Decision Tree – Classification

Decision Tree algorithm is supervised learning algorithms that can be used for solving **regression and classification problems**.

The general motive of using Decision Tree is to create a training model which can use to predict class or value of target variables by **learning decision rules** inferred from prior data (training data).

The understanding level of Decision Trees algorithm is so easy compared with other classification algorithms. The decision tree algorithm tries to solve the problem, by using tree representation. Each **internal node** of the tree corresponds to an attribute, and each **leaf node** corresponds to a class label.

Decision Tree Algorithm Pseudocode

1. Place the best attribute of the dataset at the **root** of the tree.
2. Split the training set into **subsets**. Subsets should be made in such a way that each subset contains data with the same value for an attribute.
3. Repeat step 1 and step 2 on each subset until you find **leaf nodes** in all the branches of the tree.

In decision trees, for predicting a class label for a record It start from the **root** of the tree. It compares the values of the root attribute with record's attribute. On the basis of comparison, it follows the branch corresponding to that value and jump to the next node. It continues comparing our record's attribute values with other **internal nodes** of the tree until we reach a **leaf node** with predicted class label/predicted value.

Assumptions while creating Decision Tree

The below are the some of the assumptions we make while using Decision tree:

- At the beginning, the whole training set is considered as the **root.**
- Feature values are preferred to be categorical. If the values are continuous then they are discretized prior to building the model.
- Records are **distributed recursively** on the basis of attribute values.
- Order to placing attributes as root or internal node of the tree is done by using some statistical approach.

The primary challenge in the decision tree implementation is to identify which attributes do we need to consider as the root node and each level. Handling this is know the attributes selection. We have different attributes selection measure to identify the attribute which can be considered as the root note at each level.

The popular attribute selection measures:

- Information gain
- Gini index

Attributes Selection

If dataset consists of **“n”** attributes then deciding which attribute to place at the root or at different levels of the tree as internal nodes is a complicated step. By just randomly selecting any node to be the root can’t solve the issue. If we follow a random approach, it may give us bad results with low accuracy.

For solving this attribute selection problem, researchers worked and devised some solutions. They suggested using some *criterion* like **Information gain, Gini index,** etc. These criterions will calculate values for every attribute. The values are sorted, and attributes are placed in the tree by following the order i.e., the attribute with a **high value** (in case of information gain) is placed at the root.

While using information Gain as a criterion, we assume attributes to be categorical, and for Gini index, attributes are assumed to be continuous.

Information Gain

By using information gain as a criterion, we try to estimate the information contained by each attribute. Information Gain calculates the expected reduction in entropy due to sorting on the attribute. To measure the randomness or uncertainty of a random variable X is defined by **Entropy**.

For a binary classification problem with only two classes, positive and negative class.

- If all examples are positive or all are negative then entropy will be zero i.e., low.
- If half of the records are of positive class and half are of negative class then entropy is one i.e., high.

$$H(X) = \mathbb{E}_X[I(x)] = -\sum_{x\in\mathbb{X}} p(x)\log p(x).$$

By calculating **entropy measure** of each attribute, we can calculate their **information gain**.

Example: Construct a Decision Tree by using "information gain" as a criterion

	A	B	C	D	E
1	4.8	3.4	1.9	0.2	positive
2	5	3	1.6	0.2	positive
3	5	3.4	1.6	0.4	positive
4	5.2	3.5	1.5	0.2	positive
5	5.2	3.4	1.4	0.2	positive
6	4.7	3.2	1.6	0.2	positive
7	4.8	3.1	1.6	0.2	positive
8	5.4	3.4	1.5	0.4	positive
9	7	3.2	4.7	1.4	negative
10	6.4	3.2	4.5	1.5	negative
11	6.9	3.1	4.9	1.5	negative
12	5.5	2.3	4	1.3	negative
13	6.5	2.8	4.6	1.5	negative
14	5.7	2.8	4.5	1.3	negative
15	6.3	3.3	4.7	1.6	negative
16	4.9	2.4	3.3	1	negative

We are going to use this data sample. Let's try to use information gain as a criterion. Here, we have 5 columns out of which 4 columns have continuous data and 5th column consists of class labels.

