Nonlinear Dynamics Circuit Design

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The study of chaotic systems is one of the vital and novel areas of physics. Chaotic systems can be built in the laboratory with electronic components, and one such circuit was proposed by Leon O. Chua. Here, we have demonstrated how to simulate Chua's circuit with LTspice software and obtain different dynamical patterns by changing the resistance parameter in the circuit. All the essential characteristics of a chaotic system are observed in the LTspice simulations of Chua's circuit in an easy, inexpensive manner, and this greatly helps me to explore this new and exciting area of physics.

AIM

- To study dynamics of chaotic system by constructing Chua circuit.
- To simulate inductorless Chua circuit and study all characteristics of chaos.

INTRODUCTION

Chaos theory is an interdisciplinary scientific theory and branch of mathematics focused on underlying patterns and deterministic laws highly sensitive to initial conditions in dynamical systems that were thought to have completely random states of disorder and irregularities.The butterfly effect, an underlying principle of chaos, describes how a small change in one state of a deterministic nonlinear system can result in large differences in a later state (meaning that there is sensitive dependence on initial conditions).Chaotic behavior exists in many natural systems, including fluid flow, heartbeat irregularities, weather and climate.

CHUA CIRCUIT

Chaotic systems can be built in the laboratory with electronic components and was proposed by Leon O. Chua. In Chua's circuit simulation we could display of all essential characteristics of a chaotic system by precise control of the resistance parameter with *LTspice* software . The ease of construction of the circuit has made it a ubiquitous real-world example of a chaotic system, leading some to declare it "a paradigm for chaos".

Chua's circuit is one of the simplest circuit's which exhibit chaotic behaviour. An autonomous circuit made from standard components (resistors, capacitors, inductors) must satisfy three criteria before it can display chaotic behaviour. It must contain:

1. one or more nonlinear elements
2. one or more locally active resistors
3. three or more energy-storage elements

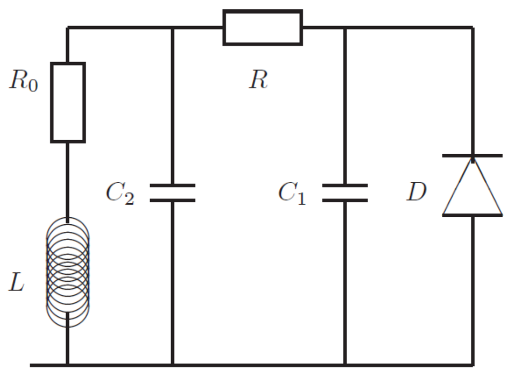

FIG. 1: Chua circuit($Kennedy\ \ 2-OpAmp$)

Chua's circuit is the simplest electronic circuit meeting these criteria. The Chua's circuit has three energy storage elements: two capacitors (labeled C_1 and C_2) and an inductor (labeled L). In this particular circuit.

Dynamics of Chua Circuit

Using KCL, the dynamics of Chua's circuit can be described by three nonlinear ordinary differential equations in the variables $V_1(t)$, $V_2(t)$, and $I(t)$. We got differential equation in Chua circuit as

$$C_1\frac{dV_1}{dt} = \frac{1}{R}(V_2 - V_1) - F(V_1) \tag{1}$$

$$C_2\frac{dV_2}{dt} = \frac{1}{R}(V_1 - V_2) + I \tag{2}$$

$$L\frac{dI}{dt} = -R_0 I - V_2 \tag{3}$$

- V_1 and V_2 are the voltage across C_1 and C_2.
- I is the current through the inductor.
- $F(V_1)$ describes the electrical response of the nonlinear resistor(Chua diode).

where $F(V)$ gives the current passing through the non-linear diode (also known as the Chua diode) for some value of voltage V. It is given by

$$F(V) = m_b V + \frac{1}{2}(m_a - m_b)[|V+B| - |V-B|] \qquad (4)$$

which has (negative) slopes m_a and m_b and breakpoints at $\pm B$. This equation tells us is that basically, there are two different resistances provided by this non-linear diode. This plot basically signify that there are two different resistances provided by this non-linear diode.

