Unison Industry Project - Portfolio Optimization

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1 Introduction

Large institutional investors own a variety of assets spanning corporate equities and debt, mortgages, commercial real estate and treasuries. We want to improve these institutional investment portfolios by providing them access to homeowner equity.

The objective of this project is to create a portfolio optimization tool which takes a sample institutional portfolio weighting and shows how diversifying into homeowner equity investments can improve risk/return of the portfolio.

The tool should be able to take a current portfolio holding of a predetermined set of investment asset classes, show expected return and risk (plus other relevant metrics such as Sharpe or VAR), and display them on an efficient frontier. Then, it should be able to show how a small re-allocation to Unison investments can improve those metrics. Finally it should also propose a reasonable allocation to the unison homeowner equity.

The project can be split into four different milestones.

- Determining a set of investment asset classes that large institutions typically invest in. Determine expected return, volatility and correlation for those asset classes using historical sample estimates.
- Estimate the expected return, volatility and correlations of the unison investments with the other asset classes. Using our estimate of Alpha and Beta of Unison vs. Case Shiller, and the above estimates, determine the return, volatility and correlation vector for Unison investments.
- Write an optimizer with proper constraints
- Build an application to show the allocation, optimization results and efficient frontier.

2 Asset Class Determination and Data Collection

In order to determine a realistic allocation of portfolios of the institutional investors, we analysed various endowment funds such as Yale, Harvard and UC; soverign funds such as GIC, ARIA, Korea Investment Fund; Pension funds such as Japan Pension fund and CalPERS. We also looked at the portfolios of JPMorgan asset management and Blackrock. From this analysis we concluded that we can broadly categorize the investment into 4 categories.

- Equities

U.S. large cap equities	MSCI USA Index
Emerging large cap equities	MSCI Emerging Markets Index
U.S. small cap equities	MSCI USA Small Cap Index
Global ex-U.S. large cap equities	MSCI World ex-US Index

- Fixed Income

U.S. large cap equities	MSCI USA Index
Emerging large cap equities	MSCI Emerging Markets Index
U.S. small cap equities	MSCI USA Small Cap Index
Global ex-U.S. large cap equities	MSCI World ex-US Index

- Private market

U.S. large cap equities	MSCI USA Index
Emerging large cap equities	MSCI Emerging Markets Index
U.S. small cap equities	MSCI USA Small Cap Index
Global ex-U.S. large cap equities	MSCI World ex-US Index

- home equity (Core Real Estate, REIT index, Case Shiller index)

U.S. large cap equities	MSCI USA Index
Emerging large cap equities	MSCI Emerging Markets Index
U.S. small cap equities	MSCI USA Small Cap Index
Global ex-U.S. large cap equities	MSCI World ex-US Index

3 Data Analysis

To understand the impact that time horizon has on returns and volatility, we analysed the annualised returns and annualised volatility for different time horizons - 1 month, 3 months, 1 year and 3 years for all the indices. The results have been summarised below:

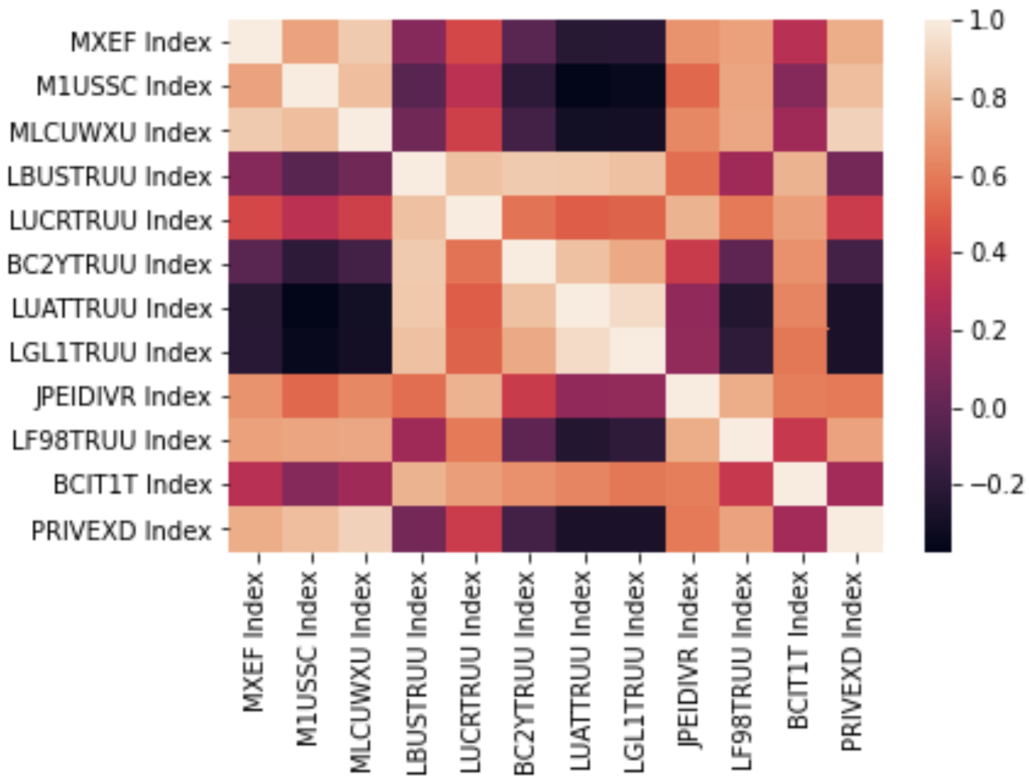
	Ret1MAnnual	Ret3MAnnual	Ret1YAnnual	Ret3YAnnual
M1US Index	9.44%	9.48%	9.01%	8.23%
MXEF Index	3.73%	3.70%	3.06%	3.28%
MLCUWXU Index	2.57%	2.50%	2.09%	2.11%
LBUSTRUU Index	5.28%	5.34%	5.42%	5.14%
LUCRTRUU Index	6.07%	6.14%	6.16%	5.84%
LUATTRUU Index	4.92%	4.99%	5.14%	4.83%
LGL1TRUU Index	7.17%	7.39%	7.76%	7.15%
JPEIDIVR Index	9.13%	9.40%	9.45%	9.10%
LF98TRUU Index	7.16%	7.18%	7.01%	6.86%
CSUSHPINSA	4.10%	4.09%	4.04%	4.04%
NAREIT	9.10%	9.14%	9.17%	9.31%

