

The main objective of this project would be to create a tool that is able to explore finite categories. This is done by feeding the tool finite categories and in return it would complete a multiplication table hence showing if the category had a valid solution or not and if it does, it would return a unique solution. As of current there are no previous work done on tools such as this but in general most research papers and articles tend to talk about category theory in general and for some, they tend to touch on the topic of finite categories but never go into the details of solving it with the multiplication tables and showing if a valid solution is feasible or not.

My initial readings were about category theory in general as a good understanding of categories was the basis for this project. Chapter 1 and 2 of Bar and Wells's "Category theory for computer science" provided a good introduction to this topic where it discusses the core basics of categories. In chapter 1, we are introduced to the concepts of sets and graphs. The paper talks about sets in mathematical settings, how it is used, its notations and specifications. It also talks about functions and its properties. For example it talks about how if a function is injective then it has a one to one relation between the domain and the codomain. It then talks about directed graphs, mainly directed multigraph with loops which is the main graph used in category theory and its properties. In chapter, 2 we are further introduced to the concept of categories which can be described as a graph of objects and arrows. The paper also talks about different terminology used and functions and laws that can be applied to categories. The part most related to the project would be in section 2.1.12 where the paper describes a category of finite sets as the category whose objects are finite sets and whose arrows are all the functions between finite sets.

Similarly, to Bar and Wells, "category theory for computer science", Bartosz Milewski's "category theory for programmers" talks first about how categories are just objects and arrows. It then goes on to explain how these arrows, which can be interpreted as functions are also called morphism and work similarly how a function would work in mathematics. This is an important part as the project deals with morphisms mainly. It also talks about the different properties that a composition of a property must satisfy. Just like the previous paper, it goes on to talk about graphs and how it is used in category theory. However it also talks about monoids and how it can be represented as a set and then how it is also a category. This would be an early introduction to monoidal category which I will talk more about later.

Fong and Spivak's "An invitation to applied category Theory" gives a better understanding of objects, identities and morphisms. It also talks about free categories which is not touched on by previous papers which refers to a category whose objects are the vertices  $V$  and whose morphisms from object 1 to object 2 are the paths from object 1 to object 2. The paper also talks about isomorphisms, which means that the morphism between 2 objects can be reversed by an inverse mapping, and also preorders as categories. Knowledge of both of which are used by the tool being developed to complete the multiplication table.

The Wikipedia article on category theory gives a better understanding of the main concepts of category theory and it was my main reference point when completing this project. It defines a category as consisting of 2 sorts of objects and the morphisms which relate the two objects. If we were to give it an analogy, in mathematics the objects would be similar to 2 sets and the morphism would be the functions relating the 2 sets. It explains the role of morphisms and functors in category theory. Functors are structure-preserving maps between categories which can be thought of as morphisms in the category of all small categories. It also talks about other concepts in category

theory such as universal constructions, equivalent categories, and higher-dimensional categories which do not really relate to the project.

Another Wikipedia article that was used to better understand the project was the article on monoidal category which was used to better understand the extensions that were added to the project. A monoidal category is a category equipped with a bifunctor that is associative up to a natural isomorphism and an object. In monoidal category, the bifunctor is called the tensor product whereby the tensor product of 2 vector spaces is the vector space to which is associated a bilinear map that maps a pair to an element. This tensor product makes vector spaces, abelian groups and  $r$ -modules into monoidal categories. The article then goes on to explain that by formal definition, a monoidal category is a monoidal structure that consists of a bifunctor, an identity object and 3 natural isomorphisms that are subject to certain coherence conditions.

Building on to that, the paper “finite tensor categories” by Pavel Etingof and Viktor Ostrik gives a deeper insight into monoidal categories. Although tensor categories are not exactly monoidal categories as stated in the textbook “Tensor Categories” by Etingof, Gelaki, Nikshych, and Ostrik, they do possess similar structures. Hence this paper did provide some insight into the topic.

Although not much past research relating to this project was found, I did find a research paper that helped with the development of the project which is the paper DisCoPy: Monoidal Categories in Python by Giovanni de Felice, Alexis Toumi, and Bob Coecke. The paper is about an open source toolbox for computing monoidal categories in python. The datastructures and methods implemented in this toolbox such as the implementation of the Diagram class in monoidal.py which is the implementation of the arrows of a free monoidal category and the Tensor class in tensor.py would mean that I would not have to reinvent the wheel, or I could just restructure my implementations similarly when I am implementing it into my project. The only downfall to this paper is that it does not touch on providing valid solutions for the categories nor does it talk about solving the multiplication tables, hence it cannot really be considered past work of the same topic but it is a good place to start for the project.