

## Binary Search Tree

In a binary tree, every node can have a maximum of two children but there is no need to maintain the order of nodes basing on their values. In a binary tree, the elements are arranged in the order they arrive at the tree from top to bottom and left to right.

A binary tree has the following time complexities...

1. **Search Operation -  $O(n)$**
2. **Insertion Operation -  $O(1)$**
3. **Deletion Operation -  $O(n)$**

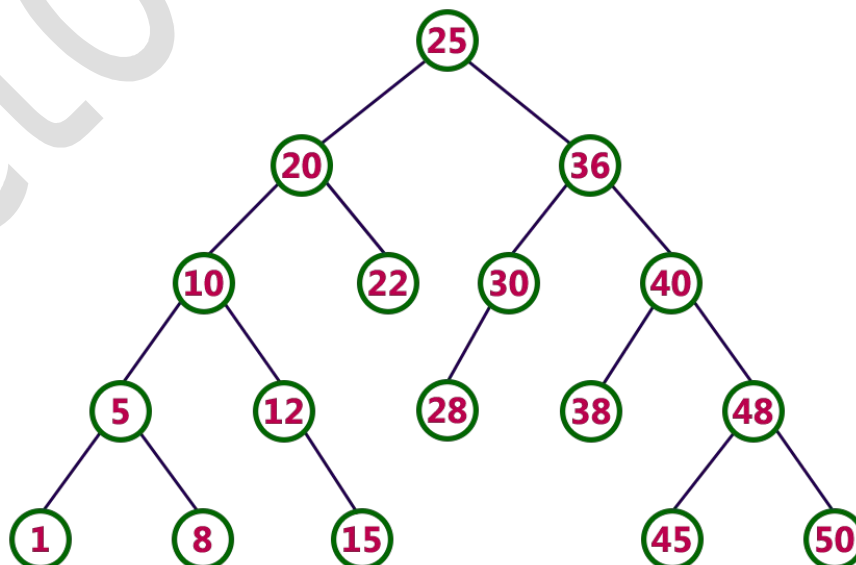
To enhance the performance of binary tree, we use a special type of binary tree known as **Binary Search Tree**. Binary search tree mainly focuses on the search operation in a binary tree. Binary search tree can be defined as follows...

***Binary Search Tree is a binary tree in which every node contains only smaller values in its left subtree and only larger values in its right subtree.***

***Or***

***In Binary Search Tree Left child is always smaller than parent and right child is always greater than parent.***

**Example :**



**Every binary search tree is a binary tree but every binary tree need not to be binary search tree.**

## Operations on a Binary Search Tree

The following operations are performed on a binary search tree...

1. **Search**
2. **Insertion**
3. **Deletion**

### Search Operation in BST

In a binary search tree, the search operation is performed with  **$O(\log n)$**  time complexity. The search operation is performed as follows...

- Step 1 - Read the search element from the user.
- Step 2 - Compare the search element with the value of root node in the tree.
- Step 3 - If both are matched, then display "Given node is found!!!" and terminate the function
- Step 4 - If both are not matched, then check whether search element is smaller or larger than that node value.
- Step 5 - If search element is smaller, then continue the search process in left subtree.
- Step 6- If search element is larger, then continue the search process in right subtree.
- Step 7 - Repeat the same until we find the exact element or until the search element is compared with the leaf node
- Step 8 - If we reach to the node having the value equal to the search value then display "Element is found" and terminate the function.
- Step 9 - If we reach to the leaf node and if it is also not matched with the search element, then display "Element is not found" and terminate the function.

### Insertion Operation in BST:

In a binary search tree, the insertion operation is performed with  $O(\log n)$  time complexity. In binary search tree, new node is always inserted as a leaf node. The insertion operation is performed as follows...

- Step 1 - Create a newNode with given value and set its **left** and **right** to **NULL**.
- Step 2 - Check whether tree is Empty.
- Step 3 - If the tree is **Empty**, then set **root** to **newNode**.

