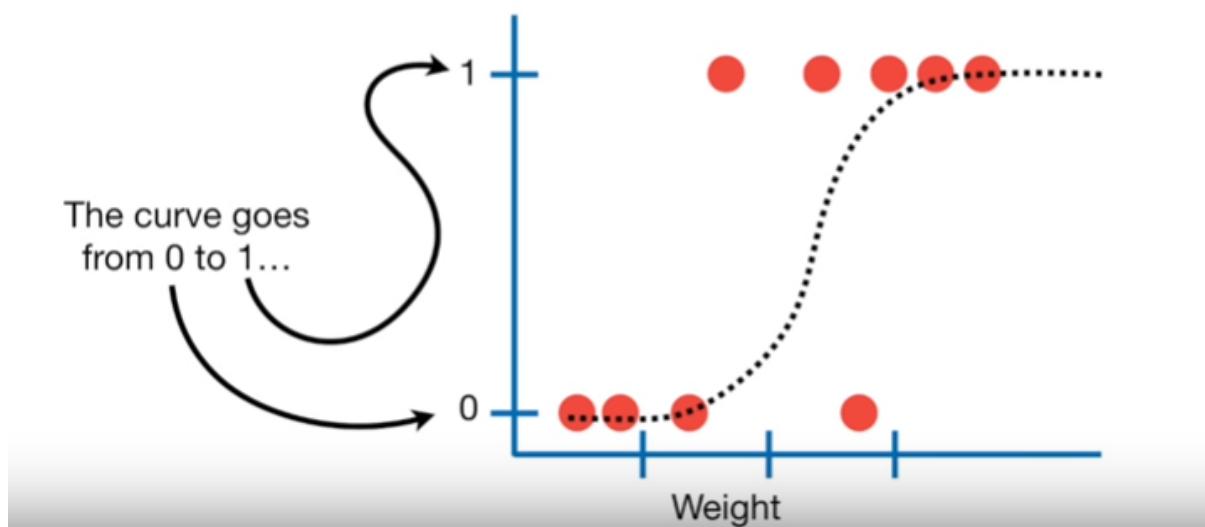
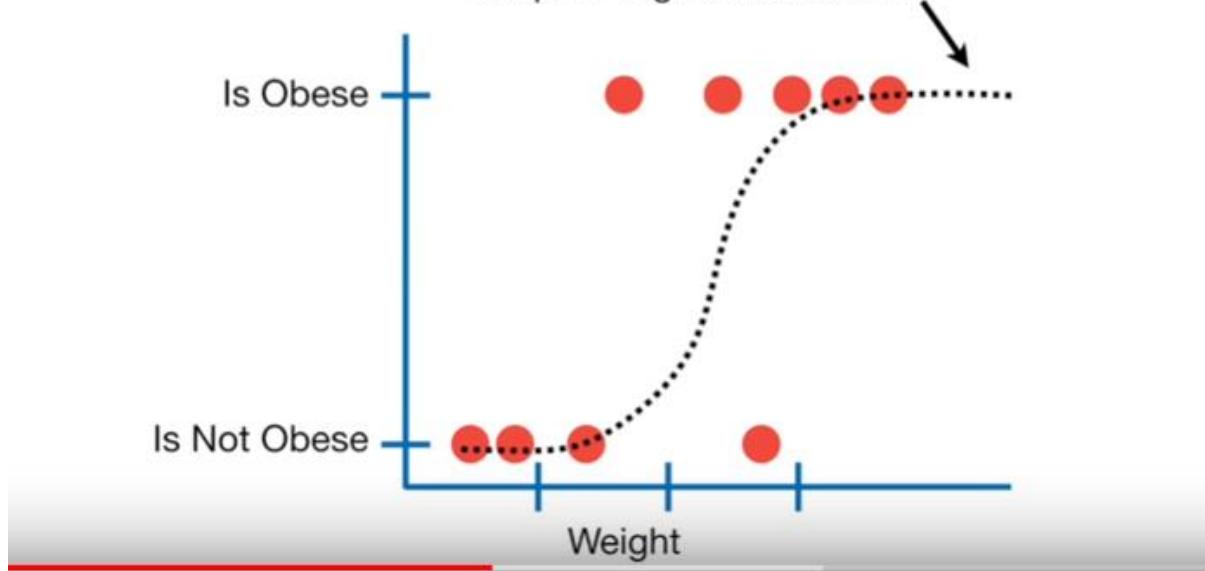


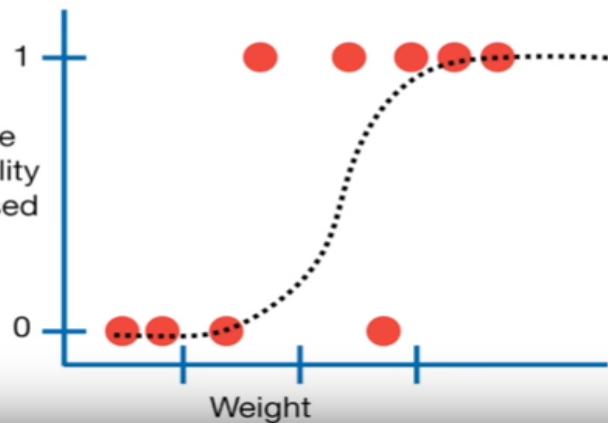
## Logistic Regression:

Logistic regression: instead of fitting a line to data, Logistic regression fits an “S” shaped “logistic function”.

...also, instead of fitting a line to the data, logistic regression fits an “S” shaped “logistic function”.



...and that means that the curve tells you the probability that a mouse is ***obese*** based on its ***weight***.



## How Gradient Boosting Classification Works

We are going to use following data:

In this StatQuest we will use this  
**Training Data...**



Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Where we have collected **Popcorn preferences** from **6 People**:

...where we have collected  
**Popcorn Preference** from **6**  
people...

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Their Age:

...their Age...

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Their Favourite **Color**:

...their **Favorite Color**...

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

And whether or not they love movie **Troll 2**:

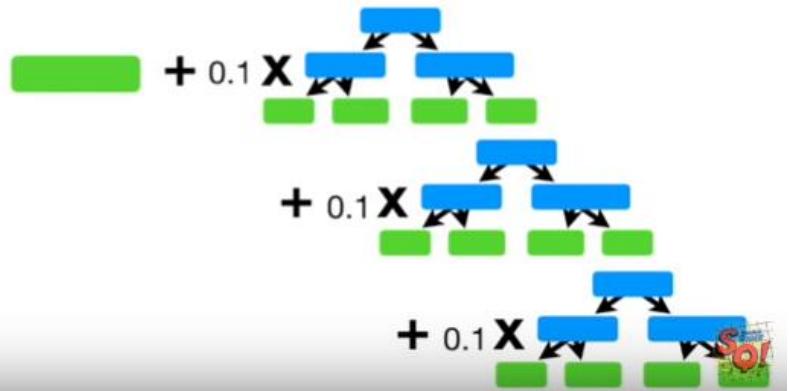
...and whether or not they  
love the movie **Troll 2**...

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

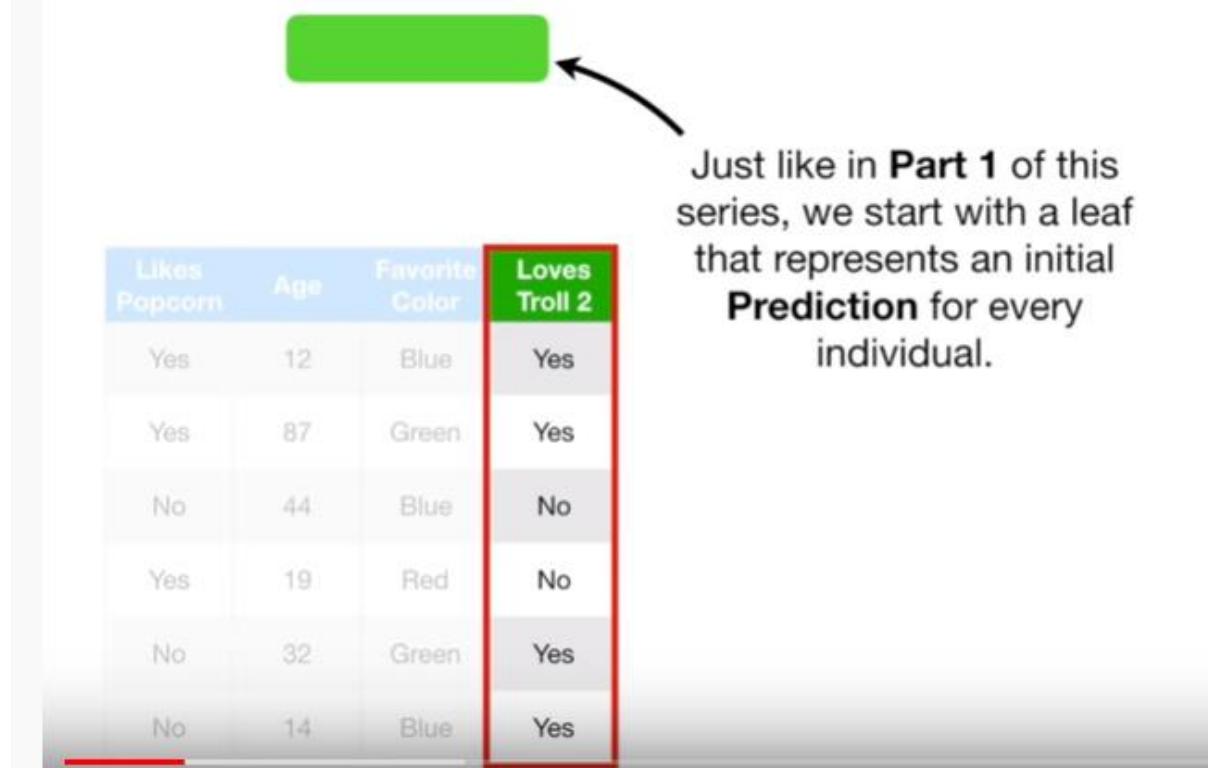
We will walk-through Step by Step to build a Model:

...and walk through, step-by-step, the most common way that **Gradient Boost** fits a model to this **Training Data**.

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes



We will start with a **Leaf** that represent Initial Prediction for every individuals:



Step 1: We will initialize an Initial Constant model ( $F_0(x)$ ) which consist of constant value for every individual. The initial Prediction for each individual will be **Log (Odds)**



When we use **Gradient Boost for Classification**, the initial **Prediction** for every individual is the **log(odds)**.

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes



When we use **Gradient Boost for Classification**, the initial **Prediction** for every individual is the **log(odds)**.

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Let's calculate the **Overall Log (Odds)** that someone loves **Troll 2**

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

So let's calculate the overall **log(odds)** that someone **Loves Troll 2**.

Logistic regression model always calculate/predict **Log (odds)/Probability** with respect to **default class**. Here default class is **Yes** (Love Troll 2). 4 People Love Troll 2 and 2 People doesn't love Troll 2.

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Since **4** people in the **Training Dataset** **Love Troll 2...**

So,  $\text{Log (odds)} = p/1-p$ ,  $p$  is probability of loving Troll 2 and  $1-p$  is the probability of not loving Troll 2.

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

← ...and 2 people do not...

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

...then the **log( odds )** that someone **Loves Troll 2** is...

$$\log\left(\frac{4}{2}\right) = 0.7$$

$$\text{Log } (p/1-p) = \text{log } (4/2) = 0.7$$

$\log(4/2) = 0.7$

...which we will put into our initial leaf.

$$\log\left(\frac{4}{2}\right) = 0.7$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

$\log(4/2) = 0.7$

← So this is the **Initial Prediction**.

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Likes_Popcorn	Age	Favorite_Color	Loves Troll 2	Initial_Prediction[Log(odds)=log(4/2=0.7)]
Yes	12	Blue	Yes	0.7
Yes	87	Green	Yes	0.7
No	44	Blue	No	0.7
Yes	19	Red	No	0.7
No	32	Green	Yes	0.7
No	14	Blue	Yes	0.7

$$\log(4/2) = 0.7$$

← So this is the **Initial Prediction**.

How do we use it for  
**Classification**?

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

$$\log(4/2) = 0.7$$

Just like with **Logistic Regression**,  
the easiest way to use the **log(odds)**  
for **Classification** is to convert it to a  
**Probability**...

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

...and we do that with a  
**Logistic Function**.

$$\text{Probability of Loving Troll 2} = \frac{e^{\log(\text{odds})}}{1 + e^{\log(\text{odds})}}$$

$$\log(4/2) = 0.7$$

So we plug the **log(odds)** into the  
**Logistic Function**...

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

$$\text{Probability of Loving Troll 2} = \frac{e^{\log(\text{odds})}}{1 + e^{\log(\text{odds})}}$$

$$\log(4/2) = 0.7$$

...do the math...

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

$$\text{Probability of Loving Troll 2} = \frac{e^{\log(4/2)}}{1 + e^{\log(4/2)}} = 0.7$$

$$\log(4/2) = 0.7$$

...and we get **0.7** as the Probability of Loving Troll 2.

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

$$\text{Probability of Loving Troll 2} = \frac{e^{\log(4/2)}}{1 + e^{\log(4/2)}} = 0.7$$

$$\log(4/2) = 0.7$$

$$\text{Probability of Loving Troll 2} = 0.7$$

And let's save that up here for now.

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

$$\text{Probability of Loving Troll 2} = \frac{e^{\log(4/2)}}{1 + e^{\log(4/2)}} = 0.7$$

And Probability equals to 0.6667 after taking 4digits after decimal:

log(4/2) = 0.7

Probability of Loving Troll 2 = 0.7

**NOTE:** These two numbers, the **log(4/2)** and the **Probability** are the same only because I'm rounding. If I allowed 4 digits passed the decimal place...

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

log(4/2) = 0.7

Probability of Loving Troll 2 = 0.7

Since the **Probability of Loving Troll 2** is greater than **0.5**, we can **Classify** everyone in the **Training Dataset** as someone who **Loves Troll 2**.

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Likes_Popcorn	Age	Favorite_Color	Loves_Troll_2	Initial_prediction[Log(odds)=log(4/2=0.7)]	Probability
Yes	12	Blue	Yes	0.7	0.7
Yes	87	Green	Yes	0.7	0.7
No	44	Blue	No	0.7	0.7
Yes	19	Red	No	0.7	0.7
No	32	Green	Yes	0.7	0.7
No	14	Blue	Yes	0.7	0.7

$\log(4/2) = 0.7$   
Probability of  
Loving Troll 2 = 0.7

Since the **Probability of Loving Troll 2** is greater than **0.5**, we can **Classify** everyone in the **Training Dataset** as someone who **Loves Troll 2**.

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

**NOTE:** While **0.5** is a very common threshold for making **Classification** decisions based on **Probability**, we could have just as easily used a different value.

For more details, check out the **StatQuest: ROC and AUC, Clearly Explained.**



$\log(4/2) = 0.7$   
Probability of  
Loving Troll 2 = 0.7

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Now, **Classifying** everyone in the **Training Dataset** as someone who **Loves Troll 2** is pretty lame, because two of the people do not love the movie.

$\log(4/2) = 0.7$   
Probability of  
Loving Troll 2 = 0.7

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

We can measure how bad the initial **Prediction** is by calculating **Pseudo Residuals**, the difference between the **Observed** and the **Predicted** values.

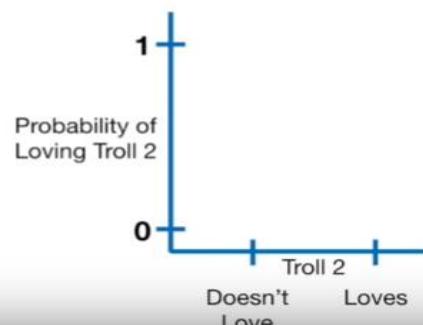
**Residual = (Observed - Predicted)**

$$\log(4/2) = 0.7$$

Probability of  
Loving Troll 2 = 0.7

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Although the math is easy, I think it's easier to grasp what's going on if we draw the **Residuals** on a graph.

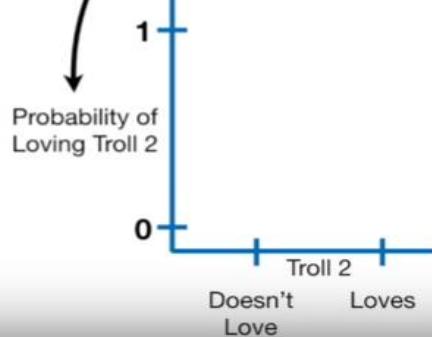


$$\log(4/2) = 0.7$$

Probability of  
Loving Troll 2 = 0.7

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

The y-axis is the **Probability of Loving Troll 2**...

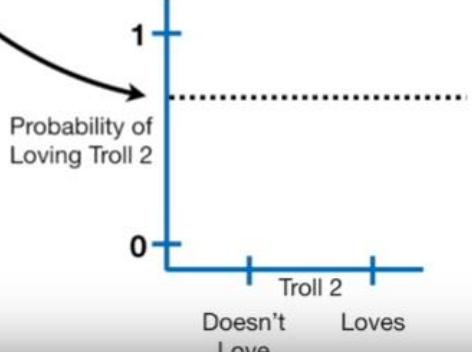


$$\log(4/2) = 0.7$$

Probability of  
Loving Troll 2 = 0.7

The **Predicted Probability of Loving Troll 2 is 0.7**.

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

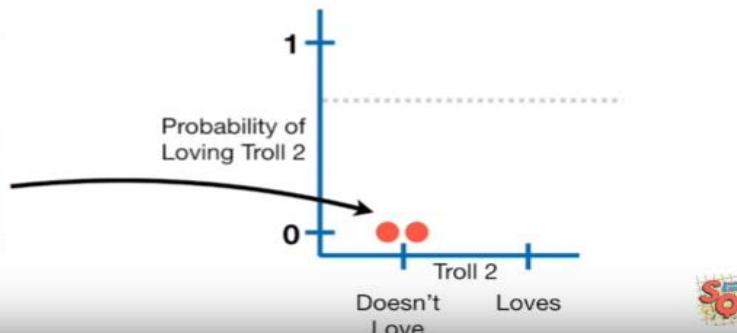


$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

The **Red Dots**, with the **Probability of Loving Troll 2 = 0**, represent the two people that **do not Love Troll 2...**

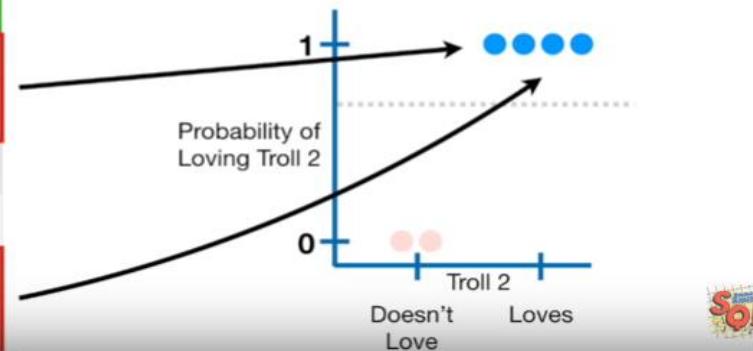


$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

...and the **Blue Dots**, with a **Probability of Loving Troll 2 = 1**, represent the four people that **Love Troll 2.**

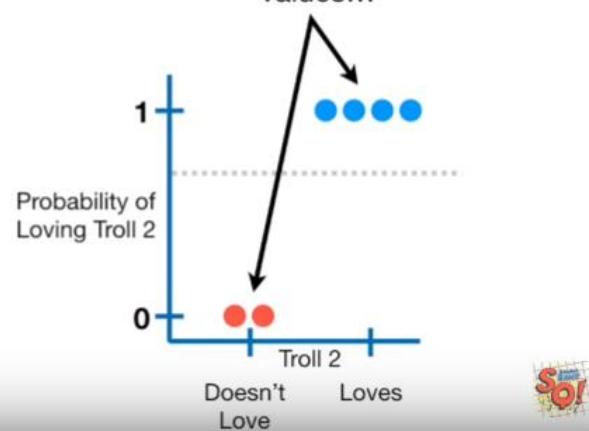


$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

In other words, the **Red** and **Blue** dots are the **Observed** values...

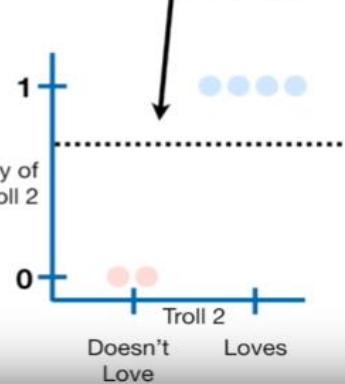


$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

...and the dotted line is the **Predicted** value.

