// Confidence levels

→ Confidence intervals

Q) let 60% of 140 million voters likely approve the president handling their job. If we sample 1000 of them, the result approval % in sample will be off. This off can be found using SE.

→ Confidence intervals give a more precise statement.

$\mu = 60\%$ & $SE = \frac{\sigma}{\sqrt{n}} = \frac{0.49}{\sqrt{1000}} = 1.6\%$ (Acc to CLT)

→ Confidence interval gives a range of plausible values for a population parameter (μ). Usually the confidence interval is centered at an estimate for μ which is an average.

$$\boxed{\text{Confidence interval} = \text{estimate} \pm \underbrace{z\,SE}_{\text{margin of error}}}$$

$$\boxed{SE = \frac{\sigma}{\sqrt{n}}}$$

Bootstrap principle: we can estimate σ by its sample version(s) and still get an approximately correct confidence level.

Estimating SE with Bootstrap principle :-

$$SE = \frac{\sigma}{\sqrt{n}} \times 100\%$$

$\sigma = \sqrt{p(1-p)}$; p = proportion of all voters who approve.

$\Rightarrow$ $S = 58\%$, $\sigma = \sqrt{0.58(1-0.58)} = 0.49$

$$58\% \pm 2 \frac{0.49}{\sqrt{1000}} \Rightarrow \underline{54.9\% \ \& \ 61.1\%}$$

(Q) Random sample = 400 ; $X = 313$; 95% CI $z \cong 2$

(A) $p = \frac{313}{400}$

$$p \pm z\sqrt{\frac{p(1-p)}{n}} \Rightarrow \underbrace{\frac{313}{400} \pm 2\sqrt{\frac{\frac{313}{400}\left(1-\frac{313}{400}\right)}{400}}}_{\text{proportion}}$$

(Q) $n = 500$; Margin error = \$5400. width of error to shrink it about \$2000?

(A) $\sqrt{n} = \sqrt{500}$. shrink ME from \$5400 to \$2000

$$\left(\frac{5400}{2000}\right)\sqrt{500} \Rightarrow \sqrt{\left(\frac{5400}{2000}\right)^2 500}$$

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// Testing hypotheses

→ we toss a coin 10 times and we get 7 tails. Is this sufficient evidence to conclude coin is biased?

null hypothesis = H_0 = nothing extraordinary is going on.

So, $P(T) = H_0 = \frac{1}{2}$

Alternative hypothesis = H_A = different chance process at work.

$P(T) = H_A \neq 1/2$

→ A company develops new drug & tests it on 1000 patients.

H_0: drug has no effect
H_A: drug has effect

→ A ~~true~~ test statistic measures how far away data is from what we would expect if H_0 is true.

$$\boxed{\text{Common test statistic: Z-statistic} = \frac{\text{observed} - \text{expected}}{SE}}$$

observed: % of tails
expected & SE are expected value & SE of this statistic computed when H_0 is true

⇒ In case of 7 tails in 10 tosses, using labels, (expected value of sum)

$\text{expected} = 10 \times P(T) \overset{H_0}{=} 5$

$SE = \sqrt{10}\sqrt{P(1-P)} = \sqrt{10}\sqrt{\frac{1}{2}\times\frac{1}{2}} = 1.58$

So, $z = \frac{7-5}{1.58} = 1.27$

If H_0 is true, z follows standard normal curve.

p-values

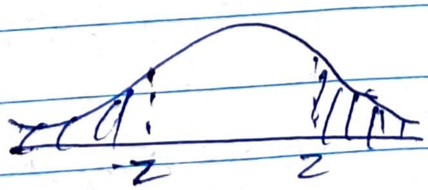
The idea of a test statistic such as z-statistic is that large values of $|z|$ are evidence against null hypothesis.

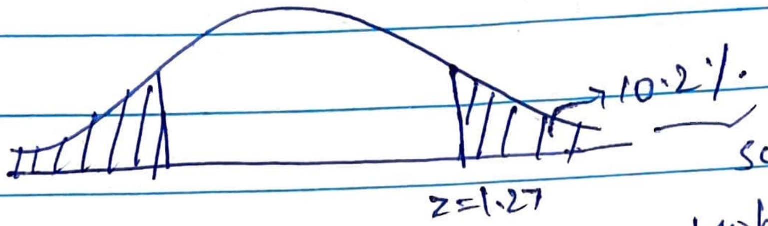
larger $|z|$, larger/stronger the evidence.

Strength of evidence is measured by p-value (observed significance level)

→ p-value is the probability of getting a value of z as extreme or more extreme than the observed z, assuming H_0 is true

if $p < 5\%$, result is statistically significant.





so p-value, the probability of getting an outcome above or below 1.27.

p-value = 20.4% is ≮ 5%.

p-value gives the probability of seeing a statistic as extreme or more extreme than observed, assuming H_0 is true.

t-test:-

→ If limit is 15 ppb and 5 samples average to 15.6 ppb, is this sufficient to conclude concentration μ is above 15 ppb?

measurement = μ + measurement error.

$H_0 = 15$ ppb $H_A > 15$ ppb

$$Z = \frac{15.6 - 15}{SE} \qquad SE = \frac{\sigma}{\sqrt{n}}$$

For small sample sizes ($n \leq 20$), normal curve is not good enough.

$$\boxed{s = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2}}$$

(only when $n \leq 20$)

$$\boxed{\bar{x} \pm t_{(n-1)}\, SE}$$

Two sample z-test :-

Q) last month rating among 1000 → 55%, this month 1500 → 58%.

Assuming H_0:

p_1 = proportion of all likely voters last month

p_2 = this month.

H_0: nothing unusual, $p_1 = p_2 \Rightarrow p_2 - p_1 = 0$

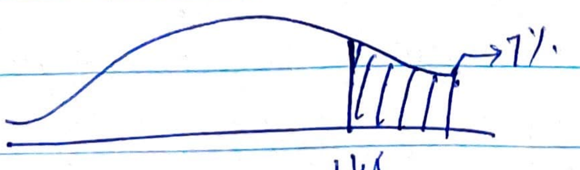
H_A: $p_2 - p_1 \neq 0$

$$Z = \frac{(\hat{p}_2 - \hat{p}_1) - (p_2 - p_1)}{\text{SE of diff}} =$$

$$SE(\hat{p}_2 - \hat{p}_1) = \sqrt{SE(\hat{p}_1)^2 + SE(\hat{p}_2)^2} = 0.0202$$

$$55 \quad = \sqrt{\sqrt{\frac{p_1(1-p_1)^2}{1000}} + \sqrt{\frac{p_2(1-p_2)^2}{1500}}} \quad 58$$

$$\therefore Z = \frac{0.03}{0.0202} = 1.48$$



p-value = 14%.

cannot reject null hypothesis.

Confidence level, $(p_2 - p_1)$

$(\hat{p}_2 - \hat{p}_1) \pm Z \, SE(\hat{p}_2 - \hat{p}_1) = [-1\%, 7\%]$, when $Z = 2$.

$$S^2_{pooled} = \frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2}{n_1 + n_2 - 2}$$