UnisonPortOpt

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1 Unison Project

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[20]:
```
import pandas as pd
import numpy as np
import math
import matplotlib.pyplot as plt
import seaborn as sns
from PIL import Image

pd.options.display.float_format = '{:.2%}'.format
```

2 Data Preprocessing

[21]:
```
data = pd.read_excel('timeseriesUpdated.xlsx', sheet_name='Sheet1').iloc[2:]
data.columns = data.iloc[0]
data = data.iloc[1:].reset_index().drop(columns = ['index'])
data.set_index('Date', inplace=True)
```

[22]:
```
pd.options.display.float_format = '{:.2f}'.format
data.head()
```

[22]:
```
2           M1US Index MXEF Index M1USSC Index MLCUWXU Index LBUSTRUU Index  \
Date
2021-03-31    11085.83    1316.43       727.88        952.58        2311.35
2021-02-26    10687.87    1339.26       713.16         932.7        2340.58
2021-01-29    10420.14    1329.57       669.87        907.54        2374.87
2020-12-31    10520.81    1291.26       645.58        919.38        2392.02
2020-11-30    10108.22    1205.07       598.89        881.32        2388.73

2           LUCRTRUU Index BC2YTRUU Index LUATTRUU Index LGL1TRUU Index  \
Date
2021-03-31         3331.52         255.07        2450.55         4251.2
2021-02-26         3385.44         256.13        2488.92        4472.86
2021-01-29         3445.22         258.32        2534.92        4733.83
2020-12-31         3486.71         257.91         2559.4         4908.6
2020-11-30         3470.86         257.14        2565.34        4966.07
```

```
2           JPEIDIVR Index  … GBIEMCOR Index BCIT1T Index BRTUTRUU Index  \
Date                        …
2021-03-31          949.99  …         141.76       351.88          130.77
2021-02-26          959.23  …         146.25       352.82          134.69
2021-01-29          984.36  …         150.13       359.43          138.36
2020-12-31          995.16  …         151.72       358.42          140.29
2020-11-30          976.62  …         146.53        354.4           138.0

2           USBMMY3M Index MXUSOINF Index HFRIAWC Index PRIVEXD Index  \
Date
2021-03-31             NaN         892.68           NaN       1973.14
2021-02-26             NaN         820.13           NaN       1865.77
2021-01-29             NaN         851.75           NaN       1809.54
2020-12-31             NaN         867.28       1519.21       1824.41
2020-11-30            0.09         867.34       1469.65       1738.74

2           M1USIRE Index CSUSHPINSA  NAREIT
Date
2021-03-31        1463.44        NaN 8962.61
2021-02-26        1400.81     238.82 8500.61
2021-01-29        1350.31     236.33 8248.44
2020-12-31        1348.44     236.31 8261.85
2020-11-30        1305.62     234.45 8040.25

[5 rows x 21 columns]
```

[23]:
```
pd.options.display.float_format = '{:.2%}'.format
```

[24]:
```
isNACount = (data.isna().sum())
isNACount
```

[24]:
```
2
M1US Index          0
MXEF Index          0
M1USSC Index       80
MLCUWXU Index       0
LBUSTRUU Index      1
LUCRTRUU Index      2
BC2YTRUU Index     74
LUATTRUU Index      1
LGL1TRUU Index      3
JPEIDIVR Index      7
LF98TRUU Index      1
GBIEMCOR Index    163
BCIT1T Index       33
BRTUTRUU Index    163
```

```
USBMMY3M Index    283
MXUSOINF Index    206
HFRIAWC Index     211
PRIVEXD Index     115
M1USIRE Index     246
CSUSHPINSA          1
NAREIT              0
dtype: int64
```

[25]:
```
#keep the indices which have atleast 90% data!
data.dropna(thresh=len(data)*0.9, axis=1, inplace=True)
data = data.astype('float')
data.isna().sum()
print("Out of", len(isNACount), "chosen indices, only", len(data.columns),␣
 ↪"remaining ",
      "after filtering out the series which have a lot of missing data.")
```

```
Out of 21 chosen indices, only 11 remaining  after filtering out the series
which have a lot of missing data.
```

[26]:
```
#interpolate the missing values, sort using index, take the log!
data = data.interpolate()
data = data.sort_index()
data = np.log(data)
```

[27]:
```
def calcReturns(df, period):
    dfRet = df.diff(period)
    return dfRet.dropna()

def logReturns(df, period):
    logRet = df.diff(period)
    return logRet.dropna()
```

[28]:
```
ret1MLog = logReturns(data, 1).dropna()
ret3MLog = logReturns(data, 3).dropna()
ret1YLog = logReturns(data, 12).dropna()
ret3YLog = logReturns(data, 36).dropna()

annualisedVol1MLog = ret1MLog.std() * np.sqrt(12)
annualisedVol3MLog = ret3MLog.std() * np.sqrt(4)
annualisedVol1YLog = ret1YLog.std() * np.sqrt(1)
annualisedVol3YLog = ret3YLog.std() * np.sqrt(1/3)

ret1MAnnulaisedLog = ret1MLog*12
ret3MAnnulaisedLog = ret3MLog*4
ret1YAnnulaisedLog = ret1YLog
ret3YAnnulaisedLog = ret3YLog*(1/3)
```

3 Average Log Annualized Return and Log Annualized Vol

[29]:
```
AnnualisedRet = pd.DataFrame()
AnnualisedRet['Ret1MAnnual'] = np.mean(ret1MAnnulaisedLog)
AnnualisedRet['Ret3MAnnual'] = np.mean(ret3MAnnulaisedLog)
AnnualisedRet['Ret1YAnnual'] = np.mean(ret1YAnnulaisedLog)
AnnualisedRet['Ret3YAnnual'] = np.mean(ret3YAnnulaisedLog)
AnnualisedRet.index = list(annualisedVol1MLog.index)
#AnnualisedRet.to_latex()

AnnualisedRet
```

[29]:

	Ret1MAnnual	Ret3MAnnual	Ret1YAnnual	Ret3YAnnual
M1US Index	9.44%	9.48%	9.01%	8.23%
MXEF Index	3.73%	3.70%	3.06%	3.28%
MLCUWXU Index	2.57%	2.50%	2.09%	2.11%
LBUSTRUU Index	5.28%	5.34%	5.42%	5.14%
LUCRTRUU Index	6.07%	6.14%	6.16%	5.84%
LUATTRUU Index	4.92%	4.99%	5.14%	4.83%
LGL1TRUU Index	7.17%	7.39%	7.76%	7.15%
JPEIDIVR Index	9.13%	9.40%	9.45%	9.10%
LF98TRUU Index	7.16%	7.18%	7.01%	6.86%
CSUSHPINSA	4.10%	4.09%	4.04%	4.04%
NAREIT	9.10%	9.14%	9.17%	9.31%

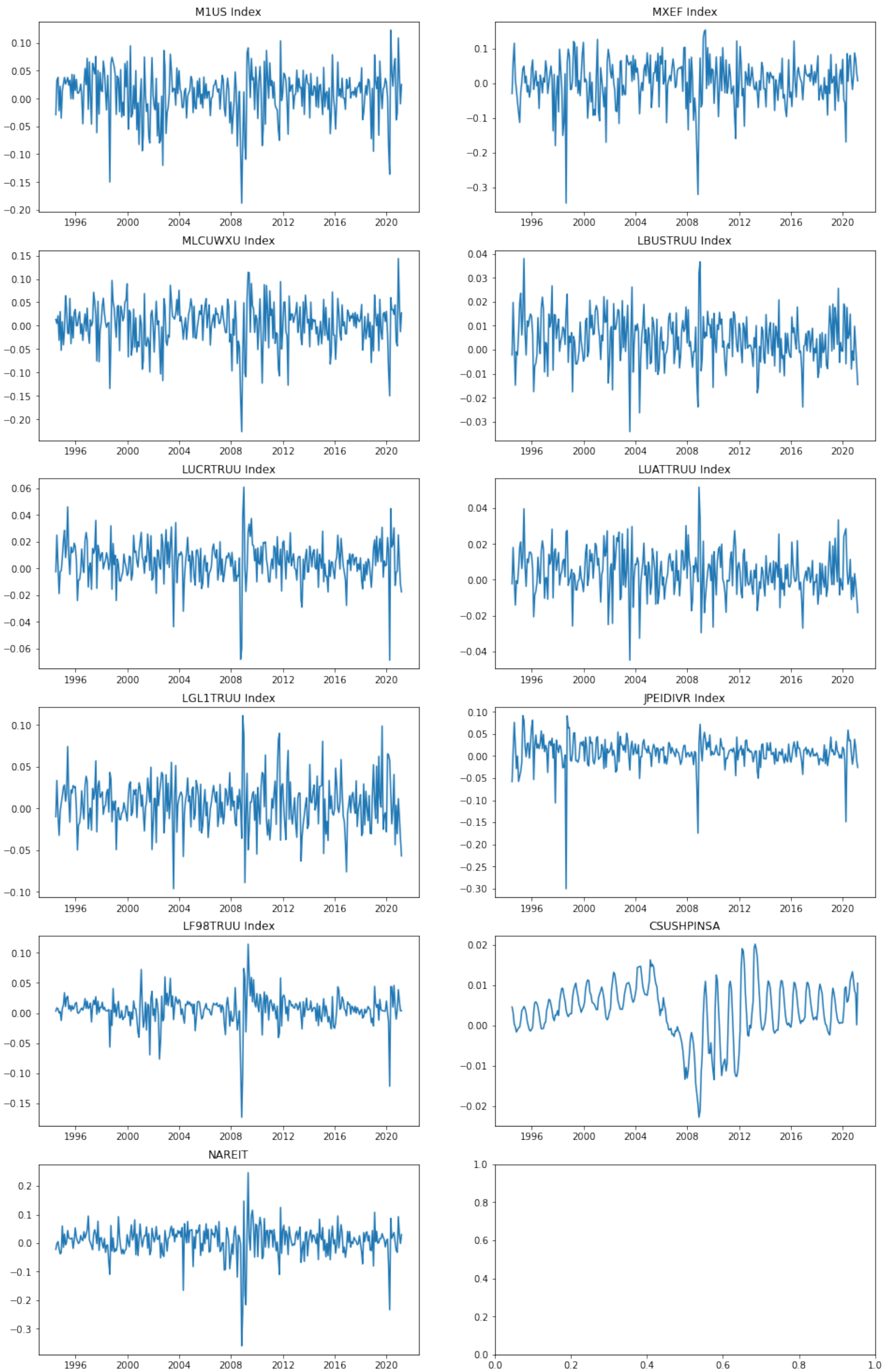
[30]:
```
AnnualisedVol = pd.DataFrame()
AnnualisedVol['Vol1MAnnual'] = annualisedVol1MLog
AnnualisedVol['Vol3MAnnual'] = annualisedVol3MLog
AnnualisedVol['Vol1YAnnual'] = annualisedVol1YLog
AnnualisedVol['Vol3YAnnual'] = annualisedVol3YLog
AnnualisedVol.index = list(annualisedVol3YLog.index)
#AnnualisedVol.to_latex()

AnnualisedVol
```

