Report

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1 Assignment 2: Fitzhugh-Nagumo Oscillator Model

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The calculated values for I1 and I2 are 0.316 and 0.682 respectively for a=0.5, b=0.1 and r=0.1. The values have been determined ny observing oscillatory region behaviour of the neuron.

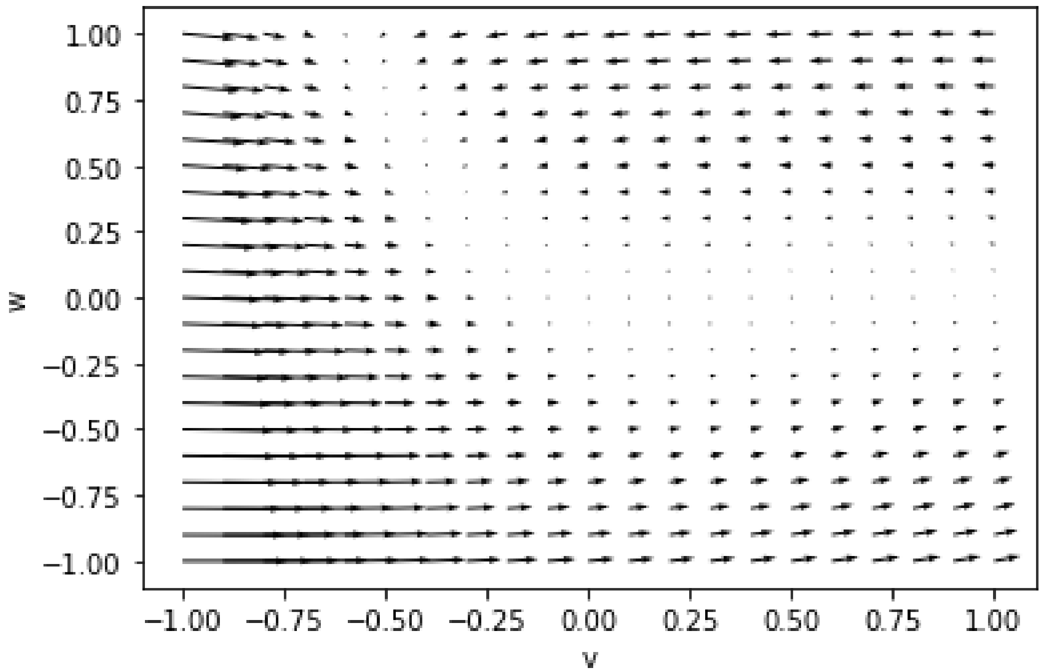
```
In [111]: import numpy as np
          import matplotlib.pyplot as plt
          import itertools

In [112]: def runFN(I, v0, w0, a, b, r, niter, dt):
              """
              Solves the state-equation for the oscillator and stores the result
              in the arrays vhist, whist and fhist respectively using Single Forward
              Euler Integration.

              Return vhist, whist, fhist and t.
              """

              t=np.arange(0, niter*dt, dt)
              vhist=np.zeros_like(t)
              whist=np.zeros_like(t)
              fhist=np.zeros_like(t)
              vhist[0]=v0
              whist[0]=w0
              fhist[0]=calcf(v0,a)
              for i in range(1,niter):
                  v_=vhist[i-1]
                  w_=whist[i-1]
                  f_=fhist[i-1]
                  v=v_+(calcf(v_, a)-w_+I)*dt
                  w=w_+(b*v_-r*w_)*dt
                  f=calcf(v,a)
                  vhist[i]=v
                  whist[i]=w
                  fhist[i]=f
```

```
    return vhist, whist, fhist, t

def calcf(v,a):
    """
    Calculates f
    """
    return -v*(v-a)*(v-1)

def v_(I, v, w, a, b, r):
    """
    Calculates dv/dt
    """
    return calcf(v, a)-w+I

def w_(I, v, w, a, b, r):
    """
    Calculates dw=dt
    """
    return b*v-r*w

def calcvnc(I, v, a, b, r):
    """
    Calculates the v-nullcline for the system
    """
    return calcf(v,a)+I

def calcwnc(I, v, a, b, r):
    """
    Calculates the w-nullcline for the system
    """
    return (b/r)*v

def phaseplot(vmin, vmax, wmin, wmax, step, I, a, b, r):
    """
    Returns the data required to plot the phase plot for the system.
    """
    v=np.arange(vmin, vmax, step)
    w=np.arange(wmin, wmax, step)
    l, m=np.meshgrid(v,w)
    vd=[]
    wd=[]
    for x, y in itertools.product(v,w):
            vd.append(v_(I, x, y, a, b, r))
            wd.append(w_(I, x, y, a, b, r))
    return m,l,vd,wd
```

