AE 618: Finite Element Method for Fluid Dynamics
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Assignment - 1

A. Consider a flow in which the velocity field is given as:

$$v_\theta = \frac{5}{r}, v_r = 0. \tag{1}$$

This corresponds to a flow due to free vortex and results in circular streamlines. Trace, numerically, the trajectory of a fluid particle released at $r = 1$ and $\theta = 0$ by following the procedure described below:

1. Write equations for $\dot{\theta}$ and $\dot{r}$. Find the analytical solution.
2. Employing the basic Euler method solve the differential equations in cylindrical/polar coordinates. Utilize a computer program and plot your results. Carry out the computations for several revolutions of the particle. Compare the results for various values of Δt. For example, for $\Delta t = 1.0$, 0.1, 0.01, 0.001.
3. Repeat the above step for the improved Euler Method.
4. Comment on the relative accuracy of the two methods and the effect of time step.

B. Repeat the above computations in Cartesian coordinates.

C. Consider the following problem:

$$u_{,xx} + f = 0 \qquad on \qquad]0,1[, \tag{2}$$
$$u(0) = g_1 \qquad , \tag{3}$$
$$u(1) = g_2 \qquad . \tag{4}$$

where f, g_1 and g_2 are constants. For the above equation:

1. Write down the weak form of the problem. Define the related solution and variational spaces.
2. Write down the Galerkin form.
3. Write down the Matrix form.