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Conditional Probability

1. Sol^n - (a) Given $P(A) = 0.3$

$P(B) = 0.4$

$P(A \cap B) = 0.2$

(a) $P(\text{exactly one of } A \text{ or } B) = P(A \,\&\, \text{not } B) + P(B \,\&\, \text{not } A)$

$= P(A) + P(B) - 2P(A \cap B)$

$= 0.3 + 0.4 - 2(0.2)$

$= 0.3$

(b) at least one of the event A or B will occur.

$P(\text{at least one of } A \text{ or } B) = P(A \text{ or } B)$

$= P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.4 - 0.2$

$= 0.5$

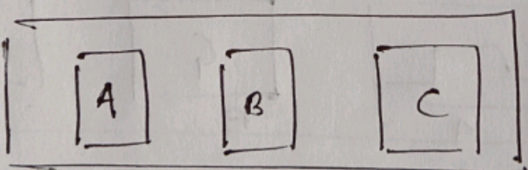
(c) $P(\text{none of } A \text{ and } B) = 1 - P(A \text{ or } B)$

$= 1 - 0.5 = 0.5$

Monty Hall Problem

2. Sol^n :-

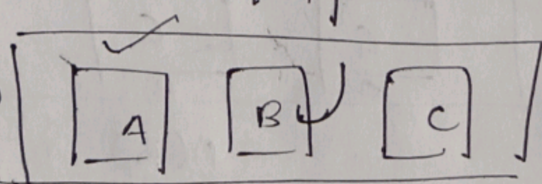
1) =)



these three are doors. & contestent chooses door A

Monty

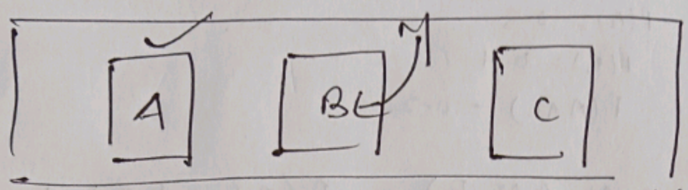
2) =)



Say Monty opened the door B

Events-(1) & -(2) happened in the past, at present we are concerned about a formulating a strategy according to which the contestent chooses to switch or move along with his former choice.

3) =)



we first find the probability that car is behind door A given Monty opened door B

let say the event in which the car is behind door A is A_{car}

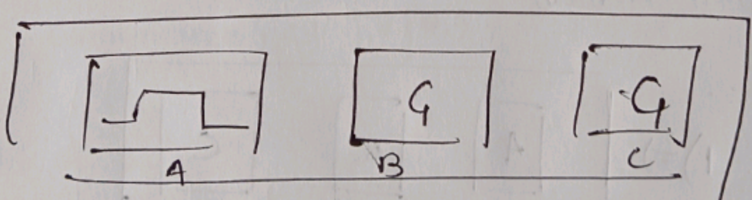
& the event in which ~~the~~ Monty opened door B is B_{open}

$$P(A_{car} \mid B_{open}) = \frac{P(A_{car})\, P(B_{open} \mid A_{car})}{P(B_{open})}$$

$$P(B_{open}) = P(A_{car}) \cdot P(B_{open} \mid A_{car}) + P(C_{car}) \cdot P(B_{open} \mid C_{car})$$

Case ①

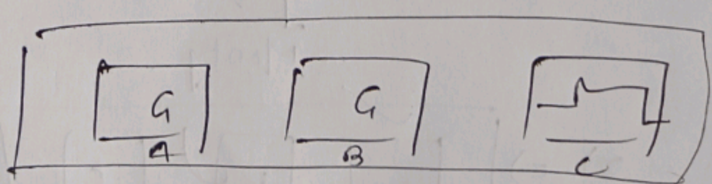
→ car is behind door A



$$P(B_{open}) = \frac{1}{2}$$

Case ②

→ Car is behind door C



$$P(B_{op}) = 1$$

$$P(B_{open}) = \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\times 1$$

$$= \frac{1}{2}$$

$$P(A_{car} \mid B_{open}) = \frac{1/3 \cdot \frac{1}{2}}{1/2} = 1/3$$

$$P(C_{car} \mid B_{open}) = 1 - 1/3 = 2/3$$

Therefore the contestent ~~stst~~ should switch the door.