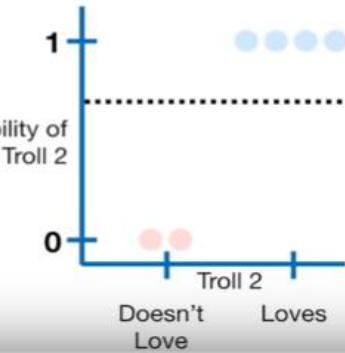


$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

So for this sample...



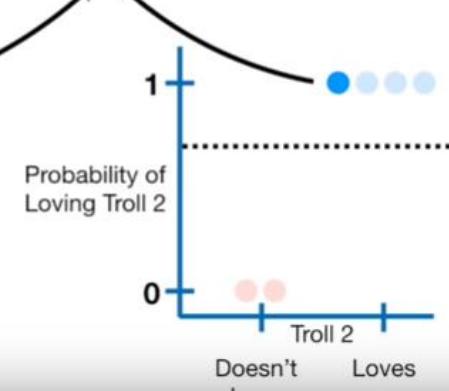
$$\log(4/2) = 0.7$$

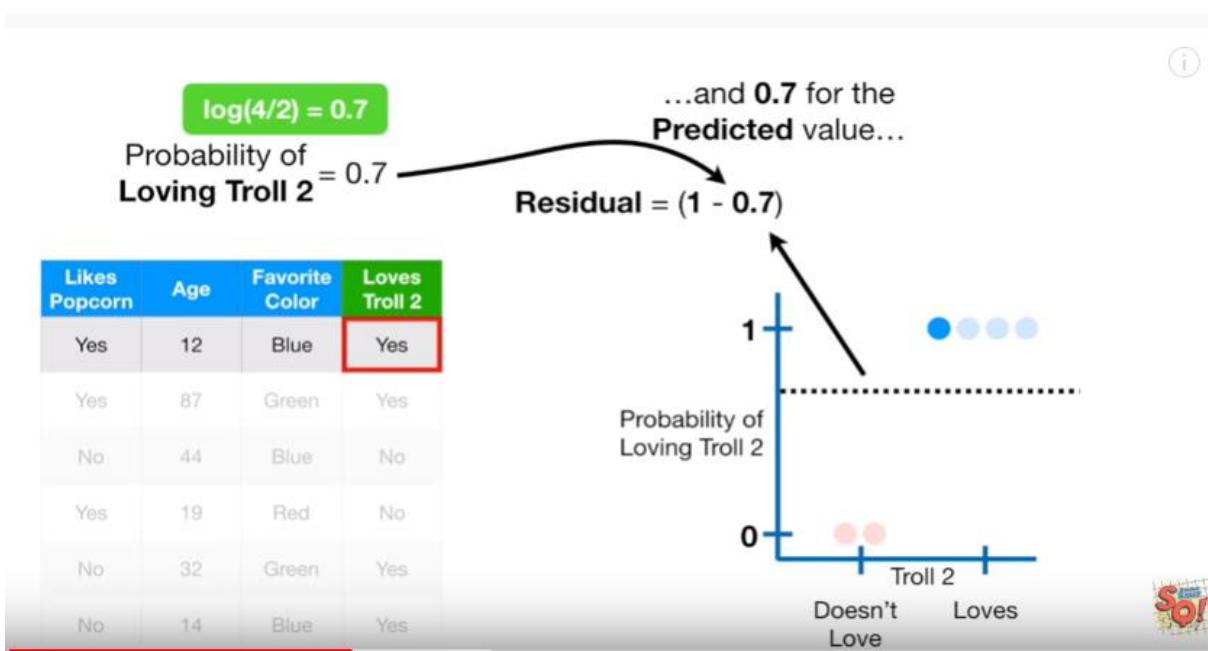
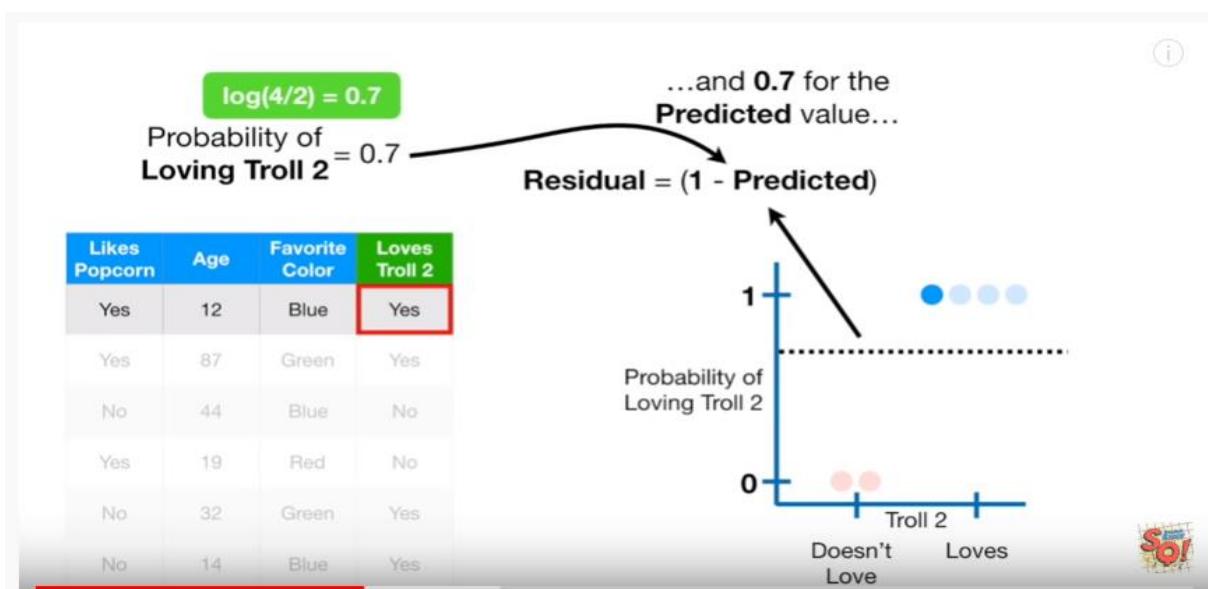
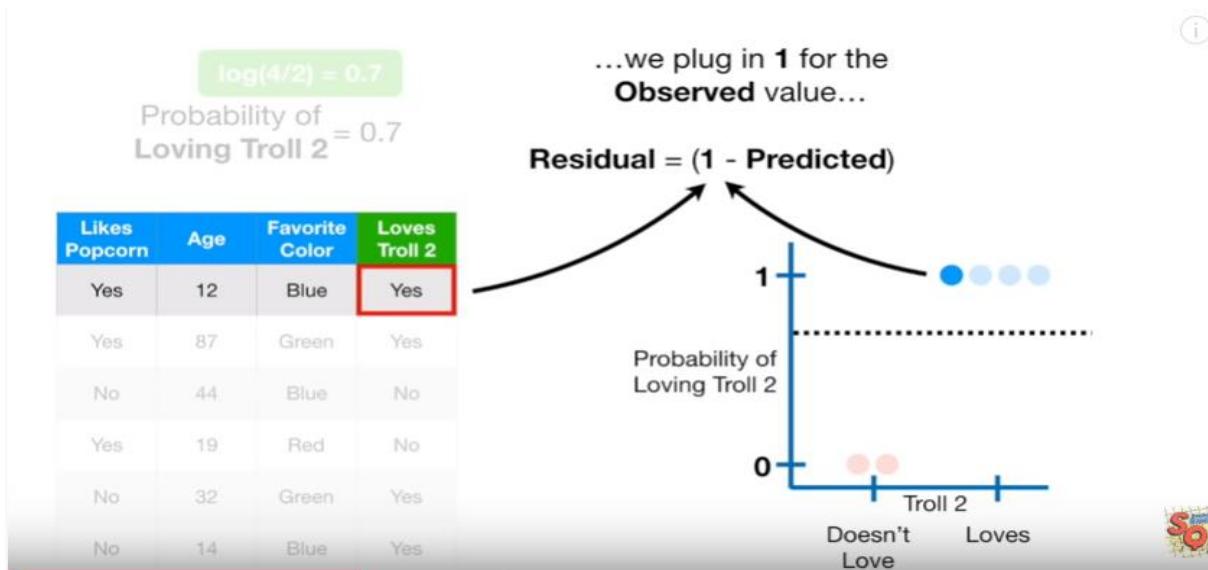
Probability of Loving Troll 2 = 0.7

...we plug in 1 for the **Observed** value...

$$\text{Residual} = (\text{Observed} - \text{Predicted})$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes





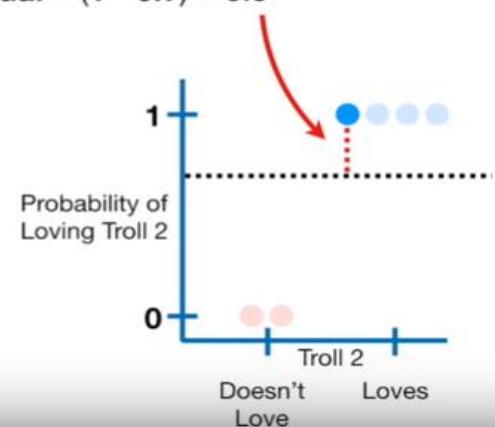
$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

...and we get **0.3...**

$$\text{Residual} = (1 - 0.7) = 0.3$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes



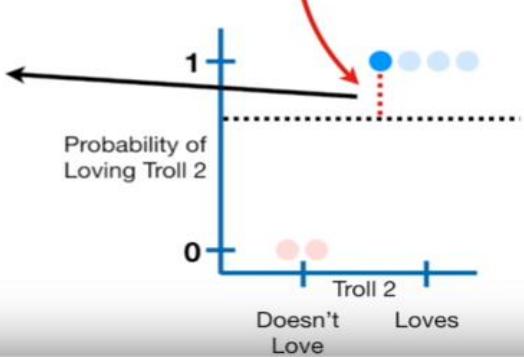
$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

...and we save the **Residual** in a new column.

$$\text{Residual} = (1 - 0.7) = 0.3$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	



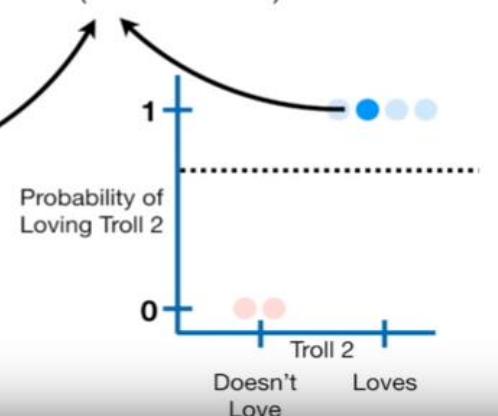
$$\log(4/2) = 0.7$$

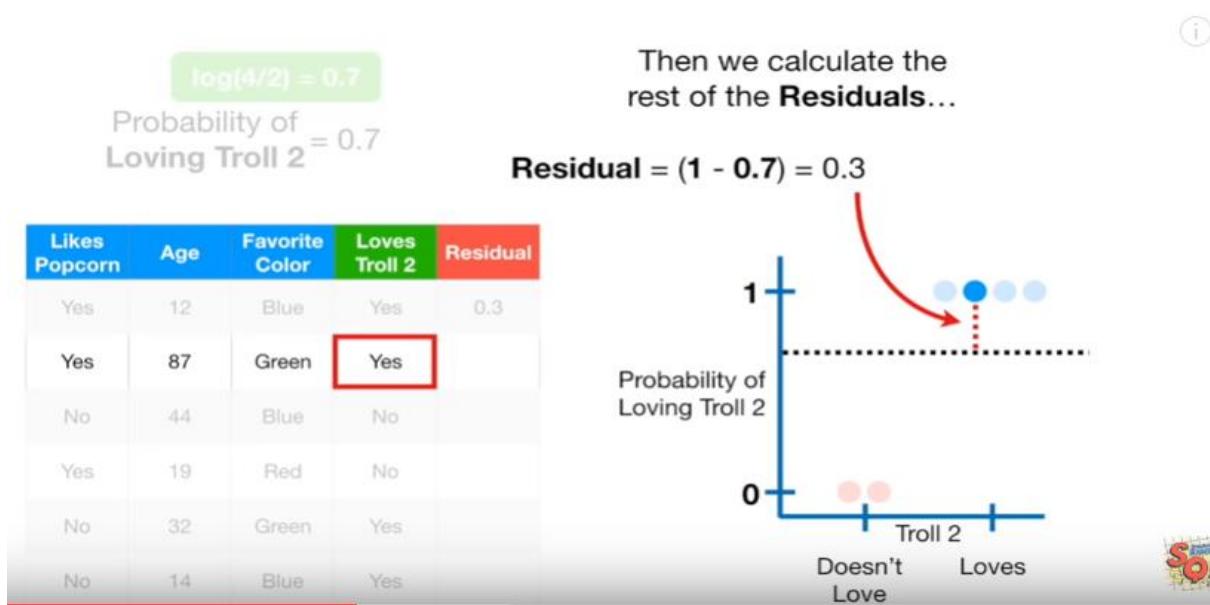
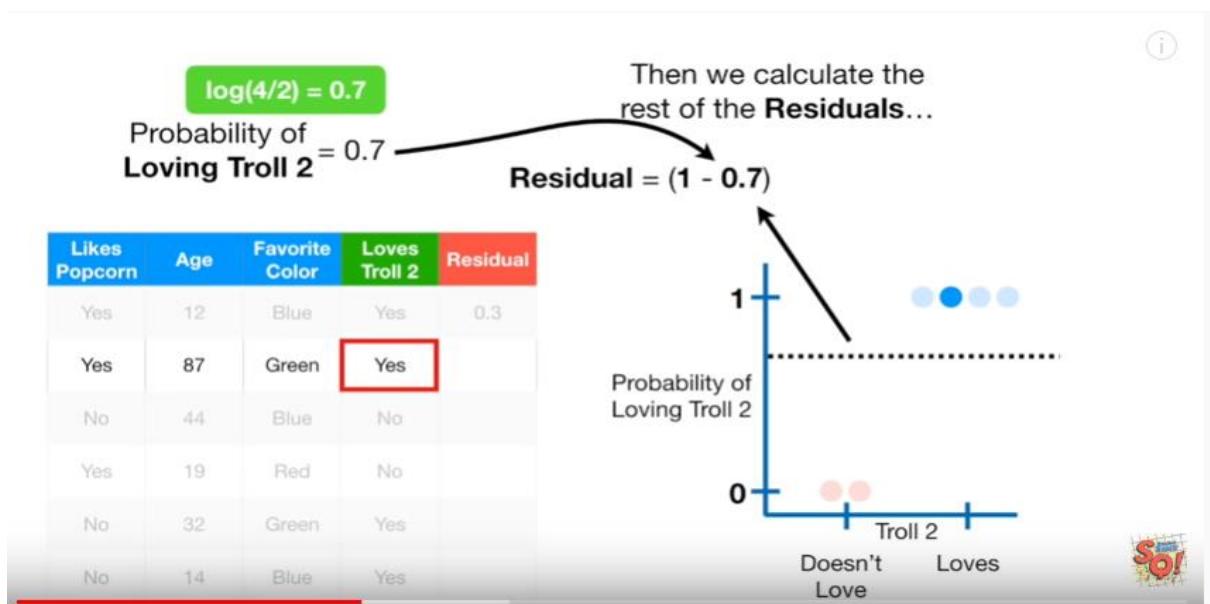
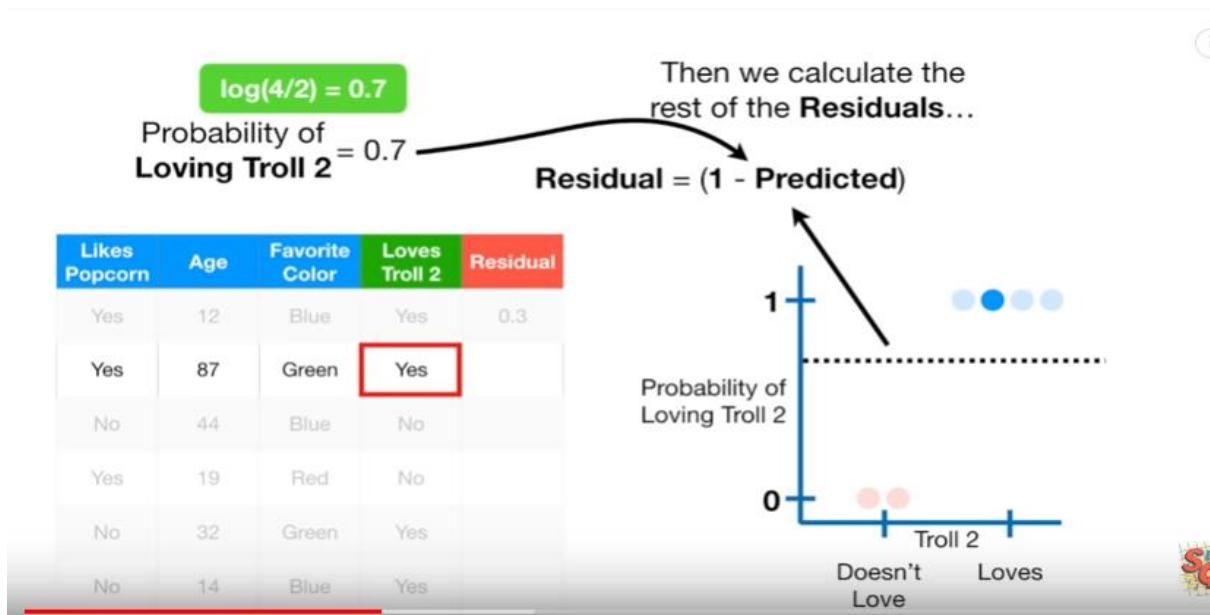
Probability of Loving Troll 2 = 0.7

Then we calculate the rest of the **Residuals**...

$$\text{Residual} = (1 - \text{Predicted})$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	





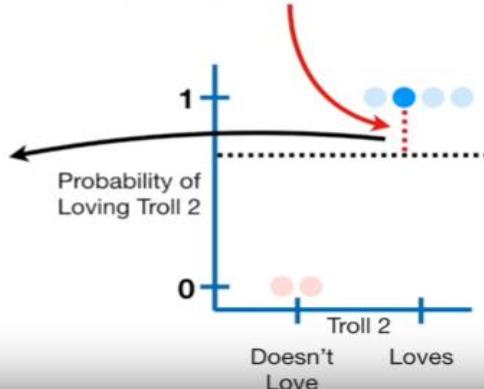
$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

$$\text{Residual} = (1 - 0.7) = 0.3$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

Then we calculate the rest of the **Residuals**...



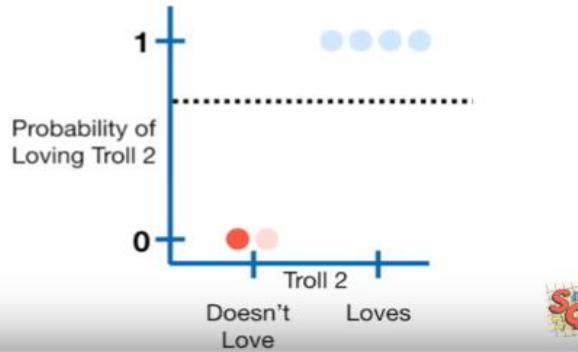
$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

$$\text{Residual} = (\text{Observed} - \text{Predicted})$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

Then we calculate the rest of the **Residuals**...



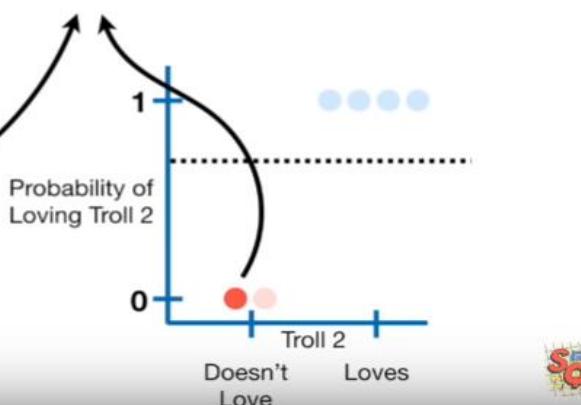
$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

$$\text{Residual} = (\text{Observed} - \text{Predicted})$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

Then we calculate the rest of the **Residuals**...

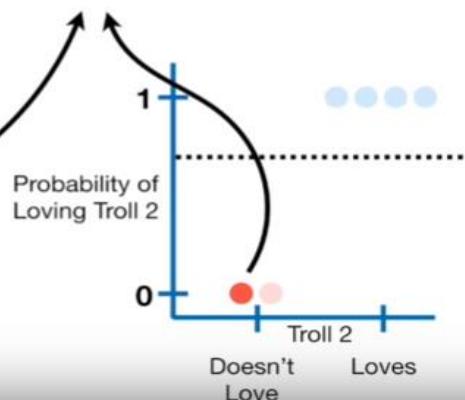


$\log(4/2) = 0.7$   
Probability of Loving Troll 2 = 0.7

Then we calculate the rest of the **Residuals**...