[30]:

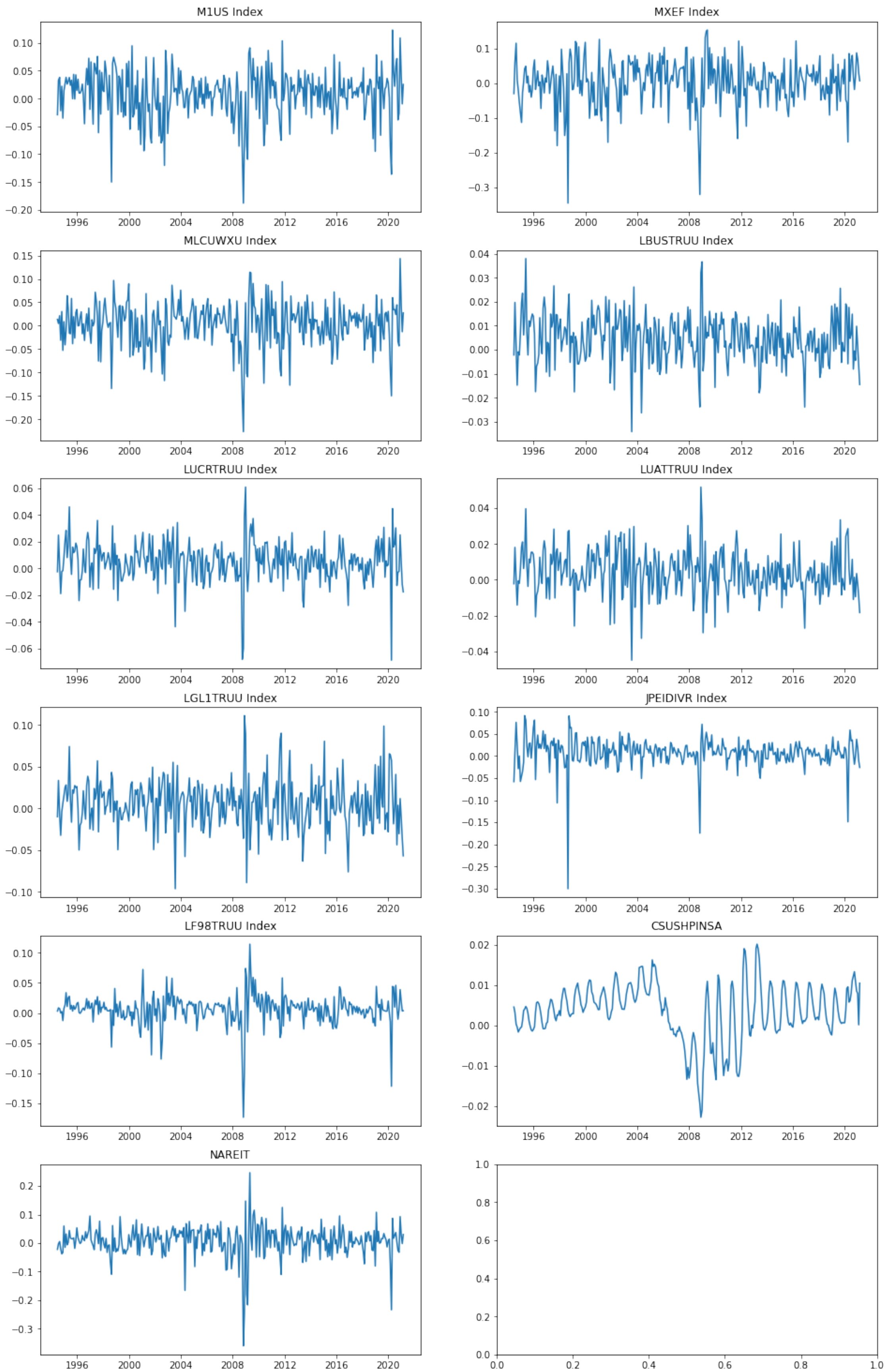
	Vol1MAnnual	Vol3MAnnual	Vol1YAnnual	Vol3YAnnual
M1US Index	15.21%	15.52%	17.05%	18.78%
MXEF Index	22.84%	25.59%	26.00%	20.99%
MLCUWXU Index	16.44%	17.59%	18.70%	16.53%
LBUSTRUU Index	3.48%	3.58%	3.76%	3.80%
LUCRTRUU Index	5.19%	5.30%	5.26%	4.36%
LUATTRUU Index	4.32%	4.44%	4.29%	4.25%
LGL1TRUU Index	10.13%	10.04%	8.80%	5.49%
JPEIDIVR Index	11.51%	11.54%	10.28%	8.44%
LF98TRUU Index	8.70%	9.96%	10.18%	8.40%
CSUSHPINSA	2.42%	3.95%	5.79%	8.88%
NAREIT	18.96%	18.95%	19.93%	18.07%

4 Log return plots

[31]:
```
fig, axes = plt.subplots(6, 2, figsize=(16,2))
axs = axes.ravel()
fig.subplots_adjust(top=10)
for i in range(0,len(ret1MLog.columns)):
    axs[i].plot(ret1MLog.iloc[:,i])
    axs[i].set_title(ret1MLog.columns[i])
```

