UAV Catch and Return

UdayGirish Maradana[*], Pradnya Sushil Shinde[†], Om Vinayak Gaikwad[‡]

December 15 2023

Abstract

The proposed project is an implementation of the Linear Quadrotor Regulator (LQR) theory for the control of a quadrotor that spans the given restricted airspace. A control model is proposed such that the quadrotor can withstand any disturbances that it may experience during trajectory traversal. Moroever, the idea focuses on the capture and retrieval of unknown aerial intruders such as a UAV. In control design approaches, control systems that provide swiftness in responses to disturbances are preferred. Various factors such as response time, launch sequence, trajectory control, capture aptness etc. determine the type of controller to be used. As for the context of this project, we have chosen a Linear Quadratic Regulator to execute the assigned goal of "capture and return".The non-linear dynamic model of the quadrotor has been derived and further linearised using a first-order Taylor series expansion. The success of the proposed controller has been defined through a series of experiments and results.

1 Introduction

Quadrotors showcase non-linear dynamic behavior that needs to be tuned with effective control strategies for the deployment of tasks that require timely response. In the context of this project, the focus is on a specific scenario where the quadrotor is tasked with tracking an unidentified UAV, capturing it, and safely returning thereafter. To elaborate further, this project presents a comprehensive approach, starting from understanding the non-linear dynamics of quadrotors to the critical step of linearization, and ultimately, the design of an effective linear quadratic controller. This strategy is pivotal for ensuring the quadrotor's ability to navigate, track, capture, and return with precision in dynamic and real-world scenarios.

While moving towards a control design approach, we will consider a set of constraints that must be observed to achieve an effective implementation of the proposed controller.

Constraints:

1. Quadrotor is not allowed to leave the airspace.
2. If the quadrotor is successful in capturing the UAV, then it should return to the home position.
3. If the UAV escapes the airspace, the quadrotor should return to the home position.
4. When the UAV is near the quadrotor, it experiences a disturbance force < 2 N and a moment < 1 Nm.

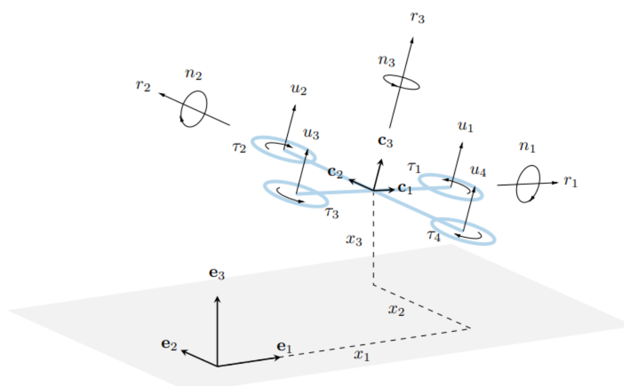


Figure 1: Free Body Diagram of Quadrotor

System Dynamics:
The corresponding basic rotation (elemental rotation: ϕ, θ, ψ) matrices that define the orientation of the body-fixed frame C with respect to the inertial reference frame E can be defined as:

$$R_3(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_1(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

Accordingly, we have

$$\mathbf{R_{C/E}} = \mathbf{R_3}(\psi)\mathbf{R_2}(\theta)\mathbf{R_1}(\phi)$$

The State Space form of the quadrotor can be described by the following set of equations:

[*] umaradana@wpi.edu
[†] pshinde1@wpi.edu
[‡] ogaikwad@wpi.edu

$$
\begin{aligned}
\dot{x} &= v, && (1)\\
\dot{\alpha} &= T^{-1}\omega, && (2)\\
\dot{v} &= -g\mathbf{e}_3 + \frac{1}{m}\mathbf{R}_{C/E}(\mathbf{u}_1 + \mathbf{u}_2 + \mathbf{u}_3 + \mathbf{u}_4)\mathbf{c}_3 \\
&\quad + \frac{1}{m}\mathbf{R}_{C/E}\mathbf{r}, && (3)\\
\dot{\omega} &= \mathbf{I}^{-1}\left((u_2 - u_4)l\mathbf{c}_1 + (u_3 - u_1)l\mathbf{c}_2 \right.\\
&\quad \left. + (u_1 - u_2 + u_3 - u_4)\boldsymbol{\sigma}\mathbf{c}_3 + \mathbf{n} - \omega \times \mathbf{I}\omega\right), && (4)
\end{aligned}
$$

where $\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}$ &$\mathbf{u_4}$ can be defined as control inputs.