**Residual = (0 - Predicted)**

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

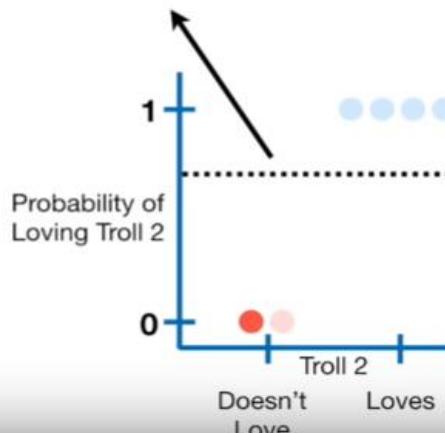


$\log(4/2) = 0.7$   
Probability of Loving Troll 2 = 0.7

Then we calculate the rest of the **Residuals**...

**Residual = (0 - Predicted)**

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

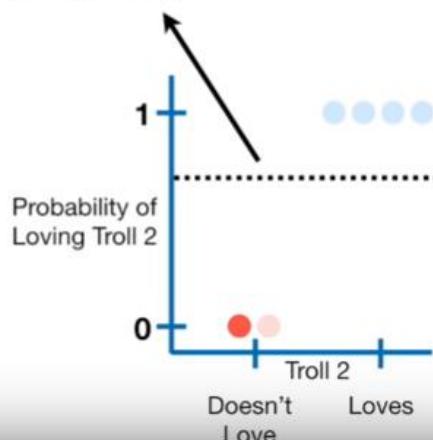


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Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	



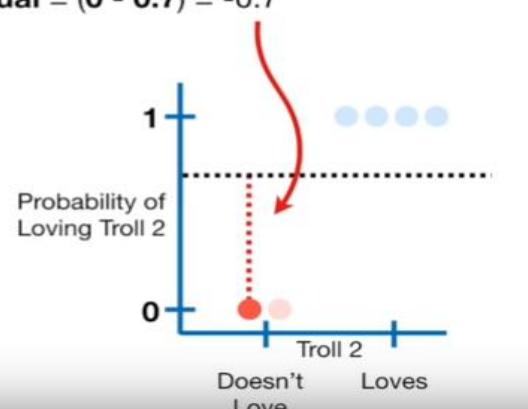
$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

$$\text{Residual} = (0 - 0.7) = -0.7$$

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Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

Then we calculate the rest of the **Residuals**...



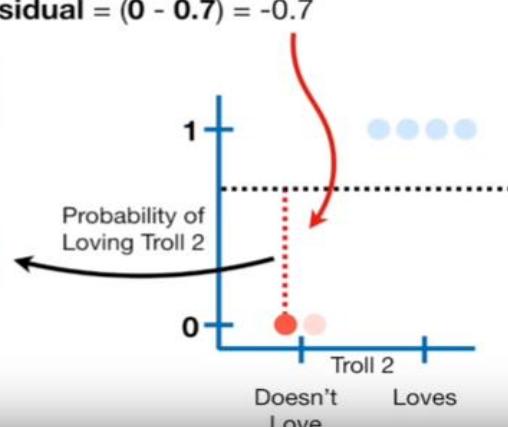
$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

$$\text{Residual} = (0 - 0.7) = -0.7$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	-0.7
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

Then we calculate the rest of the **Residuals**...



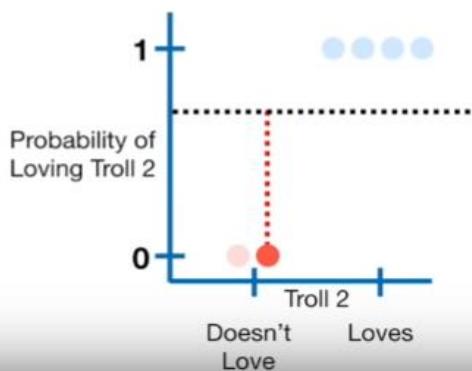
$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

$$\text{Residual} = (\text{Observed} - \text{Predicted})$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	-0.7
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

Then we calculate the rest of the **Residuals**...



$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

**Residual = (Observed - Predicted)**

Then we calculate the rest of the **Residuals**...



$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

**Residual = (Observed - Predicted)**

Then we calculate the rest of the **Residuals**...

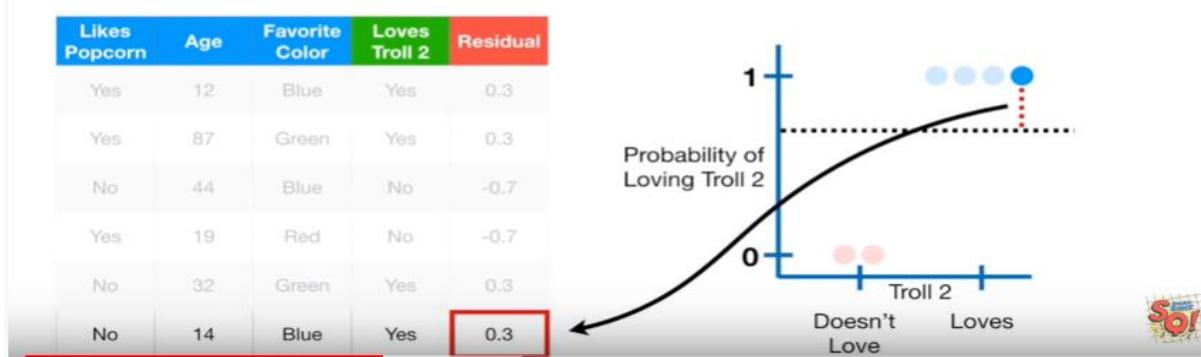


$$\log(4/2) = 0.7$$

Probability of Loving Troll 2 = 0.7

**Residual = (Observed - Predicted)**

Then we calculate the rest of the **Residuals**...



$\log(4/2) = 0.7$

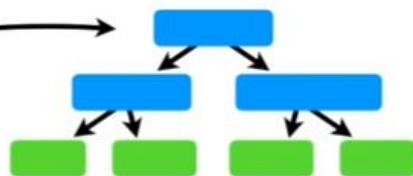
Probability of Loving Troll 2 = 0.7

Hooray! We've calculated the **Residuals** for the leaf's initial **Prediction**.

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	-0.7
Yes	19	Red	No	-0.7
No	32	Green	Yes	0.3
No	14	Blue	Yes	0.3

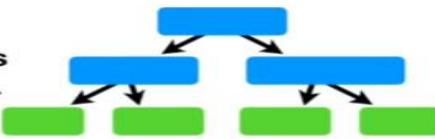
Actual_Probability	Initial_prediction[Log(odds)=log(4/2=0.7)]	Predicted_Probability(1st)	Ressiduals(Actual_Prob - Predicted_Prob)
1	0.7	0.7	1-0.7 = 0.3
1	0.7	0.7	1-0.7 = 0.3
0	0.7	0.7	0-0.7 = -0.7
0	0.7	0.7	0-0.7 = -0.7
1	0.7	0.7	1-0.7 = 0.3
1	0.7	0.7	1-0.7 = 0.3

Now we will build a **Tree**



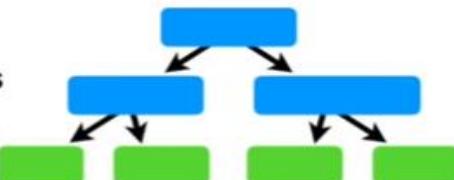
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	-0.7
Yes	19	Red	No	-0.7
No	32	Green	Yes	0.3
No	14	Blue	Yes	0.3

Now we will build a **Tree**, using **Likes Popcorn, Age and Favorite Color...**



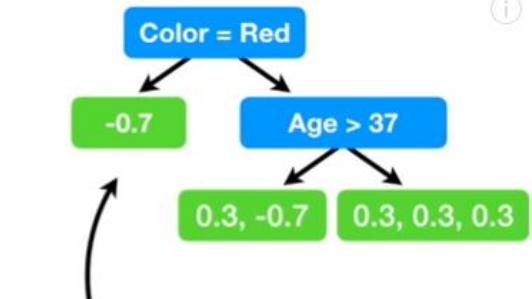
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	-0.7
Yes	19	Red	No	-0.7
No	32	Green	Yes	0.3
No	14	Blue	Yes	0.3

Now we will build a **Tree**, using **Likes Popcorn, Age and Favorite Color...**



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Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	-0.7
Yes	19	Red	No	-0.7
No	32	Green	Yes	0.3
No	14	Blue	Yes	0.3

...to Predict the Residuals.



And here's the tree!

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	-0.7
Yes	19	Red	No	-0.7
No	32	Green	Yes	0.3
No	14	Blue	Yes	0.3



Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
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Yes	87	Green	Yes	0.3
No	44	Blue	No	-0.7
Yes	19	Red	No	-0.7
No	32	Green	Yes	0.3
No	14	Blue	Yes	0.3

**NOTE:** Just like when we used **Gradient Boost for Regression**, we are limiting the number of leaves that we will allow in the tree.

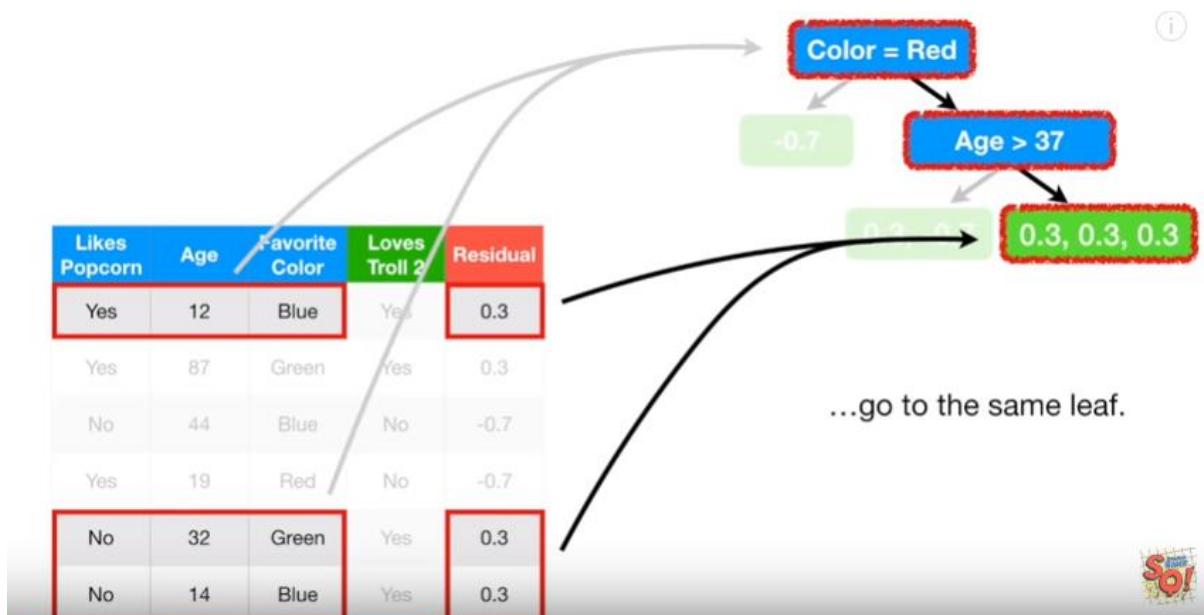
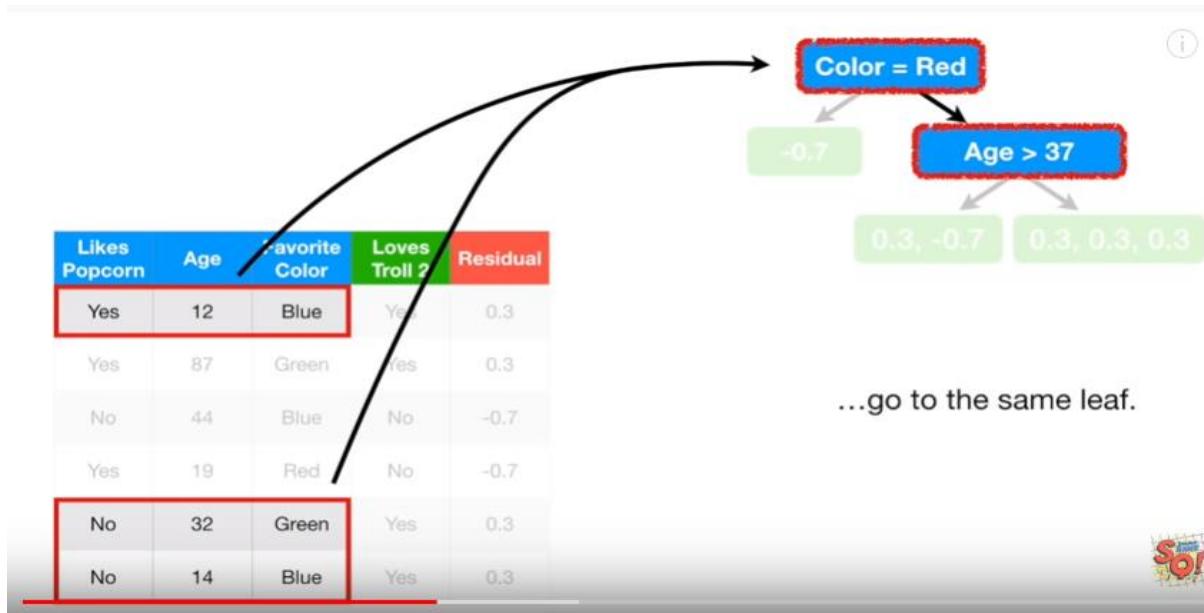
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	-0.7
Yes	19	Red	No	-0.7
No	32	Green	Yes	0.3
No	14	Blue	Yes	0.3

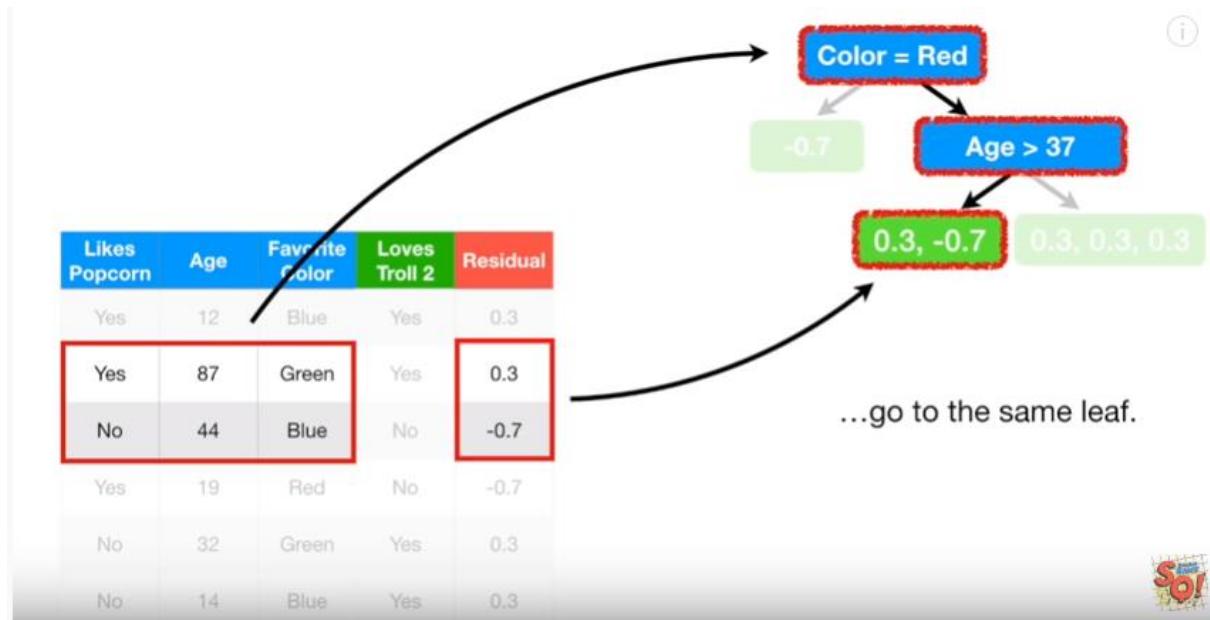
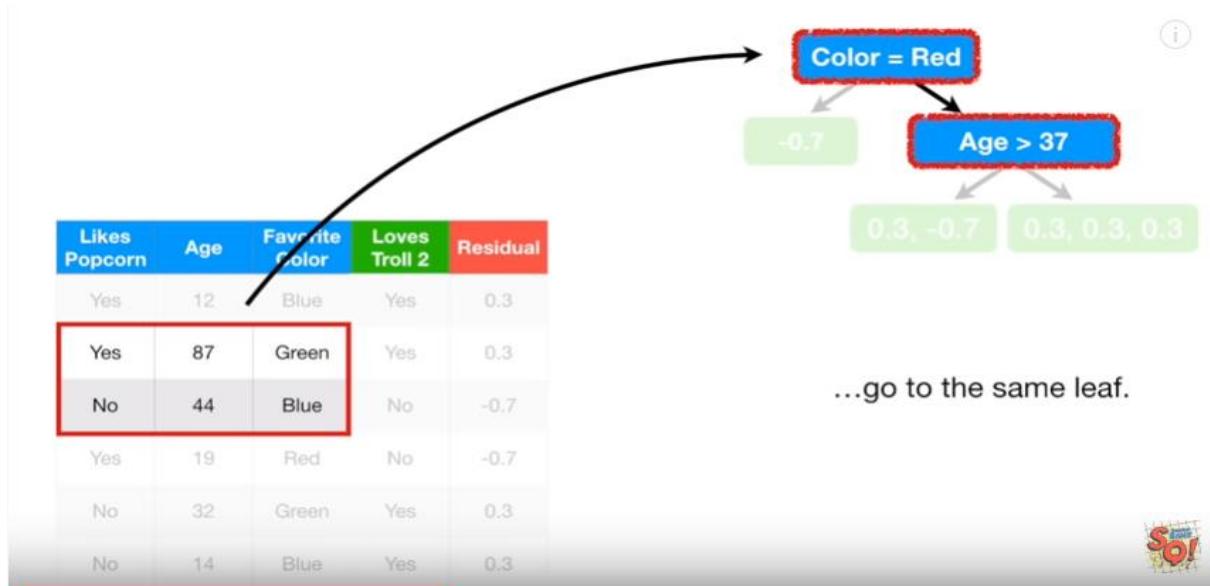
In this simple example, we are limiting the number of leaves to **3**.