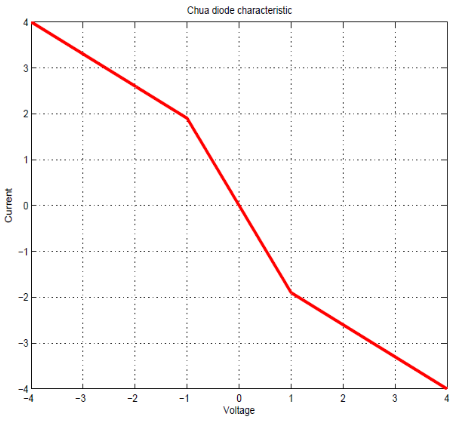

FIG. 2: $I-V$ characteristic plot

Dimensionless Equation

To get the dimensionless equations, we replace our variables as follows

$$x = \frac{V_1}{B},\ y = \frac{V_2}{B},\ z = \frac{R}{B}I,\ a = Rm_a$$
$$\alpha = \frac{C_2}{C_1},\ \beta = \frac{R^2C_2}{L},\ \gamma = \frac{RR_0C_2}{L},\ b = Rm_b$$

After changing the variables the equivalent dimensionless equations are given as

$$\begin{aligned} \dot{x} =& \alpha(y - x - F(x)) \\ \dot{y} =& x - y + z \\ \dot{z} =& -\beta y - \gamma z \end{aligned} \qquad (5)$$

Where $F(x)$ is given by

$$F(x) = bx + \frac{1}{2}(a-b)[|x+1| - |x-1|]$$

TOOLS USED FOR SIMULATION

MATHEMATICA: To visualize chaotic nature in Lorenz system, we have used it to solved coupled differential equation of Lorenz system.

PYTHON-SciPy: For solving initial value problem we have used *scipy.integrate* module.

PYTHON-Matplotlib: For visualize how system evolving with time, we have used it for various plot.

LTspice: LTspice is a high performance SPICE simulation software having schematic capture and waveform viewer with enhancements. We have used it for Chua circuit simulation.

SOLVING CHUA CIRCUIT NUMERICALLY

By solving these dimensionless Chua circuit equation(5) by numerically, we got the parametric plot as shown in $Fig.(3)$.

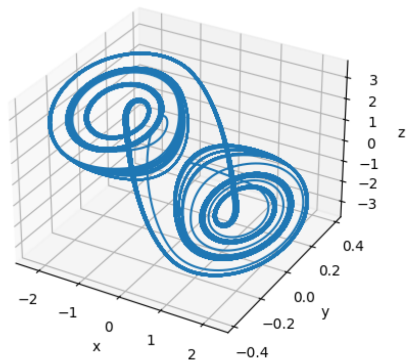

FIG. 3: 3d plot for solution of Chua equation

Also by plotting expression of $F(V)$, we got the $I-V$ characteristic plot of chua diode as shown in $Fig.(2)$.