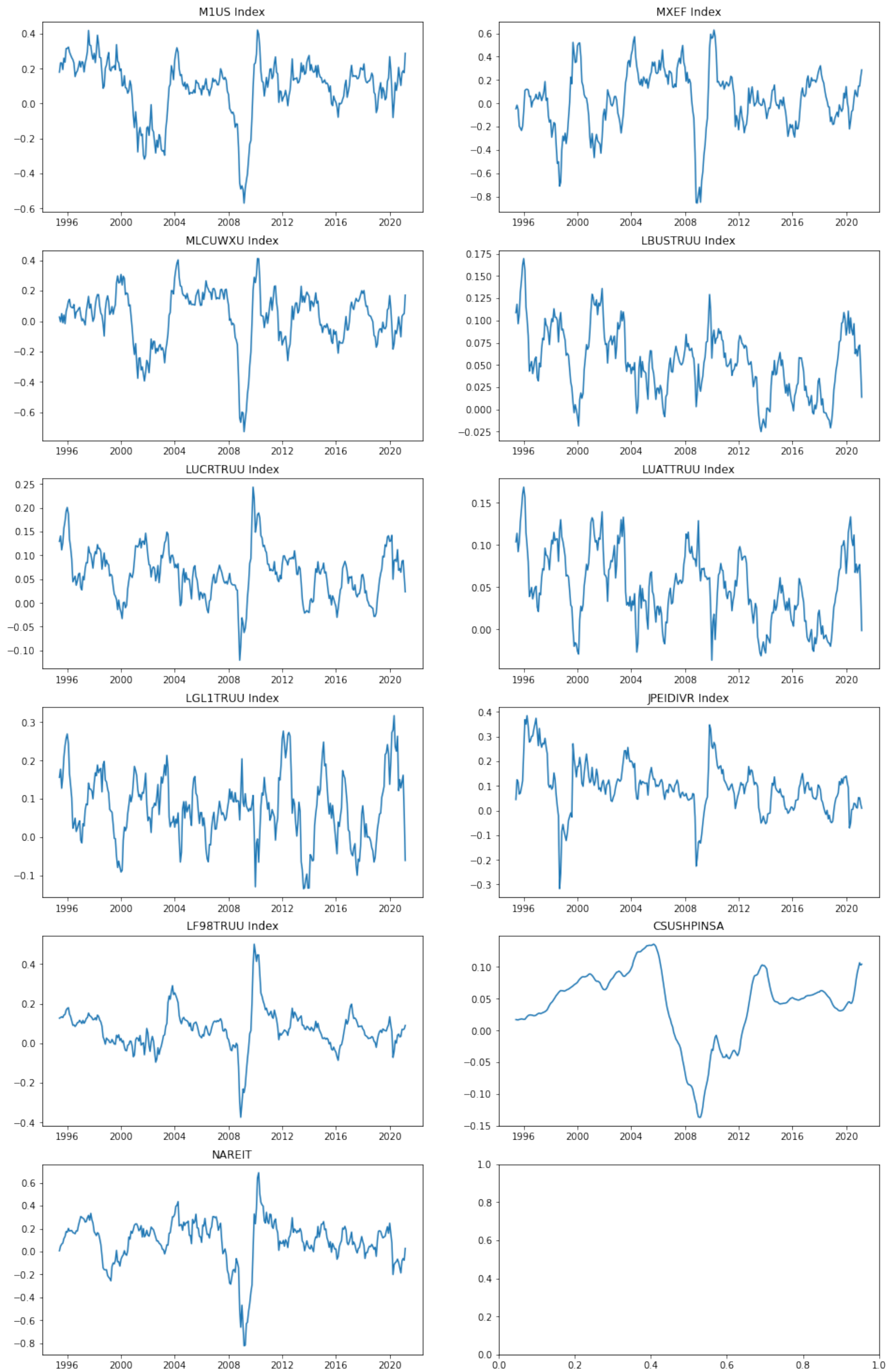

M1US Index
MXEF Index
MLCUWXU Index
LBUSTRUU Index
LUCRTRUU Index
LUATTRUU Index
LGL1TRUU Index
JPEIDIVR Index
LF98TRUU Index
CSUSHPINSA
NAREIT

[32]:
```
fig, axes = plt.subplots(6, 2, figsize=(16,2))
axs = axes.ravel()
fig.subplots_adjust(top=10)
for i in range(0,len(ret3MLog.columns)):
    axs[i].plot(ret1MLog.iloc[:,i])
    axs[i].set_title(ret1MLog.columns[i])
```


M1US Index
0.10 0.05 0.00 −0.05 −0.10 −0.15 −0.20
MXEF Index
0.1 0.0 −0.1 −0.2 −0.3
MLCUWXU Index
0.15 0.10 0.05 0.00 −0.05 −0.10 −0.15 −0.20
LBUSTRUU Index
0.04 0.03 0.02 0.01 0.00 −0.01 −0.02 −0.03
LUCRTRUU Index
0.06 0.04 0.02 0.00 −0.02 −0.04 −0.06
LUATTRUU Index
0.04 0.02 0.00 −0.02 −0.04
LGL1TRUU Index
0.10 0.05 0.00 −0.05 −0.10
JPEIDIVR Index
0.10 0.05 0.00 −0.05 −0.10 −0.15 −0.20 −0.25 −0.30
LF98TRUU Index
0.10 0.05 0.00 −0.05 −0.10 −0.15
CSUSHPINSA
0.02 0.01 0.00 −0.01 −0.02
NAREIT
0.2 0.1 0.0 −0.1 −0.2 −0.3
1996 2000 2004 2008 2012 2016 2020
1.0 0.8 0.6 0.4 0.2 0.0
0.0 0.2 0.4 0.6 0.8 1.0

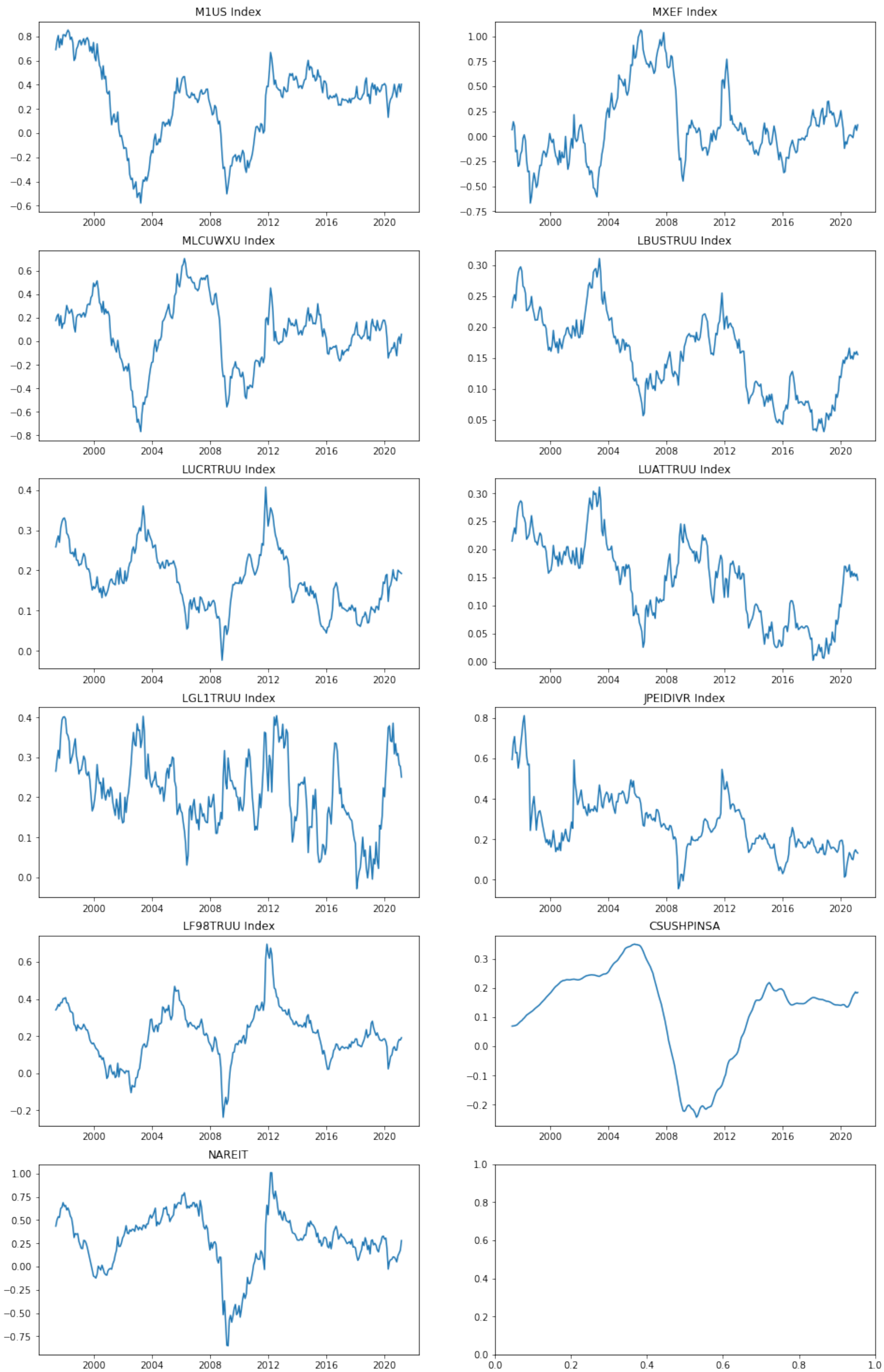
[33]:

```
fig, axes = plt.subplots(6, 2, figsize=(16,2))
axs = axes.ravel()
fig.subplots_adjust(top=10)
for i in range(0,len(ret1YLog.columns)):
    axs[i].plot(ret1YLog.iloc[:,i])
    axs[i].set_title(ret1YLog.columns[i])
```


M1US Index
MXEF Index
MLCUWXU Index
LBUSTRUU Index
LUCRTRUU Index
LUATTRUU Index
LGL1TRUU Index
JPEIDIVR Index
LF98TRUU Index
CSUSHPINSA
NAREIT

[34]:

```
fig, axes = plt.subplots(6, 2, figsize=(16,2))
axs = axes.ravel()
fig.subplots_adjust(top=10)
for i in range(0,len(ret3YLog.columns)):
    axs[i].plot(ret3YLog.iloc[:,i])
    axs[i].set_title(ret3YLog.columns[i])
```


M1US Index
MXEF Index
MLCUWXU Index
LBUSTRUU Index
LUCRTRUU Index
LUATTRUU Index
LGL1TRUU Index
JPEIDIVR Index
LF98TRUU Index
CSUSHPINSA
NAREIT