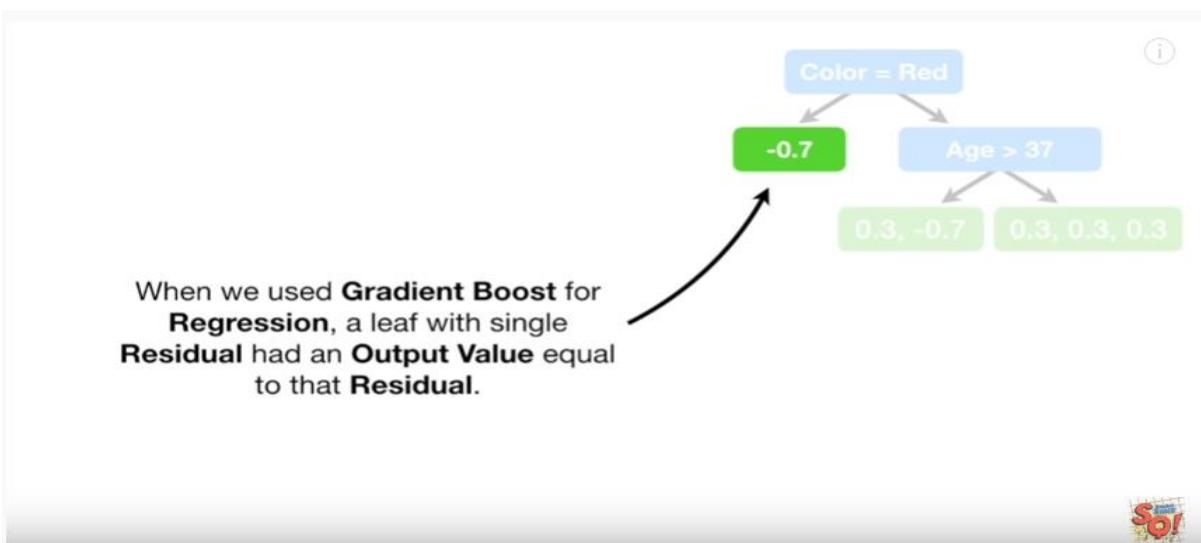
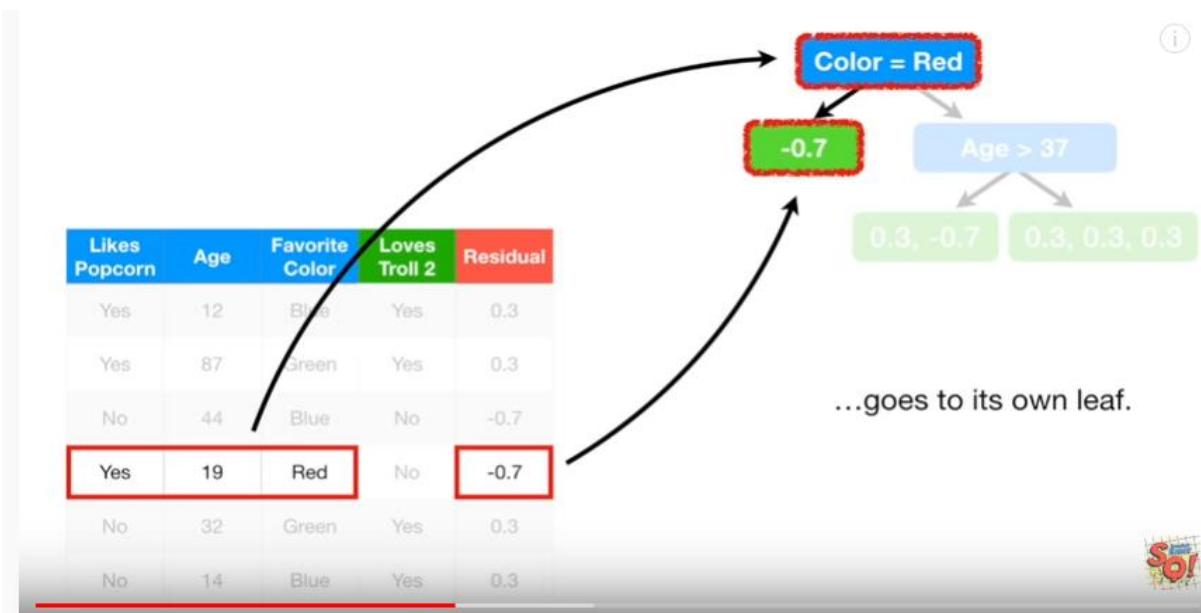
In practice people often set the maximum number of leaves to be between **8** and **32**

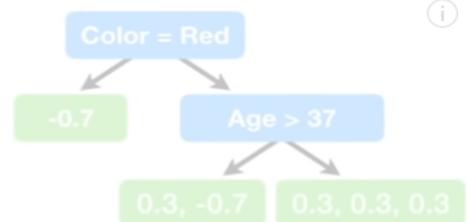
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Residual
Yes	12	Blue	Yes	0.3
Yes	87	Green	Yes	0.3
No	44	Blue	No	-0.7
Yes	19	Red	No	-0.7
No	32	Green	Yes	0.3
No	14	Blue	Yes	0.3

Now let's calculate the **Output Values** for the leaves.







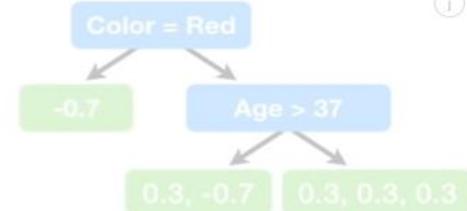


In contrast, when we use **Gradient Boost** for **Classification**, the situation is a little more complex.



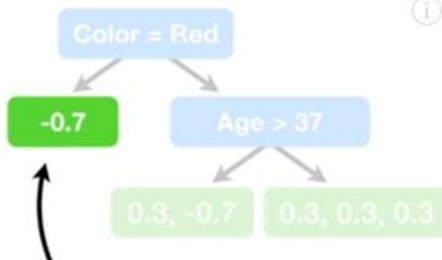
$$\log(4/2) = 0.7$$

This is because the **Predictions** are in terms of the **log(odds)**...

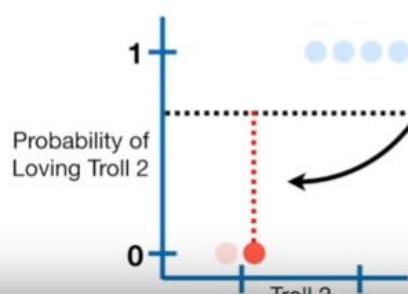


$$\log(4/2) = 0.7$$

This is because the **Predictions** are in terms of the **log(odds)**...



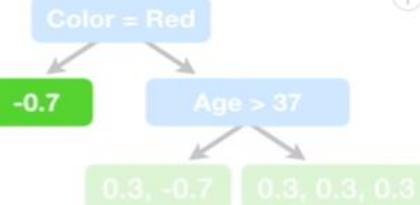
...and this leaf is derived from a **Probability**...



$$\log(4/2) = 0.7$$



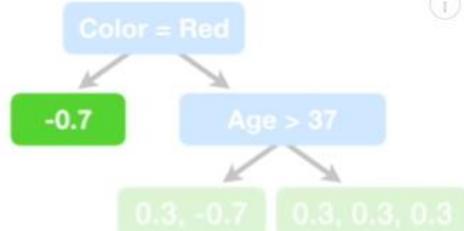
...so we can't just add them together to get a new **log(odds) Prediction** without some sort of transformation.



When we use **Gradient Boost** for **Classification**, the most common transformation is the following formula.



$$\frac{\sum \text{Residual}_i}{\sum [\text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)]}$$



The numerator is the sum of all of the **Residuals** in the leaf...



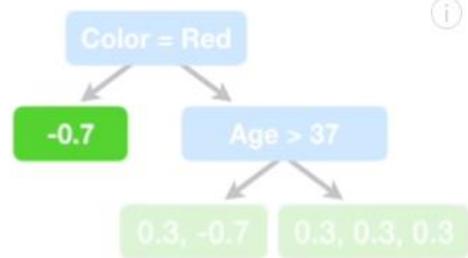
$$\frac{\sum \text{Residual}_i}{\sum [\text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)]}$$



...and the denominator is the sum of the previously predicted probabilities for each **Residual** times 1 minus the same predicted probability.



$\Sigma \text{Residual}_i$



$$\frac{\sum [\text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)]}{\Sigma \text{Residual}_i}$$

**NOTE:** The derivation of this formula is quite technical, so I'm saving it for **Part 4** of this series when we get into the nitty gritty details of **Gradient Boost for Classification**.

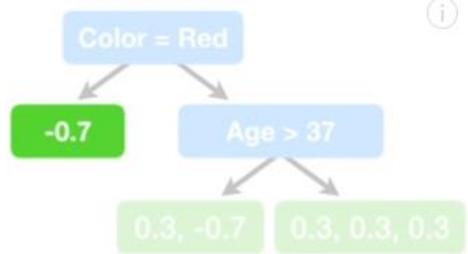


$\Sigma \text{Residual}_i$

$$\frac{\Sigma [\text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)]}{\Sigma \text{Residual}_i}$$



So for now, let's just use the formula to calculate the **Output Value** for this leaf.



$$\frac{\Sigma \text{Residual}_i}{\Sigma [\text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)]}$$

Since there is only one **Residual** in this leaf, we can ignore these summation signs for now.



$$\frac{\sum \text{Residual}_i}{\sum [\text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)]}$$



$$\log(4/2) = 0.7$$

$$\text{Probability of Loving Troll 2} = 0.7$$

So we plug in the **Residual** from the leaf...

$$\frac{\sum \text{Residual}_i}{\sum [\text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)]}$$



$$\log(4/2) = 0.7$$

$$\text{Probability of Loving Troll 2} = 0.7$$

So we plug in the **Residual** from the leaf...

$$\frac{-0.7}{\sum [\text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)]}$$



$\log(4/2) = 0.7$

Probability of Loving Troll 2 =  $0.7$

...and, since we are building the first tree, the **Previous Probability** refers to the probability from the initial leaf...

$$\sum [ \text{Previous Probability}_i \times (1 - \text{Previous Probability}_i) ]$$

$\log(4/2) = 0.7$

Probability of Loving Troll 2 =  $0.7$

...so we plug that in...

$$0.7 \times (1 - 0.7)$$

...do the math...

$$\frac{-0.7}{0.7 \times (1 - 0.7)}$$

...and we end up with -3.3 as the new **Output Value** for this leaf.

$$\frac{-0.7}{0.7 \times (1 - 0.7)} = \boxed{-3.3}$$



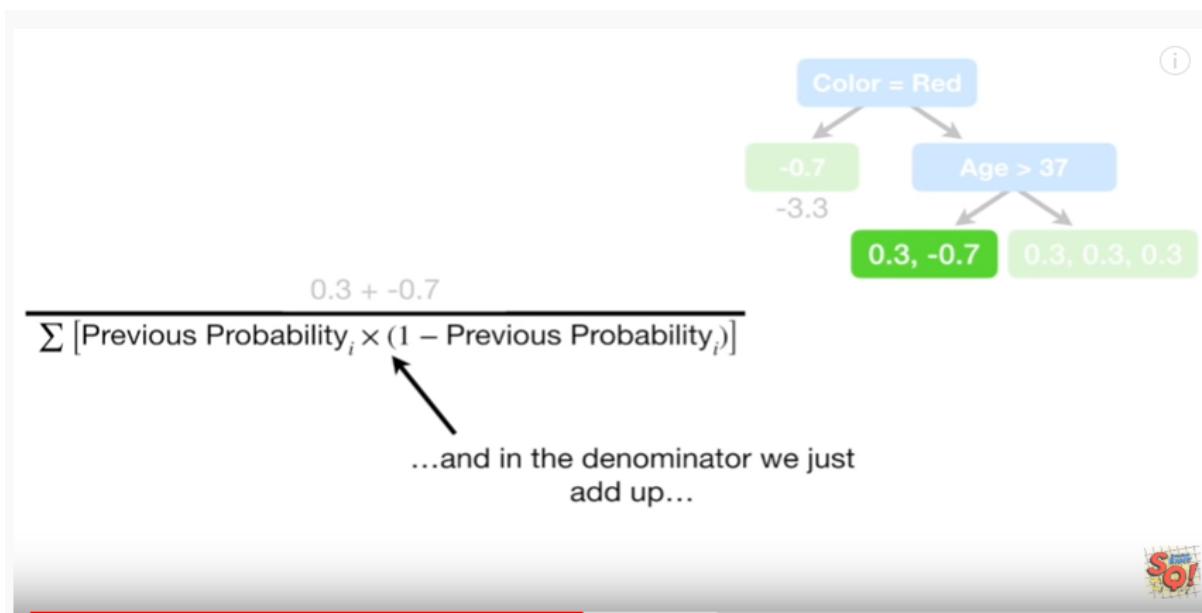
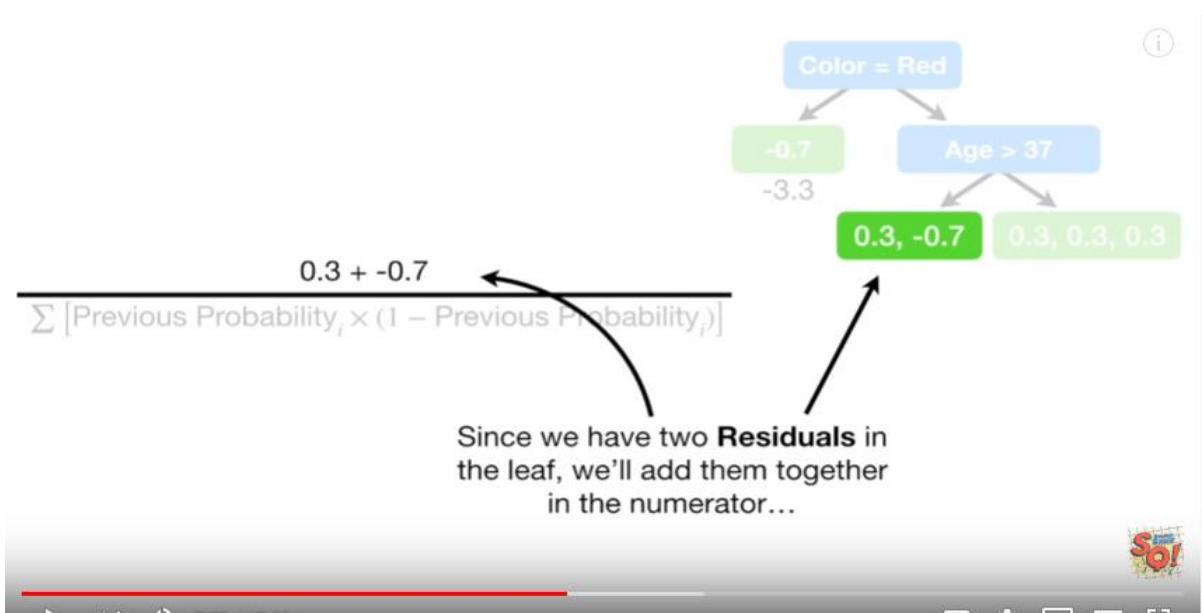
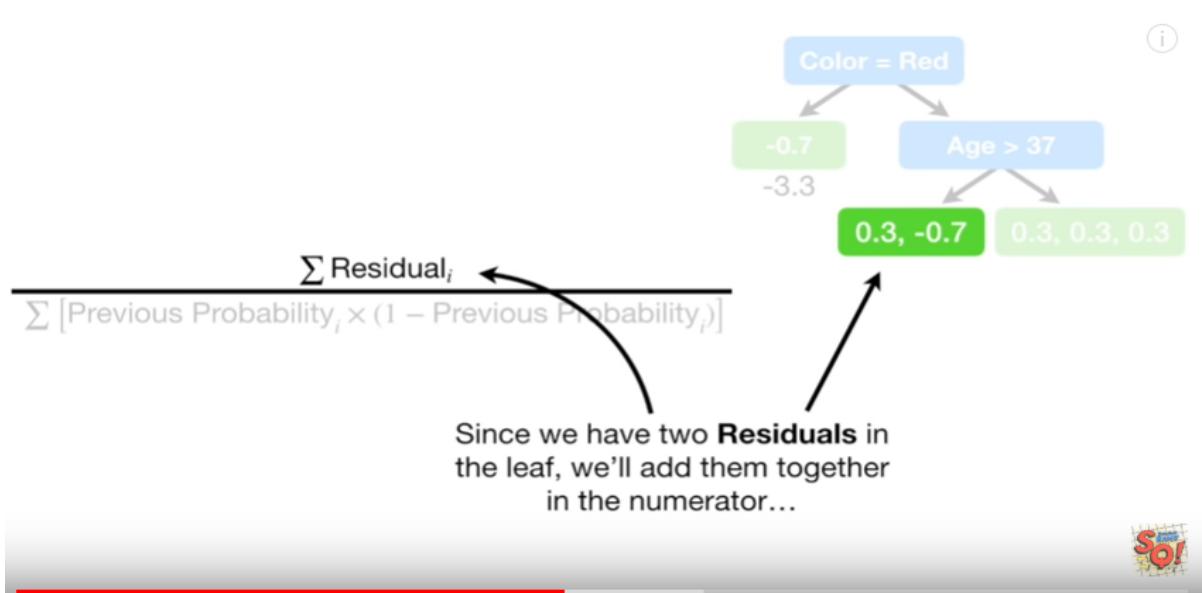
Now we need to calculate the **Output Value** for this leaf.

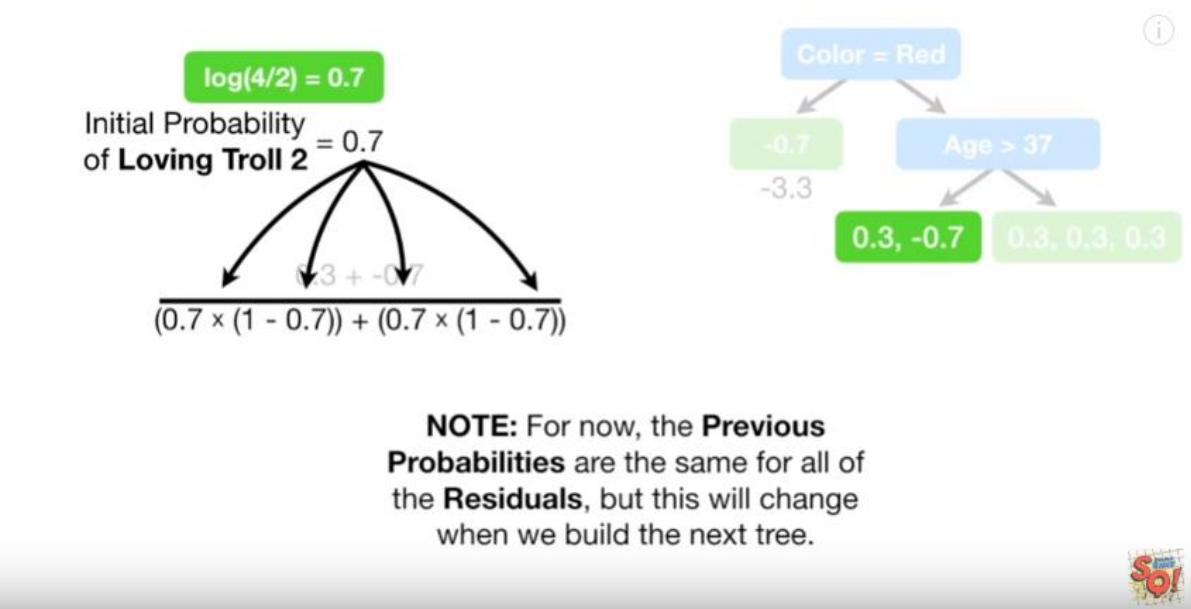
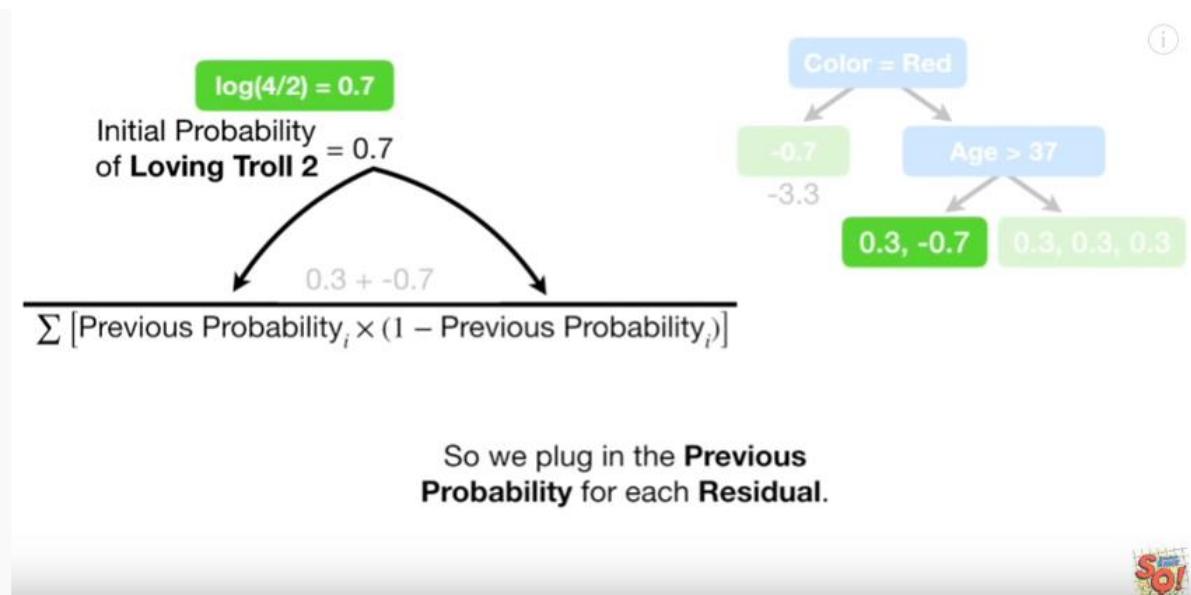
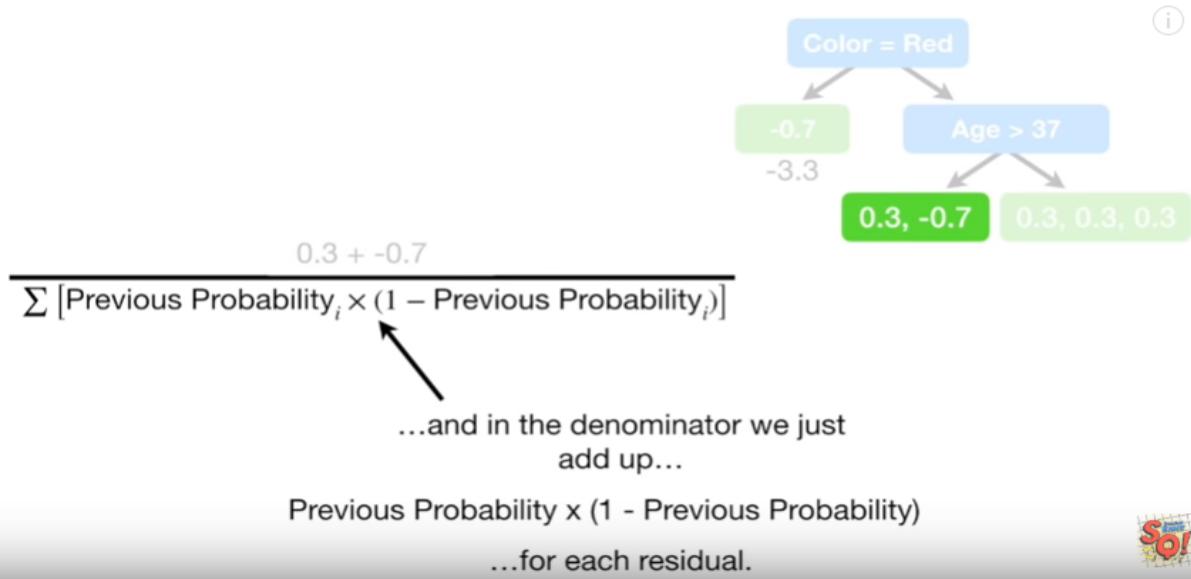


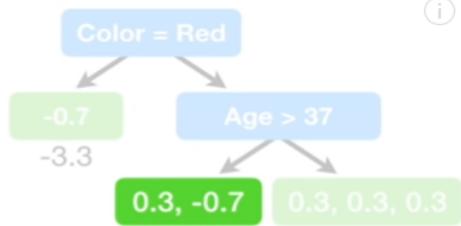
$$\frac{\sum \text{Residual}_i}{\sum [\text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)]}$$

Since we have two **Residuals** in the leaf, we'll add them together in the numerator...









$$\frac{0.3 + -0.7}{2(0.7 \times (1 - 0.7))} = -1$$

Now do the math...



