One way Analysis of variance

→ 3 groups : homework only
homework + peer grade
studying.

H_0 = nothing extraordinary = all groups are equal

If we had 2 groups, we use t-tests.

$$t = \frac{\text{difference in sample means}}{\text{SE of difference.}}$$

ANOVA → ~~Compare~~ generalize this idea to situation when we have several groups

↳ Analysis of variance.

group-1	group-2	...	group-k
y_{11}	y_{12}		y_{1k}
y_{21}	⋮		⋮
⋮			
$y_{n_1 k}$	$y_{n_2 2}$		$y_{n_k k}$

$N = n_1 + \dots n_k$; $\bar{y}_j = \frac{1}{n_j} \sum_{i=1}^{n_j} y_{ij}$

$$\bar{\bar{y}} = \frac{1}{N} \sum_{j=1}^{k} \sum_{i=1}^{n_j} y_{ij}$$

Treatment sum of squars,

$$SST = \sum_j \sum_i (\bar{y}_j - \bar{\bar{y}})^2$$

has $(k-1)$ DOF.

Treament mean squar,

$$MST = \frac{SST}{(k-1)}$$

Error of sum,

$$SSE = \sum_j \sum_i (y_{ij} - \bar{y}_j)^2 \quad \text{has } (N-k) \text{ DOF}$$

Error mean squre,

$$MSE = \frac{SSE}{N-k}$$

$$F\text{-test} = F = \frac{MST}{MSE} = \frac{\left(\frac{SST}{k-1}\right)}{\left(\frac{SSE}{N-k}\right)} \star$$

ANOVA table

Source	DOF	Sum of squares	Mean square	F	p-value
Treatment	$k-1$	SST	MST	MST/MSE	
Error	$N-k$	SSE	MSE		
Total	$N-1$	TSS			

$$\text{TSS} \hookrightarrow \sum_j \sum_i (y_{ij} - \bar{y})^2$$

if $F > 5\%$, H_0 is rejected.