\_\_\_\_ ways Frankie the frog stands at the number 0 on a number line and wants to hop to the number 8. He can hop 1, 2 or 3 units forward in a single jump. How many different ways are there for Frankie to reach the number 8?

147. \_\_\_\_\_ The median and the mean of the five integers 10, 12, 26,  $x$ ,  $x$  are equal. What is the sum of all possible values of  $x$ ?

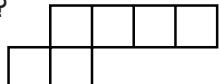
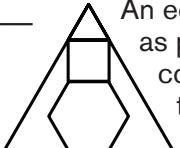
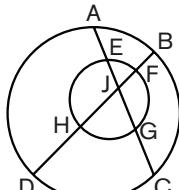
148. \_\_\_\_\_ ordered pairs How many ordered pairs of prime numbers  $(a, b)$  are there such that  $a + b = 100$ ?

149. \_\_\_\_\_ cm What is the perimeter of a right triangle with an area of  $10 \text{ cm}^2$  and a hypotenuse of length 10 cm? Express your answer in simplest radical form.

150. \_\_\_\_\_ If  $\frac{2}{x+1} + \frac{8}{y-3} = \frac{10}{3}$  and  $\frac{4}{x+1} - \frac{2}{y-3} = \frac{2}{3}$ , what is the value of  $x + y$ ?



# Warm-Up 13

151. \_\_\_\_\_ ordered triples How many ordered triples of integers  $(m, n, p)$  exist such that  $mn = p$ ,  $np = m$  and  $mp = n$ ?
152. \_\_\_\_\_  $\text{cm}^2$  What is the least possible area of a rectangle that can enclose an equilateral triangle with side length 6 cm? Express your answer in simplest radical form.
153. \_\_\_\_\_ integers How many of the first 2018 positive integers are either perfect squares or perfect cubes?
154. \_\_\_\_\_ assortments Alexander Clifton visits Sweet Dreams bakery, which sells three kinds of cookies. How many unique assortments of a dozen cookies can Alexander buy?
155. \_\_\_\_\_ dimes Gabriel and Isabel each start with a pile of 20 coins consisting of nickels, dimes and quarters. After Gabriel gives Isabel 2 coins, and Isabel gives Gabriel 5 coins, Gabriel's pile is worth twice the value of Isabel's pile. If Gabriel and Isabel have the greatest possible combined value of coins, what is the least number of dimes Isabel could end up with?
156. \_\_\_\_\_ What is the smallest positive integer multiple of 130 that is divisible by 365?
157. \_\_\_\_\_ hexominoes A *hexomino* is a planar figure formed by connecting six unit squares so that adjacent squares have a common side. One possible hexomino is shown. How many distinct hexominoes can be drawn that have exactly four squares in a row?  
Two hexominoes are distinct if one cannot be reflected or rotated to form the other hexomino.
- 
158. \_\_\_\_\_ An equilateral triangle, a square and a regular hexagon with side length 6 are stacked as pictured. A larger equilateral triangle is then drawn around the stack of polygons, completely enclosing it. The area outside the polygon stack but inside the larger triangle can be expressed in the form  $a + b\sqrt{c}$ , where  $a, b$  and  $c$  are integers and  $b\sqrt{c}$  is in simplest radical form. What is  $a + b + c$ ?
- 
159. \_\_\_\_\_ assignments Abhi, Bryan, Meghna and Noreen are each assigned a different integer from 1 to 10, inclusive. Abhi's number is prime and Noreen's number is a perfect square. Bryan's number is half of the value assigned to another person, while Meghna's number is the sum of two other assigned values. The ordered quadruple  $(2, 1, 5, 4)$  is one possible assignment. How many such assignments are there?
160. \_\_\_\_\_ degrees For the two circles shown, the measures of arcs  $CD$ ,  $GH$  and  $EF$  are 83 degrees, 98 degrees and 10 degrees, respectively. What is the measure of minor arc  $AB$ ?
- 



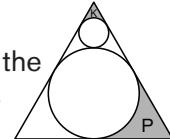
# Warm-Up 14

161. \_\_\_\_\_ If  $n$  is the product of three consecutive positive integers and  $n = 22 \times 14 \times k$ , what is the least possible value of  $k$ ?

162. \_\_\_\_\_ A regular  $5 \times 5$  *magic square* contains a permutation of the integers from 1 through 25, such that every row, every column, and the two main diagonals sum to the same value. What is the sum of the numbers missing from the magic square shown?

23	12	1		9
4	18	7	21	
10	24	13	2	16
	5	19	8	22
17		25	14	3

163. \_\_\_\_\_ Two circles are inscribed in an equilateral triangle as shown. What is the ratio of the areas of the shaded regions K to P? Express your answer as a common fraction.

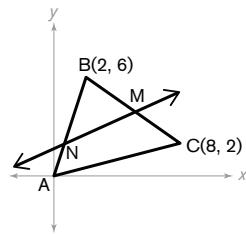


164. \_\_\_\_\_ What is the sum of the integers strictly between 1 and 100 that are multiples of neither 2 nor 3?

165. \_\_\_\_\_ James and John take turns spinning the pointer of a fair spinner that is divided into three congruent sectors. The first player whose spin lands on the WIN sector is the winner of the game. If James goes first, what is the probability that he wins the game? Express your answer as a common fraction.

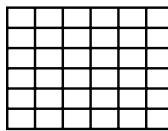


166. \_\_\_\_\_ units Triangle ABC has vertices at  $(0, 0)$ ,  $(2, 6)$  and  $(8, 2)$ . The line  $x - 3y = -7$  intersects two sides of the triangle at points M and N, as shown. What is the length of segment MN? Express your answer as a common fraction in simplest radical form.



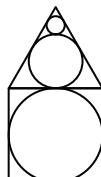
167. \_\_\_\_\_ bottles Edna enters a room with 1000 bottles lined up in a row left to right. One bottle contains a tasteless magic potion. All bottles to the left of the magic potion contain tasteless water. All bottles to the right of the magic potion contain a bitter poison. Edna can drink from no more than two bottles containing poison before becoming sick and being unable to drink anything else. She can take an unlimited number of drinks from any other bottle. What is the minimum number of bottles from which Edna may need to drink to ensure she can identify the bottle containing the magic potion no matter where it is in the lineup?

168. \_\_\_\_\_ silver rectangles In origami, a *silver rectangle* is any rectangle such that the ratio of the length of the short side to the length of the long side is exactly  $1:\sqrt{2}$ . Each of the small rectangles in the figure shown is a silver rectangle. How many silver rectangles of any size can be found in the figure?



169. \_\_\_\_\_ laps Priya, Amanda and Du simultaneously begin jogging in the same direction from the same point on a circular track. Amanda's speed is the average of Priya's and Du's speeds. Du passes Priya for the first time at the moment when Du completes his fourth lap. How many laps has Amanda completed at the moment when she passes Priya for the first time?

170. \_\_\_\_\_ An equilateral triangle is stacked above a square as shown, with a circle inscribed inside the square and two stacked circles inscribed in the triangle so that they are tangent to each other. What is the ratio of the area of the smallest circle to the area of the largest circle? Express your answer as a common fraction.





# Workout 1

171. \_\_\_\_\_ If three fair coins are simultaneously flipped, what is the probability that exactly two heads will be showing? Express your answer as a fraction in simplest form.

172. \_\_\_\_\_  $\text{m/s}^2$  An Euler Airline flight is getting ready to take off. Gary McDonald, the pilot, starts from rest at the edge of the runway. He needs to accelerate to a speed of 300 km/h in 30 seconds. Acceleration is defined as the change in speed per unit time. What is Gary's average acceleration, in meters per second per second, which is equivalent to meters per second squared, during takeoff? Express your answer as a decimal to the nearest tenth.

173. \_\_\_\_\_ pounds An object's weight on a planet is directly proportional to the mass of that planet and inversely proportional to the square of the radius of the planet. Jupiter is 318 times as massive as Earth and has a radius 11 times as large as that of Earth. If Gordon weighs 100 pounds on Earth, how many pounds would he weigh on Jupiter? Express your answer to the nearest whole number.

174. \_\_\_\_\_ million dollars The 1998 film *Armageddon* had a production budget of \$140 million. The domestic box office gross was about \$200 million, and the international box office gross was about \$350 million. The studio considers a film a financial success if the worldwide (domestic plus international) gross is at least double the sum of the production budget and the advertising budget. In millions of dollars, what was the greatest advertising budget the film could have had to be considered a financial success? Express your answer to the nearest whole number.

175. \_\_\_\_\_ days What is the mean number of days per month among all months in the year 2018? Express your answer as a decimal to the nearest tenth.

176. \_\_\_\_\_ If  $A = x^2 - 2x + 6$  and  $B = \frac{5x^2 - 1}{x + 3}$ , what is  $A + B$  if  $x = -2$ ?

177. \_\_\_\_\_ cm What is the height of a cone with a volume of  $1187.5 \text{ cm}^3$  and a base of diameter 18 cm? Express your answer to the nearest whole number.

178. \_\_\_\_\_ degrees What is the degree measure of each interior angle of a regular decagon?

179. \_\_\_\_\_ \$ Elliott's stock portfolio was valued at \$5000 on January 1. Its value decreased by 20% during January but then increased by 25% during February. What was the value of his stock portfolio at the end of February?

180. \_\_\_\_\_ hours How many hours are in the decade from January 1, 2011, through December 31, 2020?