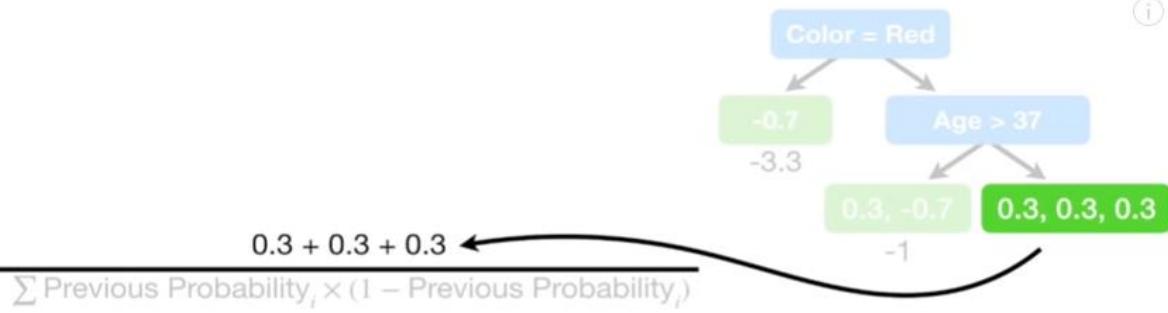
$$\frac{0.3 + -0.7}{2(0.7 \times (1 - 0.7))} = -1$$

...and the **Output Value**  
for this leaf is **-1**.

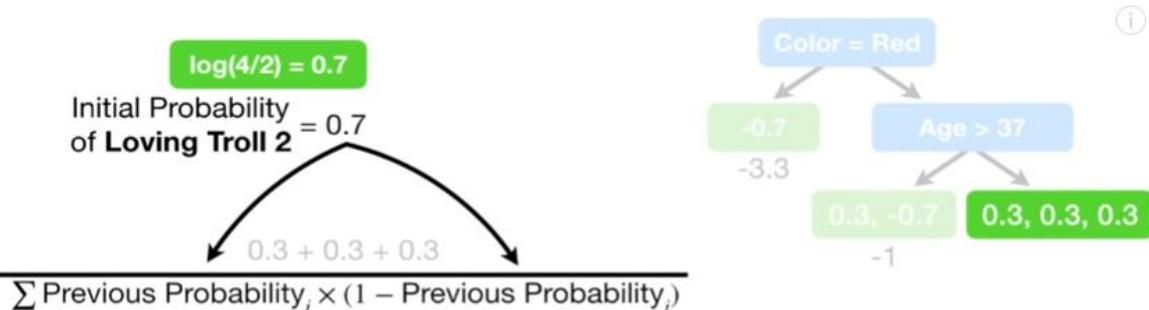


Now let's determine the  
**Output Value** for this leaf...

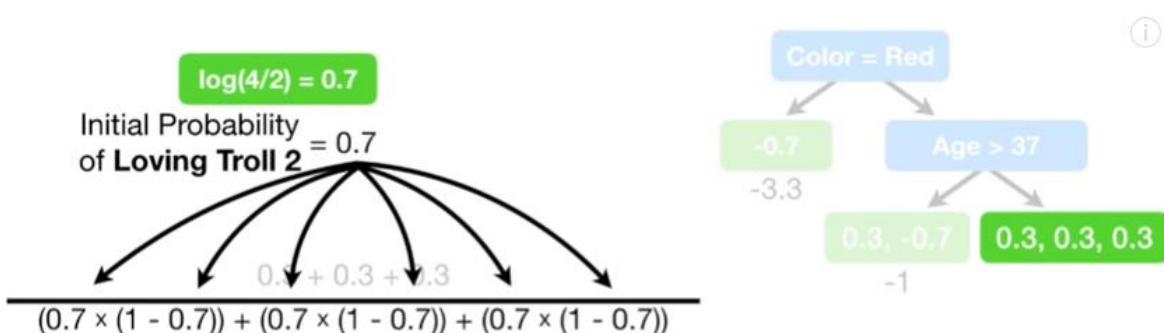




We plug the **Residuals** into the formula...

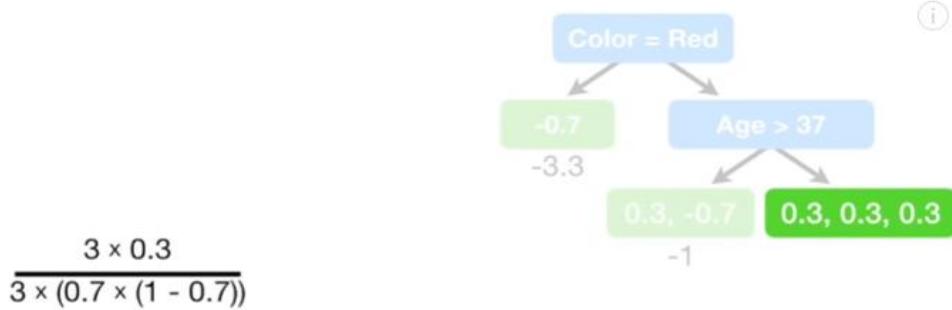


...and the **Previous Probabilities**...

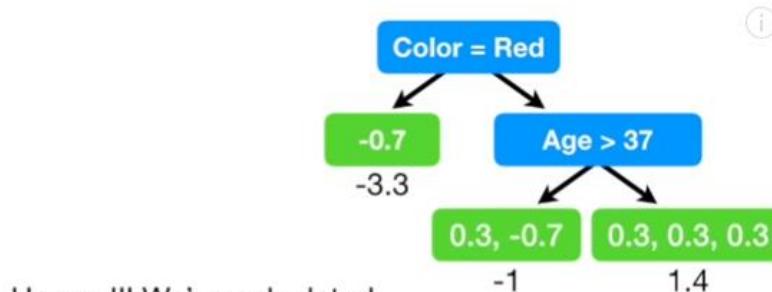
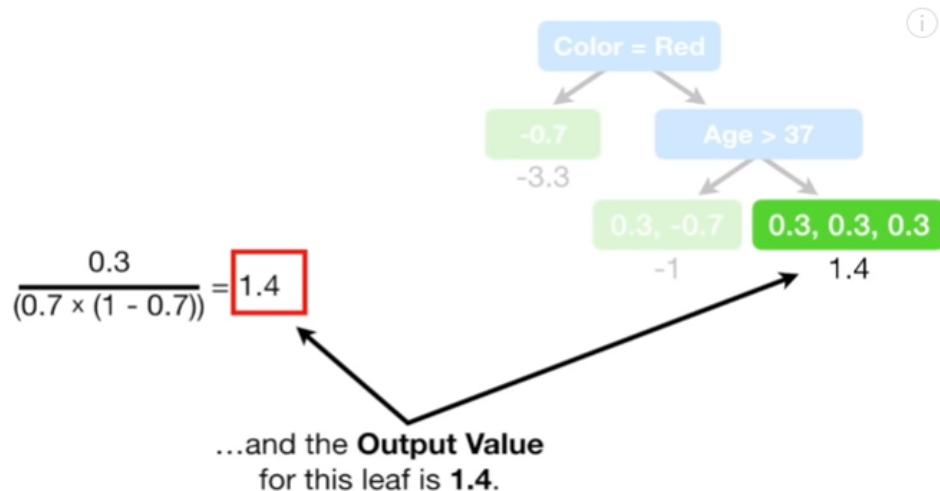


...and the **Previous Probabilities**...



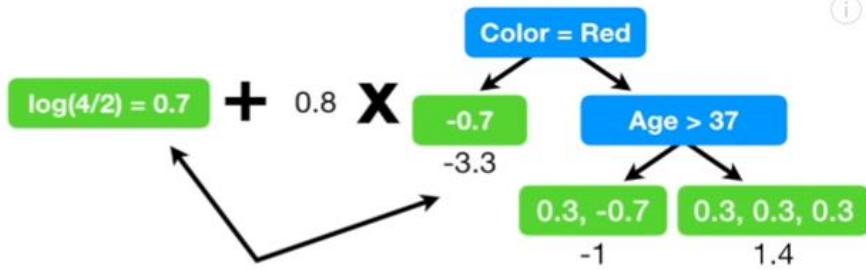


...and do the math...

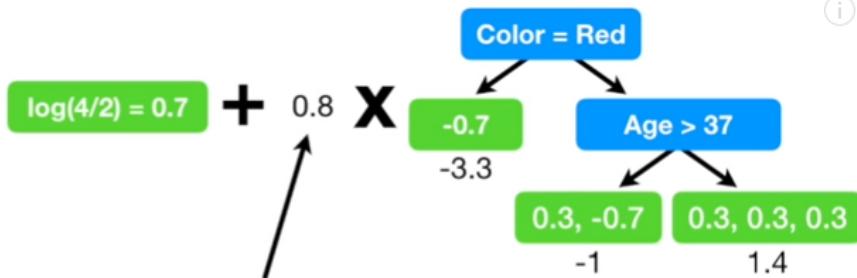


Hooray!!! We've calculated  
**Output Values** for all three  
leaves in the tree!

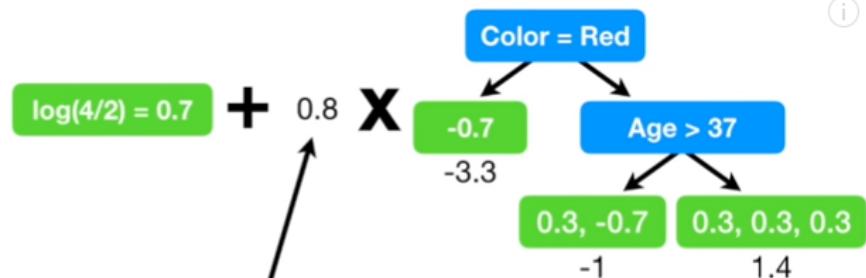




Now we are ready to update our **Predictions** by combining the initial leaf with the new tree.

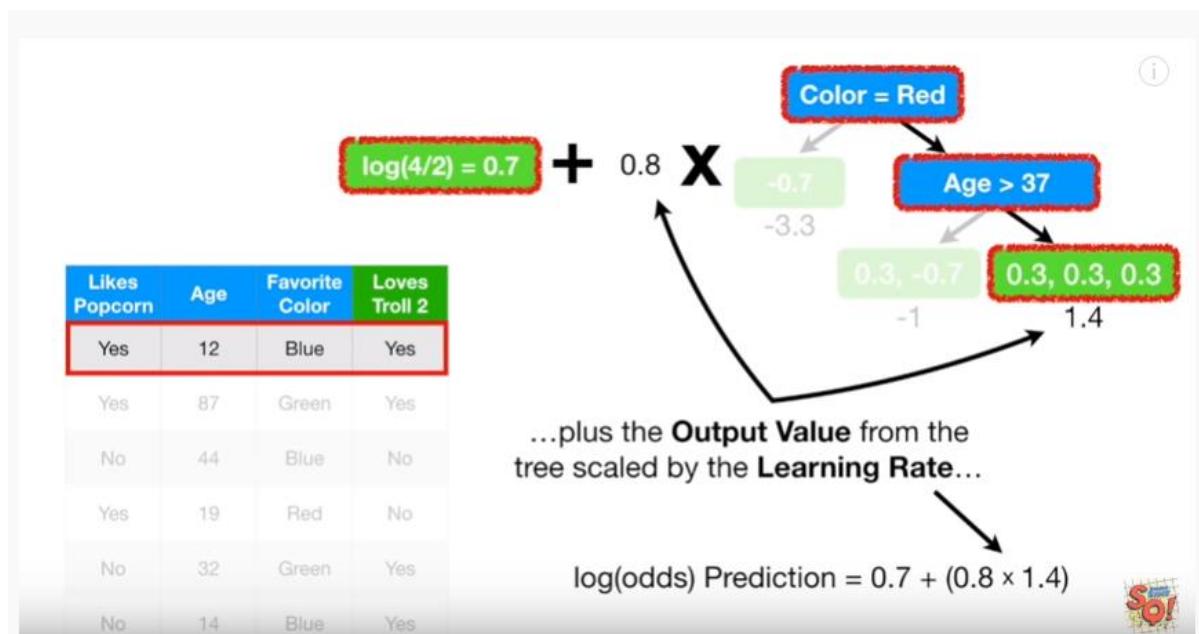
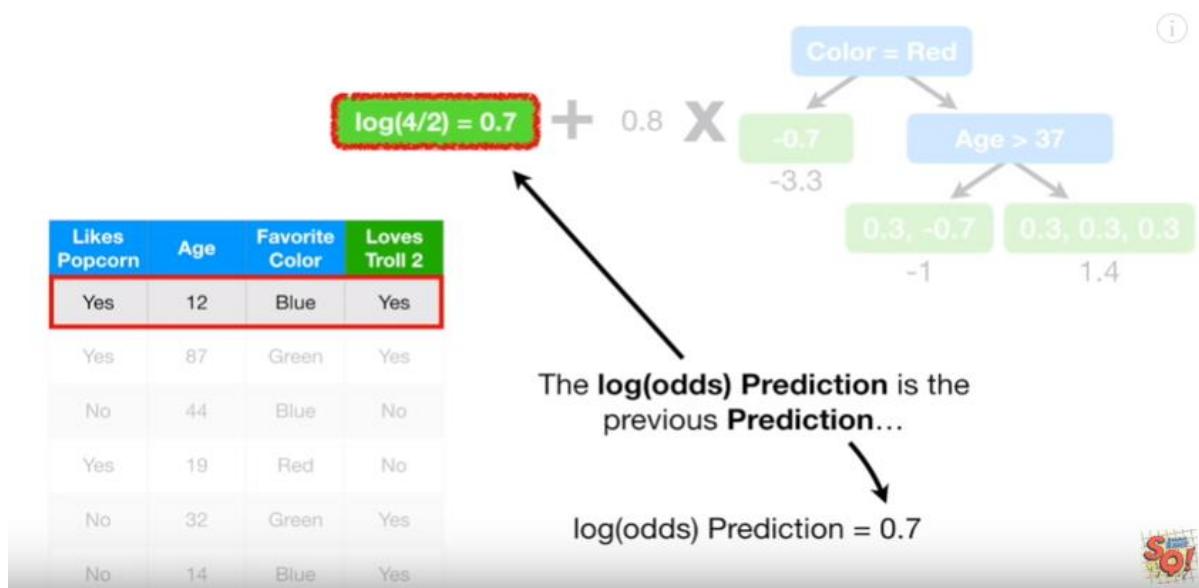
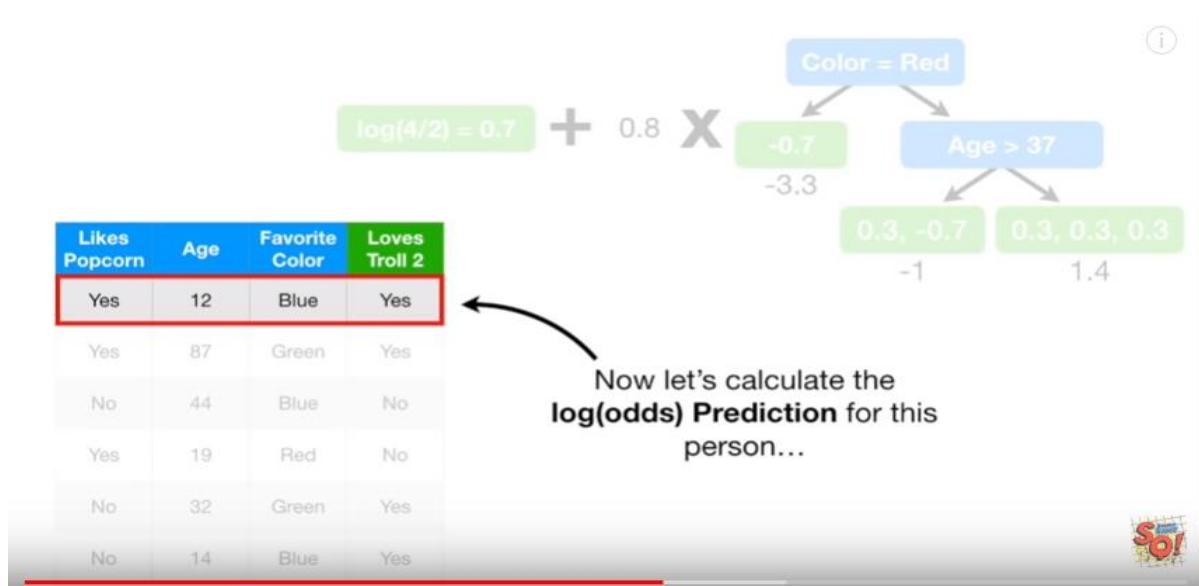


**NOTE:** Just like before, the new tree is scaled by a **Learning Rate**.



**NOTE:** Just like before, the new tree is scaled by a **Learning Rate**.

This example uses a relatively large **Learning Rate** for illustrative purposes. However, **0.1** is more common.



$\log(4/2) = 0.7$  + 0.8  $\times$ 
  
 ...and the new **log(odds)**  
**Prediction = 1.8.**

$\log(\text{odds}) \text{ Prediction} = 0.7 + (0.8 \times 1.4) = 1.8$

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Now we convert the new **log(odds)**  
**Prediction** into a **Probability**...

$\text{Probability} = \frac{e^{\log(\text{odds})}}{1 + e^{\log(\text{odds})}}$

$\log(\text{odds}) \text{ Prediction} = 0.7 + (0.8 \times 1.4) = 1.8$

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Now we convert the new **log(odds)**  
**Prediction** into a **Probability**...

$\text{Probability} = \frac{e^{1.8}}{1 + e^{1.8}}$

$\log(\text{odds}) \text{ Prediction} = 0.7 + (0.8 \times 1.4) = 1.8$

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

Now we convert the new **log(odds) Prediction** into a **Probability**...

$$\text{Probability} = \frac{e^{1.8}}{1 + e^{1.8}} = 0.9$$

$\text{log(odds) Prediction} = 0.7 + (0.8 \times 1.4) = 1.9$



Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

...and the new **Predicted Probability = 0.9**...

$$\text{Probability} = \frac{e^{1.8}}{1 + e^{1.8}} = 0.9$$

$\text{log(odds) Prediction} = 0.7 + (0.8 \times 1.4) = 1.9$



$\log(4/2) = 0.7$

Initial Probability of Loving Troll 2 = 0.7

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	12	Blue	Yes
Yes	87	Green	Yes
No	44	Blue	No
Yes	19	Red	No
No	32	Green	Yes
No	14	Blue	Yes

...so we are taking a small step in the right direction since this person **Loves Troll 2**.

$$\text{Probability} = \frac{e^{1.8}}{1 + e^{1.8}} = 0.9$$

$\text{log(odds) Prediction} = 0.7 + (0.8 \times 1.4) = 1.9$

We save the new **Predicted Probability** here.

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.
Yes	12	Blue	Yes	0.9
Yes	87	Green	Yes	
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

Probability =  $\frac{e^{1.8}}{1 + e^{1.8}} = 0.9$

log(odds) Prediction =  $0.7 + (0.8 \times 1.4) = 1.8$



**Color = Red**

$\log(4/2) = 0.7 + 0.8 \times$

**Age > 37**

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.
Yes	12	Blue	Yes	0.9
Yes	87	Green	Yes	
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

Now we calculate the new **log(odds) Prediction** for the second person...



