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Project Report

Linear Optimization Model for Phased Coal Power Phase-Out in NRW

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Abstract

This study introduces a mixed-integer linear optimization framework that uses data to help phase out coal-fired power plants in North Rhine-Westphalia (NRW) and replace them with solar, wind, or hybrid solar-wind systems. We used GAMSpy for model formulation and Gurobi as the solver. We developed and parameterized three different models with real-world datasets. These included coal capacities for each plant, renewable energy generation potentials, technology-specific costs, and schedules for decommissioning.

We applied capacity factors of 0.15 for solar energy and 0.30 for wind to determine realistic supply potentials. The demand matched the scheduled coal retirement targets. The models aim to reduce total assignment costs by efficiently allocating the demand for replacements to suitable renewable facilities. This is done while meeting requirements for demand fulfillment, minimum and maximum usage levels, and limits on the number of operational facilities in each phase. For the hybrid setup, costs are calculated based on the solar and wind contributions at each site.

Comparative analysis shows that the models using only solar or only wind tend to activate facilities in areas with the lowest costs per megawatt. However, they often need more facilities to satisfy usage requirements. On the other hand, the hybrid model can achieve similar or lower total system costs while activating fewer facilities. This is because the combined solar and wind capacity at each site leads to better coverage of demand and less underutilization. As a result, there is a more balanced distribution of renewable energy deployment across NRW, offering clear operational and cost benefits compared to single-technology methods. The framework can easily adapt to add more spatial, grid, or policy constraints, making it a useful tool for planning the energy transition

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1. Introduction

North Rhine-Westphalia (NRW), one of Germany's most coal-dependent regions is now witnessing a major energy transformation. In line with Germany's national climate commitments and the European Union's Green Deal, the country plans to phase out coal completely by 2038. Reaching this goal requires a carefully managed shift toward renewable energy sources such as solar, wind, or a combination of both that can cut greenhouse gas emissions without compromising the stability of the power supply.

This study presents an optimization-based approach to support the gradual replacement of coal power in NRW with renewable energy infrastructure. The objective is to identify deployment strategies that are both cost-effective and technically feasible, whether they rely solely on solar, solely on wind, or on a hybrid of the two, while keeping pace with the progressive closure of coal plants.

The key research question guiding this work is:

- **What renewable energy deployment strategy in NRW offers the most cost-efficient path while meeting the planned coal phase-out?**

To address the above question, we employ a **linear optimization modeling framework** using actual data from NRW's coal power plants and renewable energy potential. The model incorporates spatial allocation of facilities, capacity constraints, phased timelines, and technology specific cost parameters. **Three scenarios—solar-only, wind-only, and hybrid solar-wind—are developed to compare performance under identical demand conditions.**

2. Methodology

This section is about how we conducted the research to answer the research question. It also acts as a guide for replicating our results.

2.1 Systematic Review Process

To address our research question, first we aimed to understand the problem in depth and explore the current efforts made by the research community to solve it. We used Google Scholar to find relevant literature and identified two papers closely related to our topic. In addition, we located several websites that provided useful data for our study. The details of these sources and the process of obtaining our dataset are discussed in the following section.

2.2 Data collection and processing

To address our research question our first step was the data collection. We obtained our dataset from a publicly available website that provided information on coal power plants and lignite plants. This dataset was particularly relevant as it contained detailed information on plant locations, capacities, and demand requirements. For cost data we performed calculations which is mentioned in the Section 5. Our intention was to use this dataset to model and analyse replacement scenarios for coal plants with wind, solar, and hybrid energy systems, enabling us to draw meaningful results.

2.3 Analysis tools

To analyse the dataset and perform the optimisation, we used the Python programming language because of it is rich in ecosystem of libraries for data processing, modelling, and visualisation. Python was chosen for its flexibility and the availability of specialised optimisation tools.

The dataset generated in CSV format is processed using pandas and NumPy for cleaning, transformation, and preparation. For the optimisation stage, we used the GAMSpy framework together with the Gurobi solver to model and solve Mixed Integer Programming problems for wind, solar, and hybrid energy replacement scenarios.