# Workout 2

181. \_\_\_\_\_ inches If a television screen with a length-to-height ratio of 16:9 has an area of 576 in<sup>2</sup>, what is its perimeter?
182. \_\_\_\_\_ miles Alex Zhu bikes between home and school every day. He uses the same route to go to and from school, but it takes him 20 minutes to bike to school and only 15 minutes to bike back. If his average biking pace for the whole round-trip is 7 minutes per mile, how many miles long is the trip from home to school? Express your answer as a decimal to the nearest tenth.
183. \_\_\_\_\_ m/s Bruce and Lawson are playing ice hockey. Bruce shoots the puck at the goal 40 meters directly in front of him at a speed of 50 m/s. If Lawson is standing exactly 30 meters to the left of Bruce, what is the minimum speed at which he must skate, to reach the goal when the puck does? Express your answer as a decimal to the nearest tenth.
184. \$ \_\_\_\_\_ In 2015 the average two-adult family in a particular town paid \$619 per month for groceries, excluding sales tax. If groceries in this town were subject to a 9% sales tax, how much sales tax was paid by the average two-adult family in one month?
185. \$ \_\_\_\_\_ Linda makes 6 cakes per hour. Sara makes 4 cakes per hour. If Linda gets paid \$11 less than Sara for each cake, how much in dollars should Sara be paid for each cake for Linda and Sara to earn the same amount each hour?
186. \_\_\_\_\_ yards In a golf long-drive competition, Jason Zuback hits his first five drives 394, 401, 387, 414 and 421 yards, respectively. How long must he hit his sixth drive to ensure that the mean of his six drives is at least 400 yards?
187. \_\_\_\_\_ km A pilot flying due east is forced to make a detour from her original route to avoid turbulent weather. The pilot turns 30 degrees north of east. After traveling some distance, she turns and rejoins her original route and is 1000 km away from where she took the detour. The turn back to her original route put her at a 45 degree angle to that route. How much farther did the pilot travel due to her detour? Express your answer to the nearest whole kilometer.
188. \_\_\_\_\_ What is 12% of  $\frac{3}{4}$  of 1.8? Express your answer to the nearest thousandth.
189. \_\_\_\_\_ ways In how many different ways can four people be seated around a circular table so that no one ever has the same two neighbors more than once?
190. \_\_\_\_\_ fluid ounces A cylindrical container holds 20 fluid ounces. It has a radius of 3 inches and a height of 12 inches. How many fluid ounces will a similar container with a radius of 4.5 inches hold? Express your answer as a decimal to the nearest tenth.



# Workout 3

191. \_\_\_\_\_ seats The number of seats per row in an auditorium increases from the front to the back. The first row has 15 seats, the second row has 2 more seats than the first row, the third row has 3 more seats than the second row, the fourth row has 2 more seats than the third row, the fifth row has 3 more seats than the fourth row. This pattern continues, with successive rows alternating between 2 more seats and then 3 more seats than the previous row. How many seats are in the auditorium if there are 30 rows total?
192. \_\_\_\_\_ degrees Three interior angles of a pentagon measure 110, 120 and 130 degrees, respectively. Of the two remaining interior angles, one is three times the measure of the other. What is the measure of the pentagon's smallest interior angle?
193. \_\_\_\_\_ Hisham Dimashkieh chooses four distinct positive integers  $a$ ,  $b$ ,  $c$  and  $d$ , each less than or equal to 10. He chooses the numbers so that  $a$  is prime,  $b$  is composite,  $c$  is a perfect square and  $d$  is a perfect cube. What is the greatest possible sum of the four numbers?
194. \_\_\_\_\_ If  $p$ ,  $q$  and  $r$  are prime numbers such that  $pq + r = 73$ , what is the least possible value of  $p + q + r$ ?
195. \_\_\_\_\_ % If Bella runs 40% as fast as Thomas and 35% as fast as Tam, what percent faster than Thomas is Tam? Express your answer to the nearest percent.
196. \_\_\_\_\_ % The probability that it will rain today is 50%. The probability that it will rain tomorrow is 40%. Assuming today and tomorrow's precipitation outcomes are independent from one another, what is the percent probability that it will rain on at least one of the two days? Express your answer as a whole number.
197. \_\_\_\_\_  $\text{ft}^2$  What is the area of a 60 degree sector of a circle with radius 30 feet? Express your answer in terms of  $\pi$ .
198. \_\_\_\_\_ kg One year, the U.S. government printed \$700 million worth of paper money every day, for 365 days. Half of the total value came from \$1 bills. If a new \$1 bill weighs exactly 1 gram, what was the weight, in kilograms, of all the \$1 bills printed that year?
199. \_\_\_\_\_ trees The number of bushels of apples,  $B(n)$ , that can be harvested from an acre of land is a function of  $n$ , the number of trees planted per acre, where  $B(n) = 2025n - n^3$ . How many trees planted per acre will produce the greatest harvest? Express your answer as a whole number.
200. \_\_\_\_\_ Three days ago, there were  $p$  cupcakes on the counter. Two days ago, exactly 20% of the cupcakes were eaten. Today, there are 30% fewer cupcakes than yesterday and half as many as there were three days ago. If a whole number of cupcakes were eaten every day, what is the least possible value of  $p$ ?

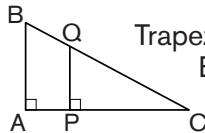


# Workout 4

201. \_\_\_\_\_ The sum of the digits of a two-digit integer is 12. The integer is equal to 15 times its units digit. What is the integer?
202. \_\_\_\_\_ If  $\frac{x^2 + 8x + 15}{x + 5} = 4.01$ , then what is the value of  $x$ ? Express your answer as a decimal to the nearest hundredth.
203. \_\_\_\_\_ units<sup>2</sup> A triangle has three vertices given by coordinates (2, 2), (2, -6) and (-5, -9). What is the area of the triangle?
204. \_\_\_\_\_ In a regular octagon, the diagonals have three possible lengths—"short," "medium," and "long." What is the ratio of the length of the medium diagonal to the long diagonal? Express your answer as a decimal to the nearest thousandth.
205. \_\_\_\_\_ % Not realizing that an 18% tip had already been added to the cost of a meal, Emalee added another 15% to the total bill. Given that there is no sales tax, what percent tip did Emalee actually pay? Express your answer as a percent to the nearest tenth.
206. \_\_\_\_\_ Penner has a deck of 40 cards composed of four suits (red, blue, green, and yellow) and cards numbered 1 through 10 in each suit. Tell secretly chooses a card. Penner then chooses the following 4 cards from the deck: Red-2, Blue-3, Green-5 and Yellow-7. For each card Penner chooses, Tell says "yes" if his secret card is of the same color or shares a common factor greater than 1 with Penner's card. Otherwise Tell says "no." Tell says "no," "yes," "no," and "yes," respectively, in response to Penner's cards. With this information, what is the best possible probability Penner has of guessing Tell's secret card? Express your answer as a common fraction.
207. \_\_\_\_\_ Henry Flannigan chooses a two-digit positive integer at random. What is the probability that the two digits have an absolute difference greater than 1? Express your answer as a common fraction.
208. \_\_\_\_\_ mi<sup>3</sup> Ngorongoro Crater is shaped approximately like a cylinder that is 10 miles across and 2000 feet deep. How many cubic miles of water would it take to fill the crater? Express your answer to the nearest whole number.
209. \_\_\_\_\_ Mady distributes  $w$  candies evenly among 20 bags. The next day, she discovers 5 more empty bags and decides to redistribute the  $w$  candies evenly into all of the bags. On the third day, Mady finds 1 more bag and redistributes the  $w$  candies evenly again. There are 2 fewer candies on the third day in each of the bags than there were in the bags on the second day. What is the value of  $w$ ?
210. \_\_\_\_\_ If  $f(x) = ax^2 + bx + c$ , with  $f(0) = 4$ ,  $f(2) = 2$  and  $f(4) - f(3) = 4$ , what is the value of  $f(1)$ ?



# Workout 5

211. \_\_\_\_\_ units<sup>2</sup>

Trapezoid  $APQB$  lies inside of right triangle  $ABC$ , as shown. If  $AP = 30$ ,  $BQ = 34$  and  $AB = 60$ , what is the area of triangle  $ABC$ ?

212. \_\_\_\_\_

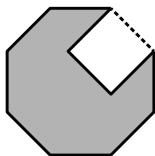
There are five two-digit positive integers arranged in decreasing order. Each digit is unique. What is the absolute difference between the greatest possible range and the least possible range of such a set of integers?

213. \_\_\_\_\_ ways

Six different donuts are lined up in a box. Six different cookies are lined up in another box. Mackenzie wants to alternate the donuts and cookies in one big long box. How many ways are there to arrange them?

214. \_\_\_\_\_ workers

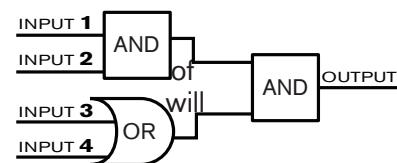
For a certain crew of workers, it takes the  $n$ th worker exactly  $n$  hours to complete a certain job alone. What is the least number of workers needed to complete an identical job in under 20 minutes by working together?

215. \_\_\_\_\_ cm<sup>2</sup>

A regular octagon has a perimeter of 64 cm. A square with one side along an edge of the octagon has been cut out of the octagon, as shown. What is the remaining area of the octagon? Express your answer in simplest radical form.

216. \_\_\_\_\_ inputs

The function machine shown here consists of three logic functions. A Boolean is a member of the set  $\{0, 1\}$ . For each function, the inputs are on the left and the output is on the right. The output of each function is a Boolean. The input of each function is a pair of Booleans. For the AND function, the output is 1 if and only if both inputs are 1. For the OR function, the output is 0 if and only if both inputs are 0. Among the 16 distinct sets inputs that can be applied on the far left, how many produce a 1 as the final output on the far right?



217. \$ \_\_\_\_\_

Joe, Bob and Randell split a restaurant bill that totaled \$80 before the tip. The group tipped 25%, Joe paid twice as much as Bob, and Randell paid the same amount as Joe. How much did Bob pay?

218. \_\_\_\_\_ pairs

How many pairs of positive integers  $a$  and  $b$  exist such that  $a^2 - b^2 = 144$ ?

219. \_\_\_\_\_ cm<sup>3</sup>

A rectangular box measures  $5 \text{ mm} \times 10 \text{ mm} \times 1 \text{ m}$ . What is the volume of the box in cubic centimeters?

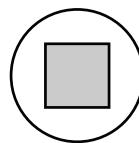
220. \_\_\_\_\_

A college has been trying to decrease the number of cars on campus and increase the number of bicycles. The price of a parking permit was tripled, and the number of cars on campus decreased 30%. Student tuition was decreased, and the number of bicycles on campus increased by 20%, producing a car to bicycle ratio of 1:3. What was the ratio of cars to bicycles before the changes occurred? Express your answer as a common fraction.



# Workout 6

221. \_\_\_\_\_  $\text{in}^2$  The gasket shown consists of a circular disk with a square removed. The square and the disk have the same center. If each corner of the square is exactly 1 inch away from the boundary of the disk, and the midpoint of each side of the square is exactly 2 inches away from the boundary of the disk, what is the area of the top face of the gasket? Express your answer as a decimal to the nearest tenth.



222. \_\_\_\_\_ % Jeffrey Pibble needs to buy 6 pairs of socks. The Sock Shop is running a limited time promotion: buy 3 pairs of socks and get 3 pairs at half off the regular price. What percent savings does Jeffrey get with the promotion compared to the regular price without the promotion?

