CS663, Assignment 5 **Yash Sanghvi, Pradyot Prakash**
Instructor: Prof. Ajit Rajwade 13D070042, 130050008
Question 5

Part (i)

Assume we take f = input row image (length N)

$$h = [1, -1]$$

$$\Rightarrow f * h = f(x) - f(x-1) = g(x)$$

Boundary Conditions:

We pad f & h with appropriate no. of zeroes & then convolve and g(x) is obtained by taking the subvector of appropriate length.

Now, we take the problem to Fourier Domain:

$$F(\omega_1) H(\omega_1) = G(\omega_1)$$

where $F(\omega_1)$, $H(\omega_1)$ are the DFTs of the signal f & kernel h after appropriate padding.

For example let $f = [1\ 2\ 3\ 4\ 5]$

$f_{padded} = [1\ 2\ 3\ 4\ 5\ 0\ 0]$

$h_{padded} = [+1\ -1\ 0\ 0\ 0\ 0\ 0]$

g (after extracting $n = 0$ to 4

$= [1\ 1\ 1\ 1\ -5]$

note $g[n] = f * h = [1\ 1\ 1\ 1\ -5]$

The natural approach to finding $f[n]$ is :

$$F^{-1}\left(\frac{G(\omega_1)}{H(\omega_1)}\right)$$

However $H(0) = +e^{-j(\omega)\frac{1}{N}} - e^{-j(0)\frac{(2)}{N}} = 0$

Fourier transform for $\omega = 0$ is not obtainable & hence one cannot obtain $f[n]$ from $g[n]$. Intuitively, we can say that the DC information is lost hence not possible to restore.

Part (ii)

A similar logic can be used to determine whether the 2D images are obtainable from its gradients

Assume the kernels used for ~~image~~ X gradient & Y gradient are
$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ & $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ respectively.

and let the X and Y gradient images be g_x, g_y ~~Now~~ respectively.

Let the 2D-DFTs be $G_x(\omega_1, \omega_2)$, $G_y(\omega_1, \omega_2)$
& 2D-DFTs of kernels after appropiate zero paddings be
$H_x(\omega_1, \omega_2)$, $H_y(\omega_1, \omega_2)$

$$H_x(0,0) = \sum \sum h_x(x,y) e^{-j\frac{(0)x}{N}} e^{-j\frac{0(y)}{N}}$$
$$= 0$$
$$H_y(0,0) = 0$$

Thus $F(\omega_1, \omega_2) = \dfrac{G_x(\omega_1, \omega_2)}{H_x(\omega_1, \omega_2)} = \dfrac{G_y(\omega_1, \omega_2)}{H_y(\omega_1, \omega_2)}$

is not defined at $(\omega_1, \omega_2) = (0,0)$
Thus, the 2D images can not be restored from ~~origino~~ gradient images.