Visualisations were created using Matplotlib and NetworkX, which enabled us to present both the allocation of demand to facilities over time and the cost implications of varying the number of operational facilities. This end-to-end Python workflow streamlined the process from data preparation to result interpretation, as discussed in the results section.

3. Literature Review

Phasing out coal-fired power in North Rhine-Westphalia (NRW) is central to Germany's energy transition. As a major historic lignite and hard coal producer, NRW must replace significant baseload capacity while maintaining grid stability, controlling costs, and meeting GHG reduction targets.

Optimisation models help design cost-effective, policy-consistent pathways by integrating economic, technical, and environmental factors. Two studies provide key methodological insights:

3.1 Gaete-Morales et al. [2019]: Linking Plans with Operations

Long-term models often neglect operational realities when integrating high shares of variable renewable energy (VRE). The FuturES framework addresses this through an iterative link between:

- **Power System Expansion (PSE):** A multi-period linear program optimising the generation/storage mix based on costs, learning rates, fuel prices, and policies such as carbon taxes.
- **Economic Dispatch (ED):** An hourly simulation using PowerGAMA to test the operational feasibility of the planned system under realistic weather and demand conditions.

This iterative process continues until convergence between the PSE plan and ED operational results is achieved. **Key findings:**

1. A fully renewable (81–100%) electricity system in Chile by 2050 was technically feasible with moderate cost increases.
2. Both short- and long-duration storage are essential for managing variability.

3. Carbon pricing accelerates investment in solar, wind, and storage technologies.

Relevance for NRW: The current model optimizes renewable siting across three phases with a cost focus, but without operational validation. Incorporating an ED-like step could help adjust capacity factors for local weather patterns, highlight risks of curtailment or load shedding, and refine siting decisions for both cost and reliability.

3.2 Arnette and Zobel [2012]: Spatial and Policy Constraints

Arnette & Zobel emphasise spatial feasibility and policy impacts. Their MOLP model jointly optimises cost and GHG reductions, using GIS-based screening to identify sites that meet physical and regulatory requirements (e.g., slope, land use, protected areas). It also simulates the impact of policies such as renewable portfolio standards, carbon taxes, and production tax credits.

Key findings:

1. Spatial constraints can significantly alter the technology mix and location of optimal sites.
2. Policy incentives can achieve substantial emissions reductions at modest additional costs.
3. Combining spatial and economic analysis yields more realistic, actionable plans than purely techno-economic approaches.

Relevance for NRW: The current model assumes uniform generation costs and applies only basic spatial limits. Incorporating site-specific costs, grid-connection expenses, land-use restrictions, and phase-specific policy measures would better reflect real-world planning conditions.

4. Mathematical Optimization Model

This study employs a mixed-integer linear programming (MILP) framework to plan the phased replacement of coal-based electricity generation with renewable facilities in North Rhine–Westphalia (NRW). The model decides, for each period, which facilities to operate and how to allocate demand from retired coal plants, minimizing total supply and delivery costs while meeting technical constraints.

4.1 Sets and Indices

- $i \in I$: candidate renewable facility locations (potential solar/wind sites).
- $j \in J$: demand nodes (coal plants scheduled for closure).
- $t \in T = \{t_1, t_2, t_3\}$: planning periods with $t_1 = 0$ –2 years, $t_2 = 2$ –4 years, $t_3 = 4$ –6 years.

4.2 Parameters

- d_{jt} : energy demand at node j in period t (coal-based energy to be replaced).
- s_{it} : available renewable supply from facility i in period t .
- c_{ijt} : cost of serving demand node j from facility i in period t .
- $l_0 \in (0, 1]$: minimum utilization fraction for viable operation (e.g., 0.80).
- p : maximum number of facilities allowed per period.

4.3 Decision Variables

- $x_{ijt} \in \{0, 1\}$: 1 if demand node j is served by facility i in period t , else 0.
- $y_{it} \in \{0, 1\}$: 1 if facility i is active in period t , else 0.

4.4 Objective Function

Minimize total allocation cost across all facilities, nodes, and periods:

$$\min Z = \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} c_{ijt} x_{ijt}. \quad (4.1)$$

4.5 Constraints

Demand satisfaction.

