Equivalence of expressions

- ✓ Any two relational expressions are said to be equivalent if resulting relation generates the same set of tuples.
- ✓ When two expressions are equivalent we can use them interchangeably i.e. we can use either of the expressions which gives the better performance.

Equivalence rule

- ✓ An equivalence rule says that expressions of two forms are equivalent.
- ✓ We can replace an expression of the first form by an expression of the second form, or vice versa i.e., we can replace an expression of the second form by an expression of the first form, since the two expressions generate the same result on any valid database.
- ✓ The optimizer uses equivalence rules to transform expressions into other logically equivalent expressions.
- ✓ We now list a number of general equivalence rules on relational-algebra expressions. We use θ1, θ2, θ3, and so on to denote predicates, L1, L2, L3, and so on to denote lists of attributes, and E1, E2, E3, and so on to denote relational-algebra expressions. A relation name r is simply a special case of a relational-algebra expression, and can be used wherever E appears.

For illustration of equivalence rule, let us consider the following relations:

Employee(emp_id,emp_name,address,position,salary,dept_id)

Department(dept_id,dept_name,HOD)

1. Conjunctive selection operations can be deconstructed into a sequence of individual selections

$$\sigma_{\theta 1 \wedge \theta 2}(E) \equiv \sigma_{\theta 1}(\sigma_{\theta 2}(E))$$

Example:

$\sigma_{\text{postion="manager"} \wedge \text{salary>50000}}(\text{Employee}) \equiv \sigma_{\text{position="manager"}}(\sigma_{\text{salary>50000}}(\text{Employee}))$

2. Selection operations are commutative:

$$\sigma_{\theta 1}(\sigma_{\theta 2}(E)) \equiv \sigma_{\theta 2}(\sigma_{\theta 1}(E))$$

Example:

$\sigma_{\text{position="manager"}}(\sigma_{\text{salary>50000}}(\text{Employee})) \equiv$

$\sigma_{\text{salary>50000}}(\sigma_{\text{position="manager"}}(\text{Employee}))$

3. Only the final operations in a sequence of projection operations is needed, the others can be omitted

$\prod_{L1}(\prod_{L2}(\ldots(\prod_{Ln}(E))\ldots)) \equiv \prod_{L1}(E)$
where $L_1 \subseteq L_2 \ldots \subseteq L_n$

Example:
$\prod_{emp\_name}(\prod_{emp\_name,address}(\prod_{emp\_name,address,position}(Employee))) \equiv \prod_{emp\_name}(Employee)$

4. Selections can be combined with Cartesian products and theta joins:
   a. $\sigma_{\theta}(E_1 \times E_2) \equiv E_1 \bowtie_{\theta} E_2$

Example:

$\sigma_{Employee.dept\_id=Department.dept\_id}(Employee \times Department) \equiv$

$Employee \bowtie_{Employee.dept\_id=Deparment.dept\_id} Department$

   b. $\sigma_{\theta 1}(E_1 \bowtie_{\theta 2} E_2) \equiv E_1 \bowtie_{\theta 1 \wedge \theta 2} E_2$

Example:

$\sigma_{Employee.postion=\text{"manager"}}(E_1 \bowtie_{Employee.dept\_id=Department.dept\_id} E_2) \equiv$

$E_1 \bowtie_{Employee.position=\text{"manager"} \wedge Employee.dept\_id=Department.dept\_id} (E_2)$

5. Theta join operations are commutative:

$E1 \bowtie_{\theta} E2 \equiv E2 \bowtie_{\theta} E1$

Example:
$Employee \bowtie_{Employee.dept\_id=Department.dept\_id} Department \equiv$
$Department \bowtie_{Employee.dept\_id=Department.dept\_id} Employee$

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Let us consider the following relation:
Sailors(**<u>sid</u>**, sname, rating, age)
Reserves(**<u>sid</u>**, **<u>bid</u>**, day)
Boats(**<u>bid,</u>** bname, color)

6. (a) Natural join operations are associative:

$(E_1 \bowtie E_2) \bowtie E_3 \equiv E_1 \bowtie (E_2 \bowtie E_3)$

Example:
(Sailors ⋈ Reserves) ⋈ Boat ≡ Sailors ⋈ (Reserves ⋈ Boat)

(b) Theta joins are associative in the following manner:

$(E_1 \bowtie_{\theta 1} E_2) \bowtie_{\theta 2 \wedge \theta 3} E_3 \equiv E_1 \bowtie_{\theta 1 \wedge \theta 3} (E_2 \bowtie_{\theta 2} E_3)$ where θ_2 involves attributes from only E_2 and E_3.