Other system parameters can be defined as:

$$
\begin{aligned}
g &= 9.81 \quad [\mathrm{m/s^2}]\\
l &= 0.2 \quad [\mathrm{m}]\\
m &= 0.5 \quad [\mathrm{kg}]\\
I &= \begin{bmatrix}1.24 & 1.24 & 2.48\end{bmatrix} \quad [\mathrm{kg\ m^2}]\\
\mu &= 3.0 \quad [\mathrm{N}]\\
\sigma &= 0.01 \quad [\mathrm{m}]\\
I_{11} &= 1.24 \quad [\mathrm{kg\ m^2}]\\
I_{22} &= 1.24 \quad [\mathrm{kg\ m^2}]\\
I_{33} &= 2.48 \quad [\mathrm{kg\ m^2}]
\end{aligned}
$$

2 Methodology

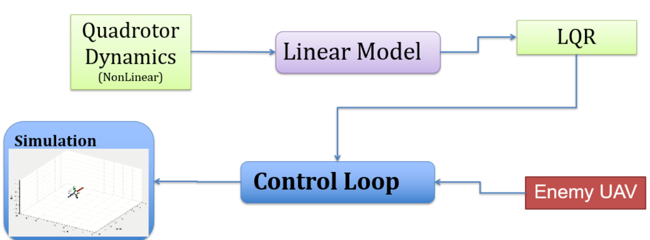


Figure 2: Control Methodology

The methodology used in this project to achieve the mentioned task is to use Linear - Quadratic - Regulator (LQR) controller to control the quadrotor trajectory and track the enemy UAV and bring it back to the nest position. To do so we have to follow the control law of the LQR controller, in which the first step is to linearize the quadrotor dynamics using a linear model. Hence, the system is represented by linear differential equations:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{5}$$

where:

- $x(t)$ is the state vector.
- $u(t)$ is the control input vector.
- A is the system matrix.
- B is the input matrix.

Now based on this we have to form a quadratic cost function and the objective is to minimize this cost function given as:

$$J = \int_0^\infty \left(x(t)^T Q x(t) + u(t)^T R u(t)\right) dt \tag{6}$$

where:

- Q is a positive semi-definite matrix, weighting the state deviations.
- R is a positive definite matrix, weighting the control efforts.

Control Law and Riccati Equation

The optimal control problem involves solving the Algebraic Riccati Equation (ARE):

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{7}$$

where P is a positive semi-definite matrix to be determined.

The optimal control law is derived as follows:

$$u(t) = -Kx(t) \tag{8}$$

where K is the optimal feedback gain, calculated as $K = R^{-1}B^T P$ which is known as the control gain.

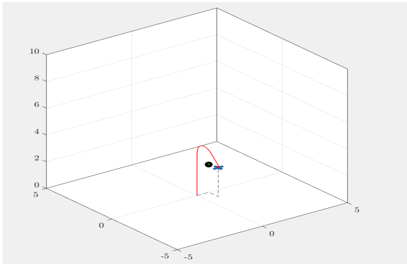
Once we solve the Algebraic Riccati Equation to find P we have to compute the optimal gain K using P. Finally, implement the control law $u(t) = -Kx(t)$ in the system.

2.0.1 Simulation

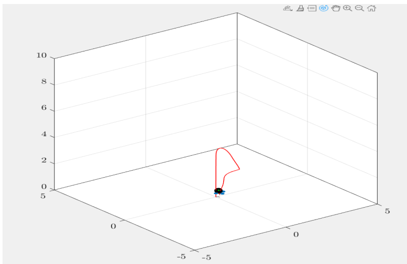
The simulation platform allocated to work on was MATLAB R2023a to mention quadrotor dynamics, calculate control gains using the "lqr" function, simulation the trajectories of the duadrotor and UAV and finally, plot the results of the position of both at certain time steps.

Advantages

- LQR provides an optimal solution for linear systems by minimizing a predefined quadratic cost function, ensuring efficient balance between system performance and control effort.
- This controller is known to be robust hence it is widely used in quadrotors as it requires more robust control
- The design of the controller is straightforward making it relatively easy to implement.