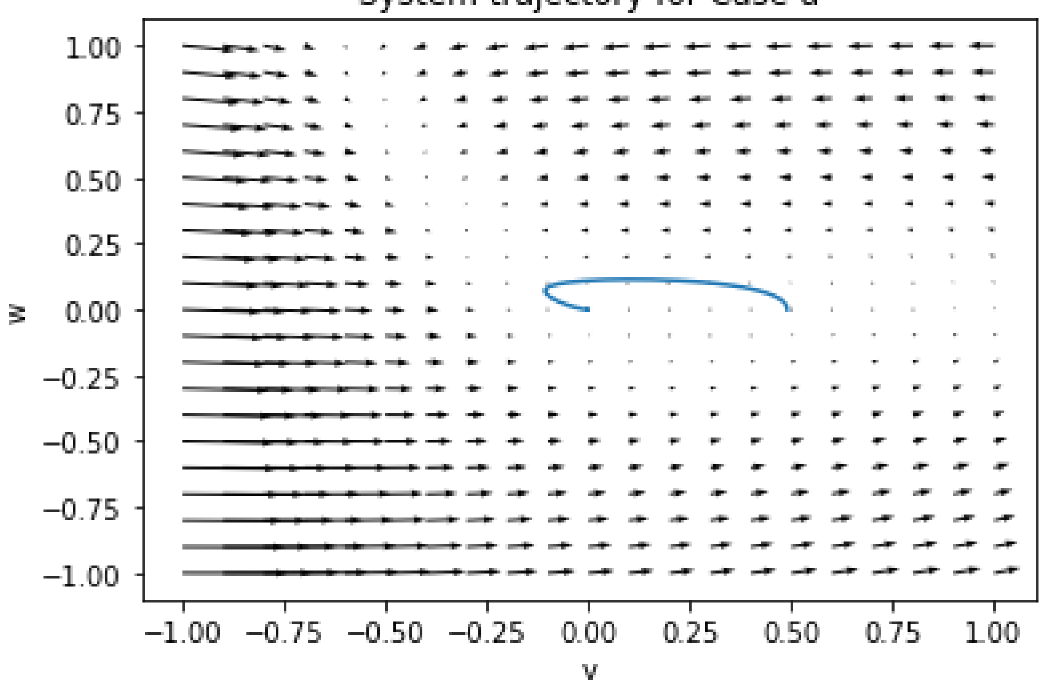
2.1 Case 1:

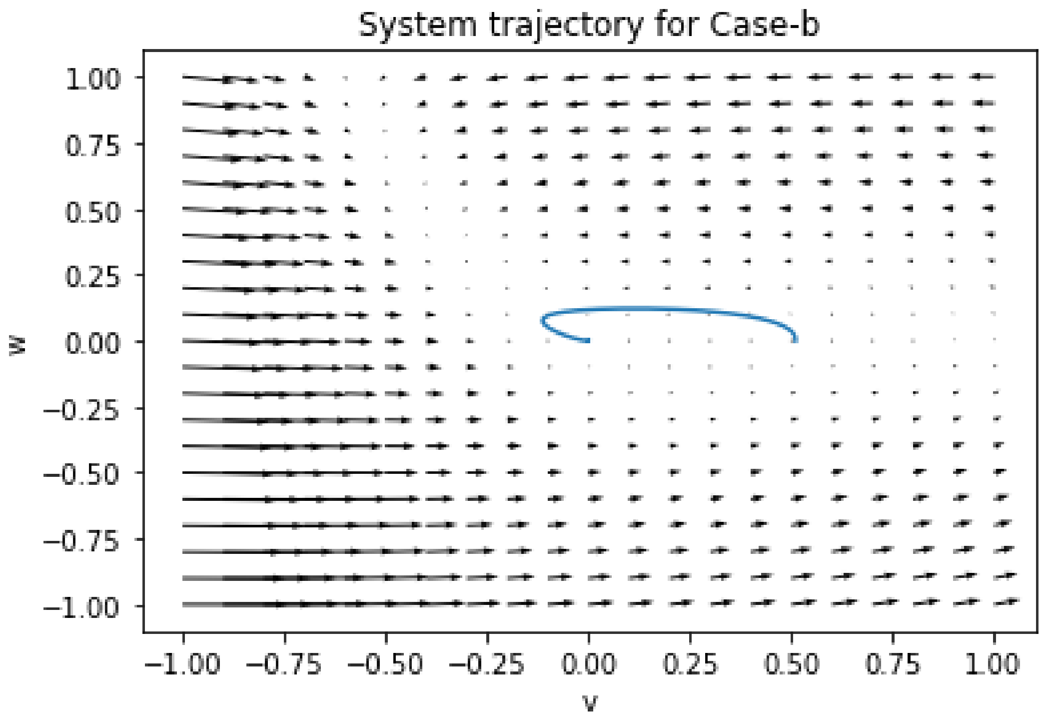
In [113]:
```
I=0
a=0.5
b=0.1
r=0.1
niter=1000
dt=0.1
w0=0

m,l,vd,wd=phaseplot(-1, 1.01, -1, 1.01, 0.1, I, a, b, r)
plt.quiver(m,l,vd,wd)
plt.xlabel("v")
plt.ylabel("w")
plt.title("Phase Plot for Case-1")
plt.show()

#Case a: v0<a; w0=0
v0=0.49
v,w,f,t=runFN(I, v0, w0, a, b, r, niter, dt)
plt.xlabel("v")
plt.ylabel("w")
plt.plot(v,w)
plt.quiver(m,l,vd,wd)
plt.title("System trajectory for Case-a")
plt.show()

#Case 2: v0>a; w0=0
v0=0.51
v,w,f,t=runFN(I, v0, w0, a, b, r, niter, dt)
plt.xlabel("v")
plt.ylabel("w")
plt.title("System trajectory for Case-b")
plt.plot(v,w)
plt.quiver(m,l,vd,wd)
plt.show()
```