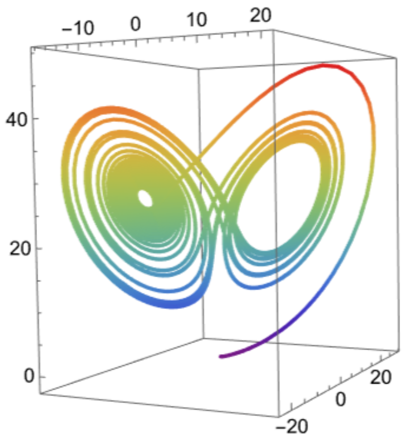

FIG. 4: 3d plot for Lorenz system

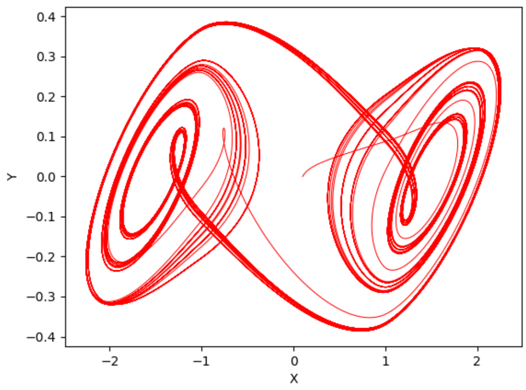

FIG. 5: x vs. y

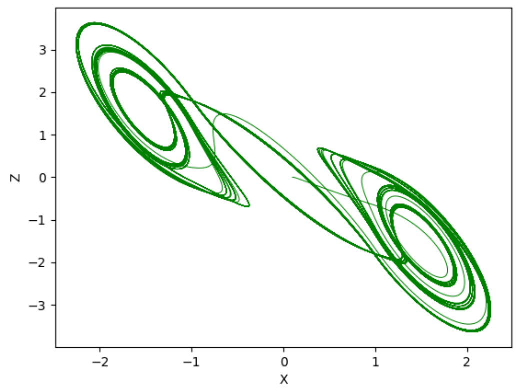

FIG. 6: x vs. z

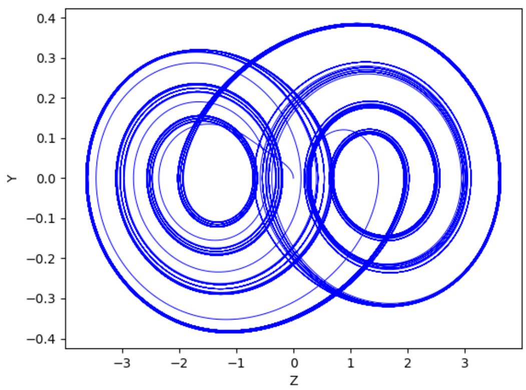

FIG. 7: y vs. z

LORENZ SYSTEM

The Lorenz system is a system of ordinary differential equations first studied by Edward Lorenz. It is notable for having chaotic solutions for certain parameter values and initial conditions. In particular, the Lorenz attractor is a set of chaotic solutions of the Lorenz system. The Lorenz equations are given as

$$\begin{aligned}\frac{dx}{dt} &= \sigma(-x+y)\\ \frac{dy}{dt} &= Rx - y - xz\\ \frac{dz}{dt} &= xy - bz\end{aligned} \tag{6}$$

where x, y, and z are dynamic variables and σ, R, and b are fixed parameters.We have solved these coupled differential equation(6) by taking initial parameter as σ=10, R=28, b=8/3 as given below.

