

VARIABLE SELECTION:-

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Variable Selection for Best Fit

All Possible Regression

Sequential Selection

All possible Regression:

we need to consider all regression equations involving zero regressors i.e. $Y = \beta_0 + \epsilon$

→ Assume that, there are $(k-1)$ Regressors (Regressors means 'x' variables)

→ As part of the model development we need to estimate 'k' parameters.

like $\beta_0, \beta_1, \beta_2, \dots, \beta_{k-1}$

∴ Total number of regressor model is 2^{k-1}

<u># of Regressors</u>	<u># of Model</u>
0	$k-1 \beta_0$
1	$k-1 \beta_1$
2	$k-1 \beta_2$
⋮	⋮
$k-1$	$k-1 \beta_{k-1}$
	2^{k-1}

Ex:- If we have 4 'x' variables, we have these following all possible Regressor models.

No	Regression Model	R ²	MS _{res}	AdjR ²	Cp
1	y = β ₀ + ε	0	226.3	0	442.92
2	y = β ₀ + β ₁ X ₁ + ε	53.4	115.06	49.2	202.55
3	y = β ₀ + β ₂ X ₂ + ε	66.6	82.39	63.6	142.49
4	y = β ₀ + β ₃ X ₃ + ε	28.6	176.3	22.1	315.16
5	y = β ₀ + β ₄ X ₄ + ε	67.5	80.35	64.5	138.73
6	y = β ₀ + β ₁ X ₁ + β ₂ X ₂ + ε	97.9	5.7	97.5	2.68
7	y = β ₀ + β ₁ X ₁ + β ₃ X ₃ + ε	54.8	122.7	45.8	198.1
8	y = β ₀ + β ₁ X ₁ + β ₄ X ₄ + ε	97.2	7.47	96.7	5.5
9	y = β ₀ + β ₂ X ₂ + β ₃ X ₃ + ε	84.7	41.54	81.7	62.44
10	y = β ₀ + β ₂ X ₂ + β ₄ X ₄ + ε	68	86.88	61.2	38.23
11	y = β ₀ + β ₃ X ₃ + β ₄ X ₄ + ε	93.5	17.57	92.2	22.37
12	y = β ₀ + β ₁ X ₁ + β ₂ X ₂ + β ₃ X ₃ + ε	98.2	5.34	97.6	3.04
13	y = β ₀ + β ₁ X ₁ + β ₂ X ₂ + β ₄ X ₄ + ε	98.2	5.33	97.6	3.02
14	y = β ₀ + β ₁ X ₁ + β ₃ X ₃ + β ₄ X ₄ + ε	98.1	5.64	97.5	3.5
15	y = β ₀ + β ₂ X ₂ + β ₃ X ₃ + β ₄ X ₄ + ε	97.3	8.2	96.3	7.34
16	y = β ₀ + β ₁ X ₁ + β ₂ X ₂ + β ₃ X ₃ + β ₄ X ₄ + ε	98.2	5.9	97.3	5

$$\therefore 2^{(k-1)} = 2^4 = 16$$

∴ k-1 = # of Regressors

∴ 16 models has developed

⇒ These (2^{k-1}) equations are evaluated by below criteria.

- ① R²
- ② Adjusted R²
- ③ MS Residual
- ④ Mallow Statistic (Cp)

Mallow Statistic [Cp] :-

Mallow statistic measures the overall bias (or) Mean Square Error in the fitted model.

\hat{y} = Predicted Response

E(y) = True Response, 'k' is # of parameters (i.e. # of β's)

$$C_p = \frac{\sum E(\hat{y} - E(y))^2}{\sigma^2}, \text{ Here we are dividing M.S.E with its S.D}$$

⇒ Standardized Mean Square Error.

Here, $\hat{y} - E(y)$ is nothing but (predicted value - population value) i.e. nothing but Bias

So, In Realtime we cannot find population value i.e. E(y) so, we can estimated by

$$C_p = \frac{SS_{Res}(k)}{MS_{Res}(Full)} - n + 2k$$

$$C_p = \frac{M_{S_{Res}}(Full)}{M_{S_{Res}}(k)}$$

where $k = \#$ of parameters including β_0

$M_{S_{Res}}(Full)$ = This is always for the full model

$SS_{Res}(k)$ = This is for ' k ' parameters (i.e., $(k-1)$ regressors)

ex:
$$\hat{C}_p = \frac{SS_{Res}(x_1)}{M_{S_{Res}}(x_1, x_2, x_3, x_4)} - 13 + 2(2)$$

$$= 202.55$$

But the question is how do you find Best C_p from all calculated ...?

→ If any calculated C_p value is equivalent to the no of parameters i.e., going to be best C_p .