

$$a) \quad w_i = a + b G_i + \varepsilon_i$$

$$b = \frac{\sum_{i=1}^{15} (w_i - \bar{w})(G_i - \bar{G})}{\sum_{i=1}^{15} (G_i - \bar{G})^2}$$

$$a = \bar{w} - b \bar{G}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{15} \varepsilon_i^2}{\sum_{i=1}^{15} (w_i - \bar{w})^2}$$

$$\varepsilon_i = w_i - a - b G_i$$

$$s = \sqrt{\frac{1}{13} \sum_{i=1}^{15} \varepsilon_i^2}$$

Observation	W	G
1	10.3	1
2	10.4	2
3	10.5	3
4	10.2	4
5	10.0	5
6	9.95	6
7	10.14	7
8	10.06	8
9	10.25	9
10	9.99	10
11	9.92	11
12	9.96	12
13	9.84	13
14	9.87	14
15	9.85	15

$$\bar{w} = 10.08$$

$$\bar{G} = 8$$

Observation	W	G	W_DEMEANED	G_DEMEANED
1	10.3	1	0.22	-7
2	10.4	2	0.32	-6
3	10.5	3	0.42	-5
4	10.2	4	0.12	-4
5	10.0	5	-0.08	-3
6	9.95	6	-0.13	-2
7	10.14	7	0.06	-1
8	10.06	8	-0.02	0
9	10.25	9	0.17	1
10	9.99	10	-0.09	2
11	9.92	11	-0.16	3
12	9.96	12	-0.12	4
13	9.84	13	-0.24	5
14	9.87	14	-0.21	6
15	9.85	15	-0.23	7

$$b = \frac{-10.64}{280} = -0.038$$

$$a = 10.08 - (-0.038) \times 8 = 10.38$$

Observation	W	G	W_DEMEANED	G_DEMEANED	E
1	10.3	1	0.22	-7	-0.042
2	10.4	2	0.32	-6	0.096
3	10.5	3	0.42	-5	0.234
4	10.2	4	0.12	-4	-0.028
5	10.0	5	-0.08	-3	-0.190
6	9.95	6	-0.13	-2	-0.202
7	10.14	7	0.06	-1	0.026
8	10.06	8	-0.02	0	-0.016
9	10.25	9	0.17	1	0.212
10	9.99	10	-0.09	2	-0.010
11	9.92	11	-0.16	3	-0.042
12	9.96	12	-0.12	4	0.036
13	9.84	13	-0.24	5	-0.046
14	9.87	14	-0.21	6	0.022
15	9.85	15	-0.23	7	0.040

$$R^2 = 1 - \frac{0.197}{0.600} = 0.672$$

$$S = \sqrt{\frac{1}{13} \times 0.197} = 0.123$$

b) $R^2 = 0.672$

67 % variance winning times explained by trend.

$e_i \approx 0.1$ games except 3, 5, 6 and 9

$e_i \approx 0.2$ games 3, 5, 6 and 9

c) 2008 $w_{16} = a + b \cdot 16 = a + 16b$

2012 $w_{17} = a + b \cdot 17 = a + 17b = w_{16} + b$

2016 $w_{18} = a + b \cdot 18 = a + 18b = w_{16} + 2b$

	Predicted	actual
$w_{16} = 10.38 - 16 \times 0.038 = 9.77$	9.77	9.69
$w_{17} = 9.77 - 0.038 = 9.73$	9.73	9.63
$w_{18} = 9.77 - 0.076 = 9.69$	9.69	

9.81

Perhaps the faster than predicted times in 2008 and 2012 can be seen as the Usain Bolt effect, whereas Bolt was older and less fast in 2016 but still was the winner as others did not yet catch up.

(The red text part was added after the 2016 Olympic Games, where Usain Bolt won again, in 9.81 sec.)