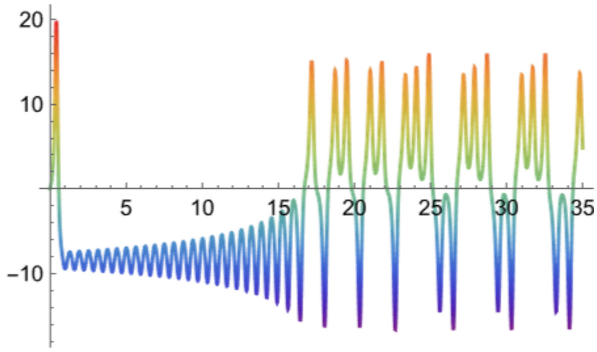

FIG. 8: $x(t)$ vs. t

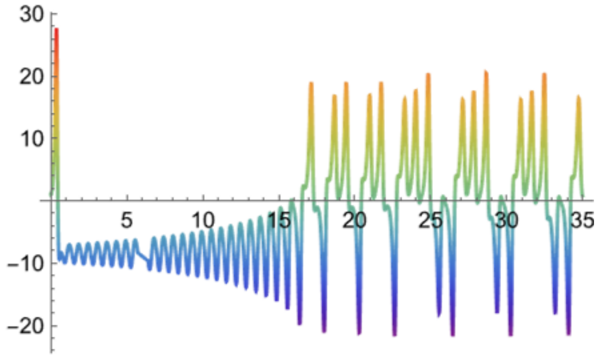

FIG. 9: $y(t)$ vs. t

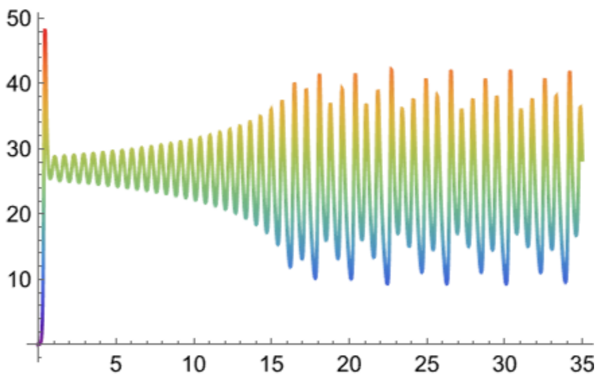

FIG. 10: $z(t)$ vs. t

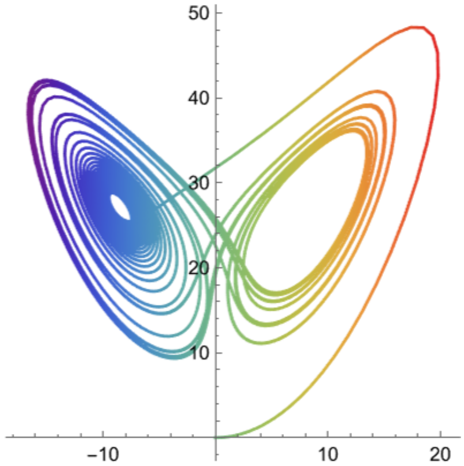

FIG. 11: x vs. z

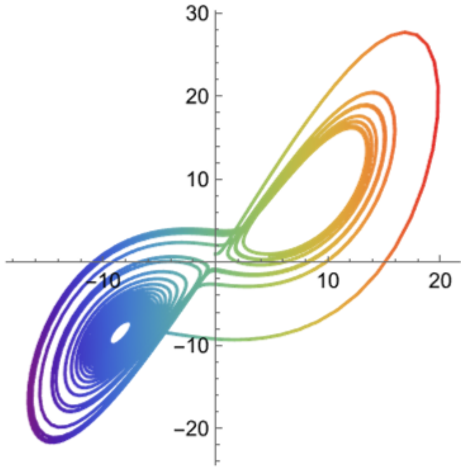

FIG. 12: x vs. y

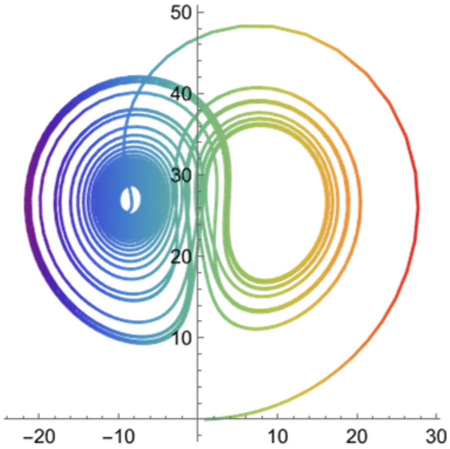

FIG. 13: y vs. z

We have used $RK4$-method to solve these coupled differential equations(6) in $PYTHON3$.