Each demand node is fully served by exactly one facility per period:

$$\sum_{i \in I} x_{ijt} = a_{jt}, \quad \forall t \in T, j \in J, \quad \text{where } a_{jt} = \begin{cases} 1, & \text{if } d_{jt} > 0, \\ 0, & \text{otherwise.} \end{cases} \quad (4.2)$$

Capacity limit.

Assigned demand to a facility cannot exceed its available supply:

$$\sum_{j \in J} d_{jt} x_{ijt} \leq s_{it} y_{it} \quad \forall t \in T, i \in I. \quad (4.3)$$

Minimum utilization.

Activated facilities must operate above a utilization threshold:

$$\sum_{j \in J} d_{jt} x_{ijt} \geq l_0 s_{it} y_{it} \quad \forall t \in T, i \in I. \quad (4.4)$$

Facility count.

At most p facilities should be running:

$$\sum_{i \in I} y_{it} \leq p \quad \forall t \in T. \quad (4.5)$$

Integrality.

Binary activation and assignment decisions:

$$x_{ijt} \in \{0, 1\}, \quad y_{it} \in \{0, 1\} \quad \forall t \in T, i \in I, j \in J. \quad (4.6)$$

5. Experiment

To address the research question of how to optimally replace coal-fired power plants in North Rhine-Westphalia (NRW) with renewable energy infrastructure, we first prepared and validated the dataset, developed three scenario-based optimization models (solar-only, wind-only, and hybrid solar-wind), and then conducted a series of numerical experiments to evaluate their performance under the same constraints.

5.1 Data Preparation

The primary dataset, `coal_mine_data_cleaned.csv`, was compiled from publicly available energy infrastructure and techno-economic sources. Data on coal power plants, their installed capacities, and geographic locations were collected from **Fraunhofer Energy Charts – Power Plant Map:** ([Charts–Interaktive-Karten \[Deutschland\]](#)). Each record corresponds to a coal plant i in a given phase t of retirement and includes:

- **Demand:** Coal capacity scheduled for replacement by renewable (denoted by `Coal MW replace(demand)`).
- **Renewable supply potential:** Solar MW (generation capacity) and Wind MW (generation capacity).
- **Assignment costs:** Total cost (€) of meeting demand from each site for solar and wind technologies.

Calculations and assumptions:

The total cost per MWh used in the model includes:

1. Initial investment (**CAPEX**)
 2. Operational expenses (**OPEX**)
 3. Grid delivery and integration costs
- Technology-specific capacity factors (CF) were applied: **Solar** $CF = 0.15$, **Wind** $CF = 0.30$. For the hybrid scenario, total effective capacity was computed as the sum of solar and wind capacities, followed by calculation of a weighted hybrid cost using each technology’s share of total capacity.

Phase-wise Delivered Cost Data

The delivered cost per MWh for each technology and phase in NRW was based on publicly available estimates from [Kost et al. \[2024\]](#):

Period/Phase	Onshore Wind (€)	Solar PV (€)
2025–2026	118–192	156–209
2027–2028	103–175	135–185
2029–2030	83–150	110–161

For modeling purposes, midpoint values were calculated:

- **Wind:** 2025–2026 \rightarrow €155/MWh, 2027–2028 \rightarrow €139/MWh, 2029–2030 \rightarrow €116/MWh
- **Solar:** 2025–2026 \rightarrow €182.5/MWh, 2027–2028 \rightarrow €160/MWh, 2029–2030 \rightarrow €135.5/MWh

Cost Calculation Formulas

Annual output (MWh) for a site i is calculated as:

$$\text{Annual Output} = \text{Power (MW)} \times 8,760 \quad (\text{hours/year})$$

The total cost of allocating demand node j to facility i in period t is:

$$\text{Total Solar Cost (€)} = \text{Solar PV (€/MWh)} \times \text{Annual Output (MWh)} \times 2$$

$$\text{Total Wind Cost (€)} = \text{Wind (€/MWh)} \times \text{Annual Output (MWh)} \times 2$$

The factor of 2 accounts for the two-year duration of each phase in the model.