**Color = Red**

$\log(4/2) = 0.7 + 0.8 \times$

**Age > 37**

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.
Yes	12	Blue	Yes	0.9
Yes	87	Green	Yes	
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

The **log(odds) Prediction** is the previous **Prediction**...

$\log(\text{odds}) \text{ Prediction} = 0.7$



$\log(4/2) = 0.7$  + 0.8 X

...plus the **Output Value** from the tree scaled by the **Learning Rate**...

log(odds) Prediction =  $0.7 + (0.8 \times -1)$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.
Yes	12	Blue	Yes	0.9
Yes	87	Green	Yes	
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

$\log(4/2) = 0.7$  + 0.8 X

...which gives us **-0.1** for the new **Prediction**.

log(odds) Prediction =  $0.7 + (0.8 \times -1) = -0.1$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.
Yes	12	Blue	Yes	0.9
Yes	87	Green	Yes	
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

Now we convert the new **log(odds) Prediction** into a **Probability**...

Probability =  $\frac{e^{\text{log(odds)}}}{1 + e^{\text{log(odds)}}}$

log(odds) Prediction =  $0.7 + (0.8 \times -1) = -0.1$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.
Yes	12	Blue	Yes	0.9
Yes	87	Green	Yes	
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.
Yes	12	Blue	Yes	0.9
Yes	87	Green	Yes	
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

Now we convert the new **log(odds) Prediction** into a **Probability**...

$$\text{Probability} = \frac{e^{-0.1}}{1 + e^{-0.1}}$$

$$\text{log(odds) Prediction} = 0.7 + (0.8 \times -1) = -0.1$$


Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.
Yes	12	Blue	Yes	0.9
Yes	87	Green	Yes	
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

Now we convert the new **log(odds) Prediction** into a **Probability**...

$$\text{Probability} = \frac{e^{-0.1}}{1 + e^{-0.1}} = 0.5$$

$$\text{log(odds) Prediction} = 0.7 + (0.8 \times -1) = -0.1$$


Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.
Yes	12	Blue	Yes	0.9
Yes	87	Green	Yes	0.5
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

...and save the new **Predicted Probability, 0.5**, here...

$$\text{Probability} = \frac{e^{-0.1}}{1 + e^{-0.1}} = 0.5$$

$$\text{log(odds) Prediction} = 0.7 + (0.8 \times -1) = -0.1$$


Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.
Yes	12	Blue	Yes	0.9
Yes	87	Green	Yes	0.5
No	44	Blue	No	
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	

**NOTE:** This new predicted probability is worse than before, and this is one reason why we build a lot of trees, and not just one.

$$\text{Probability} = \frac{e^{-0.1}}{1 + e^{-0.1}} = 0.5$$

$$\log(\text{odds}) \text{ Prediction} = 0.7 + (0.8 \times -1) = -0.1$$



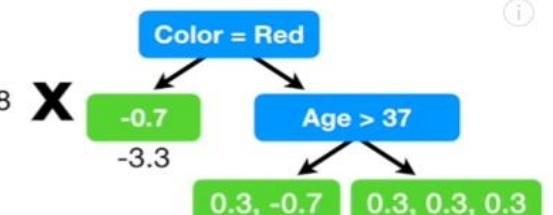
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.
Yes	12	Blue	Yes	0.9
Yes	87	Green	Yes	0.5
No	44	Blue	No	0.5
Yes	19	Red	No	
No	32	Green	Yes	
No	14	Blue	Yes	



Then we calculate the **Predicted Probabilities** for the remaining people.

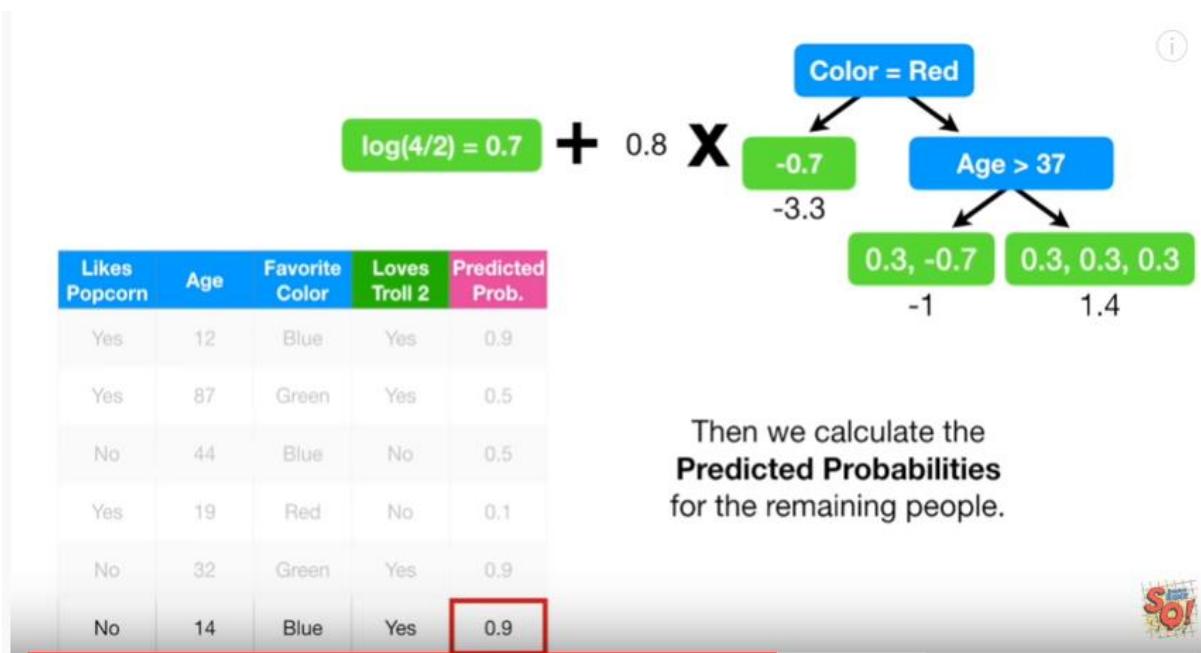
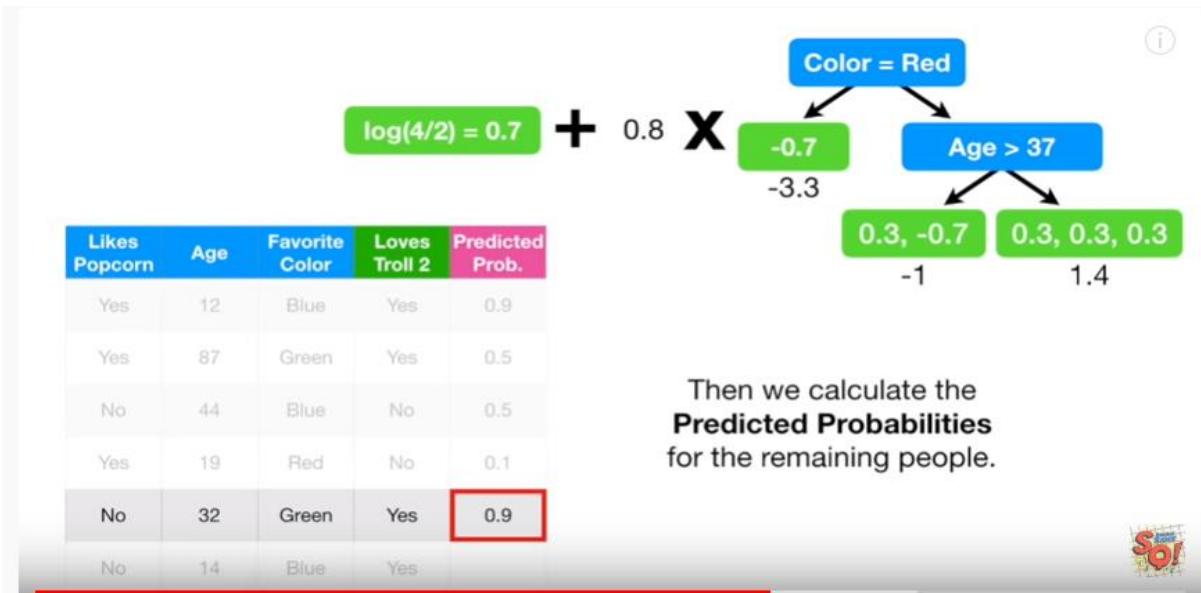


Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.
Yes	12	Blue	Yes	0.9
Yes	87	Green	Yes	0.5
No	44	Blue	No	0.5
Yes	19	Red	No	0.1
No	32	Green	Yes	
No	14	Blue	Yes	



Then we calculate the **Predicted Probabilities** for the remaining people.





Initial_prediction 1[Log(odds)=log(4/ 2=0.7)]	Predicted_Probabil ity(1st)	Residuals(Actual _Prob - Predicted Prob)	Updated_pred iction 1 [New log(odds) prediction]	New Predicted_Pro bability
0.7	0.7	1-0.7 = 0.3	1.8	0.9
0.7	0.7	1-0.7 = 0.3	-0.1	0.5
0.7	0.7	0-0.7 = -0.7	-0.1	0.5
0.7	0.7	0-0.7 = -0.7	-1.94	0.1
0.7	0.7	1-0.7 = 0.3	1.8	0.9
0.7	0.7	1-0.7 = 0.3	1.8	0.9

And now, just like before, we calculate the new **Residuals**...



Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	
Yes	87	Green	Yes	0.5	
No	44	Blue	No	0.5	
Yes	19	Red	No	0.1	
No	32	Green	Yes	0.9	
No	14	Blue	Yes	0.9	

...and just like before, **Residuals** are the difference between the **Observed and Predicted Probabilities**...   **Residual = (Observed - Predicted)**

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	
Yes	87	Green	Yes	0.5	
No	44	Blue	No	0.5	
Yes	19	Red	No	0.1	
No	32	Green	Yes	0.9	
No	14	Blue	Yes	0.9	

...and just like before, we can plot the **Observed Probabilities** on a graph... 



...and just like before, we can plot the **Observed Probabilities** on a graph...



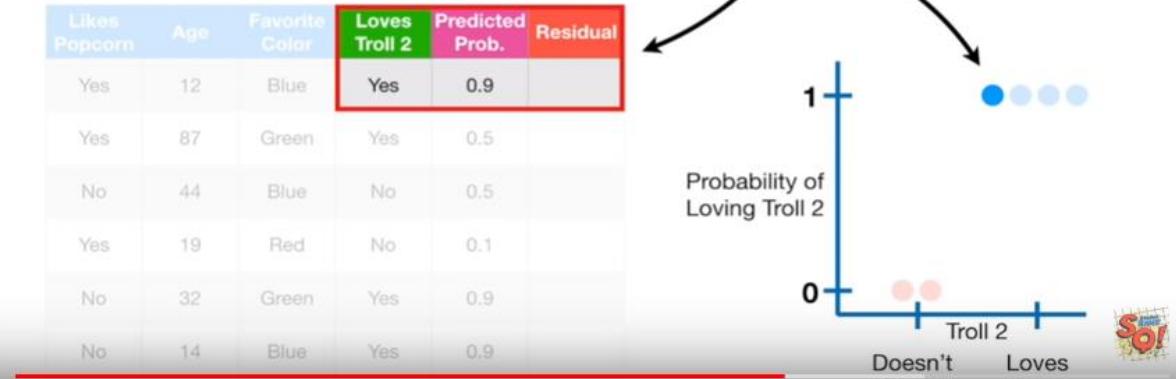
...and just like before, we can plot the **Observed Probabilities** on a graph...



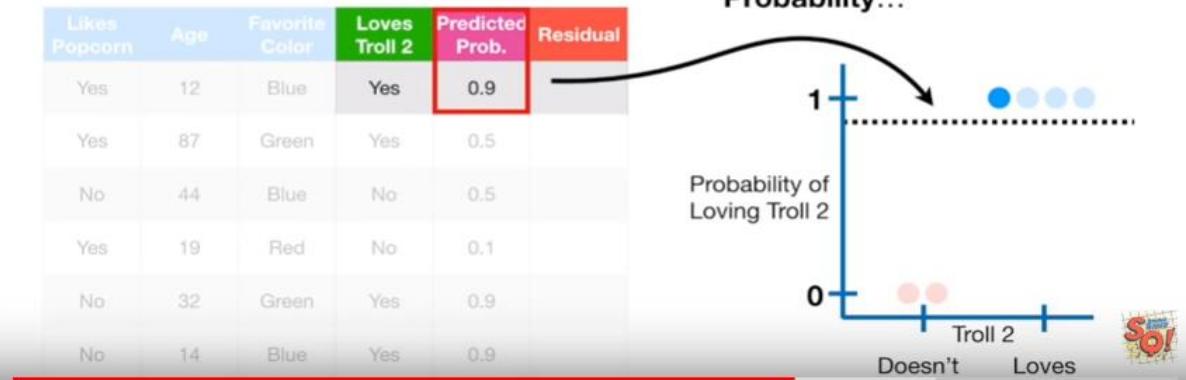
...however, now everyone has a different **Predicted Probability**.



So, to calculate the **Residual** for the first person...



We plot the **Predicted Probability**...



...and the **Residual** is the difference between the **Observed** and **Predicted** probabilities.

$$\text{Residual} = (\text{Observed} - \text{Predicted})$$



...and the **Residual** is the difference between the **Observed** and **Predicted** probabilities.

$$\text{Residual} = (1 - 0.9)$$



...and the **Residual** is the difference between the **Observed** and **Predicted** probabilities.

$$\text{Residual} = (1 - 0.9) = 0.1$$



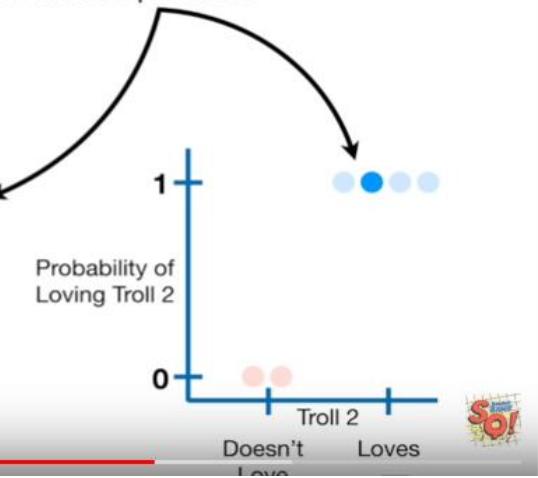
And we save that value here.

$$\text{Residual} = (1 - 0.9) = 0.1$$



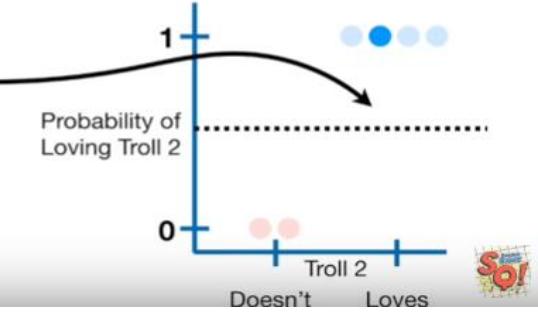
Now we calculate the **Residual**  
for the second person...

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	
No	44	Blue	No	0.5	
Yes	19	Red	No	0.1	
No	32	Green	Yes	0.9	
No	14	Blue	Yes	0.9	



We plot the **Predicted Probability**...

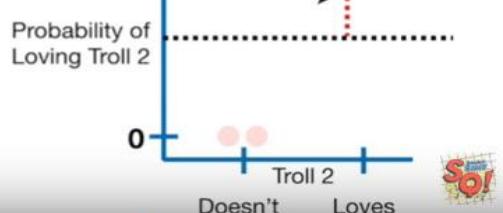
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	
No	44	Blue	No	0.5	
Yes	19	Red	No	0.1	
No	32	Green	Yes	0.9	
No	14	Blue	Yes	0.9	



...and the **Residual** is the difference.

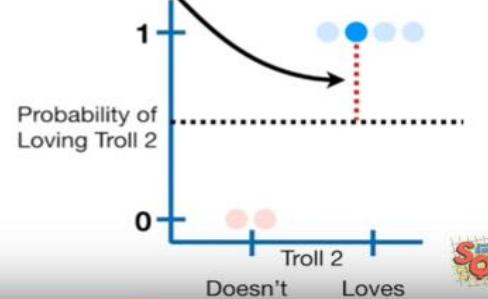
$$\text{Residual} = (1 - 0.5)$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	
No	44	Blue	No	0.5	
Yes	19	Red	No	0.1	
No	32	Green	Yes	0.9	
No	14	Blue	Yes	0.9	



...and the **Residual** is the difference.  $\rightarrow \text{Residual} = (1 - 0.5) = 0.5$

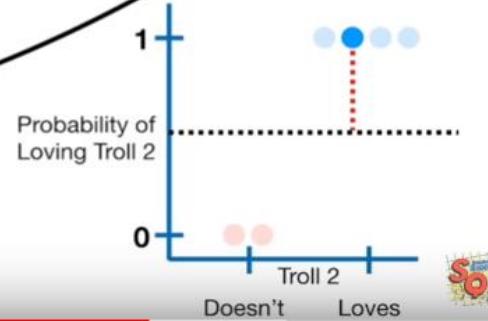
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	
No	44	Blue	No	0.5	
Yes	19	Red	No	0.1	
No	32	Green	Yes	0.9	
No	14	Blue	Yes	0.9	



And we save that value here.

$$\text{Residual} = (1 - 0.5) = 0.5$$

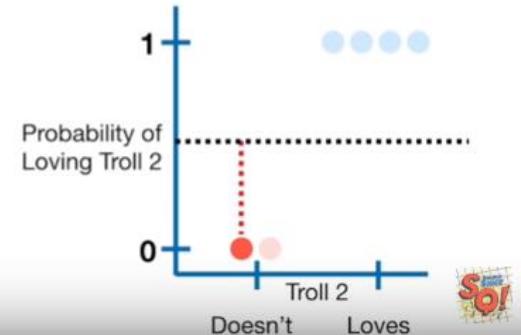
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	
Yes	19	Red	No	0.1	
No	32	Green	Yes	0.9	
No	14	Blue	Yes	0.9	



And then we just do the same thing for all of the remaining people.

$$\text{Residual} = (0 - 0.5) = -0.5$$

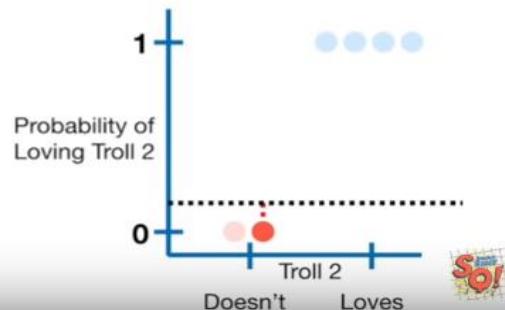
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	
No	32	Green	Yes	0.9	
No	14	Blue	Yes	0.9	



And then we just do the same thing for all of the remaining people.

$$\text{Residual} = (0 - 0.1) = -0.1$$

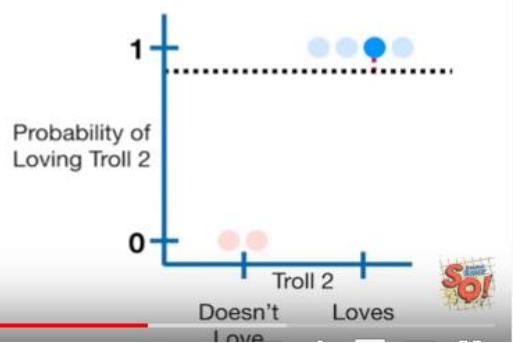
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	
No	14	Blue	Yes	0.9	



And then we just do the same thing for all of the remaining people.

$$\text{Residual} = (0 - 0.9) = 0.1$$

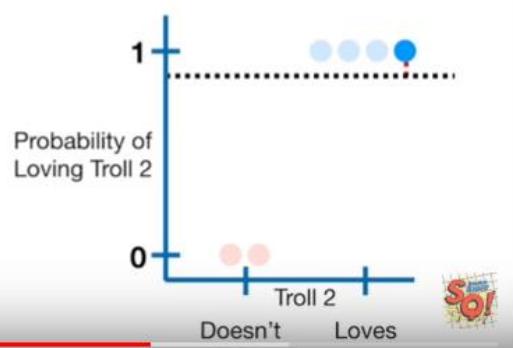
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	



And then we just do the same thing for all of the remaining people.