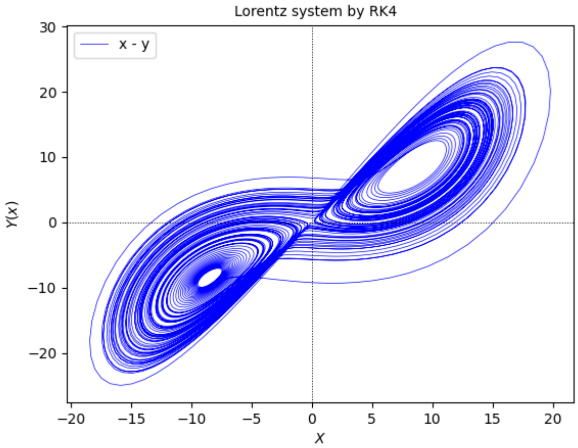

FIG. 14: x vs. y

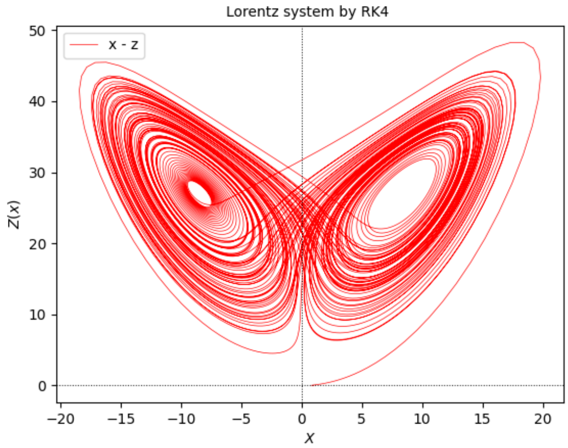

FIG. 15: x vs. z

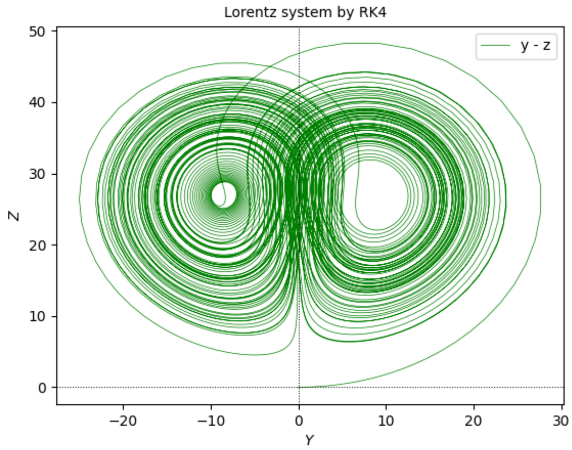

FIG. 16: y vs. z

CONSTRUCTION OF CHUA CIRCUIT

Chua's circuit composed of two capacitors C_1 and C_2, and one inductor L as energy storage elements. The circuit also comprises two resistors one linear resistor R and another non-linear resistor. The nonlinear resistor of Chua's circuit is also called 'Chua's diode'. The most robust and inexpensive realization of Chua's diode has been done by Kennedy using two op-amps $TL082$ and six linear resistors. We construct Chua's circuit with the realization of Chua's diode as proposed by Kennedy using LTspice and simulate the chaotic behavior.

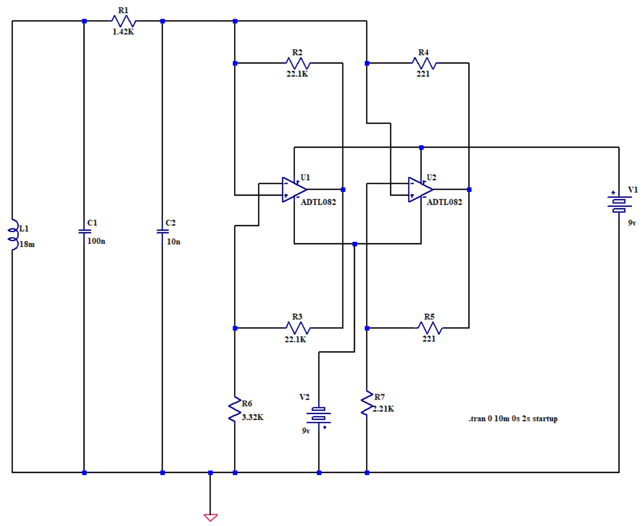

FIG. 17: Chua Circuit using $TL082$ opamp

CIRCUIT SIMULATION

In order to simulate Chua circuit, left-clicking on the Simulate menu and then again left-clicking on the Edit Simulation command is necessary. In editing the simulation command, the transient analysis is chosen. In the transient analysis, we have chosen the parameters of stop time as $10min$, time to start saving data as $0sec$, maximum time step as $2sec$, and starting external DC supply voltage as $0V$, before clicking OK. The next step is to left-click on the Run icon on the toolbar.

The voltages between the two ends of the variable resistor R are taken on the $y-axes$ and $x-axes$ respectively of the plot pane displayed on the schematic editor of $LTspice$. The plots giving characteristics of Chua circuit during simulation are displayed at different values of R as shown in next section.

CHARACTERISTICS OF CHAOS

The following characteristics are observed for Chua's circuit in the $LTspice$ simulations as the resistance R value is decreased from $2.0k\Omega$ to zero.