A, B, C, D attributes can be considered as predictors and E column class labels can be considered as a target variable. For constructing a decision tree from this data, we have to convert continuous data into categorical data.

We have chosen some random values to categorize each attribute:

A	B	C	D
>= 5	>= 3.0	>= 4.2	>= 1.4
< 5	< 3.0	< 4.2	< 1.4

There are **2 steps for calculating information gain** for each attribute:

1. Calculate entropy of Target.
2. Entropy for every attribute A, B, C, D needs to be calculated. Using information gain formula, we will subtract this entropy from the entropy of target. The result is Information Gain.

The entropy of Target: We have 8 records with negative class and 8 records with positive class. So, we can directly estimate the entropy of target as 1.

Variable E	
Positive	Negative
8	8

Calculating entropy using formula:

Calculating entropy using formula:

E(8,8) = -1*((p(+ve)*log(p(+ve)) + (p(-ve)*log(p(-ve)))
= -1*((8/16)*$\log_2$(8/16)) + (8/16) * $\log_2$(8/16))
= 1

Information gain for Var A

Var A has value >=5 for 12 records out of 16 and 4 records with value <5 value.

- For Var A >= 5 & class == positive: 5/12
- For Var A >= 5 & class == negative: 7/12
  - Entropy(5,7) = -1 * ((5/12)*log2(5/12) + (7/12)*log2(7/12)) = 0.9799
- For Var A <5 & class == positive: 3/4
- For Var A <5 & class == negative: 1/4
  - Entropy(3,1) = -1 * ((3/4)*log2(3/4) + (1/4)*log2(1/4)) = 0.81128

Entropy(Target, A) = P(>=5) * E(5,7) + P(<5) * E(3,1)
= (12/16) * 0.9799 + (4/16) * 0.81128 = 0.937745

$\text{Information Gain(IG)} = \text{E(Target) - E(Target,A)} = 1\text{- } 0.9337745 = 0.062255$

Information gain for Var B

Var B has value >=3 for 12 records out of 16 and 4 records with value <5 value.

- For Var B >= 3 & class == positive: 8/12
- For Var B >= 3 & class == negative: 4/12
  - Entropy(8,4) = -1 * ((8/12)*log2(8/12) + (4/12)*log2(4/12)) = 0.39054
- For VarB <3 & class == positive: 0/4
- For Var B <3 & class == negative: 4/4
  - Entropy(0,4) = -1 * ((0/4)*log2(0/4) + (4/4)*log2(4/4)) = 0

Entropy(Target, B) = P(>=3) * E(8,4) + P(<3) * E(0,4)
= (12/16) * 0.39054 + (4/16) * 0 = 0.292905

$\text{Information Gain(IG)} = \text{E(Target) - E(Target,B)} = 1\text{- } 0.292905 = 0.707095$

Information gain for Var C

Var C has value >=4.2 for 6 records out of 16 and 10 records with value <4.2 value.

- For Var C >= 4.2 & class == positive: 0/6
- For Var C >= 4.2 & class == negative: 6/6
    - Entropy(0,6) = 0
- For VarC < 4.2 & class == positive: 8/10
- For Var C < 4.2 & class == negative: 2/10
    - Entropy(8,2) = 0.72193

Entropy(Target, C) = P(>=4.2) * E(0,6) + P(< 4.2) * E(8,2)
= (6/16) * 0 + (10/16) * 0.72193 = 0.4512

$$\text{Information Gain(IG)} = \text{E(Target) - E(Target,C)} = \text{1- } 0.4512 = 0.5488$$

Information gain for Var D

Var D has value >=1.4 for 5 records out of 16 and 11 records with value <5 value.