- Step 4 - If the tree is **Not Empty**, then check whether the value of newNode is **smaller** or **larger** than the node (here it is root node).
- Step 5 - If newNode is **smaller** than **or equal** to the node then move to its **left** child. If newNode is **larger** than the node then move to its **right** child.
- Step 6- Repeat the above steps until we reach to the **leaf** node (i.e., reaches to NULL).
- Step 7 - After reaching the leaf node, insert the newNode as **left child** if the newNode is **smaller or equal** to that leaf node or else insert it as **right child**.

### Example

Construct a Binary Search Tree by inserting the following sequence of numbers...

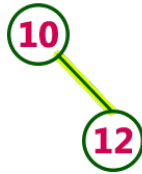
**10,12,5,4,20,8,7,15 and 13**

Above elements are inserted into a Binary Search Tree as follows...

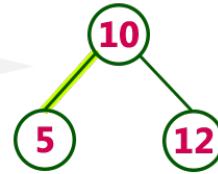
insert (10)



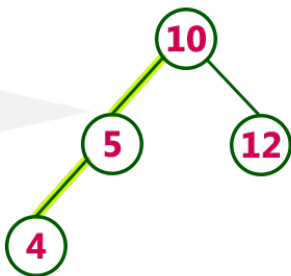
insert (12)



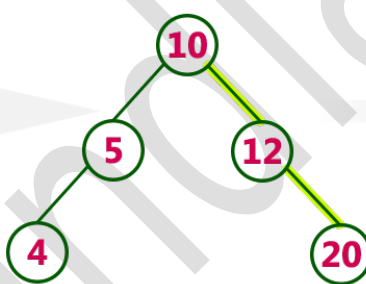
insert (5)



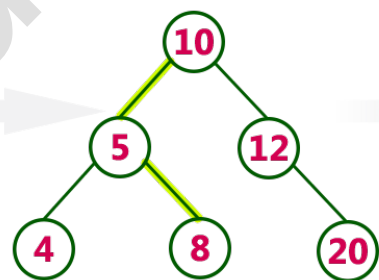
insert (4)



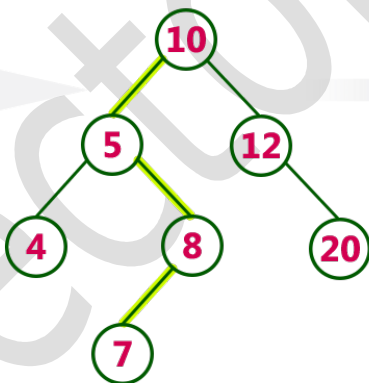
insert (20)



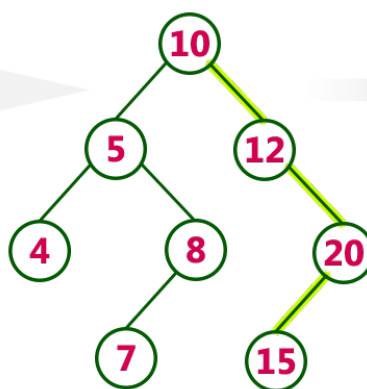
insert (8)



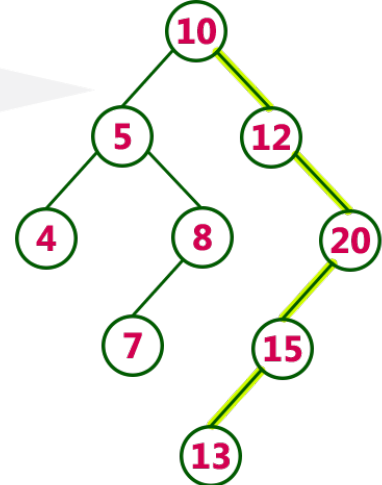
insert (7)



insert (15)



insert (13)



