Probability of Knight to remain in the N*N chessboard

DAA ASSIGNMENT-5 , GROUP 20

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***Abstract*—This Paper contains the algorithm to find out the probability that the Knight will remain inside the chessboard after exactly K steps, with the condition that it can't enter the chessboard again once it leaves the chessboard.**
One approach has been taken and we will see both the time complexity and space complexity of this algorithm.

I. Problem Statement

Given an NxN chessboard and a Knight at position (x,y). The Knight has to take exactly K steps, where at each step it chooses any of the 8 directions uniformly at random. What is the probability that the Knight remains in the chessboard after taking K steps, with the condition that it can't enter the board again once it leaves it? Solve using Dynamic programming.

II. Introduction

Dynamic Programming (DP) is an algorithmic technique for solving an optimization problem by breaking it down into simpler subproblems and utilizing the fact that the optimal solution to the overall problem depends upon the optimal solution to its subproblems.

III. Algorithmic Design

A. *Approach*

1) One thing that we can observe is that at every step the Knight has 8 choices to choose from. Suppose, the Knight has to take k steps and after taking the Kth step the knight reaches (x,y). There are 8 different positions from where the Knight can reach to (x,y) in one step, and they are: (x+1,y+2), (x+2,y+1), (x+2,y-1), (x+1,y-2), (x-1,y-2), (x-2,y-1), (x-2,y+1), (x-1,y+2).
2) The final probability after K steps will simply be equal to the (probability of reaching each of these 8 positions after K-1 steps)/8.
3) We are dividing by 8 because each of these 8 positions has 8 choices and position (x,y) is one of the choices.
4) For the positions that lie outside the board, we will either take their probabilities as 0 or simply neglect it.
5) Since we need to keep track of the probabilities at each position for every number of steps, we need Dynamic Programming to solve this problem.
6) We are going to take two vectors parentBoard and childBoard. The vector parentBoard will store the probability of reaching (x,y) after (K) number of moves.
7) Base case:If the number of steps is 0, then the probability that the Knight will remain in the board is 1.

Algorithm 1: Calculating probability of Knight to remain in board after K steps

Input: N, K, x, y
Output: Probability of Knight to remain inside chessboard

```
Function knightProbability(N,K,x,y):
    if K=0 then
        return 1.0;
    vector parentBoard , childBoard;
    rowoffset[] = {-2,-2,-1,-1,2,2,1,1};
    coloffset[] = {1,-1,2,-2,1,-1,2,-2};
    cx,cy;
    parentBoard[x][y] ← 1.0;
    for i ← 0 to K − 1 do
        for p ← 0 to N − 1 do
            for q ← 0 to N − 1 do
                moveProb ← parentBoard[p][q]/8.0;
                for w ← 0 to 7 do
                    cx ← p + rowoffset[w];
                    cy ← q + coloffset[w];
                    if cx>= 0 and cx < N and cy>=0 and cy < N then
                        childBoard[cx][cy]+← moveProb;
        parentBoard ← childBoard
        fill(childBoard.begin(),childBoard.end(),vector¡double¿(N,0.0));
    knightProb ← 0.0;
    for p ← 0 to N − 1 do
        for q ← 0 to N − 1 do
            knightProb+← parentBoard[p][q];
    return knightProb;
```

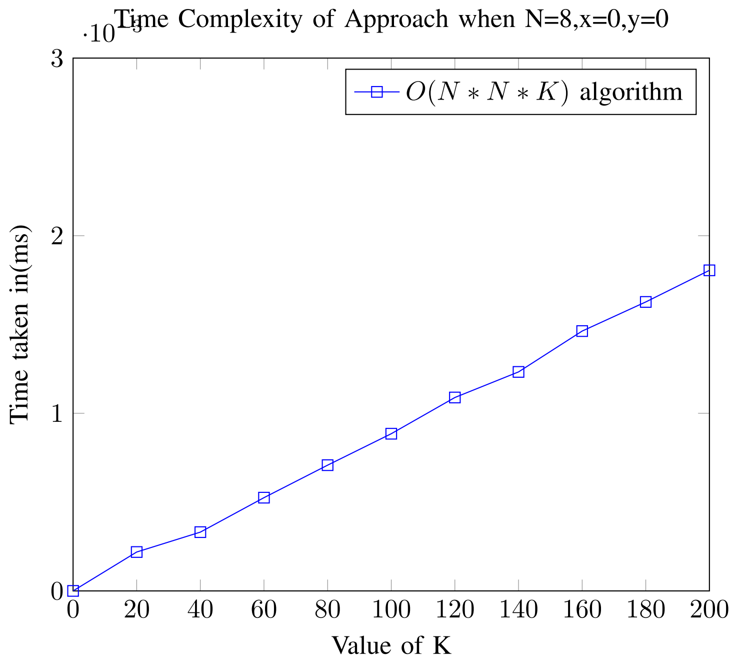
IV. Complexity Analysis

A. *Time Complexity*

In this method, we are working on n $\times$ n elements and there are k layers considered. So Time complexity would be O($n^2 \times$ k)

B. *Space Complexity*

The size of parentBoard and childBoard vectors is n$\times$ n and some constant variables are used.
Therefore, Space complexity is O(n^2)



V. Conclusion

This Dynamic Programming solution of Probability of Knight to remain in NxN chessboard after K steps has Time Complexity O($n^2 \times$k) and Space Complexity O(n^2).

VI. REFERENCES

1) 'Dynamic Programming',Wikipedia

2) 'Dynamic Programming',GeeksforGeeks

3) Avinash Kumar Saw,'Probability of knight to remain in chessboard,GeeksforGeeks,2020