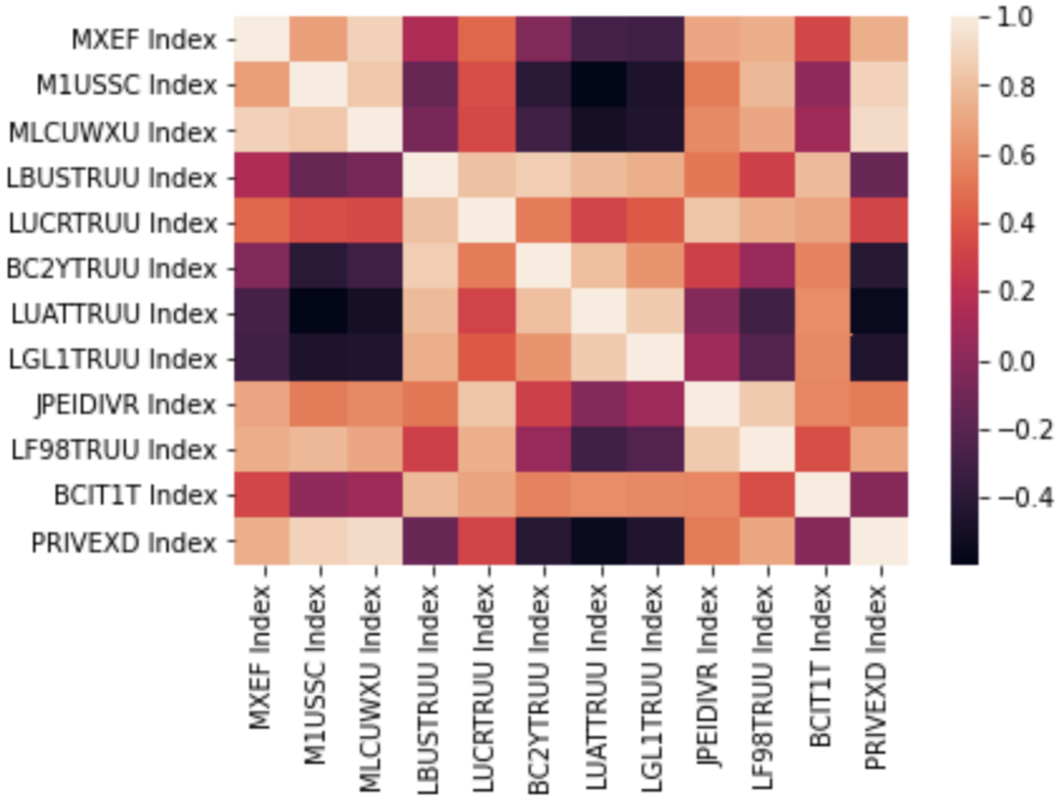
	Vol1MAnnual	Vol3MAnnual	Vol1YAnnual	Vol3YAnnual
M1US Index	15.21%	15.52%	17.05%	18.78%
MXEF Index	22.84%	25.59%	26.00%	20.99%
MLCUWXU Index	16.44%	17.59%	18.70%	16.53%
LBUSTRUU Index	3.48%	3.58%	3.76%	3.80%
LUCRTRUU Index	5.19%	5.30%	5.26%	4.36%
LUATTRUU Index	4.32%	4.44%	4.29%	4.25%
LGL1TRUU Index	10.13%	10.04%	8.80%	5.49%
JPEIDIVR Index	11.51%	11.54%	10.28%	8.44%
LF98TRUU Index	8.70%	9.96%	10.18%	8.40%
CSUSHPINSA	2.42%	3.95%	5.79%	8.88%
NAREIT	18.96%	18.95%	19.93%	18.07%

As can be seen from the results above, the returns and volatility vary across time periods. Hence, in order to include all the seasonality and cyclical shocks and to be consistent with our portfolio optimisation results, we consider a time horizon of 1 year from hereCas on.

4 Index Selection

To select the indices, we looked at the correlations between them. The heat maps for 1M and 1Y maturity have been given below.





After analysing the heat maps, 'MLCUWXU Index', 'LUCRTRUU Index', 'BC2YTRUU Index', 'LGL1TRUU Index', 'BCIT1T Index' and 'PRIVEXD Index' were dropped as they exhibited high correlation for both 1M and 1Y with one of the other indices leaving us with 'M1US Index', 'MXEF Index', 'LBUSTRUU Index', 'LUATTRUU Index', 'JPEIDIVR Index', 'LF98TRUU Index', 'CSUSHPINSA' and 'NAREIT' as the final set of indices in our portfolio.

We also use Case Shiller as a proxy to generate the time series for Unison's home-owner equity using the following equation:

$$Unison = 2.1 * (0.01 + CaseShiller) \tag{1}$$

and include this time series in our portfolio.

5 Mean-Variance Portfolio Optimisation

5.1 Without Constraints - Derivation:

Let U be the utility function with the Lagrangian incorporated for the constraint $h^T\mathbf{1}$ = 1 where h represent the weights for the assets in the portfolio. Hence U, which is the utility function that needs to be maximised, can be mathematically represented as:

$$max_h U = h^T\mu - \frac{\gamma}{2}h^T\Sigma h - \lambda * (h^T\mathbf{1} - 1) \tag{2}$$