Example:

(Sailors $\bowtie_{\text{sailors.sid=Reserves.sid}}$ Reserves) $\bowtie_{\text{Reserves.bid=Boats.bid} \wedge \text{Reserves.day=5}}$ (Boats) ≡
Sailors $\bowtie_{\text{sailors.sid = Reserves.sid} \wedge \text{Reserves.day=5}}$ (Reserves $\bowtie_{\text{Reserves.bid=Boats.bid}}$ Boats)
where bid involves attributes from only Reserves and Boats

7. The selection operation distributes over the theta join operation under the following two conditions:

(a) When all the attributes in θ_0 involve only the attributes of one of the expressions (E_1) being joined.

$\sigma_{\theta 0} (E_1 \bowtie_\theta E_2) \equiv (\sigma_{\theta 0}(E_1)) \bowtie_\theta E_2$

Example:

$\sigma_{\text{salary>55000}}$ (Employee $\bowtie_{\text{Employee.dept\_id=Department.dept\_id}}$ Department) ≡

($\sigma_{\text{salary>55000}}$(Employee)) $\bowtie_{\text{Employee.dept\_id=Deparment.dept\_id}}$ Department

Here, **salary** is the attribute involve only the attributes of one of the expressions (**Employe**e) being joined.

(b) When θ_1 involves only the attributes of E_1 and θ_2 involves only the attributes of E_2.

$\sigma_{\theta 1 \wedge \theta 2} (E_1 \bowtie_\theta E_2) \equiv (\sigma_{\theta 1}(E_1)) \bowtie_\theta (\sigma_{\theta 2}(E_2))$

Example:

$\sigma_{\text{position="manager"} \wedge \text{dept\_name="sales"}}$ (Employee $\bowtie_{\text{Employee.dept\_id=Department.dept\_id}}$ Department) ≡
($\sigma_{\text{position="manager"}}$(Employee)) $\bowtie_{\text{Employee.dept\_id=Department.dept\_id}}$ ($\sigma_{\text{dept\_name="sales"}}$(Department))

Here, When **position** involves only the attributes of Employee and **dept_name** involves only the attributes of Department.

8. The projection operation distributes over the theta join operation as follows:

(a) if θ involves only attributes from $L_1 \cup L_2$:

$$\prod_{L1 \cup L2}(E_1 \bowtie_\theta E_2) \equiv \prod_{L1}(E_1) \bowtie_\theta \prod_{L2}(E_2)$$

Example:

$$\prod_{emp\_id,emp\_name,HOD,dept\_id}(\text{Employee} \bowtie_{Employee.dept\_id=Department.dept\_id} \text{Department}) \equiv$$

$$\prod_{emp\_id,emp\_name,dept\_id}(\text{Employee}) \bowtie_{Employee.dept\_id=Department.dept\_id} \prod_{dept\_id,HOD}(\text{Department})$$

(b) In general, consider a join $E_1 \bowtie_\theta E_2$.

- Let L_1 and L_2 be sets of attributes from E_1 and E_2, respectively.
- Let L_3 be attributes of E_1 that are involved in join condition θ, but are not in $L_1 \cup L_2$, and
- let L_4 be attributes of E_2 that are involved in join condition θ, but are not in $L_1 \cup L_2$.

$$\prod_{L1 \cup L2}(E_1 \bowtie_\theta E_2) \equiv \prod_{L1 \cup L2}(\prod_{L1 \cup L3}(E_1) \bowtie_\theta \prod_{L2 \cup L4}(E_2))$$

Example:

Let us suppose, Employee is considered as E1 and Department is considered as E2

L1→emp_id, Emp_name, Position

L2→dept_name, HOD

L3→salary

L_3 (salary) be attributes of E_1(Employee) that are involved in join condition θ, but are not in $L_1 \cup L_2$