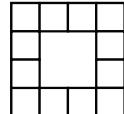
223. \_\_\_\_\_ What is the greatest possible absolute difference between the median and mean of a list of 10 positive integers that are at most 20? Express your answer as a decimal to the nearest tenth.

224. \_\_\_\_\_ tourists The tour from Ajim to the Mos Eisley Cantina can take at most 36 tourists. The price for the tour is \$520 per person until at least 15 people have signed up. After that, the price for each person, including the first 15, drops \$5 per additional tourist. If the total amount the tourists paid was \$12,740, how many tourists signed up?

225. \_\_\_\_\_ inches A rectangle has a width of 1 inch and a height 2 inches. There are two lines drawn, each connecting a vertex to the midpoint of the opposite side, and circles are inscribed in the triangles created, as shown. How far apart are the centers of the circles? Express your answer as a decimal to the nearest hundredth.



226. \_\_\_\_\_ rect-angles



How many rectangles of any size are in the figure shown?

227. \_\_\_\_\_ Let  $S$  be the set of all integers  $N$  such that both  $N$  and the number formed by reversing the digits of  $N$  are three-digit perfect squares. What is the sum of the integers in  $S$ ?

228. \_\_\_\_\_ feet A seesaw is in balance when the weight on one side of the fulcrum times its distance from the fulcrum is equal to the weight on the other side of the fulcrum times its distance from the fulcrum. Shandra weighs 96 pounds. Her little sister weighs 72 pounds. The seesaw at their playground has a beam with seats 14 feet apart. The position of the fulcrum can be adjusted as required. If each girl sits in her seat, how far should the fulcrum be from Shandra's seat to achieve perfect balance with her sister?



229. \_\_\_\_\_ Jennie Weiner has  $p$  pennies,  $n$  nickels,  $d$  dimes and  $q$  quarters with a total value of \$1.08. If the numbers  $p$ ,  $n$ ,  $d$  and  $q$  are distinct and positive, and the greatest common divisor of each pair of these numbers is 1, what is the least possible value of  $p + n + d + q$ ?

230. \_\_\_\_\_  $\text{ft}^2$  What is the total surface area of a right square pyramid with a height of 12 feet and a base with side length 10 feet?



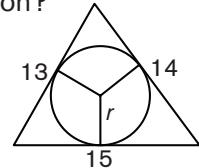
# Workout 7

231. \$ \_\_\_\_\_

Rahul Ilangovan can arrange his dad's collection of quarters as a rectangular array with 10 equal rows, 12 equal rows or 18 equal rows, using all the quarters in each arrangement. What is the least possible monetary value in dollars of the quarter collection?

232. \_\_\_\_\_ units

What is the radius of the largest circle that can be inscribed in an acute triangle with sides 13, 14 and 15 units?



233. \_\_\_\_\_

Francisco is born at 1:00 a.m. on a Tuesday and gets married exactly  $2^{18}$  hours later. On what day of the week does Francisco get married?

234. \_\_\_\_\_

Erica drew a 4 of hearts out of a standard 52-card deck, without replacement. If she draws a second card from the deck, what is the probability that her two cards will show consecutive numbers? Express your answer as a common fraction.

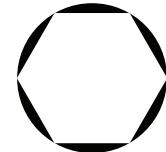
235. \_\_\_\_\_

A set  $S$  contains some, but not all, of the positive integers from 3 to 7. Some statements describing  $S$  are given below. The statement numbered  $n$  is true if the number  $n$  is in  $S$  and false if  $n$  is not in  $S$ . What is the product of the numbers that are in  $S$ ?

3. The sum of the numbers in  $S$  is odd.
4. The sum of the numbers in  $S$  is less than 15.
5.  $S$  contains exactly one composite number.
6.  $S$  contains exactly one prime number.
7. The product of the numbers in  $S$  is odd.

236. \_\_\_\_\_  $\text{m}^2$ 

The figure shows a regular hexagon of side length 12 meters inscribed in a circle. What is the total area of the shaded regions between the hexagon and the circle? Express your answer to the nearest whole number.



237. \_\_\_\_\_

Kendra starts at a positive integer  $k$  and counts up by 4s until she hits exactly 200. Mason starts at a positive integer  $m$  and counts up by 6s until he hits exactly 200. If it takes Kendra half as many steps to reach 200 as it takes Mason, what is the greatest possible value of  $k - m$ ?

238. \_\_\_\_\_

The graph of the line  $3x - 4y = 13$  is translated 2018 units to the right. What is the  $y$ -intercept of the translated line? Express your answer as a decimal to the nearest hundredth.

239. \$ \_\_\_\_\_

Ron works five days a week selling wallets in a booth at the mall. He earns a salary of \$215 per week plus 15% of his weekly sales. If he earned \$383.75 this week, what was the amount of his average daily sales for the week?

240. \_\_\_\_\_ inches

A regular hexagon is inscribed in a circle. If the area of the hexagon is  $216\sqrt{3}$  in<sup>2</sup>, what is the circumference of the circle? Express your answer in terms of  $\pi$ .



# Workout 8

241. \_\_\_\_\_ Suppose that  $A & B = k \times A^m \times B^n$ , where  $k$ ,  $m$  and  $n$  are constants. Suppose that  $5 & 3 = 18$ ,  $10 & 3 = 72$  and  $5 & 6 = 36$ . What is the value of  $10 & 6$ ?
242. \_\_\_\_\_ combinations How many different combinations of pennies, nickels, dimes and quarters are possible in the cup holder of Terry's car if he counts 15 coins total?
243. \_\_\_\_\_ in<sup>3</sup> If the average length of the edges of a right rectangular prism is 13 inches, and the dimensions of the prism are distinct integers in geometric progression, what is the sum of the volumes of the distinct prisms that meet these criteria?
244. \_\_\_\_\_ weights In the mobile shown, a beam is *in balance* when the length  $l_L$  of the left arm of the beam times the total weight  $w_L$  hanging below the left arm of the beam is equal to the length  $l_R$  of the right arm of the beam times the total weight  $w_R$  hanging below the right arm of the beam. Every beam must be in balance for the mobile to be in balance. Decorative weights are available only in powers of 2. Multiple weights can be hung in a vertical chain, one below another. If the existing weight in the figure is not removed, what is the minimum number of weights that must be added to bring the mobile shown into balance?
245. \_\_\_\_\_ % Four non-overlapping congruent circles are inscribed in a larger circle. Each small circle is shaded. Each region between two adjacent small circles and the enclosing large circle is also shaded. What percent of the figure is not shaded? Express your answer as a decimal to the nearest tenth of a percent.
246. \_\_\_\_\_ Maxine secretly chooses a positive integer between 1 and 2018, inclusive. Martin wants to identify her number with a series of guesses. Each time Martin makes a guess, Maxine tells him whether his number is correct, too high or too low. With an appropriate strategy, Martin can always identify Maxine's number after at most  $n$  guesses. What is the least value of  $n$  for which Martin can correctly guess Maxine's number?
247. \_\_\_\_\_ novels Ms. Ault's reading list has novels by only three authors: Mark Twain, Ernest Hemingway and John Steinbeck. For a summer reading assignment, Austen Mazenko must pick one of the authors and read two of that author's novels on the list. If there are exactly 100 ways for Austen to pick two novels that satisfy this requirement, what is the greatest possible total number of novels on the reading list?
248. \_\_\_\_\_ What is the sum of all prime numbers  $p$  less than 60 such that there exists a right triangle whose side lengths are all integers and whose hypotenuse has length  $p$ ?
249. \_\_\_\_\_ A line segment AB from the positive  $x$ -axis to the positive  $y$ -axis cuts off a triangle of area 54 square units in the first quadrant of the coordinate plane. The line  $y = 0.6x$  divides this triangle into two triangles of areas 40 units<sup>2</sup> and 14 units<sup>2</sup>. If the point A has coordinates  $(a, 0)$ , what is the value of  $a$ ? Express your answer in simplest radical form.
250. \_\_\_\_\_ % The Solar Sunflower is made up of super-efficient circular solar panels within a hexagonal frame. Using a two-dimensional diagram of the Solar Sunflower, as shown, with a hexagonal frame with side length 10 meters, what percent of the hexagon is covered in solar discs? Express your answer to the nearest whole number.



## OFFICIAL RULES + PROCEDURES

The following rules and procedures govern all MATHCOUNTS competitions. The MATHCOUNTS Foundation reserves the right to alter these rules and procedures at any time. **Coaches are responsible for being familiar with the rules and procedures outlined in this handbook.** Coaches should bring any difficulty in procedures or in student conduct to the immediate attention of the appropriate chapter, state or national official. Students violating any rules may be subject to immediate disqualification.

Any questions regarding the MATHCOUNTS Competition Series Official Rules + Procedures articulated in this handbook should be addressed to the MATHCOUNTS national office at (703) 299-9006 or [info@mathcounts.org](mailto:info@mathcounts.org).

## REGISTRATION

**The fastest and easiest way to register for the MATHCOUNTS Competition Series is online at [www.mathcounts.org/compreg](http://www.mathcounts.org/compreg).**

For your school to participate in the MATHCOUNTS Competition Series, a school representative is required to complete a registration form and pay the registration fees. A school representative can be a teacher, administrator or parent volunteer who has received expressed permission from his/her child's school administration to register. By completing the Competition Series Registration Form, the coach attests to the school administration's permission to register students for MATHCOUNTS.

School representatives can register online at [www.mathcounts.org/compreg](http://www.mathcounts.org/compreg) or download the Competition Series Registration Form and mail or email a scanned copy of it to the MATHCOUNTS national office. Refer to the Critical 2017-2018 Dates on pg. 10 of this handbook for contact information.

**WHAT REGISTRATION COVERS:** Registration in the Competition Series entitles a school to:

- 1) send 1-10 student(s)—depending on number registered—to the Chapter Competition. *Students can advance beyond the chapter level, but this is determined by their performance at the competition.*
- 2) receive the School Competition Kit, which includes the 2017-2018 MATHCOUNTS School Handbook, one recognition ribbon for each registered student, 10 student participation certificates and a catalog of additional coaching materials. *Mailings of School Competition Kits will occur on a rolling basis through December 31, 2017.*
- 3) receive online access to the 2018 School Competition, along with electronic versions of other competition materials at [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches). *Coaches will receive an email notification no later than November 1, 2017 when the 2018 School Competition is available online.*

Your state or chapter coordinator will be notified of your registration, and then you will be informed of the date and location of your Chapter Competition. **If you have not been contacted by mid-January with competition details, it is your responsibility to contact your local coordinator** to confirm that your registration has been properly routed and that your school's participation is expected. Coordinator contact information is available at [www.mathcounts.org/findmycoordinator](http://www.mathcounts.org/findmycoordinator).