Equating the first derivative of U with respect to h to 0, we get:

$$\frac{\partial U}{\partial h} = \mu - \gamma\Sigma h - \lambda * \mathbf{1} = 0 \tag{3}$$

$$h = \Sigma^{-1}\Big(\frac{\mu - \lambda\mathbf{1}}{\gamma}\Big) \tag{4}$$

$$\frac{\partial U}{\partial \lambda} = h^T\mathbf{1} - 1 = 0 \tag{5}$$

Substituting the value of h from equation (3) in equation (4) we get:

$$\Big(\frac{\mu^T - \lambda\mathbf{1}^T}{\gamma}\Big)\Sigma^{-1}\mathbf{1} - 1 = 0 \tag{6}$$

$$\lambda = \frac{\mu^T\Sigma^{-1}\mathbf{1} - \gamma}{\mathbf{1}^T\Sigma^{-1}\mathbf{1}} \tag{7}$$

Substituting the value of λ in h in equation (3) we get:

$$h = \frac{\Sigma^{-1}}{\gamma}\left[\mu - \frac{\mu^T\Sigma^{-1}\mathbf{1}}{\mathbf{1}^T\Sigma^{-1}\mathbf{1}}\mathbf{1}\right] + \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^T\Sigma^{-1}\mathbf{1}} \tag{8}$$

5.1.1 Results:

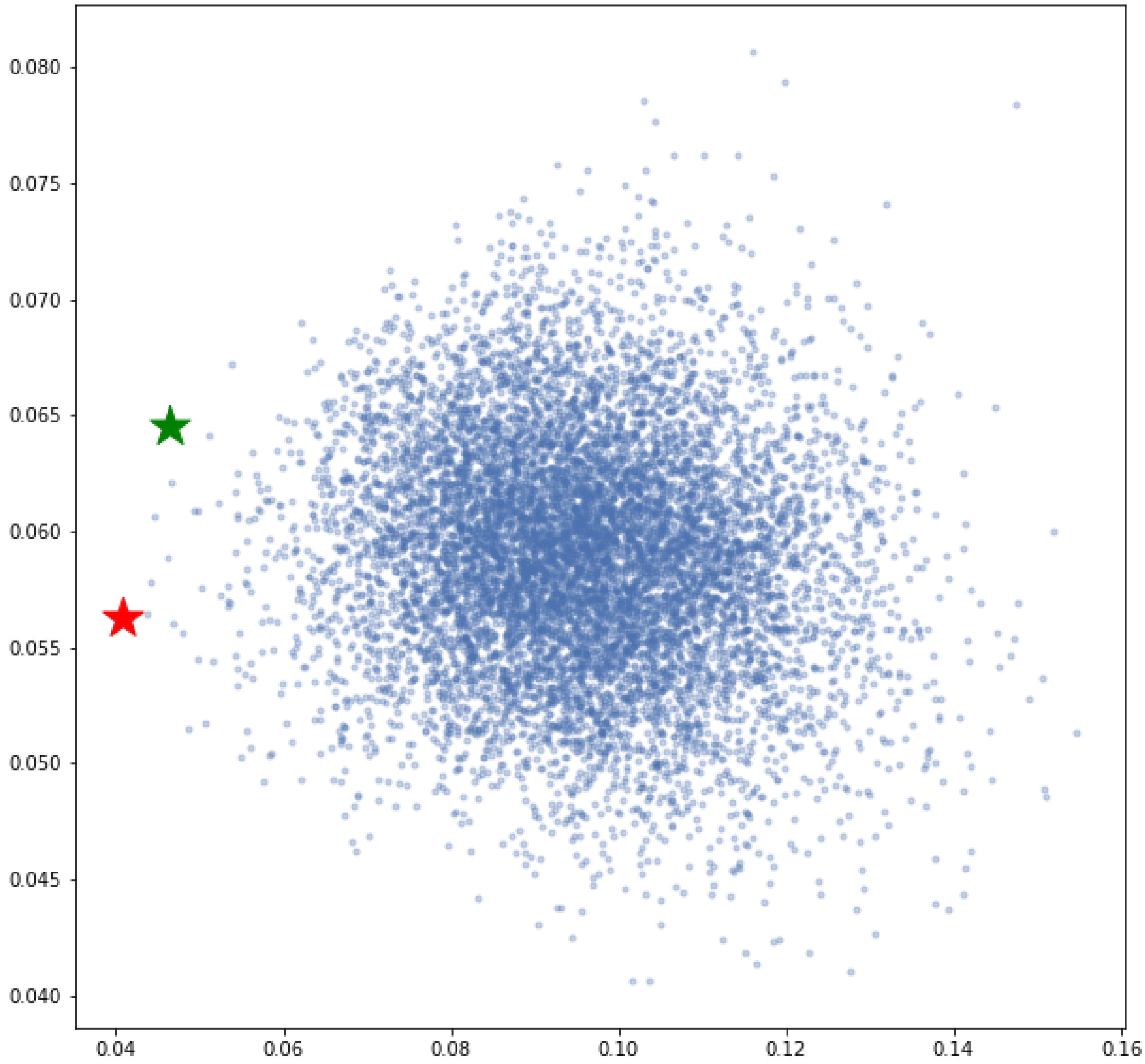
The results for the unconstrained mean-variance optimisation have been summarised below.