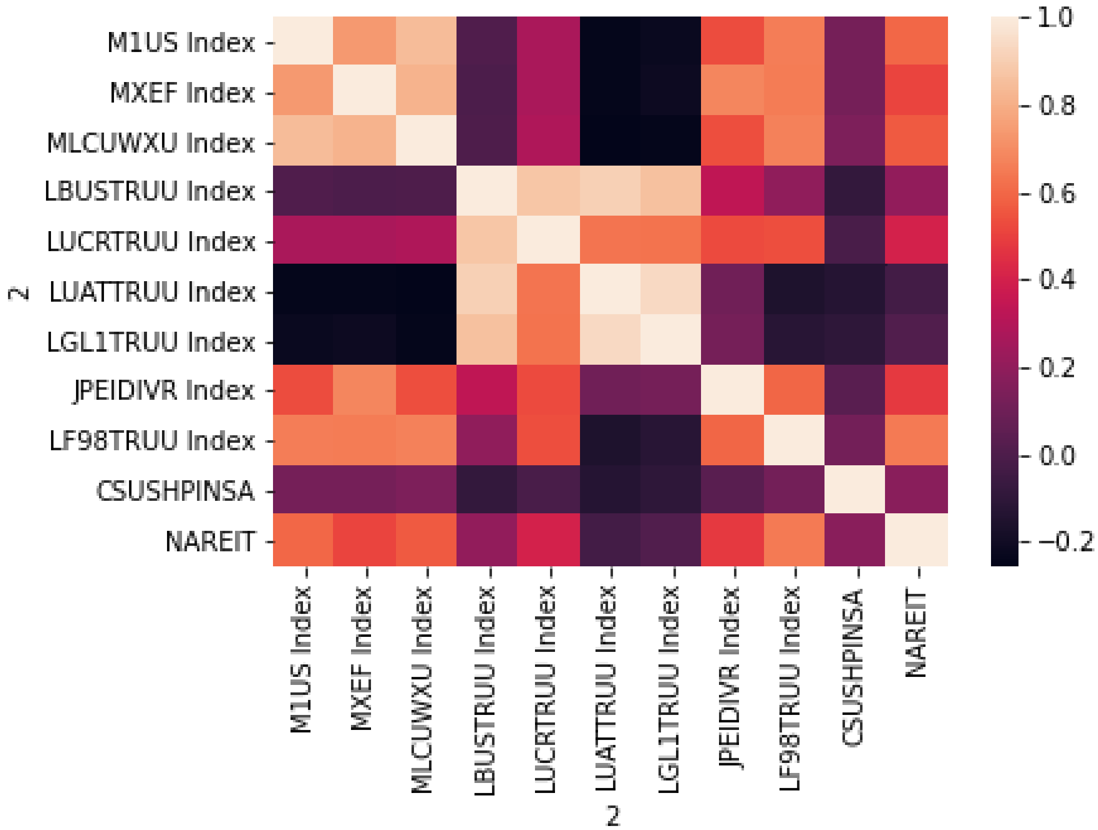
5 Correlation

[35]:
```
ret1MLog.corr()
```

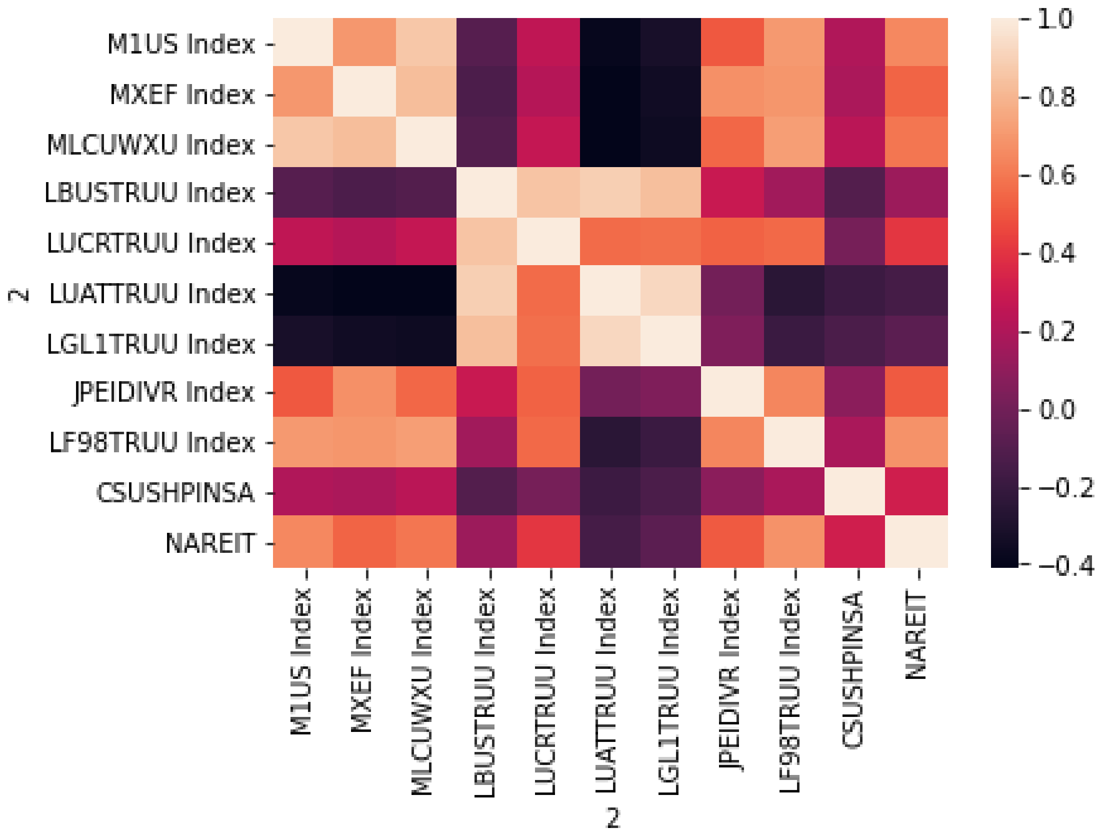
[35]:
```
2                M1US Index  MXEF Index  MLCUWXU Index  LBUSTRUU Index  \
2
M1US Index          100.00%      73.58%         84.48%           0.89%
MXEF Index           73.58%     100.00%         81.50%          -0.06%
MLCUWXU Index        84.48%      81.50%        100.00%           0.44%
LBUSTRUU Index        0.89%      -0.06%          0.44%         100.00%
LUCRTRUU Index       27.47%      27.21%         28.92%          87.50%
LUATTRUU Index      -24.10%     -24.23%        -25.27%          91.04%
LGL1TRUU Index      -21.87%     -21.22%        -24.02%          86.10%
JPEIDIVR Index       53.33%      68.58%         53.90%          33.59%
LF98TRUU Index       65.78%      65.71%         66.98%          19.99%
CSUSHPINSA           12.09%      12.29%         14.49%          -8.27%
NAREIT               60.16%      51.43%         56.73%          20.69%

2                LUCRTRUU Index  LUATTRUU Index  LGL1TRUU Index  \
2
M1US Index               27.47%         -24.10%         -21.87%
MXEF Index               27.21%         -24.23%         -21.22%
MLCUWXU Index            28.92%         -25.27%         -24.02%
LBUSTRUU Index           87.50%          91.04%          86.10%
LUCRTRUU Index          100.00%          63.47%          62.99%
LUATTRUU Index           63.47%         100.00%          93.69%
LGL1TRUU Index           62.99%          93.69%         100.00%
JPEIDIVR Index           52.54%          10.77%          12.03%
LF98TRUU Index           53.80%         -15.20%         -11.69%
CSUSHPINSA               -1.00%         -13.01%          -9.64%
NAREIT                   40.70%          -3.12%           1.26%

2                JPEIDIVR Index  LF98TRUU Index  CSUSHPINSA  NAREIT
2
M1US Index               53.33%          65.78%      12.09%  60.16%
MXEF Index               68.58%          65.71%      12.29%  51.43%
MLCUWXU Index            53.90%          66.98%      14.49%  56.73%
LBUSTRUU Index           33.59%          19.99%      -8.27%  20.69%
LUCRTRUU Index           52.54%          53.80%      -1.00%  40.70%
LUATTRUU Index           10.77%         -15.20%     -13.01%  -3.12%
LGL1TRUU Index           12.03%         -11.69%      -9.64%   1.26%
JPEIDIVR Index          100.00%          59.43%       3.72%  48.43%
LF98TRUU Index           59.43%         100.00%      11.76%  64.94%
CSUSHPINSA                3.72%          11.76%     100.00%  17.97%
```

```
NAREIT                    48.43%           64.94%        17.97% 100.00%
```