$$\text{Residual} = (0 - 0.9) = 0.1$$

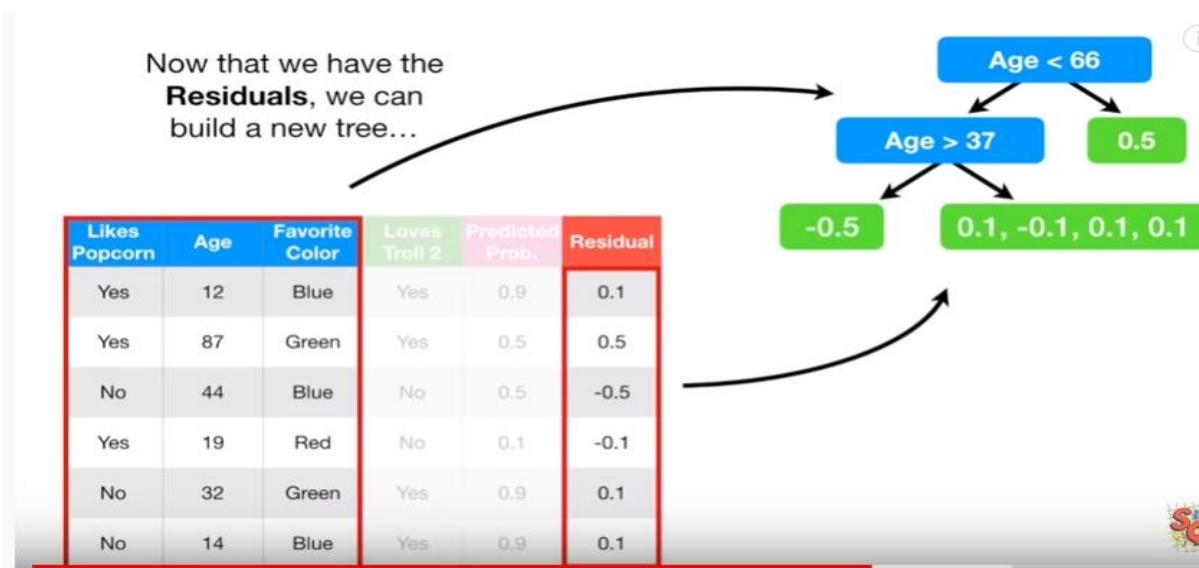
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1



Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

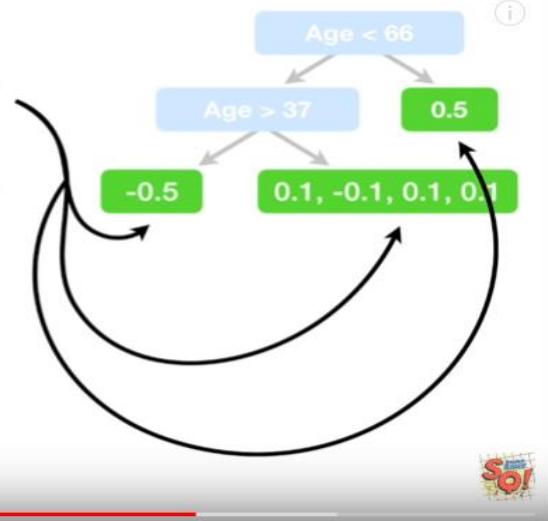
**BAM!**

Initial_prediction 1[Log(odds)=log (4/2=0.7)]	Predicted_Probability(1st)	Residuals(Actual_Prob - Predicted_Prob)	Updated_prediction 1 [New log(odds) prediction]	New Predicted_Probability	New_Residuals
0.7	0.7	1-0.7 = 0.3	1.8	0.9	1-0.9 = 0.1
0.7	0.7	1-0.7 = 0.3	-0.1	0.5	1-0.5 = 0.5
0.7	0.7	0-0.7 = -0.7	-0.1	0.5	0-0.5 = -0.5
0.7	0.7	0-0.7 = -0.7	-1.94	0.1	0-0.1 = -0.1
0.7	0.7	1-0.7 = 0.3	1.8	0.9	1-0.9 = 0.1
0.7	0.7	1-0.7 = 0.3	1.8	0.9	1-0.9 = 0.1



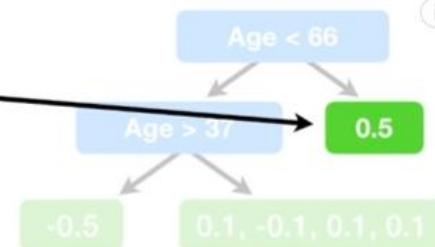
...and then we need to calculate the **Output Values** for each leaf.

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

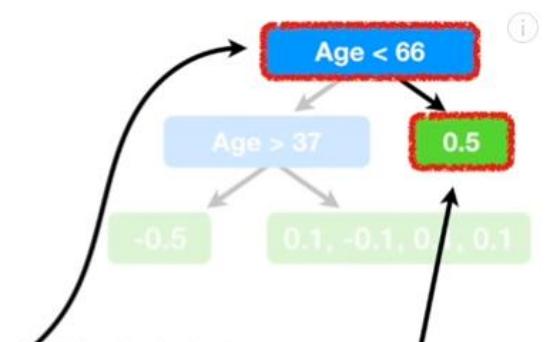


Let's start with this leaf.

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1



Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1



**NOTE:** Only the ...goes to this leaf. second person...



$\sum \text{Residual}_i$

$$\frac{\sum \text{Residual}_i}{\sum \text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)}$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

So we plug the **Residual** into the formula for the **Output Values**...

$\sum \text{Residual}_i$

$$\frac{\sum \text{Residual}_i}{\sum \text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)}$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1

...then we plug in the last **Predicted Probability**...

$\sum \text{Residual}_i$

$$\frac{\sum \text{Residual}_i}{\sum \text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)}$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

...then we plug in the last **Predicted Probability**...

$\frac{0.5}{0.5 \times (1 - 0.5)} = 2$

```

graph TD
    Root[Age < 66] --> L1[Age > 37]
    Root --> Leaf0[0.5]
    L1 --> Leaf1[-0.5]
    L1 --> Leaf2[0.1, -0.1, 0.1, 0.1]
  
```

...do the math...

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

$\frac{0.5}{0.5 \times (1 - 0.5)} = 2$

```

graph TD
    Root[Age < 66] --> L1[Age > 37]
    Root --> Leaf0[0.5]
    L1 --> Leaf1[-0.5]
    L1 --> Leaf2[0.1, -0.1, 0.1, 0.1]
  
```

...and the **Output Value** for this leaf is **2**.

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

Now let's calculate the **Output Value** for this leaf...

```

graph TD
    Root[Age < 66] --> L1[Age > 37]
    Root --> Leaf0[0.5]
    L1 --> Leaf1[-0.5]
    L1 --> Leaf2[0.1, -0.1, 0.1, 0.1]
  
```

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

**NOTE:** Only the third person... ...goes to this leaf...

$$\frac{\sum \text{Residual}_i}{\sum \text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)}$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

So we plug the **Residual** into the formula for the **Output Values**...

$$\frac{-0.5}{\sum \text{Previous Probability}_i \times (1 - \text{Previous Probability}_i)}$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

...then we plug in the last **Predicted Probability**...

$\frac{-0.5}{0.5 \times (1 - 0.5)} = -2$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

...do the math...

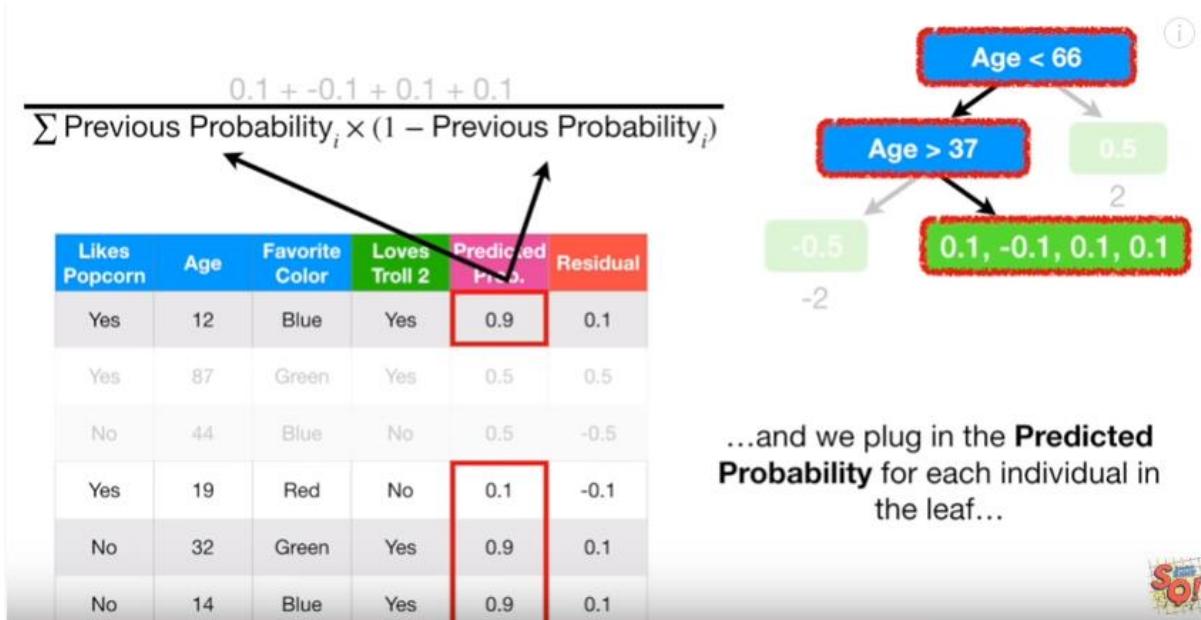
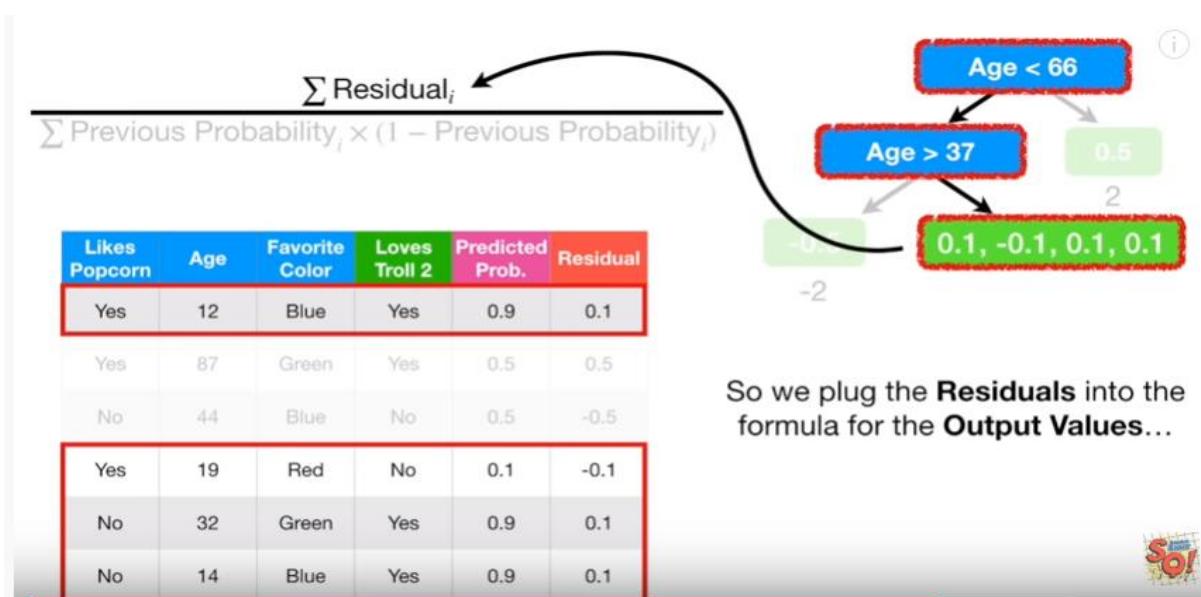
$\frac{-0.5}{0.5 \times (1 - 0.5)} = -2$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

...and the **Output Value** for this leaf is **-2**.

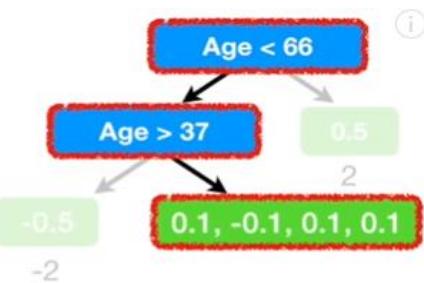
Lastly, let's calculate the **Output Value** for this leaf...

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1



$$\frac{0.1 + -0.1 + 0.1 + 0.1}{(0.9 \times (1 - 0.9))}$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

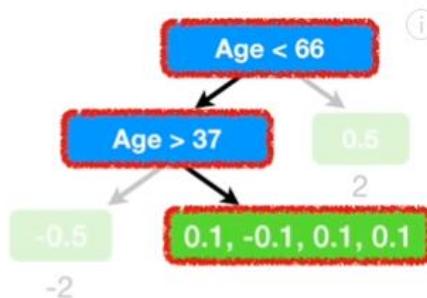


...and we plug in the **Predicted Probability** for each individual in the leaf...



$$\frac{0.1 + -0.1 + 0.1 + 0.1}{(0.9 \times (1 - 0.9)) + (0.1 \times (1 - 0.1))}$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1

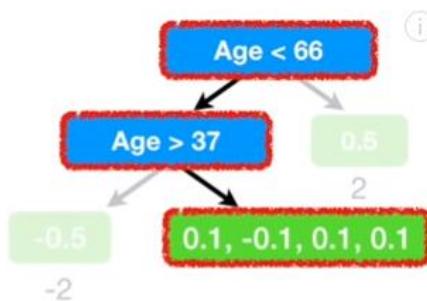


...and we plug in the **Predicted Probability** for each individual in the leaf...



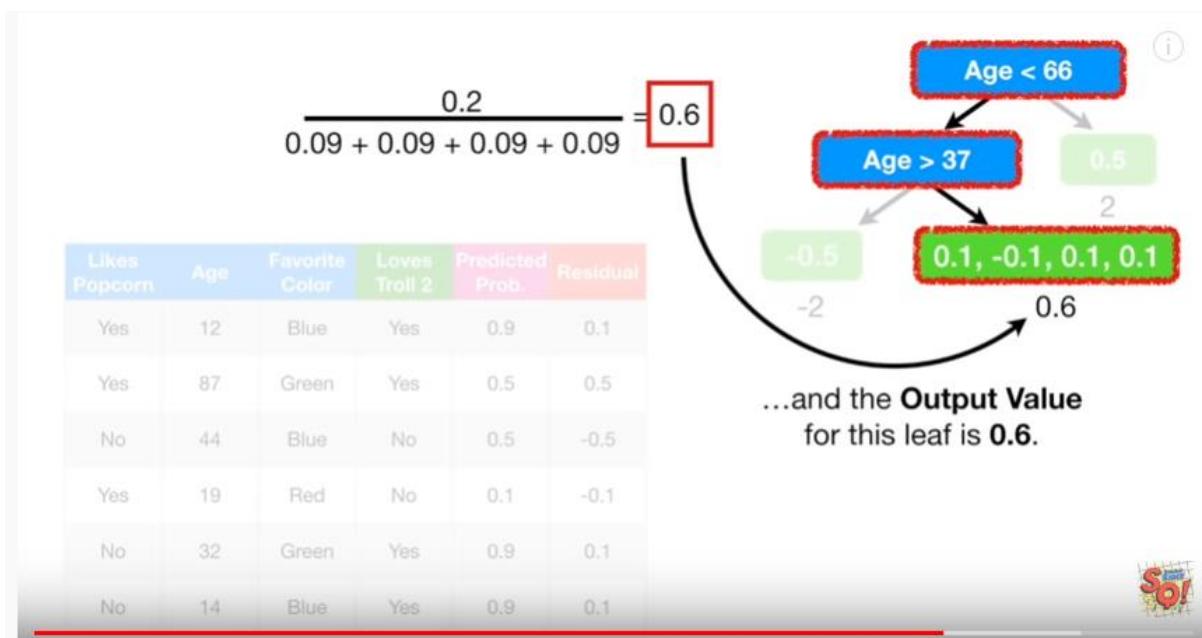
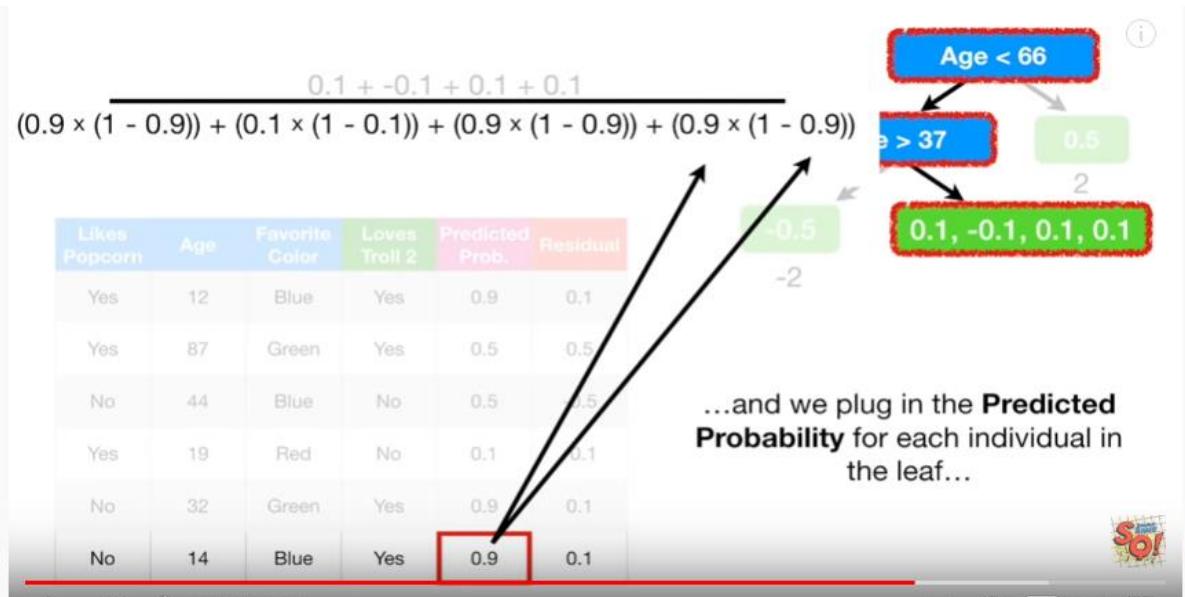
$$\frac{0.1 + -0.1 + 0.1 + 0.1}{(0.9 \times (1 - 0.9)) + (0.1 \times (1 - 0.1)) + (0.9 \times (1 - 0.9))}$$

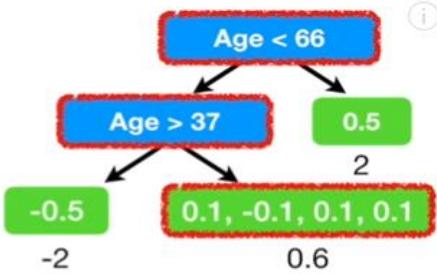
Likes Popcorn	Age	Favorite Color	Loves Troll 2	Predicted Prob.	Residual
Yes	12	Blue	Yes	0.9	0.1
Yes	87	Green	Yes	0.5	0.5
No	44	Blue	No	0.5	-0.5
Yes	19	Red	No	0.1	-0.1
No	32	Green	Yes	0.9	0.1
No	14	Blue	Yes	0.9	0.1



...and we plug in the **Predicted Probability** for each individual in the leaf...



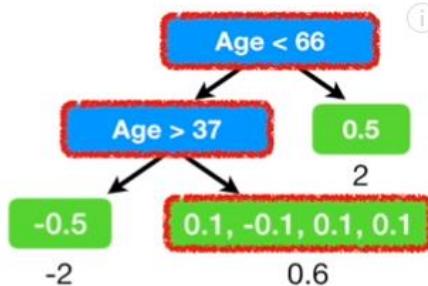




# BAM!



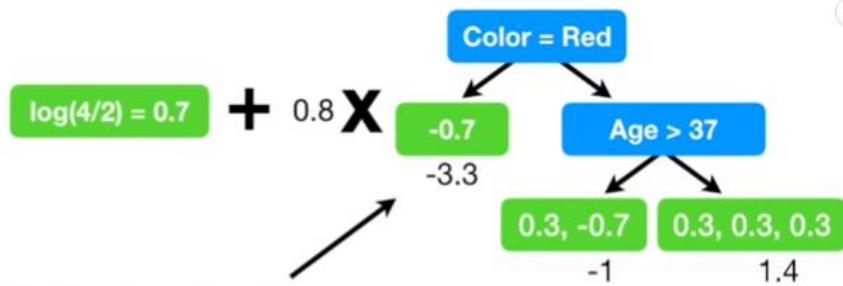
Now that we've calculated all of the **Output Values** for this tree, we can combine it with everything else we've done so far.