Index Name	Weights
M1US Index	450.57%
MXEF Index	1462.92%
LBUSTRUU Index	19917.10%
LUATTRUU Index	6391.68%
JPEIDIVR Index	-522.78%
LF98TRUU Index	2029.25%
CSUSHPINSA	-59009.09%
NAREIT	-2571.36%
Unison	34309.09%

5.2 With Constraints

5.2.1 MonteCarlo

In MonteCarlo simulation we started with assigning random weights to our assets, keeping the sum of weights equal to 1. We ran a simulation creating 10,000 portfolios which allowed us to generate an efficient frontier. Below is the image of Montecarlo results

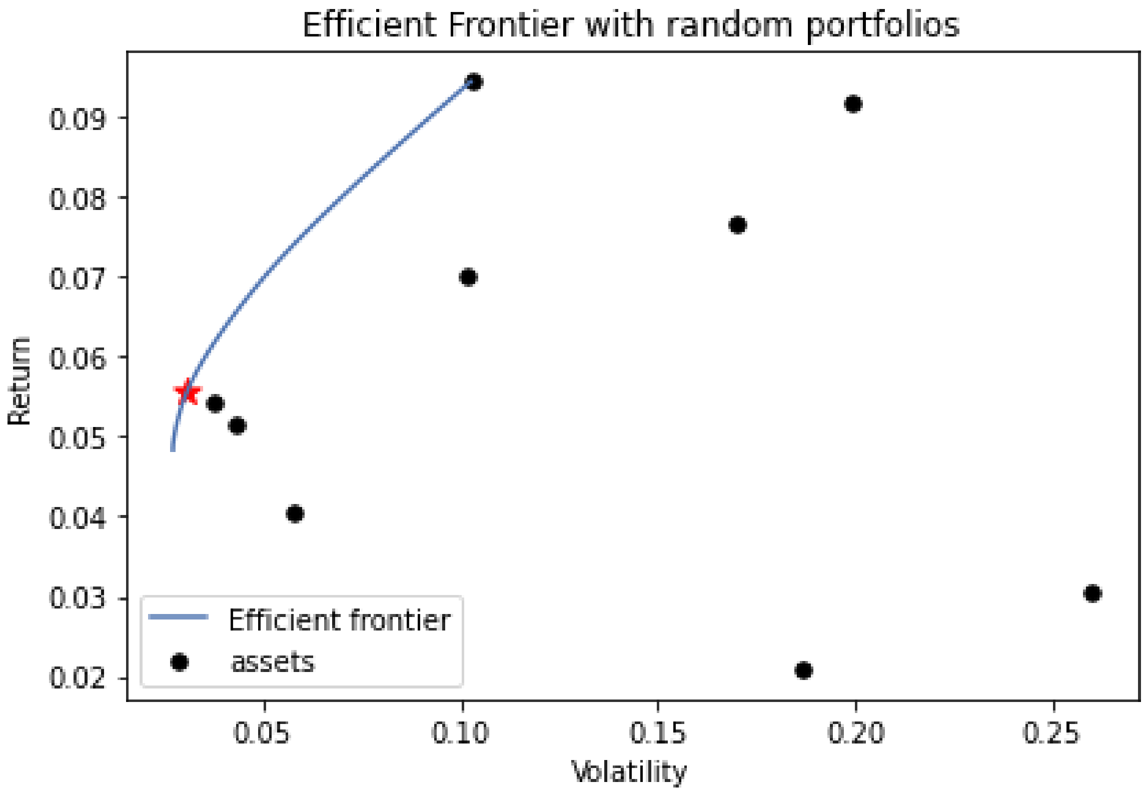


The results have been summarised below:

	Max Sharpe Portfolio	Min Variance Portfolio
Returns	6.46%	5.63%
Volatility	4.64%	4.08%
Sharpe	0.9611	0.889

5.2.2 Pypfportfolio

Montecarlo simulation though robust, is computationally very expensive. Pypfportfolio offers a better way to solve the mean variance optimization problem. Below we have run the optimization using this library on the same set of assets and under the same constraints:



The results have been summarised below:

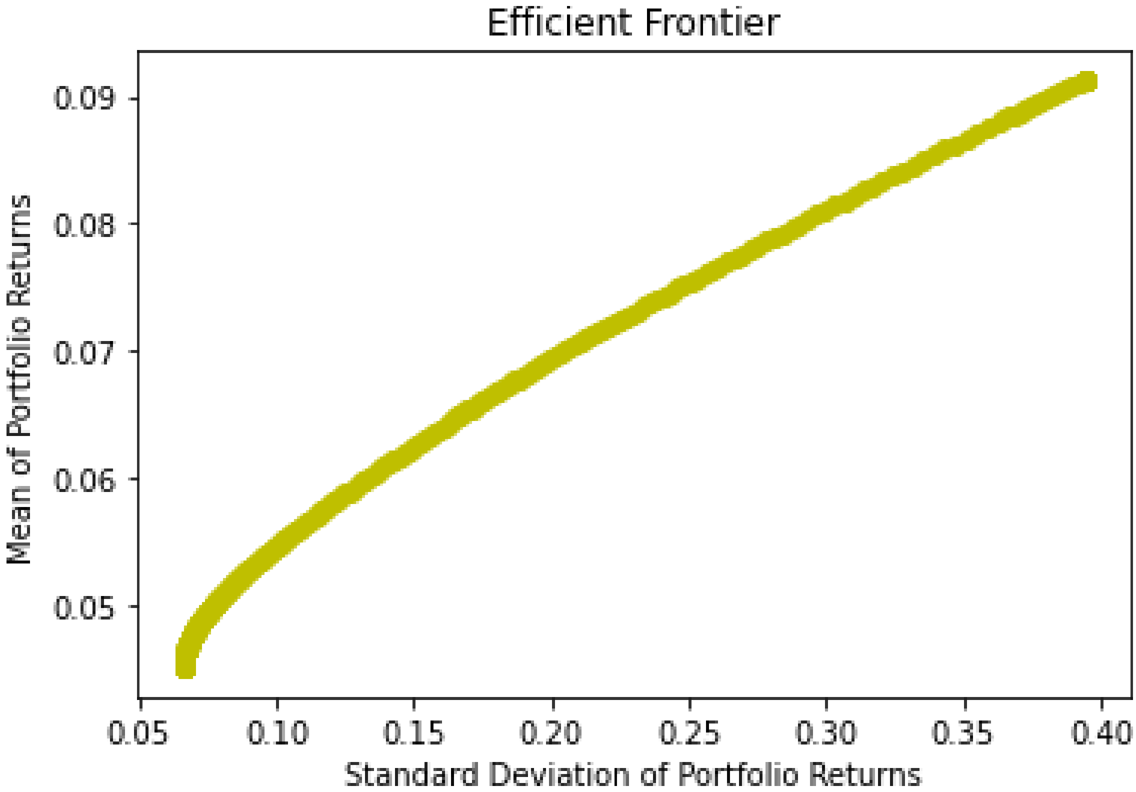
	Max Sharpe Portfolio
Returns	5.6%
Volatility	3.1%
Sharpe	1.16

As expected, results are clearly better than MonteCarlo simulation.