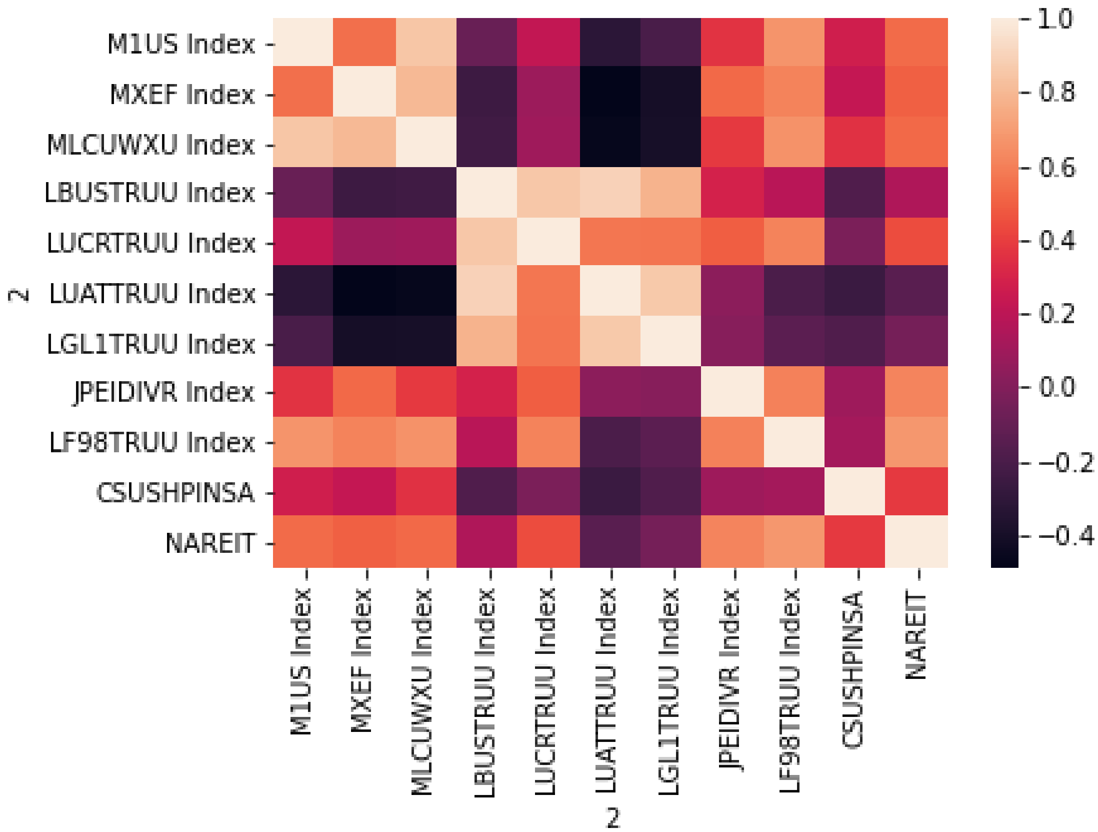
[36]:
```
heatMap1 = sns.heatmap(ret1MLog.corr())
#plt.savefig("heatmap1M.png")
```



[37]:
```
heatMap1 = sns.heatmap(ret3MLog.corr())
#plt.savefig("heatmap3M.png")
```

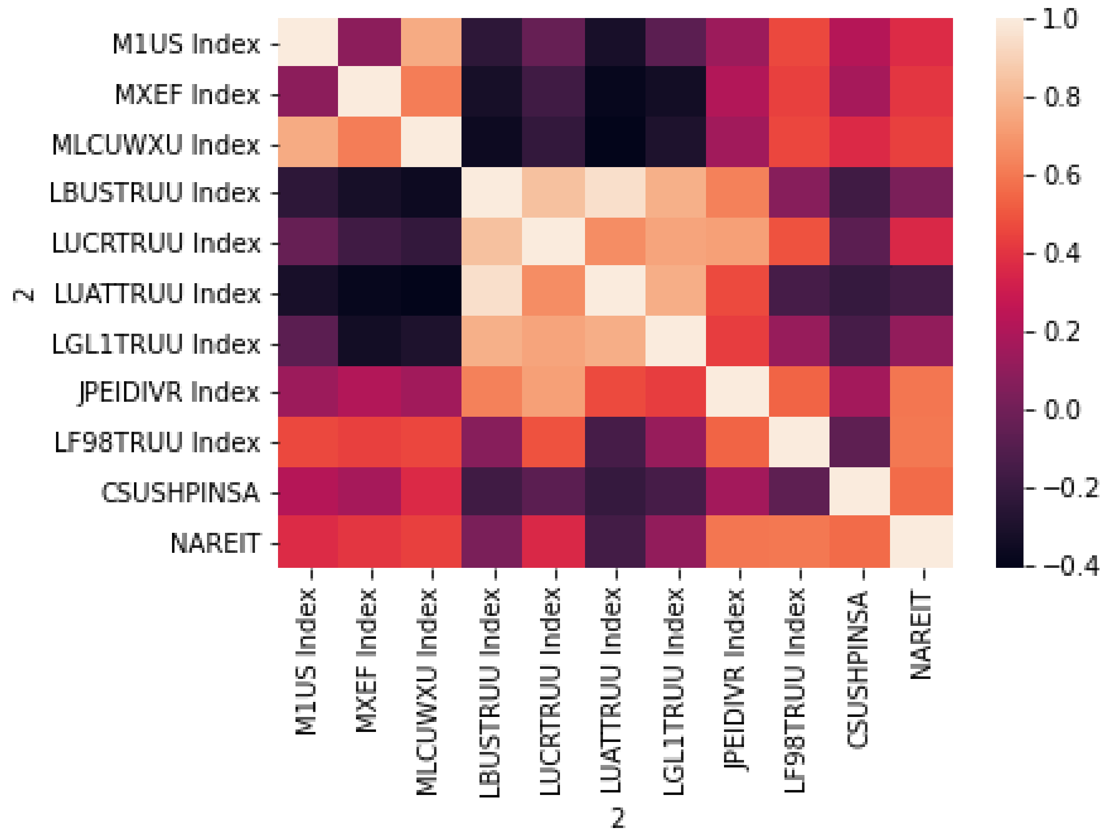



```
[38]: heatMap1 = sns.heatmap(ret1YLog.corr())
      plt.savefig("heatmap1Y.png")
```



[39]:

```
heatMap1 = sns.heatmap(ret3YLog.corr())
plt.savefig("heatmap3Y.png")
```



Let's use the 1Y return, volatility and correlation!

[40]:
```
def portfolio_annualised_performance(weights, mean_returns, cov_matrix):
    returns = np.sum(mean_returns*weights) *252
    std = np.sqrt(np.dot(weights.T, np.dot(cov_matrix, weights))) * np.sqrt(252)
    return std, returns

def random_portfolios(num_portfolios, mean_returns, cov_matrix, risk_free_rate):
    results = np.zeros((3,num_portfolios))
    weights_record = []
    for i in range(num_portfolios):
        weights = np.random.random(4)
        weights /= np.sum(weights)
        weights_record.append(weights)
        portfolio_std_dev, portfolio_return =␣
↪portfolio_annualised_performance(weights, mean_returns, cov_matrix)
        results[0,i] = portfolio_std_dev
        results[1,i] = portfolio_return
        results[2,i] = (portfolio_return - risk_free_rate) / portfolio_std_dev
    return results, weights_record
```

```
def display_simulated_ef_with_random(mean_returns, cov_matrix, num_portfolios,␣
↪risk_free_rate):
    results, weights = random_portfolios(num_portfolios,mean_returns,␣
↪cov_matrix, risk_free_rate)

    max_sharpe_idx = np.argmax(results[2])
    sdp, rp = results[0,max_sharpe_idx], results[1,max_sharpe_idx]
    max_sharpe_allocation = pd.DataFrame(weights[max_sharpe_idx],index=table.
↪columns,columns=['allocation'])
    max_sharpe_allocation.allocation = [round(i*100, 2)for i in␣
↪max_sharpe_allocation.allocation]
    max_sharpe_allocation = max_sharpe_allocation.T

    min_vol_idx = np.argmin(results[0])
    sdp_min, rp_min = results[0,min_vol_idx], results[1,min_vol_idx]
    min_vol_allocation = pd.DataFrame(weights[min_vol_idx],index=table.
↪columns,columns=['allocation'])
    min_vol_allocation.allocation = [round(i*100,2)for i in min_vol_allocation.
↪allocation]
    min_vol_allocation = min_vol_allocation.T

    print("-"*80)
    print("Maximum Sharpe Ratio Portfolio Allocation\n")
    print("Annualised Return:", round(rp, 2))
    print("Annualised Volatility:", round(sdp, 2))
    print("\n")
    print(max_sharpe_allocation)
    print("-"*80)
    print("Minimum Volatility Portfolio Allocation\n")
    print("Annualised Return:", round(rp_min, 2))
    print("Annualised Volatility:", round(sdp_min, 2))
    print("\n")
    print(min_vol_allocation)

    plt.figure(figsize=(10, 7))
    plt.scatter(results[0,:],results[1,:],c=results[2,:],cmap='YlGnBu',␣
↪marker='o', s=10, alpha=0.3)
    plt.colorbar()
    plt.scatter(sdp,rp,marker='*',color='r',s=500, label='Maximum Sharpe ratio')
    plt.scatter(sdp_min,rp_min,marker='*',color='g',s=500, label='Minimum␣
↪volatility')
    plt.title('Simulated Portfolio Optimization based on Efficient Frontier')
    plt.xlabel('annualised volatility')
    plt.ylabel('annualised returns')
    plt.legend(labelspacing=0.8)
```

```
#display_simulated_ef_with_random(list(AnnualisedRet['Ret1YAnnual']),
#                                   np.cov(ret1YLog.T),
#                                   22,
#                                  0.002)
```

Dropping Case Shiller from the dataset below

[41]:
```
stocks_to_be_dropped = ['LUCRTRUU Index','LGL1TRUU Index', 'MLCUWXU Index']
ret1YLog_NEW = ret1YLog.drop(stocks_to_be_dropped, axis=1)
ret1MAnnulaisedLog_NEW = ret1MAnnulaisedLog.drop(stocks_to_be_dropped, axis=1)
ret1MLog = ret1MLog.drop(stocks_to_be_dropped, axis=1)
annualRet_NEW = pd.DataFrame(AnnualisedRet['Ret1YAnnual']).
 ↪drop(stocks_to_be_dropped)
```