- For Var D >= 1.4 & class == positive: 0/5
- For Var D >= 1.4 & class == negative: 5/5
    - Entropy(0,5) = 0
- For Var D < 1.4 & class == positive: 8/11
- For Var D < 14 & class == negative: 3/11
    - Entropy(8,3) = -1 * ((8/11)*log2(8/11) + (3/11)*log2(3/11)) = 0.84532

Entropy(Target, D) = P(>=1.4) * E(0,5) + P(< 1.4) * E(8,3)
= 5/16 * 0 + (11/16) * 0.84532 = 0.5811575

$$\text{Information Gain(IG)} = \text{E(Target) - E(Target,D)} = \text{1- } 0.5811575 = 0.41189$$

		Target	
		Positive	Negative
A	>= 5.0	5	7
	<5	3	1
Information Gain of A = 0.062255			

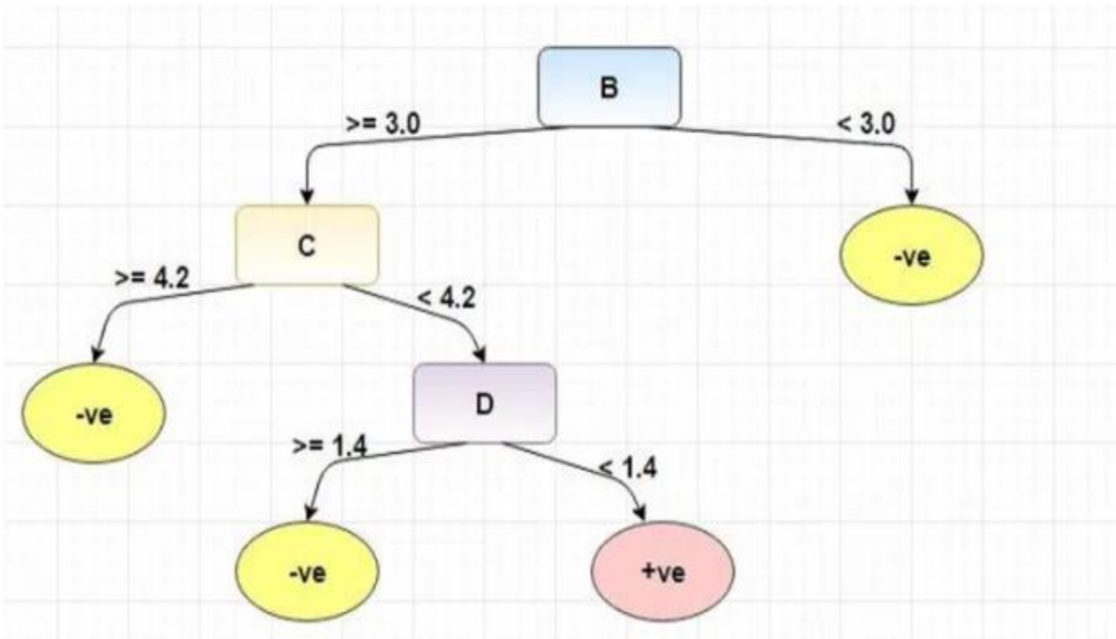
		Target	
		Positive	Negative
B	>= 3.0	8	4
	< 3.0	0	4
Information Gain of B= 0.7070795			

		Target	
		Positive	Negative
C	>= 4.2	0	6
	< 4.2	8	2
Information Gain of C= 0.5488			

		Target	
		Positive	Negative
D	>= 1.4	0	5
	< 1.4	8	3
Information Gain of D= 0.41189			

From the above Information Gain calculations, we can build a decision tree. We should place the attributes on the tree according to their values.

An Attribute with better value than other should position as root and A branch with entropy 0 should be converted to a leaf node. A branch with entropy more than 0 needs further splitting.



Gini Index

Gini Index is a metric to measure how often a randomly chosen element would be incorrectly identified. It means an **attribute with lower Gini index should be preferred**.