**DEADLINES:** The sooner your Registration Form is received, the sooner you will receive your preparation materials. To guarantee your school's participation, submit your registration by one of the following deadlines:

<i>Early Bird Discount Deadline:</i> November 3, 2017	Online registrations: submitted by 11:59 PST Emailed forms: received by 11:59 PST Mailed forms: postmarked by November 3, 2017
<i>Regular Registration Deadline:</i> December 15, 2017*	Online registrations: submitted by 11:59 PST Emailed forms: received by 11:59 PST Mailed forms: postmarked by December 15, 2017

\*Late Registrations may be accepted at the discretion of the MATHCOUNTS national office and your local coordinators, but are not guaranteed. If a school's late registration is accepted, an additional \$20 processing fee will be assessed.

**REGISTRATION FEES:** The cost of your school's registration depends on when your registration is postmarked/mailed/submitted online. The cost of your school's registration covers the students for the entire Competition Series; there are no additional registration fees to compete at the state or national level. Title I schools (as affirmed by a school's administration) receive a 50% discount off the total cost of their registration.

<i>Early Bird Registrations</i> (by November 3, 2017)	<b>\$30 per student</b> \$120 for 1 team of 4   \$300 for 1 team of 4 + 6 individuals
<i>Regular Registrations</i> (by December 15, 2017)	<b>\$35 per student</b> \$140 for 1 team of 4   \$350 for 1 team of 4 + 6 individuals
<i>Late Registrations</i> (after December 15, 2017)	<b>\$35 per student + \$20 late fee on entire order</b> \$160 for 1 team of 4   \$370 for 1 team of 4 + 6 individuals

**CANCELLATION FEES:** Registered schools that need to cancel their Competition Series registration must notify the MATHCOUNTS national office in writing via email or mail. Schools may request and receive a full refund minus a \$30 non-refundable cancellation fee to cover refund processing and the cost of materials shipped to the school. MATHCOUNTS will verify a school's non-participation with local coordinators and reserves the right to refuse a refund request. No cancellations or refund requests will be processed after February 1, 2018. *This fee does not apply to schools that reduce their number of registered students but remain registered with at least one student.*

## ELIGIBILITY REQUIREMENTS

**Eligibility requirements for the MATHCOUNTS Competition Series are different from other MATHCOUNTS programs. Eligibility for the National Math Club or the Math Video Challenge does not guarantee eligibility for the Competition Series.**

### **WHO IS ELIGIBLE:**

- U.S. students enrolled in the 6th, 7th or 8th grade can participate in MATHCOUNTS competitions.
- Schools that are the students' official school of record can register.
- Any type of school, of any size, can register—public, private, religious, charter, virtual or homeschooled—but virtual and homeschooled must fill out additional forms to participate (see pgs. 39–40).
- Schools in 50 U.S. states, District of Columbia, Guam, Puerto Rico and Virgin Islands can register.
- Overseas schools that are affiliated with the U.S. Departments of Defense and State can register.

## **WHO IS NOT ELIGIBLE:**

- Students who are not full-time 6th, 7th or 8th graders cannot participate, even if they are taking middle school math classes.
- Academic centers, tutoring centers or enrichment programs that do not function as students' official school of record cannot register. *If it is unclear whether your educational institution is considered a school, please contact your local Department of Education for specific criteria governing your state.*
- Schools located outside of the U.S. states and territories listed on the previous page cannot register.
- Overseas schools not affiliated with the U.S. Departments of Defense or State cannot register.

**NUMBER OF STUDENTS ALLOWED:** A school can register a maximum of one team of four students and six individuals; these 1-10 student(s) will represent the school at the Chapter Competition. Any number of students can participate at the school level. Prior to the Chapter Competition, coaches must notify their chapter coordinator of which students will be team members and which students will compete as individuals.

**NUMBER OF YEARS ALLOWED:** Participation in MATHCOUNTS competitions is limited to 3 years for each student, but there is no limit to the number of years a student may participate in school-based coaching.

**WHAT TEAM REGISTRATION MEANS:** Members of a school team will participate in the Target, Sprint and Team Rounds. Members of a school team also will be eligible to qualify for the Countdown Round (where conducted). Team members will be eligible for team awards, individual awards and progression to the state and national levels based on their individual and/or team performance. It is recommended that your strongest four Mathletes form your school team. Teams of fewer than four will be allowed to compete; however, the team score will be computed by dividing the sum of the team members' scores by four (see pg. 43), meaning, teams of fewer than four students will be at a disadvantage. *Only one team (of up to four students) per school is eligible to compete.*

**WHAT INDIVIDUAL REGISTRATION MEANS:** Students registered as individuals will participate in the Target and Sprint Rounds, but not the Team Round. Individuals will be eligible to qualify for the Countdown Round (where conducted). Individuals also will be eligible for individual awards and progression to the state and national levels. A student registered as an "individual" may not help his/her school's team advance to the next level of competition. *Up to six students may be registered in addition to or in lieu of a school team.*

**HOW STUDENTS ENROLLED PART-TIME AT TWO SCHOOLS PARTICIPATE:** *A student may compete only for his/her official school of record.* A student's school of record is the student's base or main school. A student taking limited course work at a second school or educational center may not register or compete for that second school or center, even if the student is not competing for his/her school of record. MATHCOUNTS registration is not determined by where a student takes his or her math course. If there is any doubt about a student's school of record, the chapter or state coordinator must be contacted for a decision before registering.

**HOW SMALL SCHOOLS PARTICIPATE:** MATHCOUNTS does not distinguish between the sizes of schools for Competition Series registration and competition purposes. Every "brick-and-mortar" school will have the same registration allowance of up to one team of four students and/or up to six individuals. A school's participants may not combine with any other school's participants to form a team when registering or competing.

**HOW HOMESCHOOLS PARTICIPATE:** Homeschools and/or homeschool groups in compliance with the homeschool laws of the state in which they are located are eligible to participate in MATHCOUNTS competitions in accordance with all other rules. Homeschool coaches must complete the 2017-2018 Homeschool + Virtual School Participation Form, verifying that students from the homeschool or homeschool group are in the 6th, 7th or 8th grade and that each homeschool complies with applicable state laws. Forms can be downloaded at [www.mathcounts.org/competition](http://www.mathcounts.org/competition) and must be submitted to the MATHCOUNTS national office in order for registrations to be processed.

**HOW VIRTUAL SCHOOLS PARTICIPATE:** Virtual schools that want to register must contact the MATHCOUNTS national office by December 8, 2017 for specific registration details. Any student registering as a virtual school student must compete in the MATHCOUNTS Chapter Competition assigned according to the student's home address. Additionally, virtual school coaches must complete the 2017-2018 Homeschool + Virtual School Participation Form, verifying that the students from the virtual school are in the 6th, 7th or 8th grade and that the virtual school complies with applicable state laws. Forms must be submitted to the national office in order for registrations to be processed; forms can be downloaded at [www.mathcounts.org/competition](http://www.mathcounts.org/competition).

**WHAT IS DONE FOR SUBSTITUTIONS OF STUDENTS:** Coaches determine which students will represent the school at the Chapter Competition. Coaches cannot substitute team members for the State Competition unless a student voluntarily releases his/her position on the school team. Additional requirements and documentation for substitutions (such as requiring parental release or requiring the substitution request be submitted in writing) are at the discretion of the State Coordinator. A student being added to a team need not be a student who was registered for the Chapter Competition as an individual. Coaches cannot make substitutions for students progressing to the State Competition as individuals. At all levels of competition, student substitutions are not permitted after on-site competition registration has been completed.

**WHAT IS DONE FOR RELIGIOUS OBSERVANCES:** A student who is unable to attend a competition due to religious observances may take the written portion of the competition up to one week in advance of the scheduled competition. In addition, all competitors from that student's school must take the Sprint and Target Rounds at the same earlier time. If the student who is unable to attend the competition due to a religious observance: (1) is a member of the school team, then the team must take the Team Round at the same earlier time; (2) is not part of the school team, then the team has the option of taking the Team Round during this advance testing or on the regularly scheduled day of the competition with the other school teams. The coordinator must be made aware of the team's decision before the advance testing takes place. *Advance testing will be done at the discretion of the chapter and state coordinators. If advance testing is deemed possible, it will be conducted under proctored conditions.* Students who qualify for an official Countdown Round but are unable to attend will automatically forfeit one place standing.

**WHAT IS DONE FOR STUDENTS WITH SPECIAL NEEDS:** Reasonable accommodations may be made to allow students with special needs to participate. However, many accommodations that are employed in a classroom or teaching environment cannot be implemented in the competition setting. Accommodations that are not permissible include, but are not limited to: granting a student extra time during any of the competition rounds or allowing a student to use a calculator for the Sprint or Countdown Rounds. *A request for accommodation of special needs must be directed to chapter or state coordinators in writing at least three weeks in advance of the Chapter or State Competition.* This written request should thoroughly explain a student's special need, as well as what the desired accommodation would entail. In conjunction with the MATHCOUNTS Foundation, coordinators will review the needs of the student and determine if any accommodations will be made. In making final determinations, the feasibility of accommodating these needs at the National Competition will be taken into consideration.

## LEVELS OF COMPETITION

There are four levels in the MATHCOUNTS Competition Series: school, chapter (local), state and national. Competition questions are written for 6th, 7th and 8th graders. The competitions can be quite challenging, particularly for students who have not been coached using MATHCOUNTS materials. All competition materials are prepared by the national office.

**SCHOOL COMPETITIONS (TYPICALLY HELD IN JANUARY 2018):** After several months of coaching, schools registered for the Competition Series should administer the 2018 School Competition to all interested

students. The School Competition should be an aid to the coach in determining competitors for the Chapter Competition. *Selection of team and individual competitors is entirely at the discretion of the coach and does not need to be based solely on School Competition scores.* School Competition materials are sent to the coach of a school, and it may be used by the teachers and students only in association with that school's programs and activities. The current year's School Competition questions must remain confidential and may not be used in outside activities, such as tutoring sessions or enrichment programs with students from other schools. For updates or edits, please check [www.mathcounts.org/coaches](http://www.mathcounts.org/coaches) before administering the School Competition.