[42]:
```
ret1MLog_wo_cs = ret1MLog.drop('CSUSHPINSA', axis=1)
```

[43]:
```
#beta = 2.1
#alpha = 0.01
#ret1MAnnulaisedLog_NEW['Unison'] = beta * (ret1MLog['CSUSHPINSA'] + alpha)
```

[44]:
```
ind_er = (1 + ret1MLog_wo_cs.mean()) ** 12 - 1
cov_matrix = ret1MLog_wo_cs.cov()
```

5.1 Unconstrained Optimization

[62]:
```
def unconstrainedPO(mu, sigma, gamma = 1):
    import numpy.linalg as LA
    a = mu.shape[1]
    sigmaInv = LA.inv(sigma)
    ones = np.ones(len(sigmaInv)).reshape((a,1))
    term2 = ((mu@sigmaInv@ones)/(ones.T@sigmaInv@ones))
    term2 = term2[0,0]
    term3 = mu.T - term2*ones
    term3 = sigmaInv@term3
    term4 = sigmaInv@ones/(ones.T@sigmaInv@ones)
    h = term3 + term4
    return h

ret1YLog['Unison'] = 2.1*(ret1YLog['CSUSHPINSA'] + 0.1)
cols = ['M1US Index', 'MXEF Index', 'LBUSTRUU Index', 'LUATTRUU Index',␣
 ↪'JPEIDIVR Index', 'LF98TRUU Index', 'NAREIT','Unison']
ret1YLogFinal = ret1YLog[cols]

cov = np.matrix(ret1YLogFinal.cov())
gamma = 1
mu = np.matrix(ret1YLogFinal.mean())

h = unconstrainedPO(mu,cov)
```

```
cols = cols
df = pd.DataFrame({'Index Name': cols, 'Weights': weights[0]})
df
```

[62]:

```
      Index Name  Weights
0      M1US Index   12.50%
1      MXEF Index   12.50%
2  LBUSTRUU Index   12.50%
3  LUATTRUU Index   12.50%
4  JPEIDIVR Index   12.50%
5  LF98TRUU Index   12.50%
6          NAREIT   12.50%
7          Unison   12.50%
```

6 MonteCarlo Implementation

[45]:

```
def montecarlo():
    p_ret = [] # Define an empty array for portfolio returns
    p_vol = [] # Define an empty array for portfolio volatility
    p_weights = [] # Define an empty array for asset weights

    num_assets = len(ret1MLog_wo_cs.columns)
    num_portfolios = 50000

    for portfolio in range(num_portfolios):
        weights = np.random.random(num_assets)
        weights = weights/np.sum(weights)
        p_weights.append(weights)
        returns = np.dot(weights, ind_er) # Returns are the product of␣
 ↪individual expected returns of asset and its
                                          # weights
        p_ret.append(returns)
        var = cov_matrix.mul(weights, axis=0).mul(weights, axis=1).sum().sum()#␣
 ↪Portfolio Variance
        sd = np.sqrt(var) # Daily standard deviation
        ann_sd = sd*np.sqrt(12) # Annual standard deviation = volatility
        p_vol.append(ann_sd)

    data = {'Returns':p_ret, 'Volatility':p_vol}

    for counter, symbol in enumerate(ret1MLog_wo_cs.columns.tolist()):
        #print(counter, symbol)
        data[symbol+' weight'] = [w[counter] for w in p_weights]

    portfolios  = pd.DataFrame(data)
```

```
    display(portfolios.head()) # Dataframe of the 10000 portfolios created

    #Plot efficient frontier
    #print("Efficient Frontier::")
    #portfolios.plot.scatter(x='Volatility', y='Returns', marker='o', s=10,␣
↪alpha=0.3, grid=True, figsize=[10,10])

    min_vol_port = portfolios.iloc[portfolios['Volatility'].idxmin()]
    # idxmin() gives us the minimum value in the column specified.              ␣
↪
    print("Min Vol Portfolio::")
    display(min_vol_port)

    rf = 0.02 # risk factor
    optimal_risky_port = portfolios.iloc[((portfolios['Returns']-rf)/
↪portfolios['Volatility']).idxmax()]
    display(optimal_risky_port)
    print("Max Sharpe Ratio:", max((portfolios['Returns']-rf)/
↪portfolios['Volatility']))

    # Plotting min vol portfolio and optimal portfolio
    print("Efficient Frontier and Min Variance Portfolio and Max Sharpe Ratio␣
↪Portfolio::")
    fig, ax = plt.subplots(figsize=(10, 10))
    ax.set_ylim(ymin=0, ymax=0.1)
    ax.set_xlim(xmin=0, xmax=0.2)
    plt.scatter(portfolios['Volatility'], portfolios['Returns'],marker='o',␣
↪s=10, alpha=0.3)
    plt.scatter(min_vol_port[1], min_vol_port[0], color='r', marker='*', s=500)
    plt.scatter(optimal_risky_port[1], optimal_risky_port[0], color='g',␣
↪marker='*', s=500)
```

[46]:
```
montecarlo()
```

```
   Returns  Volatility  M1US Index weight  MXEF Index weight  \
0    5.96%       9.74%              4.86%             30.28%
1    7.85%       8.56%             13.95%              3.14%
2    6.04%       6.23%              1.83%             13.04%
3    7.43%      11.18%              0.71%             19.70%
4    7.45%      11.60%             20.97%             20.21%

   LBUSTRUU Index weight  LUATTRUU Index weight  JPEIDIVR Index weight  \
0                 12.12%                 23.76%                 10.56%
1                 14.04%                 20.14%                 23.36%
2                 23.02%                 28.14%                  1.32%
3                  8.23%                  7.89%                 27.58%
4                 12.63%                  5.72%                 10.60%
```

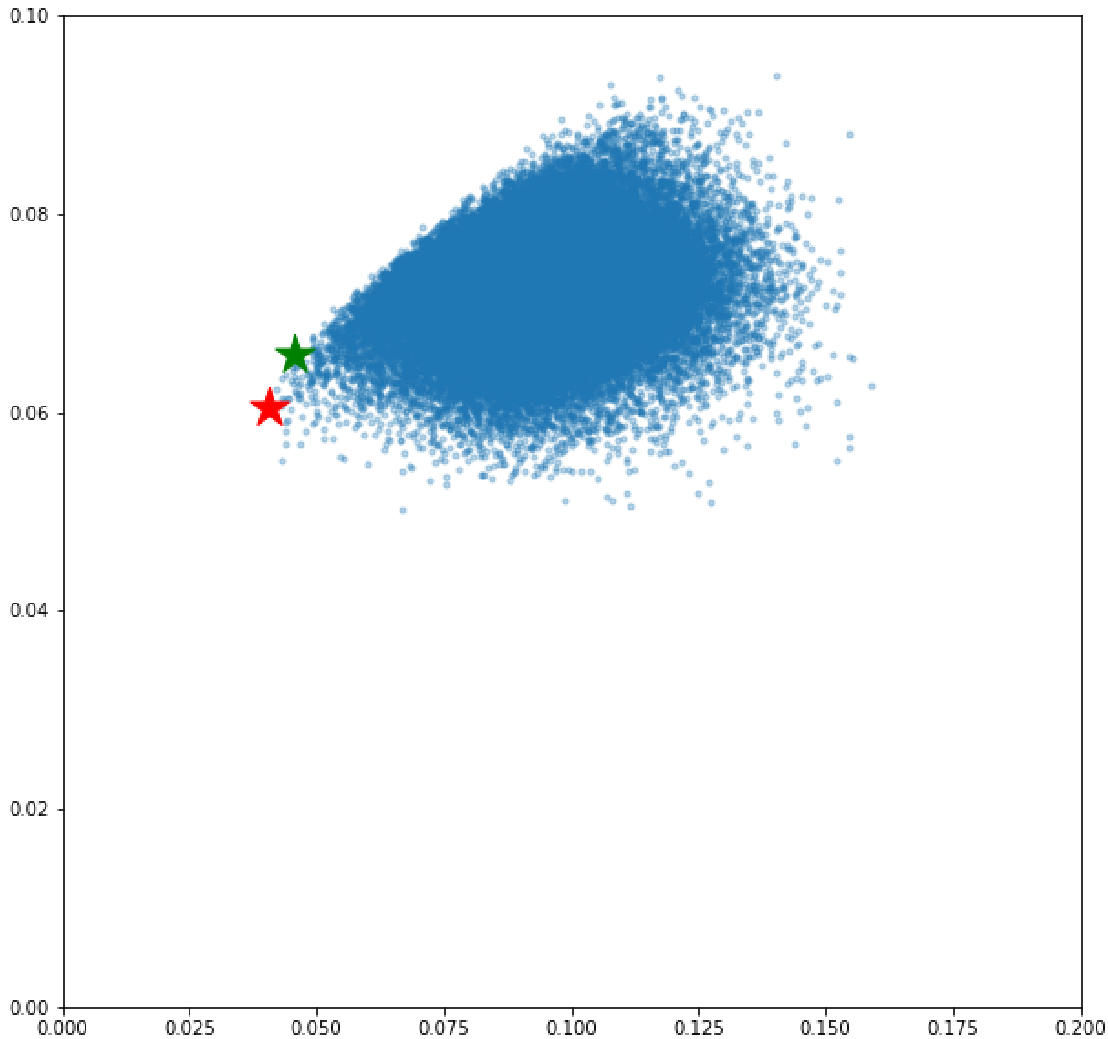
```
   LF98TRUU Index weight  NAREIT weight
0                 13.03%          5.38%
1                  2.35%         23.03%
2                 25.13%          7.50%
3                 12.23%         23.67%
4                  9.39%         20.48%
```

```
Min Vol Portfolio::
```

```
Returns                  6.04%
Volatility               4.07%
M1US Index weight        9.78%
MXEF Index weight        2.91%
LBUSTRUU Index weight   28.98%
LUATTRUU Index weight   44.98%
JPEIDIVR Index weight    5.47%
LF98TRUU Index weight    6.33%
NAREIT weight            1.55%
Name: 8661, dtype: float64
```

```
Returns                  6.59%
Volatility               4.55%
M1US Index weight       10.99%
MXEF Index weight        0.30%
LBUSTRUU Index weight   40.44%
LUATTRUU Index weight   24.77%
JPEIDIVR Index weight   12.85%
LF98TRUU Index weight    8.28%
NAREIT weight            2.38%
Name: 8863, dtype: float64
```

```
Max Sharpe Ratio: 1.0085199699760694
Efficient Frontier and Min Variance Portfolio and Max Sharpe Ratio Portfolio::
```



