Subject: MCSC 201 - Discrete Mathematics
Homework – 1 (Unit 1: Fundamentals)

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1. Use the Euclidean algorithm to compute GCD$(4389, 7293)$ and find integers s and t such that
$$\mathrm{GCD}(4389, 7293) = s\,(7293) + t\,(4389)$$

2. Show that
   (a) If GCD$(a, c) = 1$, $a \mid m$ and $c \mid m$, then $ac \mid m$.
   (b) LCM$(a, ab) = ab$.
   (c) Let $c =$ LCM(a, b). Show that if $a \mid k$ and $b \mid k$, then $c \mid k$.

3. Let a be an integer and let p be a positive integer. Prove that if $p \mid a$, then $p =$GCD(a, p).

4. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Compute each of the following
   (a) $A \odot B$ (b) $A \vee B$ (c) $A \wedge B$

5. If A, B, and C are Boolean matrices of compatible sizes, then prove that
   (a) $A \vee B = B \vee A$
   (b) $A \wedge B = B \wedge A$
   (c) $(A \vee B) \vee C = A \vee (B \vee C)$

6. **Modular arithmetic** Define $n \equiv_m n'$ (read n is congruent to n' mod m) if $(n \bmod m) = (n' \bmod m)$, or equivalently there is some $q \in \mathbb{Z}$ such that $n = n' + q\,m$.
   (a) Let $x, y \in \mathbb{Z}$ and let $m \in \mathbb{Z}^+$. Then prove that
$$x \equiv_m y \text{ if and only if } m \mid (x - y)$$
   (b) If $x \equiv_m x'$ and $y \equiv_m y'$, then $x + y \equiv_m x' + y'$.