It is important that the coach look upon coaching sessions during the academic year as opportunities to develop better math skills in all students, not just in those students who will be competing. Therefore, it is suggested that the coach postpone selection of competitors until just prior to the Chapter Competition.

**CHAPTER COMPETITIONS (HELD FEB. 1–28, 2018):** The Chapter Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be conducted. The chapter and state coordinators determine the date and location of the Chapter Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The winning team will advance to the State Competition. Additionally, the two highest-ranking competitors not on the winning team (who may be registered as individuals or as members of a team) will advance to the State Competition. This is a minimum of six advancing Mathletes (assuming the winning team has four members). Additional teams and/or individuals also may progress at the discretion of the state coordinator, but the policy for progression must be consistent for all chapters within a state.

**STATE COMPETITIONS (HELD MAR. 1– APR. 1, 2018):** The State Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be included. The state coordinator determines the date and location of the State Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The four highest-ranked Mathletes and the coach of the winning team from each State Competition will receive an all-expenses-paid trip to the National Competition.

**2018 RAYTHEON MATHCOUNTS NATIONAL COMPETITION (HELD MAY 13–14 IN WASHINGTON, DC):** The National Competition consists of the Sprint, Target, Team and Countdown Rounds (conducted officially). Expenses of the state team and coach to travel to the National Competition will be paid by MATHCOUNTS. The national program does not make provisions for the attendance of additional students or coaches. All national competitors will receive a plaque and other items in recognition of their achievements. Winning teams and individuals also will receive medals, trophies and college scholarships.

## COMPETITION COMPONENTS

The four rounds of a MATHCOUNTS competition, each described below, are designed to be completed in approximately three hours:

**TARGET ROUND** (approximately 30 minutes): In this round eight problems are presented to competitors in four pairs (six minutes per pair). The multi-step problems featured in this round engage Mathletes in mathematical reasoning and problem-solving processes. *Problems assume the use of calculators.*

**SPRINT ROUND** (40 minutes): Consisting of 30 problems, this round tests accuracy, with the time period allowing only the most capable students to complete all of the problems. *Calculators are not permitted.*

**TEAM ROUND** (20 minutes): In this round, interaction among team members is permitted and encouraged as they work together to solve 10 problems. *Problems assume the use of calculators.*

Note: The order in which the written rounds (Target, Sprint and Team) are administered is at the discretion of the competition coordinator.

**COUNTDOWN ROUND:** A fast-paced oral competition for top-scoring individuals (based on scores on the Target and Sprint Rounds), this round allows pairs of Mathletes to compete against each other and the clock to solve problems. Calculators are not permitted.

At Chapter and State Competitions, a Countdown Round (1) may be conducted officially, (2) may be conducted unofficially (for fun) or (3) may be omitted. However, the use of an official Countdown Round must be consistent for all chapters within a state. In other words, *all* chapters within a state must use the round officially in order for *any* chapter within a state to use it officially. All students, whether registered as part of a school team or as individual competitors, are eligible to qualify for the Countdown Round.

An official Countdown Round determines an individual's final overall rank in the competition. If a Countdown Round is used officially, the official procedures as established by the MATHCOUNTS Foundation must be followed, as described below.\*

- The top 25% of students, up to a maximum of 10, are selected to compete. These students are chosen based on their Individual Scores.
- The two lowest-ranked students are paired; a question is read and projected, and students are given 45 seconds to solve the problem. A student may buzz in at any time, and if s/he answers correctly, a point is scored. If a student answers incorrectly, the other student has the remainder of the 45 seconds to answer.
- Three total questions are read to the pair of students, one question at a time, and the student who scores the higher number of points (not necessarily 2 out of 3) progresses to the next round and challenges the next-higher-ranked student.
- If students are tied in their matchup after three questions (at 1-1 or 0-0), questions should continue to be read until one is successfully answered. The first student who answers an additional question correctly progresses to the next round.
- This procedure continues until the 4th-ranked Mathlete and his/her opponent compete. For the final four matchups, the first student to correctly answer three questions advances.
- The Countdown Round proceeds until a 1st place individual is identified. More details about Countdown Round procedures are included in the 2018 School Competition.

\*Rules for the Countdown Round change for the National Competition.

An unofficial Countdown Round does not determine an individual's final overall rank in the competition, but is done for practice or for fun. The official procedures do not have to be followed. Chapters and states choosing not to conduct the round officially must determine individual winners solely on the basis of students' scores in the Target and Sprint Rounds of the competition.

## SCORING

**MATHCOUNTS Competition Series scores do not conform to traditional grading scales.  
Coaches and students should view an Individual Score of  
23 (out of a possible 46) as highly commendable.**

**INDIVIDUAL SCORE:** calculated by taking the sum of the number of Sprint Round questions answered correctly and twice the number of Target Round questions answered correctly. There are 30 questions in the Sprint Round and eight questions in the Target Round, so the maximum possible Individual Score is  $30 + 2(8) = 46$ . If used officially, the Countdown Round yields final individual standings.

**TEAM SCORE:** calculated by dividing the sum of the team members' Individual Scores by four (even if the team has fewer than four members) and adding twice the number of Team Round questions answered correctly. The highest possible Individual Score is 46. Four students may compete on a team, and there are 10 questions in the Team Round. Therefore, the maximum possible Team Score is  $((46 + 46 + 46 + 46) \div 4) + 2(10) = 66$ .

**TIEBREAKING ALGORITHM:** used to determine team and individual ranks and to determine which individuals qualify for the Countdown Round. In general, questions in the Target, Sprint and Team Rounds increase in difficulty so that the most difficult questions occur near the end of each round. In a comparison of questions to break ties, generally those who correctly answer the more difficult questions receive the higher rank. The guidelines provided below are very general; competition officials receive more detailed procedures.

- *Ties between individuals:* the student with the higher Sprint Round score will receive the higher rank. If a tie remains after this comparison, specific groups of questions from the Target and Sprint Rounds are compared.
- *Ties between teams:* the team with the higher Team Round score, and then the higher sum of the team members' Sprint Round scores, receives the higher rank. If a tie remains after these comparisons, specific questions from the Team Round will be compared.

## RESULTS DISTRIBUTION

Coaches should expect to receive the scores of their students, as well as a list of the top 25% of students and top 40% of teams, from their competition coordinators. In addition, single copies of the blank competition materials and answer keys may be distributed to coaches after all competitions at that level nationwide have been completed. Before distributing blank competition materials and answer keys, coordinators must wait for verification from the national office that all such competitions have been completed. Both the problems and answers from Chapter and State Competitions will be posted on the MATHCOUNTS website following the completion of all competitions at that level nationwide, replacing the previous year's posted tests.

Student competition papers and answers will not be viewed by or distributed to coaches, parents, students or other individuals. Students' competition papers become the confidential property of MATHCOUNTS.

## ADDITIONAL RULES

**All answers must be legible.**

**Pencils and paper** will be provided for Mathletes by competition organizers. However, students may bring their own pencils, pens and erasers if they wish. They may not use their own scratch paper or graph paper.

**Use of notes or other reference materials** (including dictionaries and translation dictionaries) is prohibited.

**Specific instructions stated in a given problem** take precedence over any general rule or procedure.

**Communication with coaches is prohibited during rounds but is permitted during breaks.** All communication between guests and Mathletes is prohibited during competition rounds. Communication between teammates is permitted only during the Team Round.

**Calculators are not permitted in the Sprint and Countdown Rounds, but they are permitted in the Target, Team and Tiebreaker (if needed) Rounds.** When calculators are permitted, students may use any calculator (including programmable and graphing calculators) that does not contain a QWERTY (typewriter-like) keypad. Calculators that have the ability to enter letters of the alphabet but do not have a keypad in a standard typewriter arrangement are acceptable. Smart phones, laptops, tablets, iPods®, personal

digital assistants (PDAs) and any other “smart” devices are not considered to be calculators and may not be used during competitions. Students may not use calculators to exchange information with another person or device during the competition.

**Coaches are responsible for ensuring their students use acceptable calculators, and students are responsible for providing their own calculators.** Coordinators are not responsible for providing Mathletes with calculators or batteries before or during MATHCOUNTS competitions. Coaches are strongly advised to bring backup calculators and spare batteries to the competition for their team members in case of a malfunctioning calculator or weak or dead batteries. Neither the MATHCOUNTS Foundation nor coordinators shall be responsible for the consequences of a calculator’s malfunctioning.

**Pagers, cell phones, tablets, iPods® and other MP3 players should not be brought into the competition room.** Failure to comply could result in dismissal from the competition.

**Should there be a rule violation or suspicion of irregularities, the MATHCOUNTS coordinator or competition official has the obligation and authority to exercise his/her judgment regarding the situation and take appropriate action, which might include disqualification of the suspected student(s) from the competition.**

# FORMS OF ANSWERS

The following rules explain acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Competition answers will be scored in compliance with these rules for forms of answers.

**Units of measurement are not required in answers, but they must be correct if given.** When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lb 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

**All answers must be expressed in simplest form.** A “common fraction” is to be considered a fraction in the form  $\pm \frac{a}{b}$ , where  $a$  and  $b$  are natural numbers and  $\text{GCF}(a, b) = 1$ . In some cases the term “common fraction” is to be considered a fraction in the form  $\frac{A}{B}$ , where  $A$  and  $B$  are algebraic expressions and  $A$  and  $B$  do not have a common factor. A simplified “mixed number” (“mixed numeral,” “mixed fraction”) is to be considered a fraction in the form  $\pm N\frac{a}{b}$ , where  $N$ ,  $a$  and  $b$  are natural numbers,  $a < b$  and  $\text{GCF}(a, b) = 1$ . Examples:

Problem: What is $8 \div 12$ expressed as a common fraction?	Answer: $\frac{2}{3}$	Unacceptable: $\frac{4}{6}$
Problem: What is $12 \div 8$ expressed as a common fraction?	Answer: $\frac{3}{2}$	Unacceptable: $\frac{12}{8}, 1\frac{1}{2}$
Problem: What is the sum of the lengths of the radius and the circumference of a circle of diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of $\pi$ ?	Answer: $\frac{1+2\pi}{8}$	
Problem: What is $20 \div 12$ expressed as a mixed number?	Answer: $1\frac{2}{3}$	Unacceptable: $1\frac{8}{12}, \frac{5}{3}$

**Ratios should be expressed as simplified common fractions** unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}, \frac{3}{\pi}, \frac{4-\pi}{6}$	Unacceptable: $3\frac{1}{2}, \frac{1}{3}, 3.5, 2:1$
--	---

**Radicals must be simplified.** A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form?	Answer: $5\sqrt{3}$	Unacceptable: $\sqrt{75}$
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**Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., “How many dollars...,” “How much will it cost...,” “What is the amount of interest...”) should be expressed in the form (\$)*a.bc*, where *a* is an integer and *b* and *c* are digits.** The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* are both zero, in which case they both may be omitted. Answers in the form (\$)*a.bc* should be rounded to the nearest cent, unless otherwise specified. Examples:  
Acceptable Forms: 2.35, 0.38, .38, 5.00, 5      Unacceptable: 4.9, 8.0

**Do not make approximations for numbers** (e.g.,  $\pi$ ,  $\frac{2}{3}$ ,  $5\sqrt{3}$ ) in the data given or in solutions unless the problem says to do so.