L4→dept_id

L_4 (dept_id) be attributes of E_2(Department) that are involved in join condition θ, but are

not in $L_1 \cup L_2$

$$\prod_{emp\_id,emp\_name,position,dept\_name,HOD}(\text{Employee} \bowtie_{Employee.salary>50000 \wedge Employee.dept\_id=Department.dept\_id} \text{Department})$$

$$\equiv$$

$$\prod_{emp\_id,emp\_name,position,dept\_name,HOD}(\prod_{emp\_id,emp\_name,position,salary}(\text{Employee}) \bowtie_{Employee.salary>50000 \wedge Employee.dept\_id=Department.dept\_id} \prod_{salary,dept\_id}(\text{Department}))$$

Let us consider the following relation,

Civil_depart(emp_id,emp_name,address)

Comp_depart(emp_id,emp_name,address)

IT_depart(emp_id,emp_name,address)

9. The set operations union and intersection are commutative.

$E_1 \cup E_2 \equiv E_2 \cup E_1$

Example:

$\prod_{emp\_name}$(civil_depart) U $\prod_{emp\_name}$(computer_depart) ≡

$\prod_{emp\_name}$(computer_depart) U $\prod_{emp\_name}$(civil_depart)

$E_1 \cap E_2 \equiv E_2 \cap E_1$

Example:

$\prod_{emp\_name}$(civil_depart) $\cap$ $\prod_{emp\_name}$(computer_depart) ≡

$\prod_{emp\_name}$(computer_depart) $\cap$ $\prod_{emp\_name}$(civil_depart)

(Set difference is not commutative)

10. Set union and intersection are associative.

$(E_1 \cup E_2) \cup E_3 \equiv E_1 \cup (E_2 \cup E_3)$

Example:

($\prod_{emp\_name}$(civil_depart) U $\prod_{emp\_name}$(computer_depart)) U $\prod_{emp\_name}$(IT_depart) ≡

$\prod_{emp\_name}$(civil_depart) U**(** $\prod_{emp\_name}$(computer_depart) U $\prod_{emp\_name}$(IT_depart))

$(E_1 \cap E_2) \cap E_3 \equiv E_1 \cap (E_2 \cap E_3)$

Similar to union

11. The selection operation distributes over the union, intersection, and set-difference operations.

a. $\sigma_\theta (E_1 \cup E_2) \equiv \sigma_\theta (E_1) \cup \sigma_\theta(E_2)$

Example:

$\sigma_{address=\text{"pokhara"}}$ (civil_depart $\cup$ computer_depart) ≡

$\sigma_{address=\text{"pokhara"}}$ (civil_depart) $\cup$ $\sigma_{address=\text{"pokhara"}}$(computer_depart)

b. $\sigma_\theta (E_1 \cap E_2) \equiv \sigma_\theta (E_1) \cap \sigma_\theta(E_2)$

Example:

$\sigma_{address="pokhara"}$ (civil_depart $\cap$ computer_depart) $\equiv$

$\sigma_{address="pokhara"}$ (civil_depart) $\cap$ $\sigma_{address="pokhara"}$(computer_depart)

c. $\sigma_\theta (E_1 - E_2) \equiv \sigma_\theta (E_1) - \sigma_\theta(E_2)$

Example:

$\sigma_{address="pokhara"}$ (civil_depart – computer_depart) $\equiv$

$\sigma_{address="Pokhara"}$ (civil_depart) – $\sigma_{address="Pokhara"}$(computer_depart)

d. $\sigma_\theta (E_1 \cap E_2) \equiv \sigma_\theta(E_1) \cap E_2$

e. $\sigma_\theta (E_1 - E_2) \equiv \sigma_\theta(E_1) - E_2$

12. The projection operation distributes over the union operation.

$\Pi_L(E_1 \cup E_2) \equiv (\Pi_L(E_1)) \cup (\Pi_L(E_2))$

Example:

$\Pi_{address="pokhara"}$(civil_depart $\cup$ computer_depart) $\equiv$

($\Pi_{address="Pokhara"}$(civil_depart)) $\cup$ ($\Pi_{address="Pokhara"}$(computer_depart))