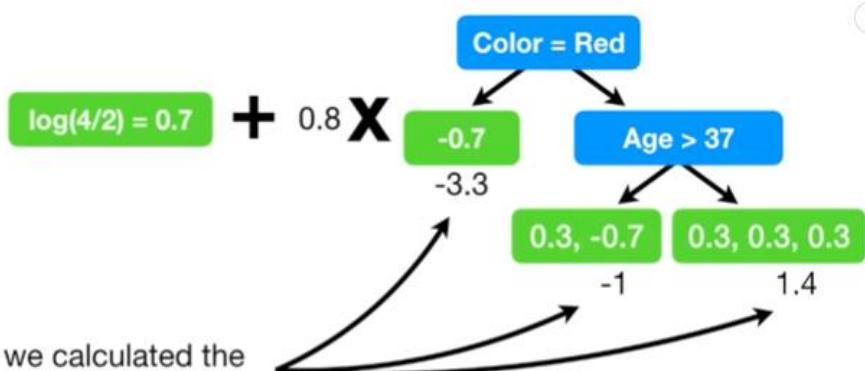
$$\log(4/2) = 0.7$$



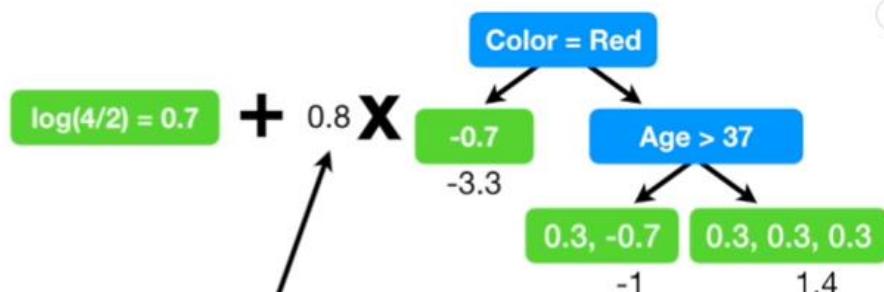
We started with just a leaf, which made one **Prediction** for every individual.



Then we built a tree based on the **Residuals**, the difference between the **Observed** values and the single value **Predicted** by the leaf...

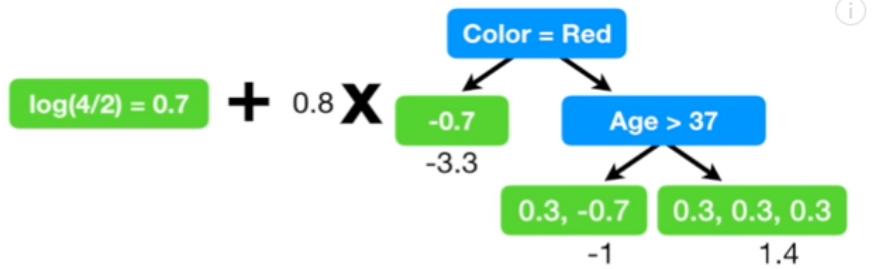


...then we calculated the **Output Values** for each leaf...

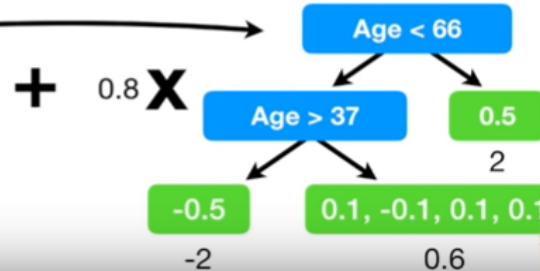


...and we scaled it with a **Learning Rate**.

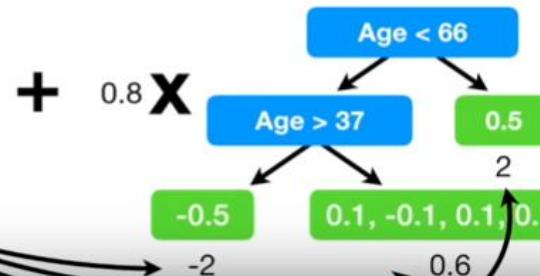




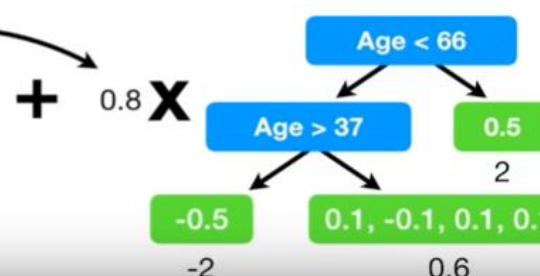
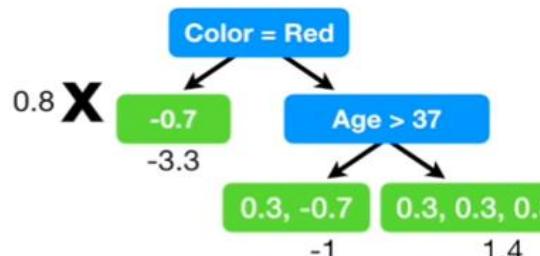
Then we built another tree based on the new **Residuals**, the difference between the **Observed** values and the values **Predicted** by the leaf **and** the first tree...

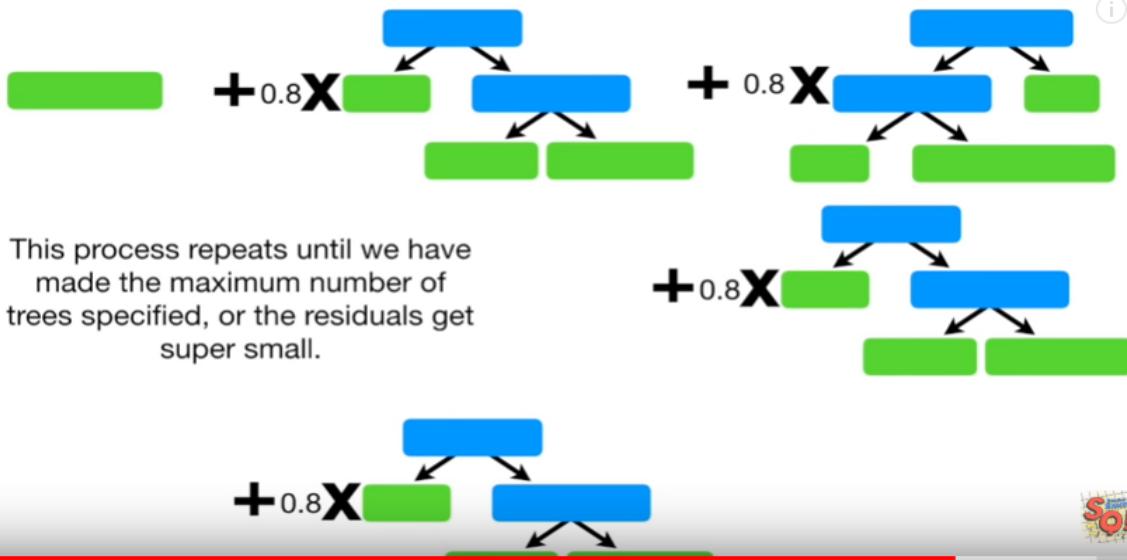


...then we calculated the **Output Values** for each leaf...

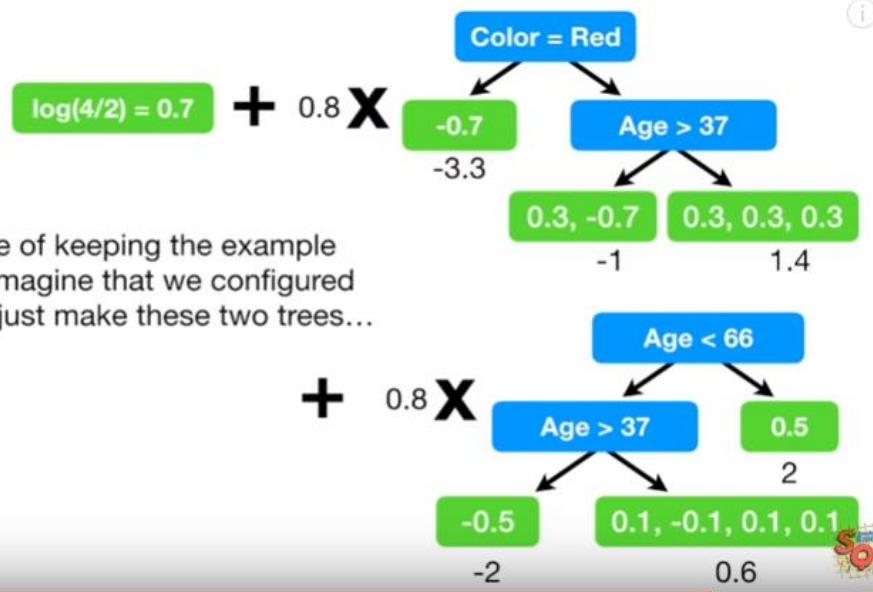


...and we scaled the this new tree with the **Learning Rate** as well.



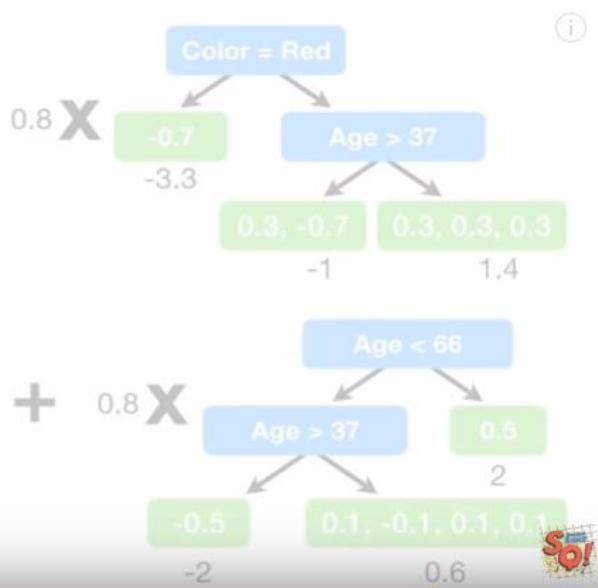


Now, for the sake of keeping the example relatively simple, imagine that we configured **Gradient Boost** to just make these two trees...



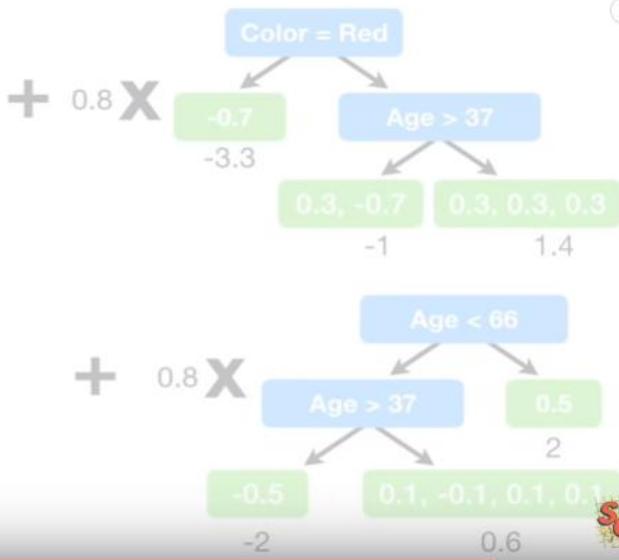
...and we needed to **Classify** a new person as someone who **Loves Troll 2** or **does not Love Troll 2**.

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	25	Green	???



The **Prediction** starts with the leaf...

$$\log(4/2) = 0.7$$



**Log(odds) Prediction**  
that someone Loves = 0.7

Troll 2:

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	25	Green	???



...then we run the data down the first tree...

**Log(odds) Prediction**  
that someone Loves = 0.7

Troll 2:

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	25	Green	???



...and we add the scaled Output Value...

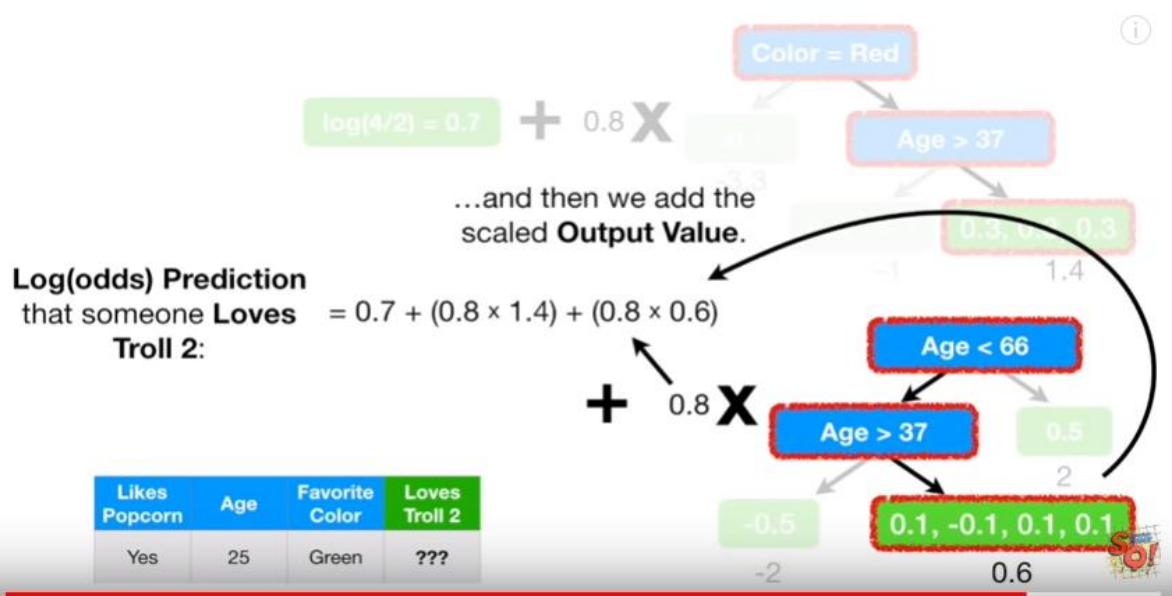
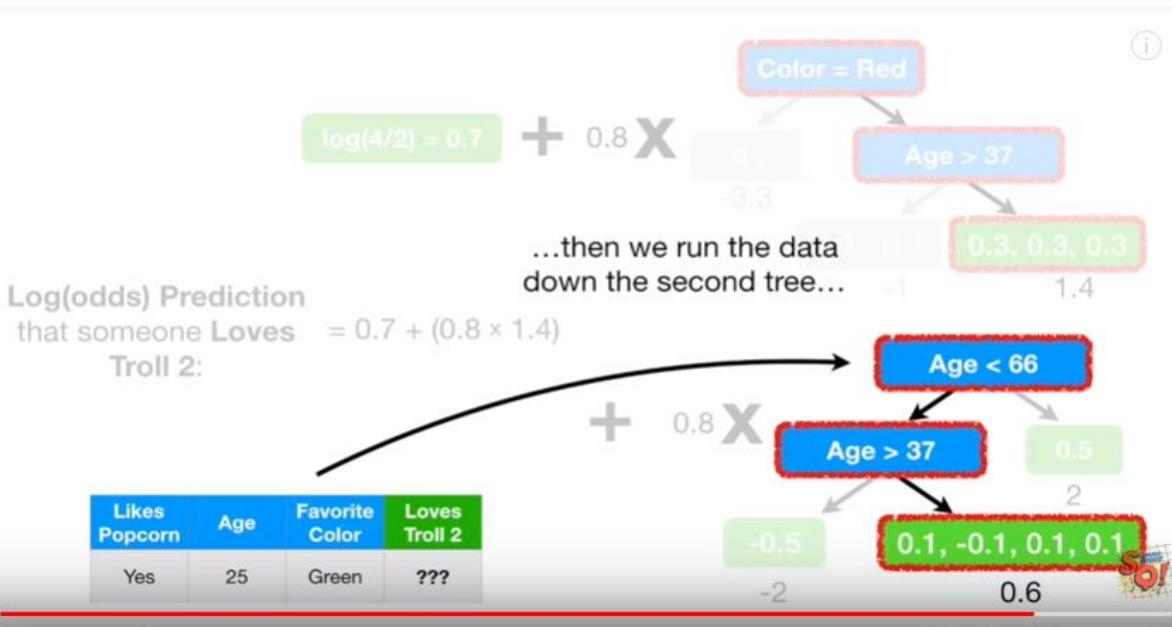
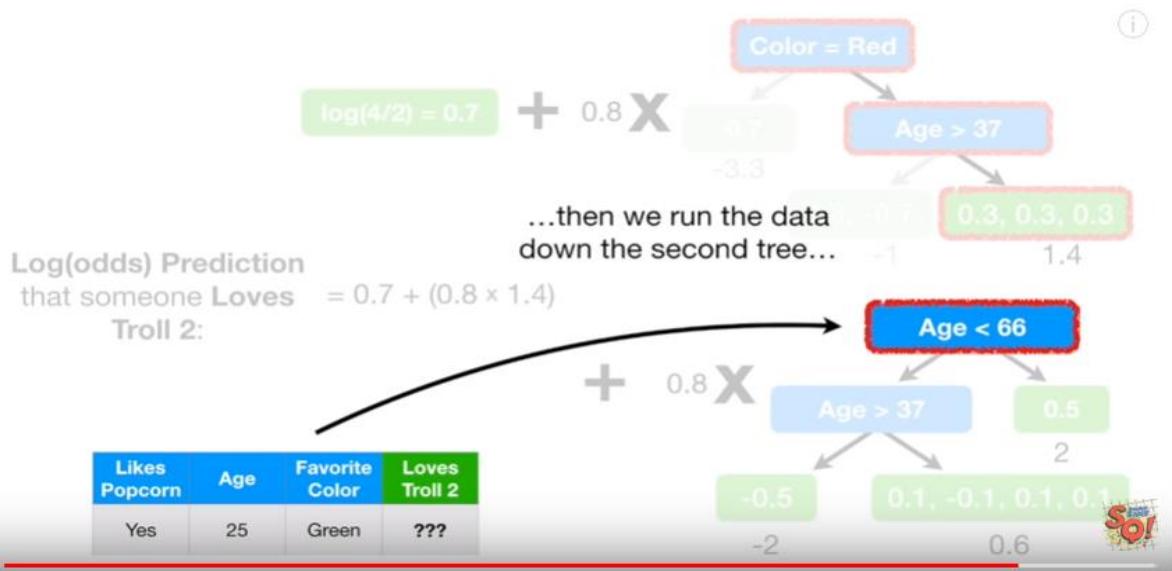
**Log(odds) Prediction**

$$\text{that someone Loves} = 0.7 + (0.8 \times 1.4)$$

Troll 2:

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	25	Green	???





**Log(odds) Prediction**  
that someone Loves =  $0.7 + (0.8 \times 1.4) + (0.8 \times 0.6) =$

**Troll 2:**

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	25	Green	???

**Log(odds) Prediction**

$$\text{that someone Loves} = 0.7 + (0.8 \times 1.4) + (0.8 \times 0.6) = 2.3$$

**Troll 2:**

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	25	Green	???

Now we need to convert this Log(odds) into a Probability.

**Log(odds) Prediction**

$$\text{that someone Loves} = 0.7 + (0.8 \times 1.4) + (0.8 \times 0.6) = 2.3$$

**Troll 2:**

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	25	Green	???

So we plug the **Log(odds)** into the **Logistic Function**.

### Log(odds) Prediction

that someone **Loves** =  $0.7 + (0.8 \times 1.4) + (0.8 \times 0.6) = 2.3$

**Troll 2:**

$$\text{Probability} = \frac{e^{2.3}}{1 + e^{2.3}}$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	25	Green	???



...do the math...

### Log(odds) Prediction

that someone **Loves** =  $0.7 + (0.8 \times 1.4) + (0.8 \times 0.6) = 2.3$

**Troll 2:**

$$\text{Probability} = \frac{e^{2.3}}{1 + e^{2.3}} = 0.9$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	25	Green	???



...and the **Predicted Probability** that this individual will **Love Troll 2** is **0.9**.

### Log(odds) Prediction

that someone **Loves** =  $0.7 + (0.8 \times 1.4) + (0.8 \times 0.6) = 2.3$

**Troll 2:**

$$\text{Probability} = \frac{e^{2.3}}{1 + e^{2.3}} = 0.9$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	25	Green	???



i

Since we are using **0.5** as our threshold for deciding how to **Classify** people, and **0.9 > 0.5...**

### Log(ods) Prediction

that someone Loves =  $0.7 + (0.8 \times 1.4) + (0.8 \times 0.6) = 2.3$

Troll 2:

$$\text{Probability} = \frac{e^{2.3}}{1 + e^{2.3}} = 0.9$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	25	Green	???



i

...we will **Classify** this person as someone who **Loves Troll 2.**

### Log(ods) Prediction

that someone Loves =  $0.7 + (0.8 \times 1.4) + (0.8 \times 0.6) = 2.3$

Troll 2:

$$\text{Probability} = \frac{e^{2.3}}{1 + e^{2.3}} = 0.9$$

Likes Popcorn	Age	Favorite Color	Loves Troll 2
Yes	25	Green	YES!!!



i

**NOTE:** Before we go, I want to remind you that **Gradient Boost** usually uses trees with between **8 to 32** to leaves.

We used small trees in this **StatQuest** because our **Training Dataset** was super small.