Phase Plot for Case-1



System trajectory for Case-a



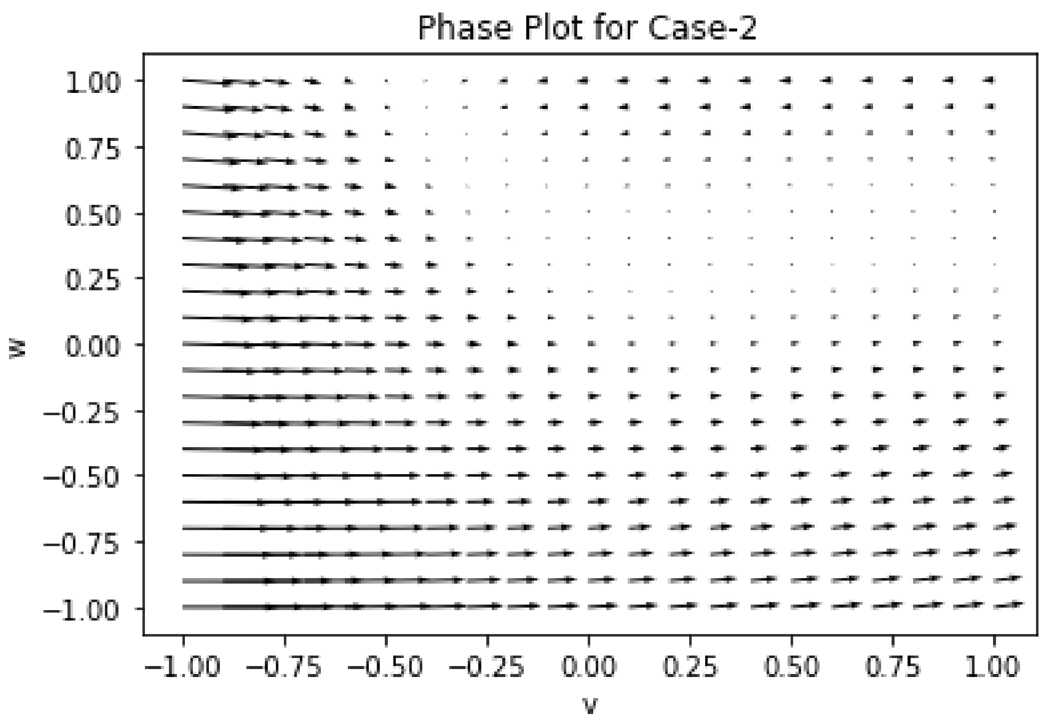


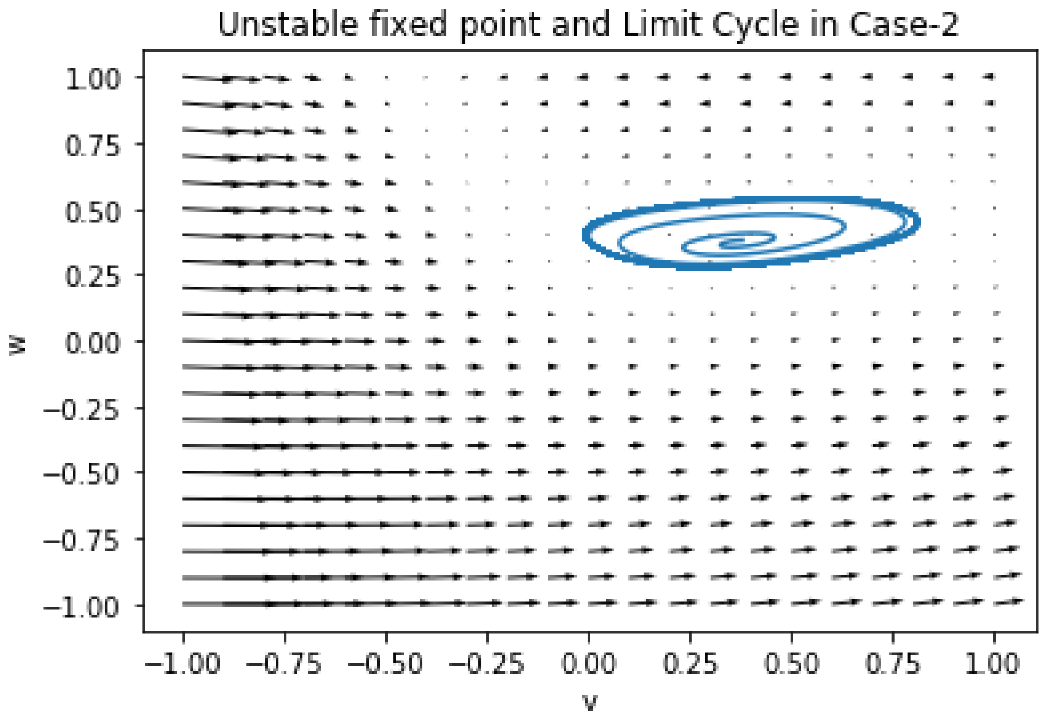
2.2 Case-2:

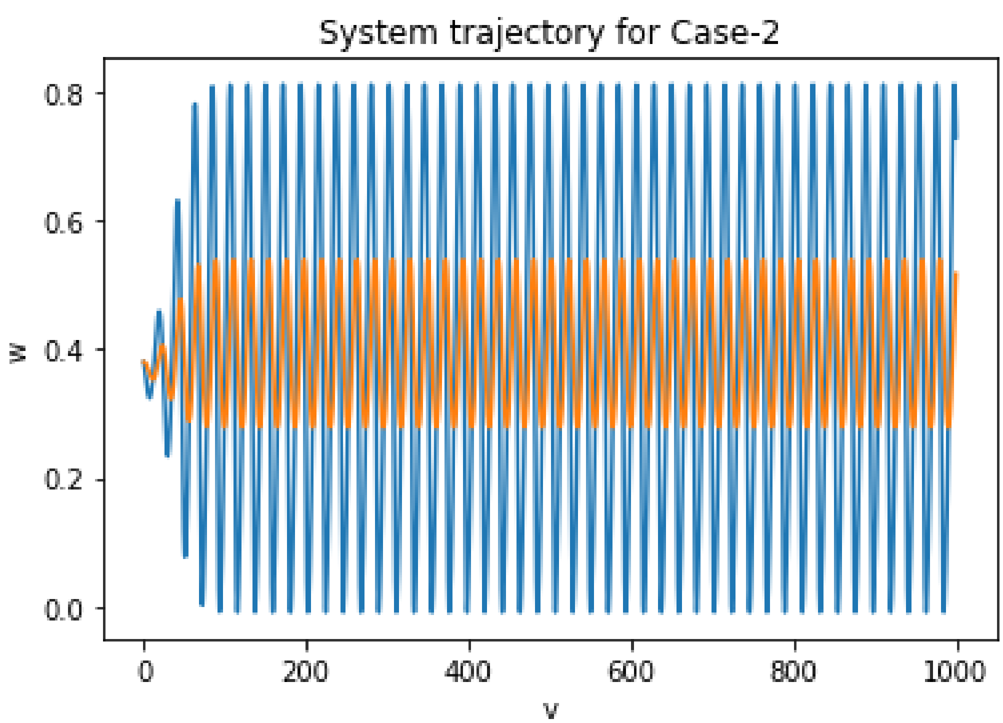
1. In this case, the fixed point is at (v,w)= (0.369, 0.369). This is analytically determined.
2. I is chosen to be 0.4