5.2 Mathematical Modelling

We already discussed in detail in Section 4

5.3 Experimental Procedure

The experiment was conducted in two main stages for each scenario:

1. **Base Run:** Solved the model with p set to the maximum feasible number of facilities, producing:
 - The list of open facilities per phase ($y_{i,t} = 1$).
 - Demand-to-supply assignments ($x_{i,j,t} = 1$).
 - Phase-specific assignment network diagrams.
2. **Scenario Sweep:** Varied p from 0 to 39 and re-solved the model each time to record:
 - The total assignment cost (objective value).
 - The relationship between allowed facility count and cost savings.

5.4 Result Generation

For each run, we generated:

- **Facility schedules:** Which facilities are open in each phase.
- **Assignment matrices:** Mapping demand nodes to facilities.
- **Cost vs. Facility Count curves:** Showing diminishing marginal cost savings beyond a “knee point” in p .
- **Spatial allocation diagrams:** Visualizing how the model consolidates or disperses demand depending on capacity, cost, and the utilization threshold.

6. Results

Based on the experiments we obtained interesting insights on the data. This section describes our findings for the experiments performed in the section 5

6.1 Wind Plant

The optimization model was run for three time periods: t1, t2, and t3. For each period, a network graph was created to show the assignments of demand locations (coal plants being replaced) to supply locations (wind generation facilities). Refer Figure 6.1 for the graphs.

- **t1 Assignments:** The network graph displays multiple demand nodes linked to their assigned wind facilities. Most facilities are self-supplied, indicated by loops, while some have cross-assignments with different plants.
- **t2 Assignments:** Like t1, most facilities meet their own demand, though a few plants supply others. The connectivity pattern varies slightly, showing shifts in the supply-demand balance during this period.
- **t3 Assignments:** The graph is sparser than in t1 and t2 as most of the facilities do not have demand and supply during third phase. Many nodes have self-loops, and there are fewer cross-connections, showing less interdependence between sites.

The model was also examined to see how the total assignment cost changes with the number of facilities. The resulting plot shows: Refer Figure 6.2 for the plot.

1. For 0 to 17 facilities, the total cost remains at 0. This means no feasible assignments incur costs in this range due to the absence of active facilities.
2. At 18 facilities, there is a sharp increase in cost to over 4×10^{10} € indicating the minimum threshold needed to meet demand with available capacity.



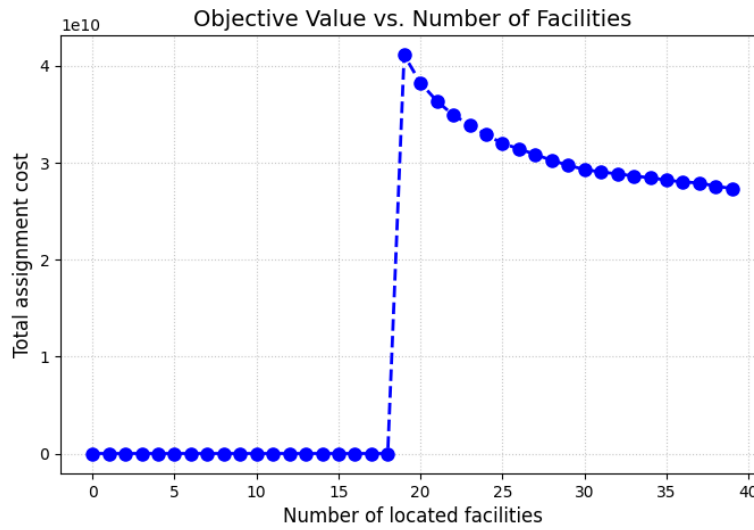


Figure 6.2: Assignment cost v/s Running facilities Plot of Wind Plant.

3. Beyond 18 facilities, the cost gradually decreases as more facilities are added. This shows economies of scale, but the rate of decrease slows after about 25 facilities.

These results point to a crucial facility threshold for feasibility and demonstrate how increasing facility availability impacts total cost. The network graphs further illustrate how facility use patterns change over time.

6.2 Solar Plant

The model aimed to find the lowest total cost for replacing coal power with solar power while meeting demand at different times.