## Deletion Operation in BST

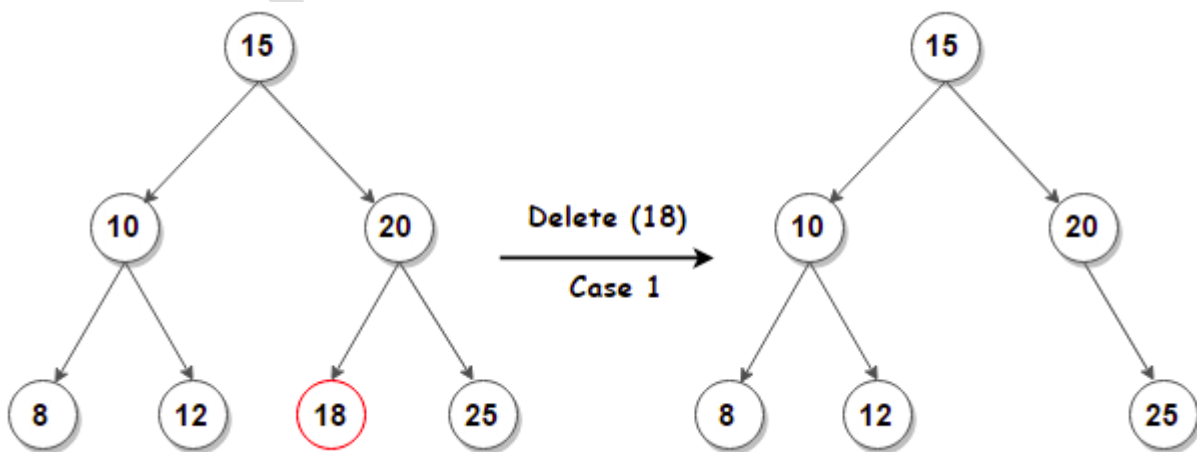
In a binary search tree, the deletion operation is performed with  **$O(\log n)$**  time complexity. Deleting a node from Binary search tree includes following three cases...

- **Case 1: Deleting a Leaf node (A node with no children)**
- **Case 2: Deleting a node with one child**
- **Case 3: Deleting a node with two children**

### Case 1: Deleting a leaf node

We use the following steps to delete a leaf node from BST...

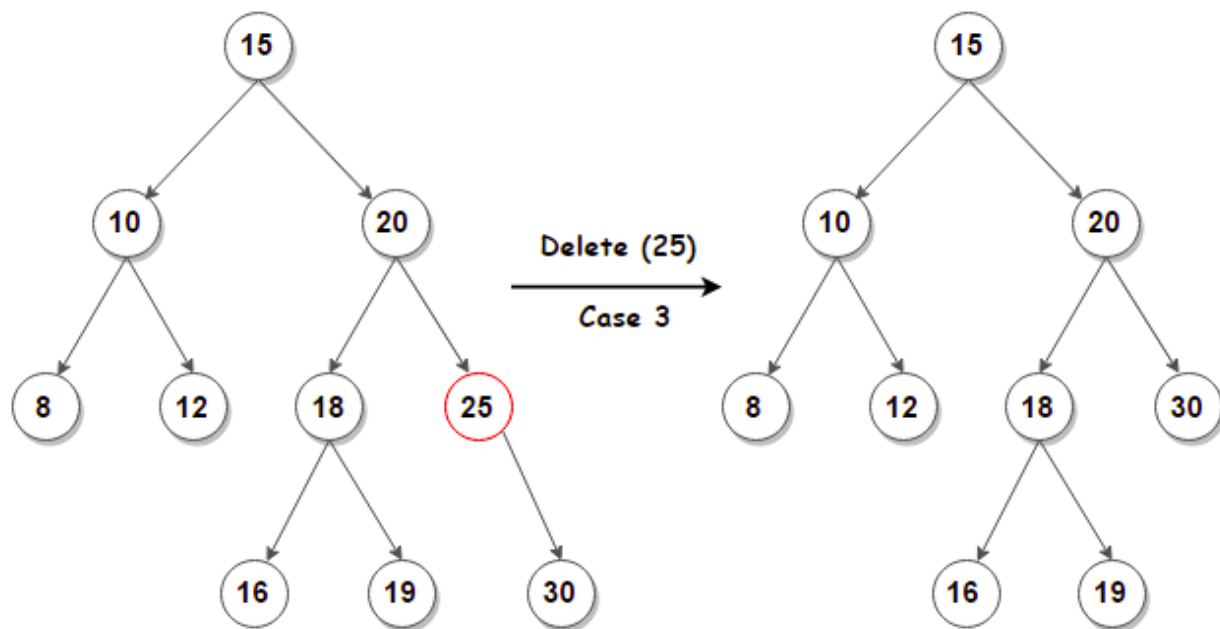
- Step 1 - **Find** the node to be deleted using **search operation**
- Step 2 - Delete the node using **free** function (If it is a leaf) and update NULL in the parent reference and terminate the function.