In [114]:
```
I=0.4
a=0.5
b=0.1
r=0.1
niter=10000
dt=0.1
w0=0

m,l,vd,wd=phaseplot(-1, 1.01, -1, 1.01, 0.1, I, a, b, r)
plt.title("Phase Plot for Case-2")
plt.xlabel("v")
plt.ylabel("w")
plt.quiver(m,l,vd,wd)
plt.show()

#Slightly perturbed state v0 and w0
v0=0.38
w0=0.38
```

```
m,l,vd,wd=phaseplot(-1, 1.01, -1, 1.01, 0.1, I, a, b, r)
v,w,f,t=runFN(I, v0, w0, a, b, r, niter, dt)
plt.title("Unstable fixed point and Limit Cycle in Case-2")
plt.xlabel("v")
plt.ylabel("w")
plt.plot(v,w)
plt.quiver(m,l,vd,wd)
plt.show()

v,w,f,t=runFN(I, v0, w0, a, b, r, niter, dt)
plt.title("System trajectory for Case-2")
plt.xlabel("v")
plt.ylabel("w")
plt.plot(t,v)
plt.plot(t,w)
plt.show()
```



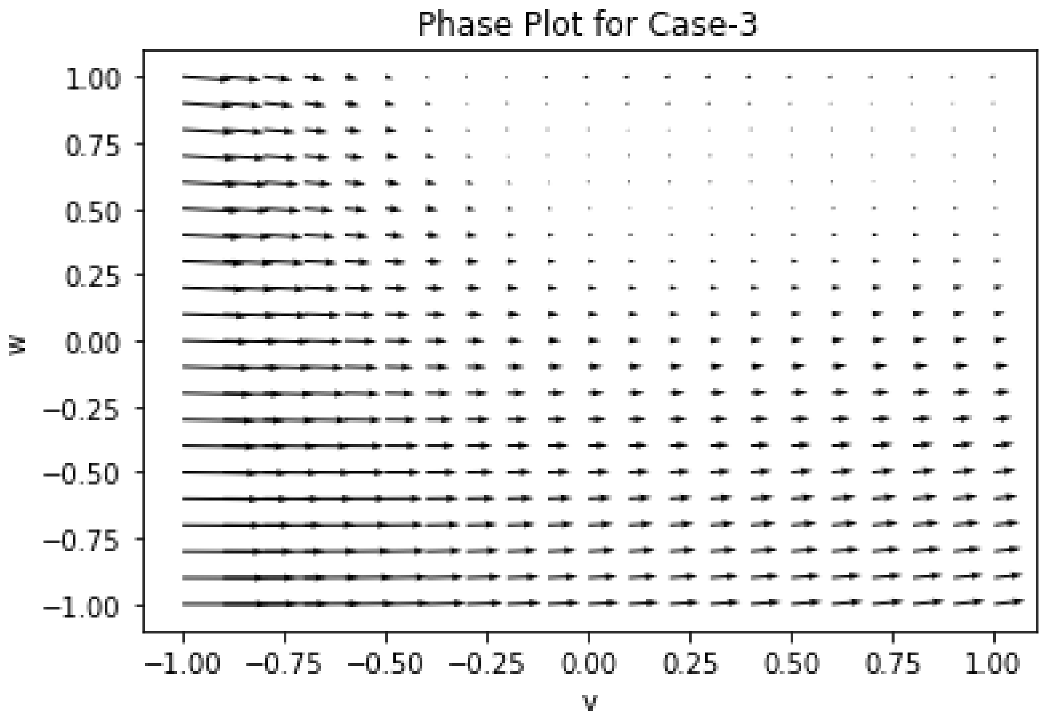

Unstable fixed point and Limit Cycle in Case-2
w
v
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00