6 Adding Unison Home Equity to the portfolio

As we saw earlier, convex optimization packages have clear advantages over Monte Carlo simulation. To be able to incorporate all the constraints we intend to use in this project we have decided to go ahead with CVXOPT package.

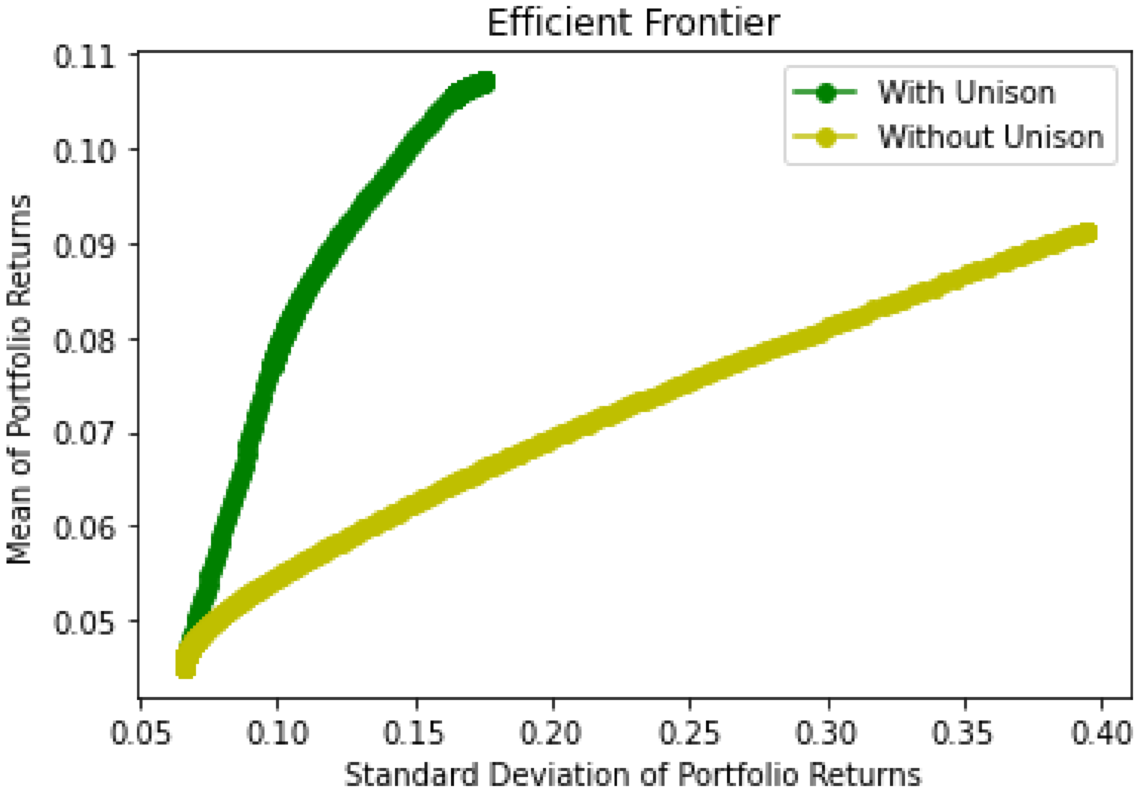
6.1 Efficient Frontier including all shortlisted assets without Unison



The results have been summarised below:

	Max Sharpe Portfolio
Returns	4.63%
Volatility	6.68%
Sharpe	0.69

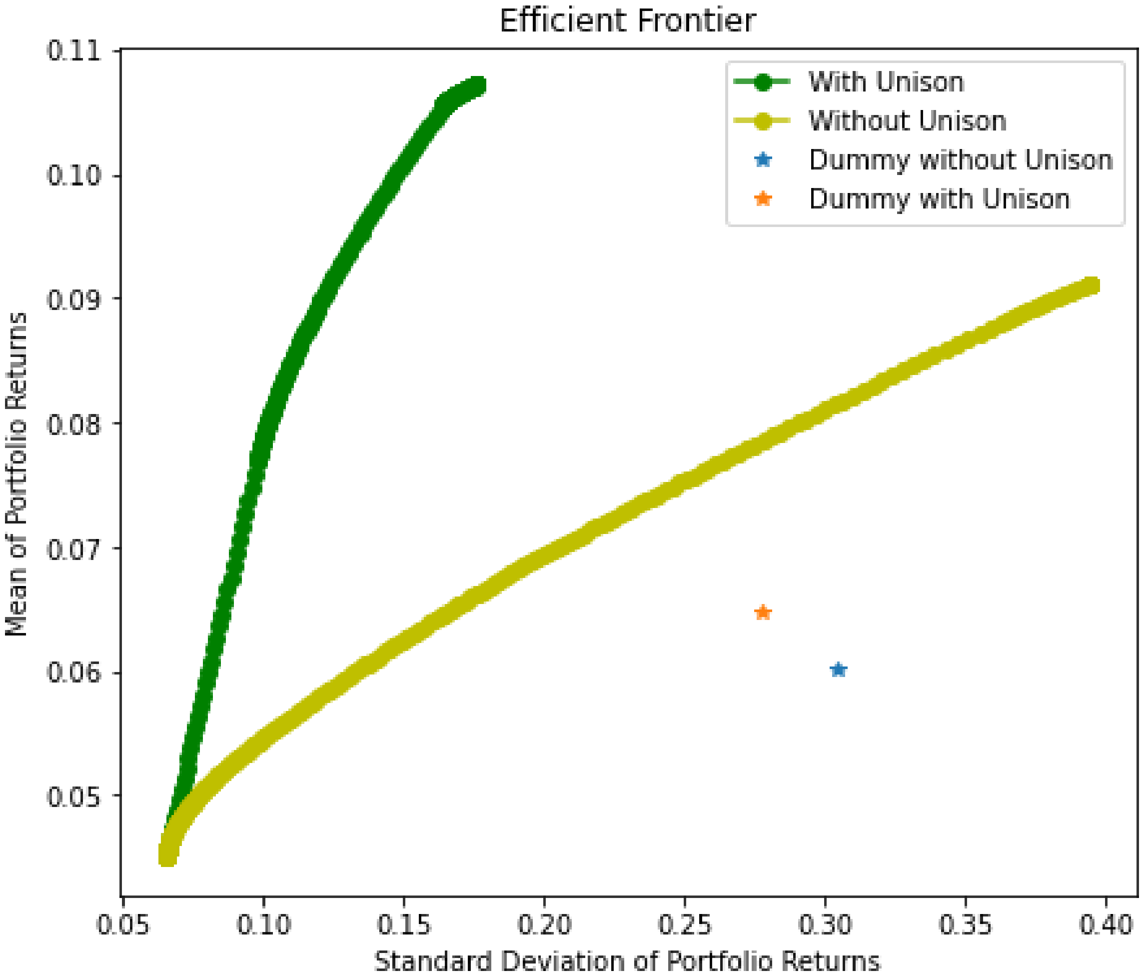
6.2 Efficient Frontier including all shortlisted assets with Unison



The results have been summarised below:

	Max Sharpe Portfolio
Returns	7.85%
Volatility	9.98%
Sharpe	0.79

To show the benefit of adding Unison Home Equity to the client portfolio we have created a dummy portfolio, consisting equal weights of all the selected assets. To this we add 10% Unison home equity and proportionately reducing weight from other assets.


Efficient Frontier
With Unison
Without Unison
Dummy without Unison
Dummy with Unison
Mean of Portfolio Returns
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
Standard Deviation of Portfolio Returns

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