**Do not do any intermediate rounding** (other than the “rounding” a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

**Scientific notation** should be expressed in the form  $a \times 10^n$  where  $a$  is a decimal,  $1 \leq |a| < 10$ , and  $n$  is an integer. Examples:

Problem: What is 6895 expressed in scientific notation?	Answer: $6.895 \times 10^3$
Problem: What is 40,000 expressed in scientific notation?	Answer: $4 \times 10^4$ or $4.0 \times 10^4$

**An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form.** Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

**The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.**

# COMPETITION COACH TOOLKIT

This is a collection of lists, formulas and terms that Mathletes frequently use to solve problems like those found in this handbook. There are many others we could have included, but we hope you find this collection useful.

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50
$\frac{1}{3}$	0. $\bar{3}$	33. $\bar{3}$
$\frac{1}{4}$	0.25	25
$\frac{1}{5}$	0.2	20
$\frac{1}{6}$	0.1 $\bar{6}$	16. $\bar{6}$
$\frac{1}{8}$	0.125	12.5
$\frac{1}{9}$	0. $\bar{1}$	11. $\bar{1}$
$\frac{1}{10}$	0.1	10
$\frac{1}{11}$	0.0 $\bar{9}$	9.0 $\bar{9}$
$\frac{1}{12}$	0.08 $\bar{3}$	8. $\bar{3}$

n	$n^2$	$n^3$
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375

## Common Arithmetic Series

$$1 + 2 + 3 + 4 + \dots + n = \frac{n(n + 1)}{2}$$

$$1 + 3 + 5 + 7 + \dots + (2n - 1) = n^2$$

$$2 + 4 + 6 + 8 + \dots + 2n = n^2 + n$$

## Prime Numbers

2	43
3	47
5	53
7	59
11	61
13	67
17	71
19	73
23	79
29	83
31	89
37	97
41	

## Combinations & Permutations

$${}_nC_r = \frac{n!}{r!(n - r)!} \quad {}_nP_r = \frac{n!}{(n - r)!}$$

## Divisibility Rules

- 2: units digit is 0, 2, 4, 6 or 8
- 3: sum of digits is divisible by 3
- 4: two-digit number formed by tens and units digits is divisible by 4
- 5: units digit is 0 or 5
- 6: number is divisible by both 2 and 3
- 8: three-digit number formed by hundreds, tens and units digits is divisible by 8
- 9: sum of digits is divisible by 9
- 10: units digit is 0

## Geometric Mean

$$\frac{a}{x} = \frac{x}{b} \quad \text{and} \quad x = \sqrt{ab}$$

## Distance Traveled

$$\text{Distance} = \text{Rate} \times \text{Time}$$

## Quadratic Formula

For  $ax^2 + bx + c = 0$ , where  $a \neq 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Pythagorean Triples

- |               |               |                |
|---------------|---------------|----------------|
| $(3, 4, 5)$   | $(5, 12, 13)$ | $(7, 24, 25)$  |
| $(8, 15, 17)$ | $(9, 40, 41)$ | $(12, 35, 37)$ |

## Equation of a Line

Standard Form

$$Ax + By = C$$

Slope-Intercept Form

$$y = mx + b$$

$m$  = slope

$b$  =  $y$ -intercept

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

$m$  = slope

$(x_1, y_1)$  = point on the line

## Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

## Sum and Difference of Cubes

$$a^3 + b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 - b^3 = (a + b)(a^2 - ab + b^2)$$

### Circles

Circumference $2 \times \pi \times r = \pi \times d$	Area $\pi \times r^2$
---	--------------------------

For radius  $r$

Arc Length $\frac{x}{360} \times 2 \times \pi \times r$	Sector Area $\frac{x}{360} \times \pi \times r^2$
--	--

For central angle  
of  $x$  degrees

### Pythagorean Theorem



$$a^2 + b^2 = c^2$$

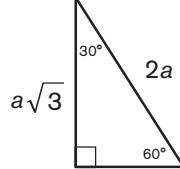
### Given $A(x_1, y_1)$ and $B(x_2, y_2)$

$$\text{Distance from } A \text{ to } B = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

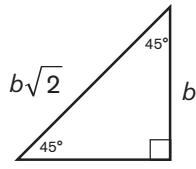
$$\text{Midpoint of } \overline{AB} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{Slope of } \overline{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

### Special Right Triangles



30-60-90  
Right Triangle



45-45-90  
Right Triangle

### Area of Polygons

Square	side length $s$	$s^2$
Rectangle	length $l$ , width $w$	$l \times w$
Parallelogram	base $b$ , height $h$	$b \times h$
Trapezoid	bases $b_1, b_2$ , height $h$	$\frac{1}{2}(b_1 + b_2) \times h$
Rhombus	diagonals $d_1, d_2$	$\frac{1}{2} \times d_1 \times d_2$
Triangle	base $b$ , height $h$	$\frac{1}{2} \times b \times h$
Triangle	semi-perimeter $s$ , side lengths $a, b, c$	$\sqrt{s(s-a)(s-b)(s-c)}$
Equilateral Triangle	side length $s$	$\frac{s^2\sqrt{3}}{4}$

### Polygon Angles $(n$ sides)

Sum of the interior angle measures:

$$180 \times (n - 2)$$

Central angle measure of a regular polygon:

$$\frac{360}{n}$$

Interior angle measure of a regular polygon:

$$\frac{180 \times (n - 2)}{n} \quad \text{or} \quad 180 - \frac{360}{n}$$

Solid	Dimensions	Surface Area	Volume
Cube	side length $s$	$6 \times s^2$	$s^3$
Rectangular Prism	length $l$ , width $w$ , height $h$	$2 \times (l \times w + w \times h + l \times h)$	$l \times w \times h$
Circular Cylinder	base radius $r$ , height $h$	$2 \times \pi \times r \times h + 2 \times \pi \times r^2$	$\pi \times r^2 \times h$
Circular Cone	base radius $r$ , height $h$	$\pi \times r^2 + \pi \times r \times \sqrt{r^2 + h^2}$	$\frac{1}{3} \times \pi \times r^2 \times h$
Sphere	radius $r$	$4 \times \pi \times r^2$	$\frac{4}{3} \times \pi \times r^3$
Pyramid	base area $B$ , height $h$		$\frac{1}{3} \times B \times h$

## Vocabulary & Terms

The following list is representative of terminology used in the problems but **should not** be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference	geometric sequence	rate
absolute value	hemisphere	ratio
acute angle	image(s) of a point(s) <i>(under a transformation)</i>	rational number
additive inverse ( <i>opposite</i> )	improper fraction	ray
adjacent angles	infinite series	real number
apex	inscribe	reciprocal ( <i>multiplicative inverse</i> )
arithmetic mean	integer	reflection
arithmetic sequence	interior angle of a polygon	regular polygon
base 10	intersection	relatively prime
binary	inverse variation	revolution
bisect	irrational number	right angle
box-and-whisker plot	isosceles	right polyhedron
center	lateral edge	rotation
chord	lateral surface area	scalene triangle
circumscribe	lattice point(s)	scientific notation
coefficient	LCM	sector
collinear	median of a set of data	segment of a circle
common divisor	median of a triangle	segment of a line
common factor	mixed number	semicircle
common fraction	mode(s) of a set of data	semiperimeter
complementary angles	multiplicative inverse <i>(reciprocal)</i>	sequence
congruent	natural number	set
convex	obtuse angle	significant digits
coordinate plane/system	ordered pair	similar figures
coplanar	origin	slope
counting numbers	palindrome	space diagonal
counting principle	parallel	square root
diagonal of a polygon	Pascal's Triangle	stem-and-leaf plot
diagonal of a polyhedron	percent increase/decrease	supplementary angles
digit-sum	perpendicular	system of equations/inequalities
direct variation	planar	tangent figures
divisor	polyhedron	tangent line
domain of a function	polynomial	term
edge	prime factorization	transformation
equiangular	principal square root	translation
equidistant	proper divisor	triangular numbers
expected value	proper factor	trisect
exponent	proper fraction	twin primes
exterior angle of a polygon	quadrant	union
factor	quadrilateral	unit fraction
finite	random	variable
frequency distribution	range of a data set	whole number
frustum	range of a function	y-intercept
function		
GCF		

# ANSWERS

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3 - One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.

4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary.

6/7 - Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.

## Probability Stretch

Answer	Difficulty			
1. 15.38	(2)	6. 50	(4)	
2. $\frac{1}{4}$	(3)	7. 20	(2)	
3. 0.2	(4)	8. $\frac{9}{16}$	(4)	
4. $\frac{2}{5}$	(3)	9. $\frac{1}{5}$	(3)	
5. $\frac{3}{8}$	(3)	10. 0.17	(4)	

## Warm-Up 1

Answer	Difficulty			
31. -19	(1)	36. 18	(2)	
32. 14	(2)	37. $\frac{1}{2}$	(2)	
33. 2112	(2)	38. 0	(3)	
34. $\frac{1}{221}$	(3)	39. 452	(4)	
35. $60\sqrt{14}$	(3)	40. 47	(4)	

## Patterns Stretch

Answer	Difficulty			
11. 1480	(4)	16. 2601	(3)	
12. 423	(3)	17. 2	(4)	
13. 1457	(3)	18. $\frac{4}{3}$	(4)	
14. 36	(4)	19. 125	(4)	
15. 1365	(3)	20. 243	(5)	

## Warm-Up 2

Answer	Difficulty			
41. 5	(2)	46. $\frac{5}{18}$	(3)	
42. 32	(4)	47. $\frac{9}{2}$	(4)	
43. 81	(4)	48. 25	(3)	
44. 600	(2)	49. 16	(3)	
45. 4	(3)	50. $4\pi$	(4)	

## Travel Stretch

Answer	Difficulty			
21. $2\frac{2}{3}$	(5)	26. 3	(3)	
22. 2:35	(4)	27. $7\frac{1}{2}$	(5)	
23. 880	(3)	28. $1\frac{1}{4}$	(3)	
24. 3	(4)	29. 5	(4)	
25. 2	(4)	30. $\frac{5}{8}$	(3)	

## Warm-Up 3

Answer	Difficulty			
51. -1009	(3)	56. $\frac{13}{18}$	(3)	
52. 12	(3)	57. 25	(4)	
53. $\frac{6}{25}$	(5)	58. 348	(3)	
54. 10,227	(3)	59. 1,000,000	(3)	
55. $2\sqrt{5}$	(4)	60. $4\frac{1}{20}$	(3)	

\* The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.