Bayes Theorem :-

Sol^n - 3 :- $S = [6]$

let A be the event that we draw ~~two~~ 3 Red balls & B be the event that ~~five ba~~ All six balls are Red.

we need to find $P(B|A)$

$$P(B|A) = \frac{P(B)\cdot P(A|B)}{P(A)}$$

$$P(A) = \frac{^3C_3}{^6C_3}\times\frac{1}{4} + \frac{^4C_3}{^6C_3}\times\frac{1}{4} + \frac{^5C_3}{^6C_3}\times\frac{1}{4} + \frac{^6C_3}{^6C_3}\times\frac{1}{4}$$

$$= \frac{1}{4}\left(\frac{1}{20} + \frac{4}{20} + \frac{10}{20} + \frac{20}{20}\right)$$

$$= \frac{35}{80} = \frac{7}{16}$$

$$\boxed{P(B/A) = \frac{1/\cancel{4}\times\frac{20}{20}}{7/\cancel{16}\,4} = \frac{4}{7}}$$

Random Variables

Que - 4

Sol^n:-

- $P(X<0.5) = 0.1+0.2 = 0.3$
- $P(0.25<X<0.75) = 0.2+0.2 = 0.4$
- $P(X=0.2 \mid X<0.6) = \dfrac{P(x=0.2)}{P(x=0.2)+P(x=0.4)+P(x=0)}$

$$= \frac{0.1}{0.1+0.2+0.2} = \frac{1}{5} = 0.2$$

Que - 5 (Random Variables)

Sol^n:-

$$F(x) = \begin{cases} 0 & x<0 \\ 2/3 & 0 \le x < 1 \\ \dfrac{7-6c}{6} & 1 \le x < 2 \\ \dfrac{4c^2-9c+6}{4} & 2 \le x \le 3 \\ 1 & x>3 \end{cases}$$

c is real constant

(1) By Right hand continuity.

$$\frac{4c^2-9c+6}{4} = 1$$

$$\Rightarrow 4c^2-9c+6 = 4$$

$$\Rightarrow 4c^2-9c+2 = 0$$

$$\Rightarrow 4c^2-8c-c+2 = 0$$

$$\Rightarrow 4c(c-2)-1(c-2) = 0$$

$$(4c-1)(c-2) = 0$$

$c = 1/4$ or $c = 2$

$c = 2$ is not possible as $f(x) > 0$

(2) $P(1 < X < 2) = F(2^-) - F(1) = 0$

$$= \cancel{\frac{4(1/4)^2 - 9(1/4) + 6}{4} - \frac{7 - 6(1/4)}{6}}$$

$$= \cancel{\frac{\frac{1}{4} - \frac{9}{4} + 6}{4} - \frac{7 - \frac{6}{4}}{6}}$$

$$= \cancel{1 - \frac{22}{24} = \frac{2}{24} = \frac{1}{12}}$$

$P(2 \leq X < 3) = F(3^-) - F(2^-)$

$$= \cancel{1 - 1 = 0}$$

$$= \frac{4c^2 - 9c + 6}{4} - \frac{7 - 6c}{6} = \frac{1}{12}$$

$P(0 < x \leq 1) = F(1) - F(0)$

$$= \frac{22}{24} - 2/3$$

$$= \frac{22 - 16}{24} = \frac{6}{24} = \frac{\cancel{3}}{\cancel{12}}$$

$$= \frac{1}{6}$$

$P(X \geq 3) = F(3^-)$

$= 1$

:- Expectation Calculation

Soln - 6 :-

$p(x) = 1 \quad x \in [0,1]$

$p(x) = 0 \quad x < 0 \text{ or } x > 1$

~~E[X^~~

- $E[X] = \int_0^1 p(x) \cdot x \, dx = \frac{1}{2}$

- $Var[x] = E[x^2] - (E[x])^2$

$= \int_0^1 x^2 \cdot p(x) \, dx - 1/4$

$= 1/3 - 1/4 = \boxed{1/12}$

- $E[Y]$:

$E[x^2 + Y^2] = 1$

$\Rightarrow E[x^2] + E[Y^2] = 1$

$\Rightarrow E[Y^2] = 2/3$

Now, $Var[y] = E[y^2] - (E[Y])^2$

$\frac{5}{9} = 2/3 - (E[y])^2$

$\Rightarrow E[y] = 1/3$

- $E[X+Y] = E[X] + E[Y] = \frac{1}{2} + 1/3$

$= 5/6$