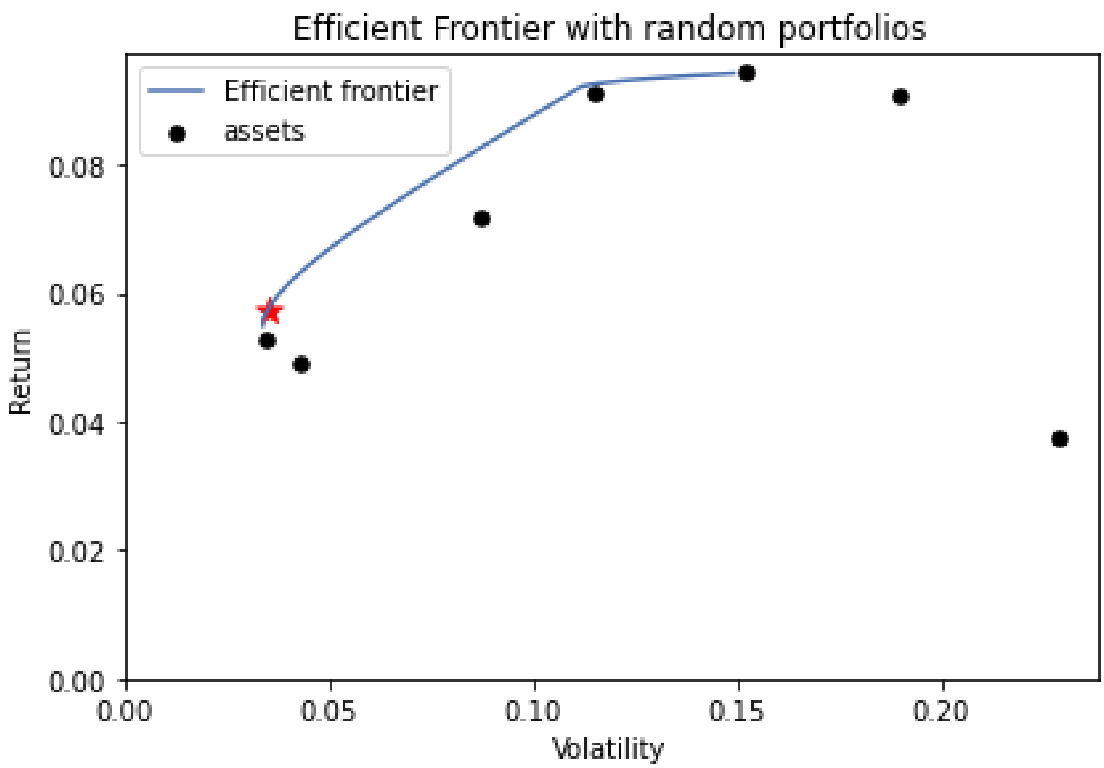
6.1 PyPfPortfolio Implementation

[47]:
```
def pyEffPortFlio(ret):
    from pypfopt import plotting
    import pandas as pd
    from pypfopt.efficient_frontier import EfficientFrontier
    from pypfopt import risk_models
    from pypfopt import expected_returns
    mu = expected_returns.mean_historical_return(ret, returns_data=True,␣
↪frequency=12, compounding=False)
    S = risk_models.sample_cov(ret, returns_data=True, frequency=12)

    # Optimize for maximal Sharpe ratio
    ef = EfficientFrontier(mu, S)
```

```
    #weights = ef.max_sharpe(risk_free_rate = factors['RF'].mean()*12/100)
    fig, ax = plt.subplots()
    #mkt_std = (mkt.std()*np.sqrt(12)/100)
    #mkt_ret = (mkt.mean()*12/100)
    #xvalues=[0,mkt_std]
    #yvalues=[0.04457,mkt_ret]
    #plt.plot(xvalues,yvalues)
    #xvalues=[0,0.218]
    #yvalues=[0.04457,0.199]
    #plt.plot(xvalues,yvalues)
    plotting.plot_efficient_frontier(ef, ax=ax, show_assets=True)
    ef.max_sharpe()
    ret_tangent, std_tangent, _ = ef.portfolio_performance(verbose=True)
    ax.scatter(std_tangent, ret_tangent, marker="*", s=100, c="r", label="Max␣
↪Sharpe")

    # Output
    ax.set_title("Efficient Frontier with random portfolios")
    ax.set_ylim(ymin=0)
    ax.set_xlim(xmin=0)
    plt.show()
```

[48]:
```
pyEffPortFlio(ret1MLog_wo_cs)
```

```
Expected annual return: 5.8%
Annual volatility: 3.5%
Sharpe Ratio: 1.07
```



7 CVXOPT Implementation

[49]:
```
import numpy as np
import cvxopt as opt
from cvxopt import blas, solvers
import matplotlib.pyplot as plt
```

[50]:
```
def optimal_portfolio(returns):
    # Turn off progress printing
    solvers.options['show_progress'] = False
    returns = np.asmatrix(returns.T)                # -> (n_assets,␣
↪n_observations)
    n_assets = len(returns)
    # Vector of desired returns
    N = n_assets*(int(1e+2))
    mus = [10 ** (5.0 * t / N - 1.0) for t in range(N)]


    # Obtain expected returns and covariance
    m1 = np.mean(returns, axis=1)                       # Mean returns
```

```
    c1 = np.cov(returns, bias=True)                        # Volatility (in terms␣
↪of standard deviation)
    # Convert to cvxopt matrices
    pbar = opt.matrix(m1)
    S = opt.matrix(c1)

    # Limits for each stock
    lower_bound = 0.0 #5%
    upper_bound = 1
    # Check error
    if n_assets*lower_bound > 1:
        print('Too many stocks for the lower bound limit.')
        lower_bound = round(1.00/n_assets, 3)
        print('New lower band: ', lower_bound)

    upper_bound_array = np.ones((n_assets, 1)) * upper_bound


    # Create constraint matrices
    G = opt.matrix(np.vstack((-np.eye(n_assets), np.eye(n_assets))))
    #h = opt.matrix(np.vstack((-lower_bound*np.ones((n_assets, 1)),␣
↪upper_bound*np.ones((n_assets, 1)), 1.2*Benchmark_weights*np.ones((11, 1)))))
    h = opt.matrix(np.vstack((-lower_bound*np.ones((n_assets, 1)),␣
↪upper_bound_array)))
    A = opt.matrix(1.0, (1, n_assets))
    b = opt.matrix(1.0)

    # Calculate efficient frontier weights using quadratic programming
    portfolios = [solvers.qp(mu * S, -pbar, G, h, A, b)['x'] for mu in mus]
    sol = solvers.qp(S, -pbar, G, h, A, b)

    ## CALCULATE RISKS AND RETURNS FOR FRONTIER
    weights = [np.asarray(x) for x in portfolios]
    returns = [blas.dot(pbar, x) for x in portfolios]
    risks = [np.sqrt(blas.dot(x, S * x)) for x in portfolios]

    return weights, np.asarray(returns), np.asarray(risks), sol, returns, risks
```