Example: Construct a Decision Tree by using "gini index" as a criterion

	A	B	C	D	E
1	4.8	3.4	1.9	0.2	positive
2	5	3	1.6	0.2	positive
3	5	3.4	1.6	0.4	positive
4	5.2	3.5	1.5	0.2	positive
5	5.2	3.4	1.4	0.2	positive
6	4.7	3.2	1.6	0.2	positive
7	4.8	3.1	1.6	0.2	positive
8	5.4	3.4	1.5	0.4	positive
9	7	3.2	4.7	1.4	negative
10	6.4	3.2	4.5	1.5	negative
11	6.9	3.1	4.9	1.5	negative
12	5.5	2.3	4	1.3	negative
13	6.5	2.8	4.6	1.5	negative
14	5.7	2.8	4.5	1.3	negative
15	6.3	3.3	4.7	1.6	negative
16	4.9	2.4	3.3	1	negative

We are going to use same data sample that we used for information gain example. Let's try to use gini index as a criterion. Here, we have 5 columns out of which 4 columns have continuous data and 5th column consists of class labels.

A, B, C, D attributes can be considered as predictors and E column class labels can be considered as a target variable. For constructing a decision tree from this data, we have to convert continuous data into categorical data.

We have chosen some random values to categorize each attribute:

A	B	C	D
>= 5	>= 3.0	>=4.2	>= 1.4
< 5	< 3.0	< 4.2	< 1.4

Gini Index for Var A

Var A has value >=5 for 12 records out of 16 and 4 records with value <5 value.

- For Var A >= 5 & class == positive: 5/12
- For Var A >= 5 & class == negative: 7/12
    - gini(5,7) = 1- ((5/12)2 + (7/12)2) = 0.4860
- For Var A <5 & class == positive: 3/4
- For Var A <5 & class == negative: 1/4
    - gini(3,1) = 1- ((3/4)2 + (1/4)2) = 0.375

By adding weight and sum each of the gini indices:

$$\mathrm{gini}(\mathrm{Target},\ \mathrm{A}) = (12/16)\ *\ (0.486) + (4/16)\ *\ (0.375)\ = 0.45825$$

Gini Index for Var B

Var B has value >=3 for 12 records out of 16 and 4 records with value <5 value.

- For Var B >= 3 & class == positive: 8/12
- For Var B >= 3 & class == negative: 4/12
  - gini(8,4) = 1- ((8/12)2 + (4/12)2) = 0.446
- For Var B <3 & class == positive: 0/4
- For Var B <3 & class == negative: 4/4
  - gin(0,4) = 1- ((0/4)2 + (4/4)2) = 0

$$\mathrm{gini(Target,\ B)} = (12/16)\ *\ 0.446 + (4/16)\ *\ 0 = 0.3345$$

Gini Index for Var C

Var C has value >=4.2 for 6 records out of 16 and 10 records with value <4.2 value.

- For Var C >= 4.2 & class == positive: 0/6
- For Var C >= 4.2 & class == negative: 6/6
  - gini(0,6) = 1- ((0/8)2 + (6/6)2) = 0
- For Var C < 4.2& class == positive: 8/10
- For Var C < 4.2 & class == negative: 2/10
  - gin(8,2) = 1- ((8/10)2 + (2/10)2) = 0.32

$$\mathrm{gini(Target,\ C)} = (6/16)\ *\ 0 + (10/16)\ *\ 0.32 = 0.2$$

Gini Index for Var D

Var D has value >=1.4 for 5 records out of 16 and 11 records with value <1.4 value.

- For Var D >= 1.4 & class == positive: 0/5
- For Var D >= 1.4 & class == negative: 5/5
  - gini(0,5) = 1- ((0/5)2 + (5/5)2) = 0
- For Var D < 1.4 & class == positive: 8/11
- For Var D < 1.4 & class == negative: 3/11
  - gin(8,3) = 1- ((8/11)2 + (3/11)2) = 0.397

$$\mathrm{gini(Target,\ D)} = (5/16)\ *\ 0 + (11/16)\ *\ 0.397 = 0.273$$

		wTarget	
		Positive	Negative
A	>= 5.0	5	7
	<5	3	1
Ginin Index of A = 0.45825			

		Target	
		Positive	Negative
B	>= 3.0	8	4
	< 3.0	0	4
Gini Index of B= 0.3345			

		Target	
		Positive	Negative
C	>= 4.2	0	6
	< 4.2	8	2
Gini Index of C= 0.2			

		Target	
		Positive	Negative
D	>= 1.4	0	5
	< 1.4	8	3
Gini Index of D= 0.273