Assignment Cost vs. Running Facilities plot of Solar Plant. Refer Figure 6.4 for the plot.

1. The first graph, “Objective Value vs. Number of Facilities,” illustrates how the total cost varies as the number of facilities increases.
2. When there are fewer than 15 facilities, the total cost is zero since no facilities are selected.
3. At 15 facilities, the total cost reaches its peak, which is a little over 5×10^{10} €
4. As more facilities are added, the cost gradually decreases. After about 30 facilities, the cost stabilizes around 3.1×10^{10} €

Assignments in Each Time Period The network diagrams display which facilities are used to meet demand in each period. Refer Figure 6.3 for the graphs.

- **t1 Assignments:** Many facilities are active, and most demand points connect to a facility.

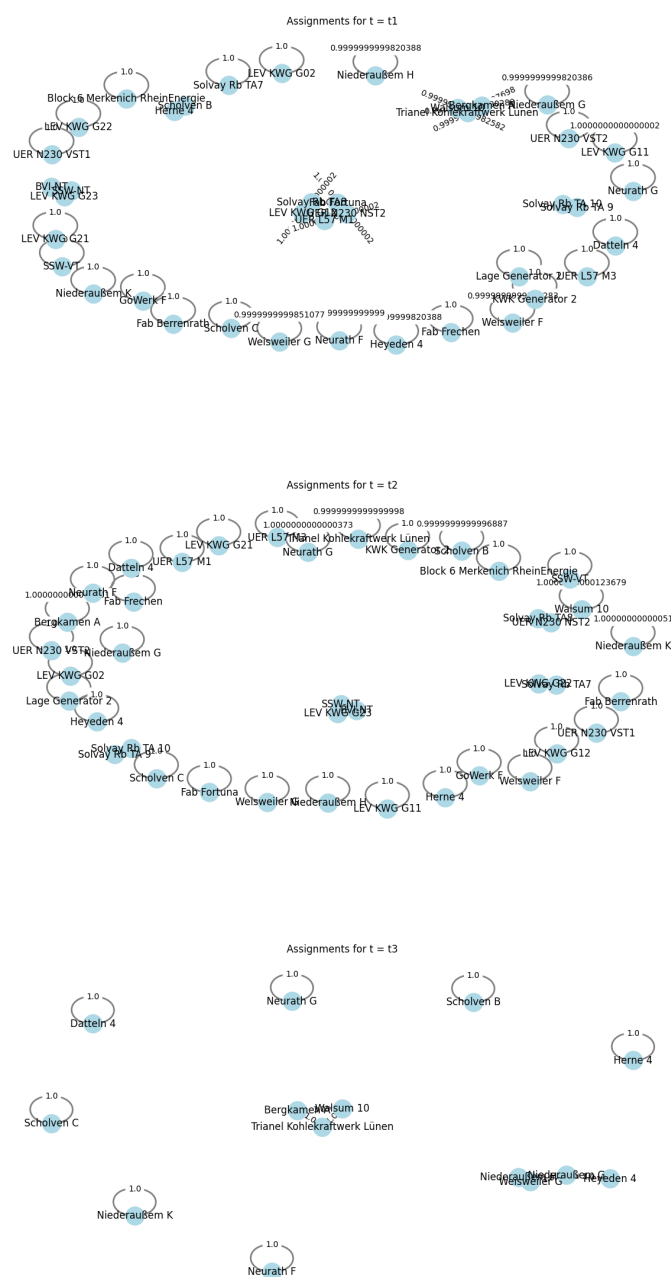


Figure 6.3: Phase-wise assignments of facilities of solar.