7.1 Optimal Portfolio without Case-Shiller

[51]:
```
ret = ret1MLog_wo_cs
weights, returns, risks, sol, plot_return_wo_cs, plot_risk_wo_cs =␣
 ↪optimal_portfolio(ret)
ann_returns_wo_cs = (1 + returns)**12 - 1
ann_risks_wo_cs = np.sqrt(12) * risks
sharpe1 = (ann_returns_wo_cs - 0.02)/ann_risks_wo_cs
ind_opt = np.argmax(sharpe1)            # Index of selected portfolio
```

```python
opt_portfolio = {}
opt_portfolio['return'] = returns[ind_opt] * 12
opt_portfolio['risk'] = risks[ind_opt] * np.sqrt(12)
opt_portfolio['sharpe'] = sharpe1[ind_opt]

wt = weights[ind_opt]/sum(weights[ind_opt])
ind_w = np.flip(np.argsort(wt, axis=0), axis=0)
opt_portfolio['weights'] = wt[ind_w]
ind_w = ind_w.ravel().tolist()
sym1 = pd.DataFrame(list(ret))

sym=sym1.loc[ind_w]

#sym = [str(sym[k][0][0]) for k in range(len(sym))]
opt_portfolio['stocks'] = sym

output = pd.DataFrame(columns=["Ticker","Weights%"])
output["Ticker"] = sym[0]
output["Weights%"] = wt[ind_w]
output = output.reset_index(drop=True)
display(output)
print(opt_portfolio)
```

	Ticker	Weights%
0	LBUSTRUU Index	85.19%
1	M1US Index	7.85%
2	LF98TRUU Index	5.97%
3	JPEIDIVR Index	0.99%
4	LUATTRUU Index	0.00%
5	NAREIT	0.00%
6	MXEF Index	0.00%

```
{'return': 0.05756605592770696, 'risk': 0.03517106208680689, 'sharpe':
1.1119782375301672, 'weights': array([[[8.51884331e-01]],

       [[7.85272883e-02]],

       [[5.96615896e-02]],

       [[9.91016888e-03]],

       [[1.57891218e-05]],

       [[6.14080622e-07]],

       [[2.18675002e-07]]]), 'stocks':                  0
2  LBUSTRUU Index
```

```
0       M1US Index
5   LF98TRUU Index
4   JPEIDIVR Index
3   LUATTRUU Index
6           NAREIT
1       MXEF Index}
```

7.2 Optimal Portfolio with Case-Shiller

[52]:
```python
ret = ret1MLog #ret1MLog*12
weights, returns, risks, sol, plot_return_cs, plot_risk_cs =␣
↪optimal_portfolio(ret)
ann_returns_cs = (1 + returns)**12 - 1
ann_risks_cs = np.sqrt(12) * risks
sharpe1 = (ann_returns_cs - 0.02)/ann_risks_cs
ind_opt = np.argmax(sharpe1)            # Index of selected portfolio

opt_portfolio = {}
opt_portfolio['return'] = returns[ind_opt] * 12
opt_portfolio['risk'] = risks[ind_opt] * np.sqrt(12)
opt_portfolio['sharpe'] = sharpe1[ind_opt]

wt = weights[ind_opt]/sum(weights[ind_opt])
ind_w = np.flip(np.argsort(wt, axis=0), axis=0)
opt_portfolio['weights'] = wt[ind_w]
ind_w = ind_w.ravel().tolist()
sym1 = pd.DataFrame(list(ret))

sym=sym1.loc[ind_w]

#sym = [str(sym[k][0][0]) for k in range(len(sym))]
opt_portfolio['stocks'] = sym

output = pd.DataFrame(columns=["Ticker","Weights%"])
output["Ticker"] = sym[0]
output["Weights%"] = wt[ind_w]
output = output.reset_index(drop=True)
display(output)
print(opt_portfolio)
```

	Ticker	Weights%
0	CSUSHPINSA	54.28%
1	LBUSTRUU Index	41.12%
2	M1US Index	2.89%
3	LF98TRUU Index	0.93%
4	JPEIDIVR Index	0.78%
5	LUATTRUU Index	0.00%
6	NAREIT	0.00%

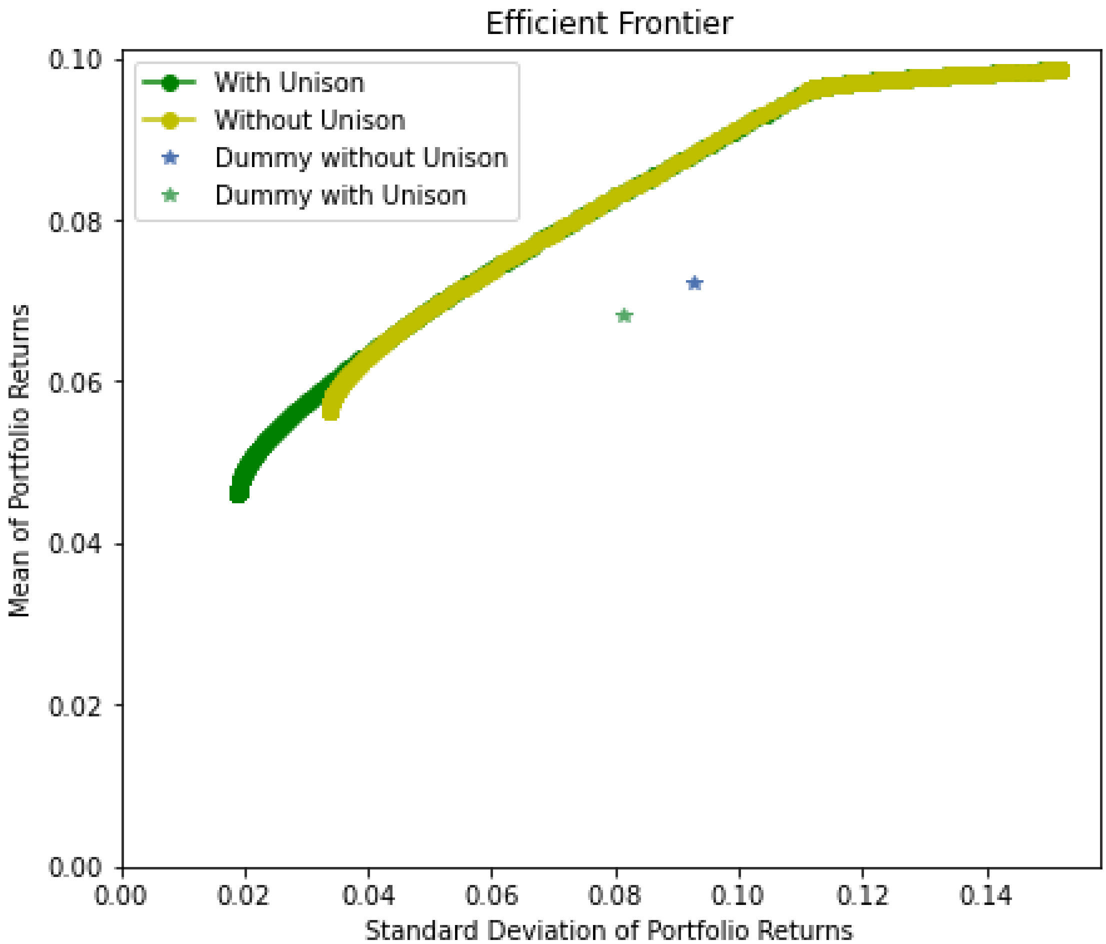
```
7      MXEF Index     0.00%
```

```
{'return': 0.04808291963245642, 'risk': 0.020157701669695136, 'sharpe':
1.4464373588108888, 'weights': array([[[5.42797510e-01]],

       [[4.11185230e-01]],

       [[2.89008908e-02]],

       [[9.30195696e-03]],

       [[7.80931010e-03]],

       [[3.38432855e-06]],

       [[1.12253013e-06]],

       [[5.94927122e-07]]]), 'stocks':                  0
6      CSUSHPINSA
2  LBUSTRUU Index
0      M1US Index
5  LF98TRUU Index
4  JPEIDIVR Index
3  LUATTRUU Index
7          NAREIT
1      MXEF Index}
```

7.3 Efficient Frontier plots for comparision

[53]:
```
#Plot Efficient Frontier
weights = np.ones((1,7)) * 1/7
returns = np.dot(weights, ind_er)
risk = np.sqrt(weights @ ret1MLog_wo_cs.cov() @ weights.T).values *np.sqrt(12)
ind_er_uni = (1 + ret1MLog.mean())**12 - 1
weights = np.ones((1,8)) * 1/8
returns_uni = np.dot(weights, ind_er_uni)
risk_uni = np.sqrt(weights @ ret1MLog.cov() @ weights.T).values *np.sqrt(12)
fig, ax = plt.subplots(figsize=(7,6))
plt.plot(ann_risks_cs, ann_returns_cs, 'g-o', label='With Unison')
plt.plot(ann_risks_wo_cs, ann_returns_wo_cs, 'y-o', label='Without Unison')
plt.plot(risk,returns,'*', label='Dummy without Unison')
plt.plot(risk_uni,returns_uni,'*', label='Dummy with Unison')
plt.title('Efficient Frontier')
plt.ylabel('Mean of Portfolio Returns')
plt.xlabel('Standard Deviation of Portfolio Returns')
ax.set_ylim(ymin=0)
ax.set_xlim(xmin=0)
plt.legend()
```

```
plt.show()
```



7.3.1 Sharpe of Dummy Portfolio without Unison

[54]:
```
returns/risk
```

[54]: array([[0.78009566]])

7.3.2 Sharpe of Dummy Portfolio with Unison

[55]:
```
returns_uni/risk_uni
```

[55]: array([[0.83990393]])