

Resolution for Unicorns

Part A: CNF

In the syntax of Propositional Logic, the knowledge base for *unicorn-kb* can be represented as

$(\text{Mythical} \Rightarrow \text{Immortal}) \wedge$
 $(\neg \text{Mythical} \Rightarrow (\neg \text{Immortal} \wedge \text{Mammal})) \wedge$
 $((\text{Immortal} \vee \text{Mammal}) \Rightarrow \text{Horned}) \wedge$
 $(\text{Horned} \Rightarrow \text{Magical}).$

We apply the following steps to this knowledge base to convert it into **CNF**.

a) Eliminate \Leftrightarrow

\rightarrow The knowledge base does not contain any \Leftrightarrow

b) Eliminate \Rightarrow

$(\neg \text{Mythical} \vee \text{Immortal}) \wedge$
 $(\neg (\neg \text{Mythical}) \vee ((\neg \text{Immortal}) \wedge \text{Mammal})) \wedge$
 $((\neg (\text{Immortal} \vee \text{Mammal})) \vee \text{Horned}) \wedge$
 $(\neg \text{Horned} \vee \text{Magical})$

c) Move \neg inwards

$(\neg \text{Mythical} \vee \text{Immortal}) \wedge$
 $(\text{Mythical} \vee (\neg \text{Immortal} \wedge \text{Mammal})) \wedge$
 $((\neg \text{Immortal} \wedge \neg \text{Mammal}) \vee \text{Horned}) \wedge$
 $(\neg \text{Horned} \vee \text{Magical})$

d) Distributing \vee over \wedge wherever possible to get the CNF

$(\neg \text{Mythical} \vee \text{Immortal}) \wedge$
 $(\text{Mythical} \vee \neg \text{Immortal}) \wedge$
 $(\text{Mythical} \vee \text{Mammal}) \wedge$
 $(\text{Horned} \vee \neg \text{Immortal}) \wedge$
 $(\text{Horned} \vee \neg \text{Mammal}) \wedge$
 $(\neg \text{Horned} \vee \text{Magical})$

Part B: Resolution

From Part 2 of this Lab, (`tt-entails unicorn-kb 'Horned`) returned `#t`. Since *unicorn-kb* entails *Horned*, we know that $(\text{unicorn-kb} \wedge \neg \text{Horned})$ is unsatisfiable. So, we will apply resolution to $(\text{unicorn-kb} \wedge \neg \text{Horned})$. If we get the empty clause while resolving $(\text{unicorn-kb} \wedge \neg \text{Horned})$, we can conclude that *unicorn-kb* entails *Horned*.

Resolving $\neg \text{Horned}$ with $(\text{Horned} \vee \neg \text{Immortal})$, we get $\neg \text{Immortal}$. We add $\neg \text{Immortal}$ to *unicorn-kb*. Now, resolving $\neg \text{Immortal}$ and $(\neg \text{Mythical} \vee \text{Immortal})$, we get $\neg \text{Mythical}$. Add $\neg \text{Mythical}$ to *unicorn-kb*.

Resolving $\neg \text{Horned}$ with $(\text{Horned} \vee \neg \text{Mammal})$, we get $\neg \text{Mammal}$. We add $\neg \text{Mammal}$ to *unicorn-kb*. Now, resolving $\neg \text{Mammal}$ and $(\text{Mythical} \vee \text{Mammal})$, we get *Mythical*. Add *Mythical* to *unicorn-kb*.

Notice that at this point *unicorn-kb* contains both $\neg \text{Mythical}$ and *Mythical*. Finally, resolving $\neg \text{Mythical}$ and *Mythical* gives us the empty clause. This implies that $(\text{unicorn-kb} \wedge \neg \text{Horned})$ is unsatisfiable and hence, *unicorn-kb* entails *Horned*.