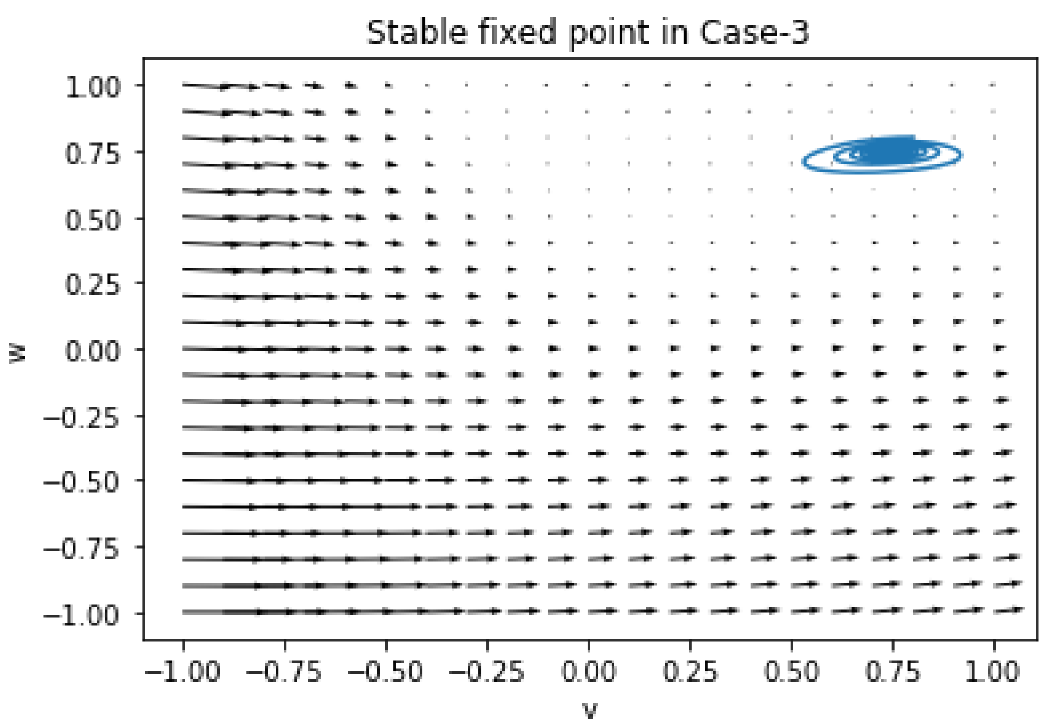

System trajectory for Case-2
w
v
0.8
0.6
0.4
0.2
0.0
0
200
400
600
800
1000

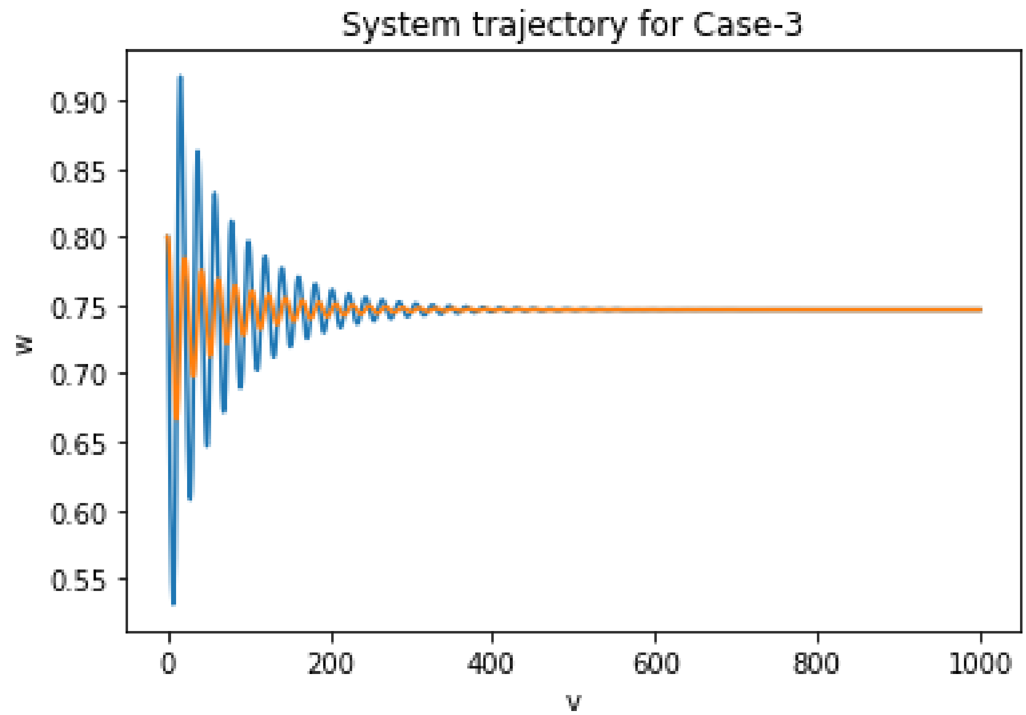
3 Case-3:

In this case, the fixed point is at (v,w)=(0.746, 0.746)

```
In [115]: I=0.7
          a=0.5
          b=0.1
          r=0.1
          niter=10000
          dt=0.1
          w0=0

          m,l,vd,wd=phaseplot(-1, 1.01, -1, 1.01, 0.1, I, a, b, r)
          plt.title("Phase Plot for Case-3")
          plt.xlabel("v")
          plt.ylabel("w")
          plt.quiver(m,l,vd,wd)
          plt.show()

          #Slightly perturbed state v0 and w0
          v0=0.80
          w0=0.80

          m,l,vd,wd=phaseplot(-1, 1.01, -1, 1.01, 0.1, I, a, b, r)
          v,w,f,t=runFN(I, v0, w0, a, b, r, niter, dt)
          plt.xlabel("v")
          plt.ylabel("w")
          plt.plot(v,w)
          plt.title("Stable fixed point in Case-3")
          plt.quiver(m,l,vd,wd)
          plt.show()

          v,w,f,t=runFN(I, v0, w0, a, b, r, niter, dt)
          plt.title("System trajectory for Case-3")
          plt.xlabel("v")
          plt.ylabel("w")
          plt.plot(t,v)
          plt.plot(t,w)
          plt.show()
```


Phase Plot for Case-3
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
w
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
v


Stable fixed point in Case-3
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
w
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00
v

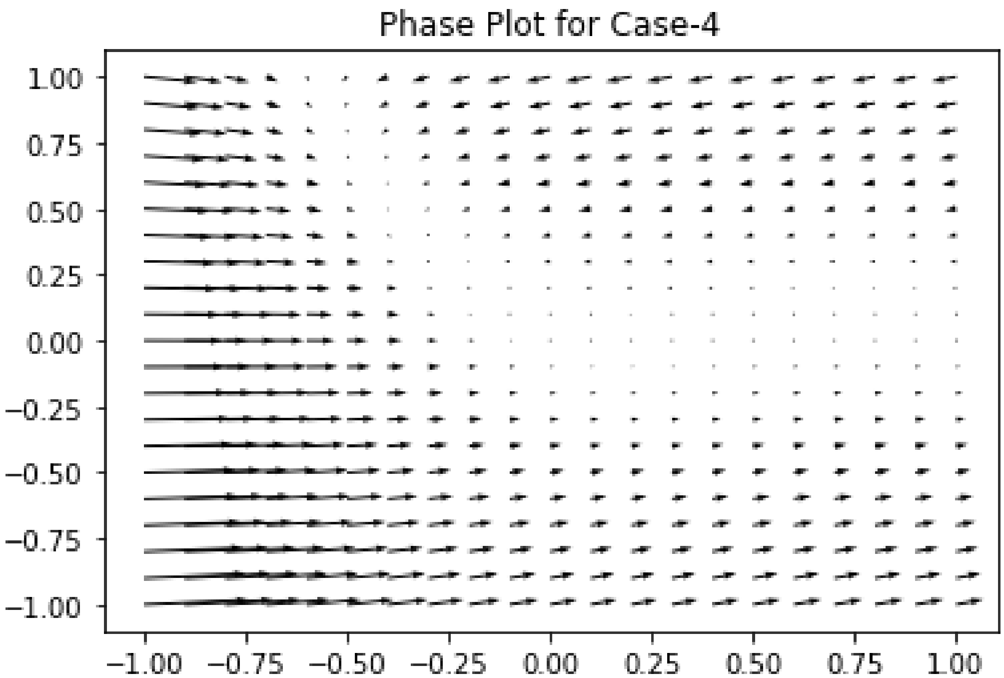


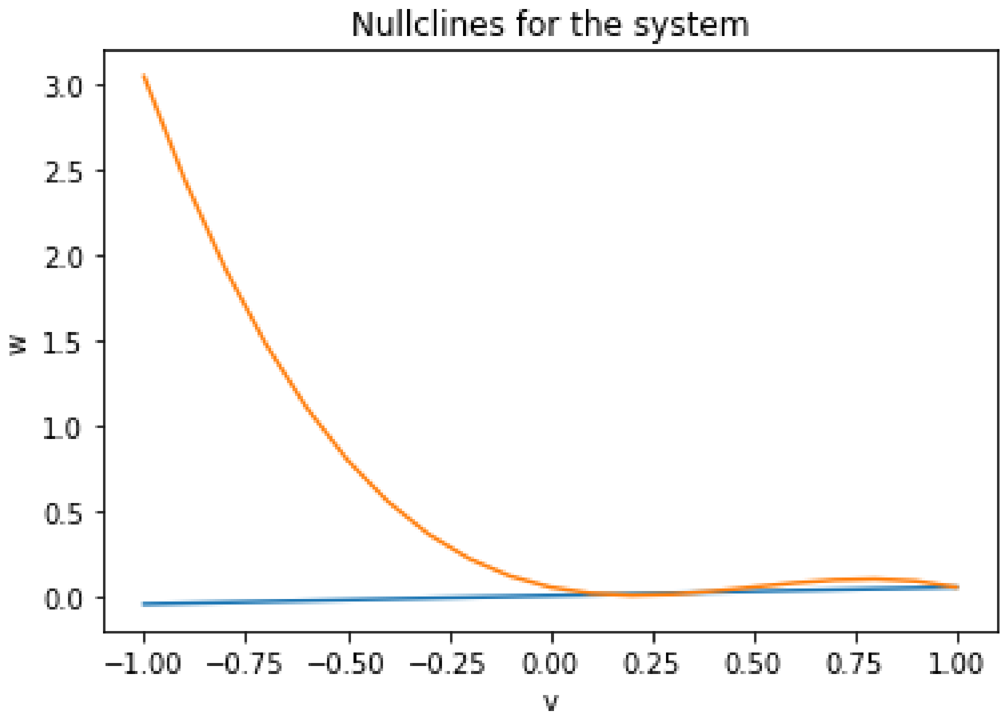
4 Case-4:

1. Analytically, the intersections are (0.138, 0.007), (0.362, 0.018) and (1.0, 0.050)
2. (a,b,r) have been chosen as (0.5, 0.01, 0.2).

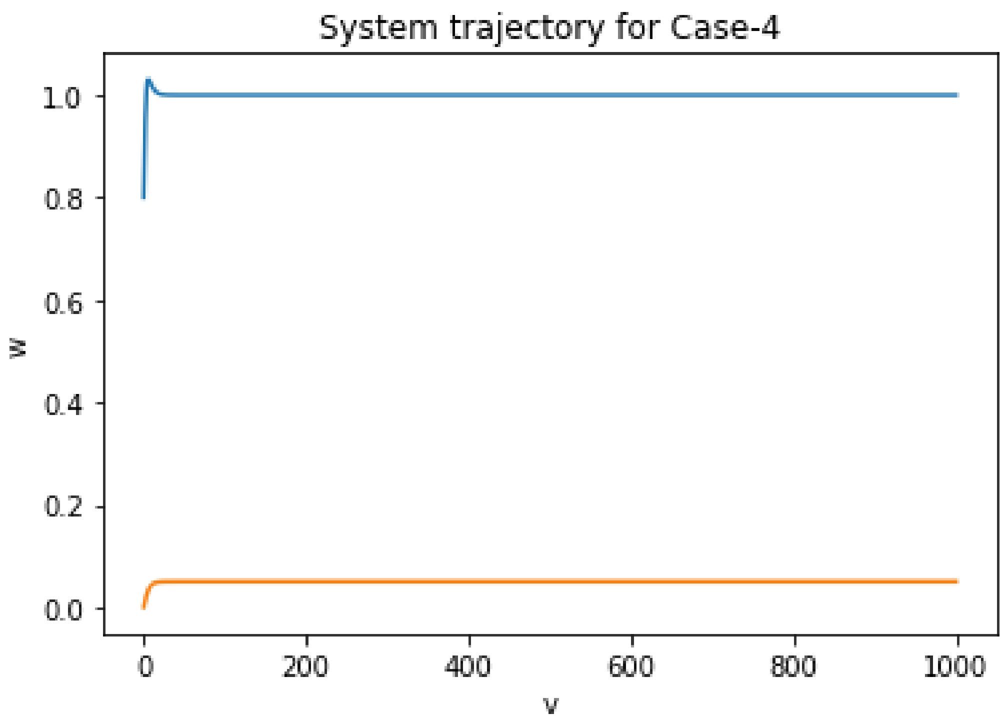
In [116]:
```
I=0.05
a=0.5
b=0.01
r=0.2
niter=10000
dt=0.1
w0=0

m,l,vd,wd=phaseplot(-1, 1.01, -1, 1.01, 0.1, I, a, b, r)
plt.title("Phase Plot for Case-4")
plt.quiver(m,l,vd,wd)
plt.show()

v=np.arange(-1, 1.01, 0.1)
vhist=[calcvnc(I, x, a, b, r) for x in v]
whist=[calcwnc(I, x, a, b, r) for x in v]
plt.title("Nullclines for the system")
plt.xlabel("v")
```

```
plt.ylabel("w")
plt.plot(v, whist, label="w-nc")
plt.plot(v, vhist, label='v-nc')
plt.show()

v,w,f,t=runFN(I, v0, w0, a, b, r, niter, dt)
plt.title("System trajectory for Case-4")
plt.xlabel("v")
plt.ylabel("w")
plt.plot(t,v)
plt.plot(t,w)
plt.show()
```





Nullclines for the system
w
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
v



System trajectory for Case-4
w
1.0
0.8
0.6
0.4
0.2
0.0
0
200
400
600
800
1000
v