## Case 2: Deleting a node with one child

We use the following steps to delete a node with one child from BST...

- Step 1 - **Find** the node to be deleted using **search operation**
- Step 2 - If it has only one child then create a link between its parent node and child node.
- Step 3 - Delete the node using **free** function and terminate the function.

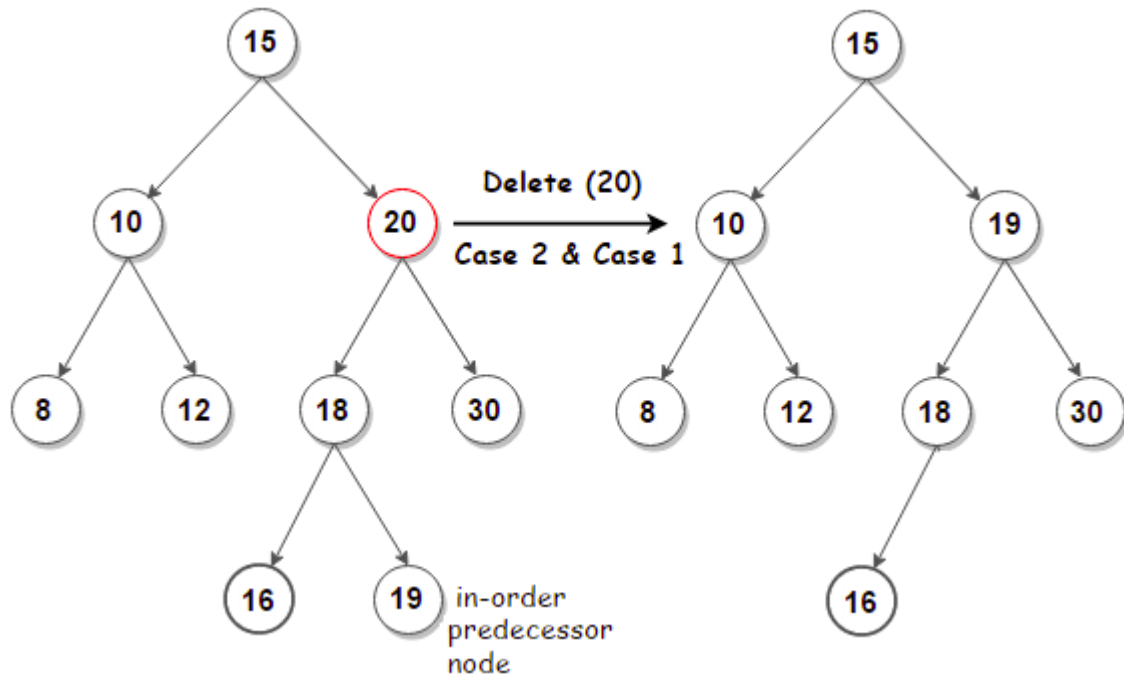


## Case 3: Deleting a node with two children

We use the following steps to delete a node with two children from BST...

- Step 1 - **Find** the node to be deleted using **search operation**
- Step 2 - If it has two children, then find the **largest** node in its **left subtree** (OR) the **smallest** node in its **right subtree**.
- Step 3 - **Swap** both **deleting node** and node which is found in the above step.
- Step 4 - Then check whether deleting node came to **case 1** or **case 2** or else goto step 2

- Step 5 - If it comes to **case 1**, then delete using case 1 logic.
- Step 6- If it comes to **case 2**, then delete using case 2 logic.
- Step 7 - Repeat the same process until the node is deleted from the tree.



Note :

Reference Books : Taken contents and diagrams from various websites.