## **Warm-Up 4**

<b>Answer</b>	<b>Difficulty</b>		
61. $\frac{1}{2}$	(3)	66. $3\pi$	(4)
62. 900	(4)	67. 432	(3)
63. 50	(3)	68. 4	(4)
64. 21	(4)	69. 27	(3)
65. 191	(4)	70. 48	(5)

## **Warm-Up 7**

<b>Answer</b>	<b>Difficulty</b>		
91. 12	(2)	96. $\sqrt{3}/4$	(5)
92. 160	(4)	97. 9	(4)
93. $5\sqrt{2}$	(5)	98. 11	(4)
94. 161	(3)	99. 3	(4)
95. $\frac{7}{36}$	(3)	100. 1000	(3)

## **Warm-Up 5**

<b>Answer</b>	<b>Difficulty</b>		
71. 17	(3)	76. $\frac{1}{3}$	(2)
72. 6	(2)	77. 15	(4)
73. 120	(5)	78. 63	(3)
74. 9	(3)	79. 4	(3)
75. 4	(3)	80. 1	(5)

## **Warm-Up 8**

<b>Answer</b>	<b>Difficulty</b>		
101. 68	(3)	106. $\frac{11}{18}$	(4)
102. 40	(3)	107. $\sqrt{2} - 1$	(5)
103. 144	(4)	108. 4	(5)
104. $\frac{1}{7}$	(3)	109. 173	(5)
105. 81	(3)	110. $\frac{1}{4}$	(5)

## **Warm-Up 6**

<b>Answer</b>	<b>Difficulty</b>		
81. 1320	(2)	86. $\frac{3}{8}$	(4)
82. 128	(4)	87. 100	(2)
83. 5	(3)	88. 80	(2)
84. 3.81	(2)	89. 315	(4)
85. 126	(5)	90. $\frac{1}{4}$	(3)

## **Warm-Up 9**

<b>Answer</b>	<b>Difficulty</b>		
111. $\frac{79}{111}$	(2)	116. 180	(3)
112. $\frac{8}{11}$	(5)	117. 6	(4)
113. $\frac{17}{24}$	(5)	118. 36	(7)
114. $100\sqrt{10}$	(4)	119. 85	(2)
115. 305	(4)	120. 148	(4)

## **Warm-Up 10**

<b>Answer</b>	<b>Difficulty</b>		
121. 23	(3)	126. $6\sqrt{7}$	(6)
122. 100	(3)	127. 9	(4)
123. $\frac{3}{8}$	(5)	128. 105	(5)
124. 5	(3)	129. $6/\pi$	(6)
125. 6720	(5)	130. 30	(4)

## **Warm-Up 13**

<b>Answer</b>	<b>Difficulty</b>		
151. 5	(4)	156. 9490	(5)
152. $18\sqrt{3}$	(3)	157. 13	(4)
153. 53	(3)	158. 105	(5)
154. 91	(4)	159. 17	(5)
155. 3	(4)	160. 25	(4)

## **Warm-Up 11**

<b>Answer</b>	<b>Difficulty</b>		
131. 20	(4)	136. 132	(6)
132. 76	(4)	137. 350	(4)
133. $\frac{3}{8}$	(5)	138. 28	(4)
134. 25	(4)	139. 18	(3)
135. $\frac{175}{256}$	(5)	140. $\frac{8}{27}$	(5)

## **Warm-Up 14**

<b>Answer</b>	<b>Difficulty</b>		
161. 30	(4)	166. $\frac{(11\sqrt{10})}{8}$	(5)
162. 52	(2)	167. 45	(6)
163. $\frac{1}{9}$	(5)	168. 140	(4)
164. 1632	(4)	169. 7	(5)
165. $\frac{3}{5}$	(5)	170. $\frac{1}{27}$	(5)

## **Warm-Up 12**

<b>Answer</b>	<b>Difficulty</b>		
141. 867	(3)	146. 81	(5)
142. 58	(5)	147. 58	(4)
143. 241	(5)	148. 12	(3)
144. 300	(3)	149. $10+2\sqrt{35}$	(6)
145. 0	(3)	or $2\sqrt{35}+10$	
		150. 8	(5)

## Workout 1

Answer	Difficulty		
171. $\frac{3}{8}$	(2)	176. 33	(3)
172. 2.8	(4)	177. 14	(3)
173. 263	(5)	178. 144	(4)
174. 135	(3)	179. 5000 or 5000.00	(3)
175. 30.4	(2)	180. 87,672	(3)

## Workout 5

Answer	Difficulty		
211. 3375	(5)	216. 3	(3)
212. 57	(4)	217. 20 or 20.00	(3)
213. 1,036,800	(5)	218. 4	(4)
214. 11	(5)	219. 50	(2)
215. $64 + 128\sqrt{2}$ or $128\sqrt{2} + 64$	(5)	220. $\frac{4}{7}$	(4)

## Workout 2

Answer	Difficulty		
181. 100	(3)	186. 383	(2)
182. 2.5	(5)	187. 250	(5)
183. 62.5	(5)	188. 0.162	(2)
184. 55.71	(2)	189. 3	(4)
185. 33	(4)	190. 67.5	(4)

## Workout 6

Answer	Difficulty		
221. 37.9	(5)	226. 52	(5)
222. 25	(2)	227. 2996	(3)
223. 7.6	(4)	228. 6	(3)
224. 28	(3)	229. 11	(4)
225. 1.47	(5)	230. 360	(4)

## Workout 3

Answer	Difficulty		
191. 1530	(3)	196. 70	(3)
192. 45	(3)	197. $150\pi$	(3)
193. 34	(3)	198. 127,750,000	(2)
194. 44	(4)	199. 26	(4)
195. 14	(3)	200. 70	(3)

## Workout 7

Answer	Difficulty		
231. 45	(4)	236. 78	(5)
232. 4	(6)	237. 128	(5)
233. Thursday	(4)	238. $-1516.75$	(4)
234. $\frac{8}{51}$	(3)	239. 225 or 225.00	(3)
235. 72	(5)	240. $24\pi$	(5)

## Workout 4

Answer	Difficulty		
201. 75	(2)	206. $\frac{1}{3}$	(4)
202. 1.01	(3)	207. $\frac{32}{45}$	(3)
203. 28	(3)	208. 30	(2)
204. 0.924	(5)	209. 1300	(3)
205. 35.7	(3)	210. 2	(4)

## Workout 8

Answer	Difficulty		
241. 144	(4)	246. 11	(5)
242. 816	(5)	247. 26	(5)
243. 1729	(6)	248. 195	(6)
244. 5	(4)	249. $3\sqrt{7}$	(6)
245. 4.7	(6)	250. 87	(6)

# **MATHCOUNTS Problems Mapped to Common Core State Standards (CCSS)**

Forty-two states, the District of Columbia, four territories and the Department of Defense Education Activity (DoDEA) have voluntarily adopted the Common Core State Standards (CCSS). As such, MATHCOUNTS considers it beneficial for teachers to see the connections between the 2017-2018 *MATHCOUNTS School Handbook* problems and the CCSS. MATHCOUNTS not only has identified a general topic and assigned a difficulty level for each problem but also has provided a CCSS code in the Problem Index (pages 54-55). A complete list of the Common Core State Standards can be found at [www.corestandards.org](http://www.corestandards.org).

The CCSS for mathematics cover K-8 and high school courses. MATHCOUNTS problems are written to align with the NCTM Standards for Grades 6-8. As one would expect, there is great overlap between the two sets of standards. MATHCOUNTS also recognizes that in many school districts, algebra and geometry are taught in middle school, so some MATHCOUNTS problems also require skills taught in those courses.

In referring to the CCSS, the Problem Index code for each of the Standards for Mathematical Content for grades K-8 begins with the grade level. For the Standards for Mathematical Content for high school courses (such as algebra or geometry), each code begins with a letter to indicate the course name. The second part of each code indicates the domain within the grade level or course. Finally, the number of the individual standard within that domain follows. Here are two examples:

- 6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6
- G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry

Some math concepts utilized in MATHCOUNTS problems are not specifically mentioned in the CCSS. Two examples are the Fundamental Counting Principle (FCP) and special right triangles. In cases like these, if a related standard could be identified, a code for that standard was used. For example, problems using the FCP were coded 7.SP.8, S-CP.8 or S-CP.9 depending on the context of the problem; SP → Statistics and Probability (the domain), S → Statistics and Probability (the course) and CP → Conditional Probability and the Rules of Probability. Problems based on special right triangles were given the code G-SRT.5 or G-SRT.6, explained above.

There are some MATHCOUNTS problems that either are based on math concepts outside the scope of the CCSS or based on concepts in the standards for grades K-5 but are obviously more difficult than a grade K-5 problem. When appropriate, these problems were given the code SMP for Standards for Mathematical Practice. The CCSS include the Standards for Mathematical Practice along with the Standards for Mathematical Content. The SMPs are (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure and (8) Look for and express regularity in repeated reasoning.

# PROBLEM INDEX

It is difficult to categorize many of the problems in the *MATHCOUNTS School Handbook*. It is very common for a MATHCOUNTS problem to straddle multiple categories and cover several concepts. This index is intended to be a helpful resource, but since each problem has been placed in exactly one category and mapped to exactly one Common Core State Standard (CCSS), the index is not perfect. In this index, the code **9 (3) 7.SP.3** refers to problem 9 with difficulty rating 3 mapped to CCSS 7.SP.3. For an explanation of the difficulty ratings refer to page 49. For an explanation of the CCSS codes refer to page 53.