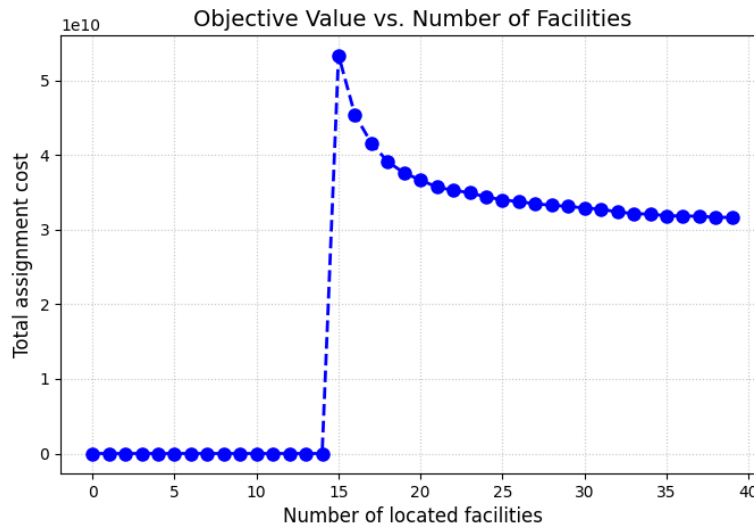


Figure 6.4: Assignment cost v/s Running facilities Plot of Solar Plant.

- **t2 Assignments:** A similar number of facilities are active, with slightly different links between demand points and facilities.
- **t3 Assignments:** Far fewer facilities are active compared to t1 and t2 as most of the facilities do not have demand and supply during third phase. Only a small group of facilities is linked to demand points.

These results highlight how the selection and number of facilities change over time and how this impacts the total cost.

6.3 Hybrid Plant

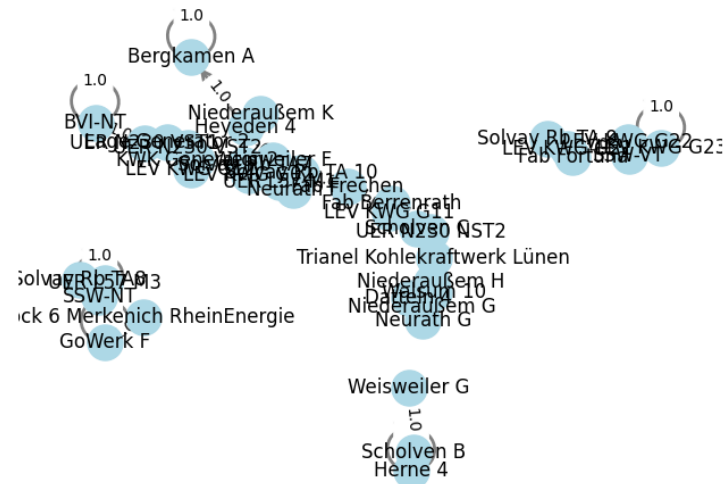
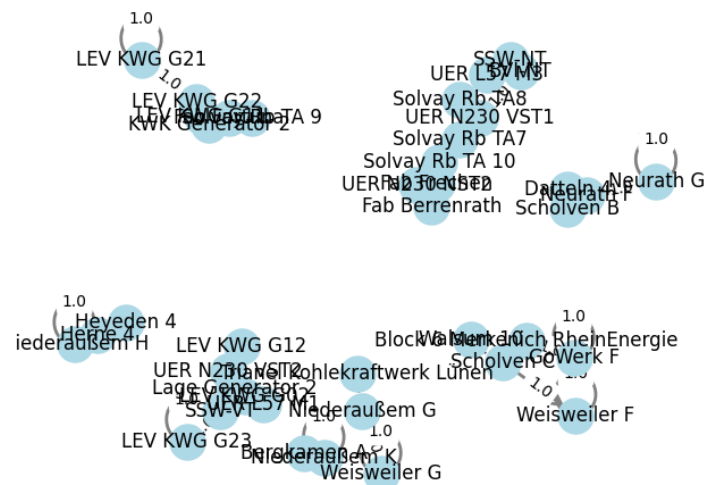
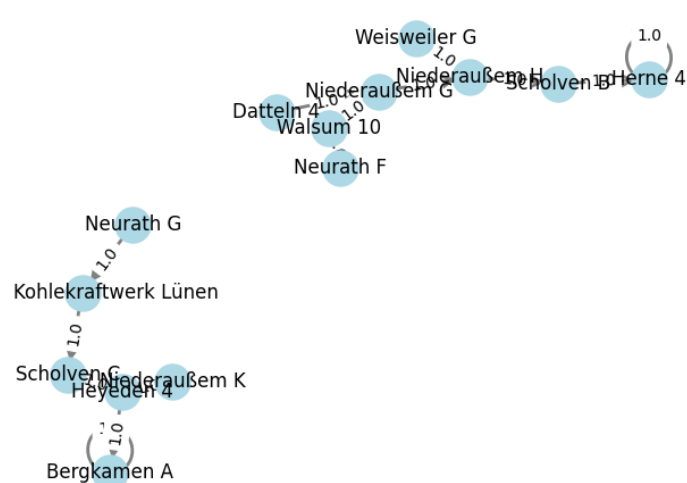
The results from the hybrid facility location model are presented in four main visual outputs.