(a) Capture

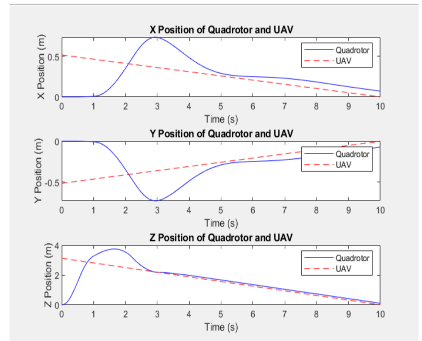


(b) Return

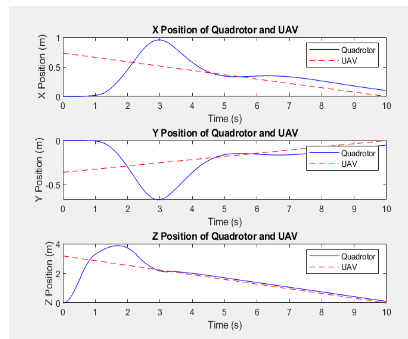
Figure 3: Simulation Results: UAV Capture & Return

2.0.2 Disadvantages

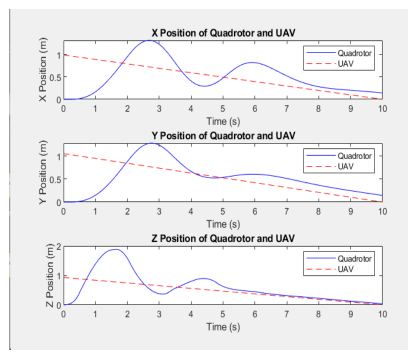
- LQR controllers are mainly designed of linear systems and work pretty well on them and hence the performance can degrade for non-linear systems if not linearized correctly.
- Selecting appropriate weighting matrices (Q and R) often reuqiries trial-and-error approach, which can be time-consuming and may not yield the best results for all scenarios.
- It explicitly account for external disturbances or model uncertainties. While it can be robust to small disturbances, its performance might be suboptimal for variations.



(a) Trajectory 1



(b) Trajectory 2



(c) Trajectory 3

Figure 4: Random Trajectory Plots

3 Results

In the presented results, we observe the X, Y, and Z position trajectories of both a quadrotor and an Unmanned Aerial Vehicle (UAV) across three distinct, randomly generated flight paths executed by the UAV. These trajectories are depicted in a series of plots, where the dashed lines illustrate the UAV's random flight paths over a simulation duration of 10 seconds. In contrast, the solid lines represent the corresponding trajectories of the quadrotor.

A notable aspect of these trajectories is that, after an initial period, there is a visible convergence between the paths of the quadrotor and the UAV. This convergence becomes more pronounced with time, eventually leading to a complete alignment of their trajectories. The significance of this convergence lies in its representation of the quadrotor successfully "catching" or intercepting the UAV. Following this interception, the alignment of their paths symbolizes the joint "return" of both the quadrotor and the UAV, indicating a successful completion of the simulated mission.

This behavior, captured in the plots, is important in understanding the effectiveness of the control algorithm used in guiding the quadrotor to intercept and subsequently bring back the UAV to the nest position.

References

[1] T. Huang, D. Huang, Z. Wang, N. Qin and A. Shah, "Robust control for a quadrotor UAV based on linear quadratic regulator," 2020 39th Chinese Control Conference (CCC), Shenyang, China, 2020, pp. 6893-6898, doi: 10.23919/CCC50068.2020.9188679.

[2] B. Fan, J. Sun and Y. Yu, "A LQR controller for a quadrotor: Design and experiment," 2016 31st Youth Academic Annual Conference of Chinese Association of Automation (YAC), Wuhan, China, 2016, pp. 81-86, doi: 10.1109/YAC.2016.7804869.

[3] Rinaldi, F., Chiesa, S. & Quagliotti, F. Linear Quadratic Control for Quadrotors UAVs Dynamics and Formation Flight. J Intell Robot Syst 70, 203–220 (2013).

[4] Luís Martins, Carlos Cardeira, Paulo Oliveira, Linear Quadratic Regulator for Trajectory Tracking of a Quadrotor, IFAC-PapersOnLine, Volume 52, Issue 12, 2019.

[5] B. D. O. Anderson, "Controller design: moving from theory to practice," in IEEE Control Systems Magazine, vol. 13, no. 4, pp. 16-25, Aug. 1993, doi: 10.1109/37.229554.