<b>NUMBER THEORY</b>  45 (3) 4.OA.4 56 (3) 4.OA.4 59 (3) SMP 68 (4) N-RN.1 69 (3) S-CP.9 78 (3) SMP 84 (2) SMP 109 (5) SMP 110 (5) 8.EE.2 111 (2) 7.NS.2 119 (2) 4.OA.4 121 (3) 6.NS.4 130 (4) SMP 134 (4) 6.NS.4 139 (3) SMP 148 (3) 7.NS.3 151 (4) SMP 153 (3) 6.EE.2 156 (5) 6.NS.4 161 (4) F-BF.2 193 (3) SMP 194 (4) SMP 201 (2) 7.NS.3 227 (3) 8.EE.2 229 (4) 6.NS.4 231 (4) 6.NS.4 233 (4) SMP 237 (5) 6.NS.4 241 (4) N-RN.2 246 (5) SMP	<b>LOGIC</b>  83 (3) S-CP.9 92 (4) SMP 131 (4) SMP 145 (3) SMP 167 (6) SMP 216 (3) S-CP.9 235 (5) SMP	<b>PLANE GEOMETRY</b>  32 (2) 4.G.2 43 (4) 7.G.6 53 (5) 7.G.4 61 (3) 7.G.6 66 (4) G-C.2 82 (4) 7.G.6 106 (4) 7.G.4 107 (5) 7.G.6 113 (5) 8.G.8 118 (7) G-SRT.4 123 (5) G-SRT.6 133 (5) 7.G.6 136 (6) G-SRT.5 149 (6) A-REI.4 152 (3) G-SRT.6 158 (5) G-SRT.6 160 (4) G-C.2 163 (5) G-SRT.5 170 (5) G-SRT.5 178 (4) 8.G.5 187 (5) G-SRT.6 192 (3) 8.G.5 197 (3) G-C.2 204 (5) 7.G.1 211 (5) G-SRT.5 215 (5) G-SRT.6 221 (5) G-SRT.6 225 (5) G-SRT.6 232 (6) G-C.2 236 (5) G-C.2 240 (5) 7.G.6 245 (6) G-SRT.6 250 (6) G-SRT.6	<b>MEASUREMENT</b>  40 (4) 8.G.5 49 (3) 5.MD.1 50 (4) G-SRT.5 55 (4) 8.G.8 67 (3) 6.G.1 96 (5) 7.G.6 126 (6) 8.G.8 132 (4) 8.G.5 143 (5) 8.G.7 155 (4) 7.RP.3 172 (4) 6.RP.3 181 (3) 6.EE.7 183 (5) 8.G.7 198 (2) 6.RP.3	<b>SOLID GEOMETRY</b>  60 (3) 7.G.6 74 (3) G-GMD.3 129 (6) G-GMD.3 177 (3) 8.G.9 190 (4) 8.G.9 208 (2) 8.G.9 219 (2) 7.G.6 230 (4) G-GMD.3	<b>COORDINATE GEOMETRY</b>  79 (3) G-C.2 93 (5) 8.G.8 112 (5) 8.F.3 166 (5) 8.F.3 203 (3) 6.G.1 238 (4) 8.G.1 249 (6) 8.G.8

<sup>1</sup> CCSS 7.SP.8 & S-CP.9

<sup>2</sup> CCSS F-BF.2

# SOLUTIONS

ONLY AVAILABLE TO  
REGISTERED COACHES IN  
THE COMPETITION  
SERIES.

**Registered Coaches:** Log in at  
*[www.mathcounts.org/coaches](http://www.mathcounts.org/coaches)*  
to access the electronic version of the  
School Handbook with solutions

# OTHER MATHCOUNTS PROGRAMS

MATHCOUNTS was founded in 1983 as a way to provide new avenues of engagement in math for middle school students. MATHCOUNTS began solely as a competition, but has grown to include 3 unique but complementary programs: the **MATHCOUNTS Competition Series**, the **National Math Club** and the **Math Video Challenge**. Your school can participate in all 3 MATHCOUNTS programs!



The **National Math Club** is a free enrichment program that provides teachers and club leaders with resources to run a math club. The materials provided through the National Math Club are designed to engage students of all ability levels—not just the top students—and are a great supplement for classroom teaching. This program emphasizes collaboration and provides students with an enjoyable, pressure-free atmosphere in which they can learn math at their own pace.

Active clubs also can earn rewards by having a minimum number of club members participate (based on school/organization/group size). **There is no cost to sign up for the National Math Club**, and registration is open to schools, organizations and groups that consist of at least 4 students in 6th, 7th and/or 8th grade and have regular in-person meetings. More information can be found at [www.mathcounts.org/club](http://www.mathcounts.org/club), and the 2017-2018 School Registration Form is included on the next page.



The **Math Video Challenge** is an innovative program that challenges students to work in teams of 4 to create a video explaining the solution to a MATHCOUNTS handbook problem and demonstrating its real-world application. This project-based activity builds math, communication and collaboration skills.

Students post their videos to the contest website, where the general public votes for the best videos. The 100 videos with the most votes advance to judging rounds, in which 20 semifinalists and, later, 4 finalists are selected. This year's finalists will present their videos to the students competing at the 2018 Raytheon MATHCOUNTS National Competition, and the 224 Mathletes will vote to determine the winner. Members of the winning team receive college scholarships. **Registration is completely free** and open to all 6th, 7th and 8th grade students. More information can be found at [videochallenge.mathcounts.org](http://videochallenge.mathcounts.org).

! The fastest way to register is online  
at [www.mathcounts.org/clubreg](http://www.mathcounts.org/clubreg)!



## 2017-2018 SCHOOL REGISTRATION FORM

*This registration form is for U.S. middle schools only. To register a non-school group (such as a Girl Scout troop, Boys and Girls Club Chapter or math circle) for the National Math Club, please go to [www.mathcounts.org/club](http://www.mathcounts.org/club) to review eligibility requirements and register.*

\*indicates required information

### STEP 1 Tell Us About Your School

#### U.S. School with Students in Grades 6-8

*One school can have multiple clubs, as long as each club has a different club leader.*

School Name\* \_\_\_\_\_

School Type (check one)\*     Public     Charter     Private     Homeschool     Virtual

Title I School? (check one)\*     Yes     No

*Overseas U.S. schools must provide additional information below:*

*My school is sponsored by the U.S. Department of:     Defense (DoDDS)     State*

Country \_\_\_\_\_

**Approximate Total Number of Students Participating in Club (Minimum 4)\*** \_\_\_\_\_

Club Leader First & Last Name\* \_\_\_\_\_

Club Leader Email\* \_\_\_\_\_

Club Leader Alternate Email\* \_\_\_\_\_

*Your email and alternate email address will not be made public or shared and will only be used by MATHCOUNTS.*

Club Street Address\* \_\_\_\_\_

City, State and ZIP Code\* \_\_\_\_\_

#### I am a MATHCOUNTS alumnus/a. → Please tell us more information below!

When? (for example, 1999-02) \_\_\_\_\_ Where? (state/territory) \_\_\_\_\_

Which Program(s)?     MATHCOUNTS Competition Series → Highest Level Reached: \_\_\_\_\_

The National Math Club (formerly MATHCOUNTS Club Program)

Math Video Challenge (formerly Reel Math Challenge)

### STEP 3

Turn in Your Form

! **IMPORTANT!** By submitting this form you attest your group consists of at least 4 U.S. students in grades 6-8 who meet in person regularly, and is therefore eligible to participate in the National Math Club. The club leader will receive an emailed confirmation once this registration has been processed.

**Mail or email a scanned copy of this completed form to:**

Address: MATHCOUNTS Registration | 1420 King Street | Alexandria, VA 22314

Email: [reg@mathcounts.org](mailto:reg@mathcounts.org)



## 2017–2018 ADDITIONAL STUDENTS REGISTRATION FORM

**Step 1: Tell us about your school so we can find your original registration (please print legibly).**

Coach Name _____	School Name _____
Coach Email Address _____	City, State ZIP _____
School Address _____	

**Step 2: Tell us how many students you are adding to your school's registration. Following the instructions below.**

Please circle the number of additional students you will enter in the Chapter Competition and the associated cost below (depending on the date your registration is postmarked).

# of Students You Are Adding	1	2	3	4	5	6	7	8	9
Early Bird Rate <small>(postmarked by Nov. 3, 2017)</small>	\$30	\$60	\$90	\$120	\$150	\$180	\$210	\$240	\$270
Regular Rate <small>(postmarked by Dec. 15, 2017)</small>	\$35	\$70	\$105	\$140	\$175	\$210	\$245	\$280	\$315
Late Registration <small>(postmarked after Dec. 15, 2017)</small>	\$55	\$90	\$125	\$160	\$195	\$230	\$265	\$300	\$335

My school qualifies for the **50% Title I discount**, so the Amount Due in Step 4 will be half the amount I circled above. Principal signature required below to verify Title I eligibility.

Principal Name \_\_\_\_\_ Principal Signature \_\_\_\_\_

**Step 3: Tell us what your school's FINAL registration should be (including all changes/additions).**

Total # of Registered Students	1 (1 individual)	2 (2 ind)	3 (3 ind)	4 (1 team)	5 (1 tm, 1 ind)	6 (1 tm, 2 ind)	7 (1 tm, 3 ind)	8 (1 tm, 4 ind)	9 (1 tm, 5 ind)	10 (1 tm, 6 ind)
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**Step 4: Almost done... just fill in payment information and turn in your form!**

Amount Due \$ \_\_\_\_\_  Credit Card  Check (payable to MATHCOUNTS Foundation)  Money Order  Purchase Order # \_\_\_\_\_ (must include copy of P.O.)  
Do NOT include any credit card information on this form. Within 5 business days you will receive an email invoice enabling you to pay by credit card.

**IMPORTANT!** By submitting this form you (1) agree to adhere to the rules of the MATHCOUNTS Competition Series; (2) attest you have the school administration's permission to register students for this program under this school's name; and (3) affirm the above named school is a U.S. school eligible for this program and not an academic or enrichment center.  
The coach will receive an emailed confirmation and receipt once this additional students registration has been processed.



# ACKNOWLEDGMENTS

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provides engaging math programs for U.S. middle school students of all ability levels to build confidence and improve attitudes towards math and problem solving.

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