Objective Value vs. Number of Facilities. Refer Figure 6.6 for the plot.

1. The plot shows how the total assignment cost changes with the number of facilities. Initially, when there are fewer than six facilities, the cost stays very low indicating that there is not enough coverage or no allocations at all.
2. At six facilities the cost sharply increases and reaches its peak. After this point, the cost gradually decreases as more facilities are added and eventually levels off after about 20 facilities.
3. This trend suggests that beyond a certain number of facilities, adding more has little to no effect on lowering the total cost.

Assignments in Each Time Period.

The network diagrams display which facilities are used to meet demand in each period. Refer Figure 6.5 for the graphs.

Assignments for $t = t_1$ Assignments for $t = t_2$ Assignments for $t = t_3$ 

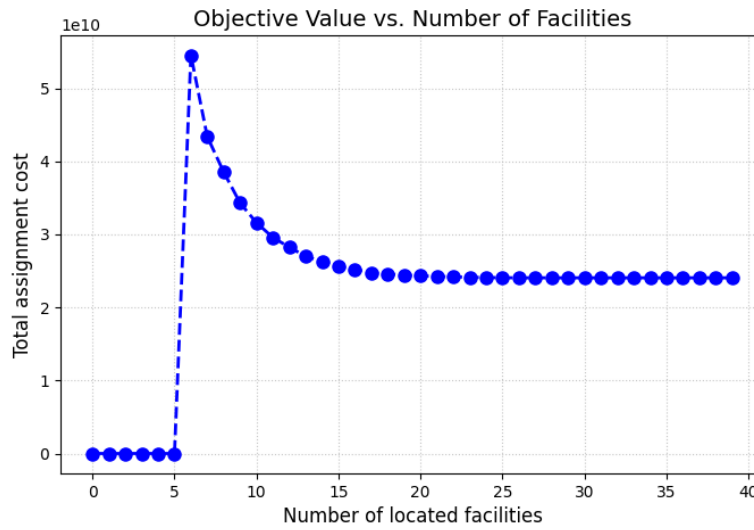


Figure 6.6: Assignment cost v/s Running facilities Plot of Hybrid Plant.

t1 Assignment: For the first time period (t1), the network diagram shows the connections between demand nodes and their assigned facilities. Each arrow represents an allocation, and all shown allocations have a weight of 1. The diagram displays multiple active facilities serving various demand points, with several facilities linked to multiple nodes.

t2 Assignment: In the second time period (t2), the network shows a denser pattern of connections compared to t1. More facilities are active, and there are more allocation lines, which indicates higher activity or more demand being met during this period.

t3 Assignment: For the third time period (t3), the network appears sparser than t2. Fewer facilities are active and the allocations are more spread out, indicating a different operational pattern that is because most of the facilities do not have demand and supply during third phase.

Overall, the model outputs show how the total cost changes as facilities increase and how allocations vary significantly between time periods. This gives insight into the trade-offs in cost efficiency and the operational changes across different phases.

7. Conclusion

Phase-wise replacement of coal based facilities can be framed as Optimization Problem. Our work presents this replacement with Solar, Wind and Hybrid (solar and wind) energies. Utilizing mixed integer programming using solver we have optimized cost for replacing coal with renewable energies. We found out that it is feasible to replace a coal facilities based on various constraints as detailed in our work. These optimization has given us estimates for costs, thus helping us to understand how would replacement economically work.

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