CS663, Assignment 5 **Yash Sanghvi, Pradyot Prakash**
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Question 5

(I)

Assume we take f = input row image (length N) Let,

$$h = [1 - 1]$$

$$f * h = f(x) - f(x-1) = g(x)$$

For the boundary condition, we pad f and h with appropriate number of zeros and then convolve the two functions. g is obtained by taking the sub-vector of appropriate length.

Now taking the problem to Fourier domain,

$$G(w) = H(w)F(w)$$

where H and F were computed from the appropriately padded images. Solving in Fourier domain, we get,

$$f = F^{-1}(\frac{G}{H})$$

But, we see that $H(0) = h(0) + h(1) = 0$. So, inverse Fourier transform does not exist for $w = 0$. This implies that we cannot obtain the image back. This problem is not restricted to the gradient operator that we used. For other operators, such as [1 -2 1], we still face the issue of $H(0) = 0$.

Intuitively, we can say that the DC information of the original signal f is lost and hence it is not possible to restore it.

(II)

A similar logic extends to the 2D case as well. Suppose we use the following Prewitt operators for calculating gradients along the X and Y directions, respectively,

$$h_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$h_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Let the 2D DFT of gradient images be $G_x(w_1, w_2)$ and $G_y(w_1, w_2)$, and the 2D DFT of the differential operators (after appropriate zero padding) be $H_x(w_1, w_2)$ and $H_y(w_1, w_2)$.

Note that,

$H_x(0,0) = \Sigma\Sigma h_x(x,y)e^{\frac{-j(0)x}{N}}e^{\frac{-j(0)y}{N}} = 0$

Similarly,

$H_y(0,0) = \Sigma\Sigma h_y(x,y)e^{\frac{-j(0)x}{N}}e^{\frac{-j(0)y}{N}} = 0$

Thus $F(w1, w2) = \frac{G_x(w_1,w_2)}{H_x(w_1,w_2)} = \frac{G_y(w_1,w_2)}{H_y(w_1,w_2)}$ is not defined at $(w_1, w_2) = (0, 0)$ Thus the 2D images can not restored from the gradient images.

The same analysis works for any operator which sums to 0 including but not limited to Sobel operators.