I. Period doubling or Bifurcation

A limit cycle is a stable orbit closing on itself after some time. A period-1 limit cycle closes on itself in a time of one time period. After this stage, another limit cycle appears as closing on itself in the time period which is equal to twice the time required for period-1 limit cycle. This is called periodic doubling or bifurcation of the chaotic system. As the resistance parameter value gradually decreases, this periodic doubling is continued until the period reaches infinity.

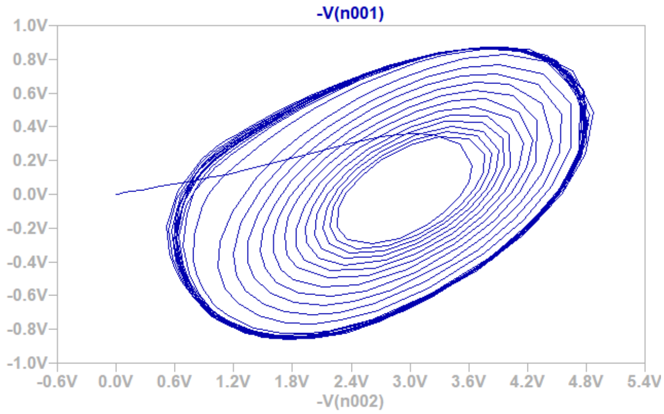

FIG. 18: Period-1 at $R = 1.84k\Omega$

Period-1 limit cycle occurs at the resistance value of $1.84k\Omega$ for the values of components fixed in the simulations (the thick line at the boundary in $Fig.(18)$.

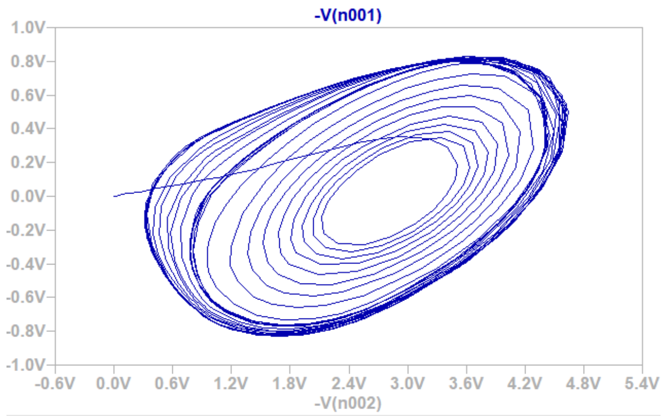

FIG. 19: Period-2 at $R = 1.82k\Omega$

As the resistance value goes on decreasing, period-2 limit cycle appears at $1.82k\Omega$ value of the resistance (the two thick orbits at the boundary). Similarly, period-4 orbit occurs at the resistance value of $1.79k\Omega$ (the four orbits near the boundary with closely spaced lines).

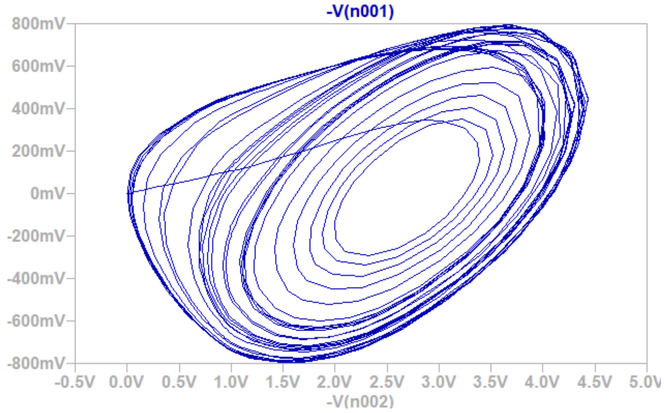

FIG. 20: Period-3 at $R = 1.80k\Omega$

II. Periodic windows

Between the chaotic regimes, there is a range of values of resistance over which stable periodic motion occurs. These regions of periodicity are called the periodic windows. A period-3 window appears at the resistance value of $1.80k\Omega$.

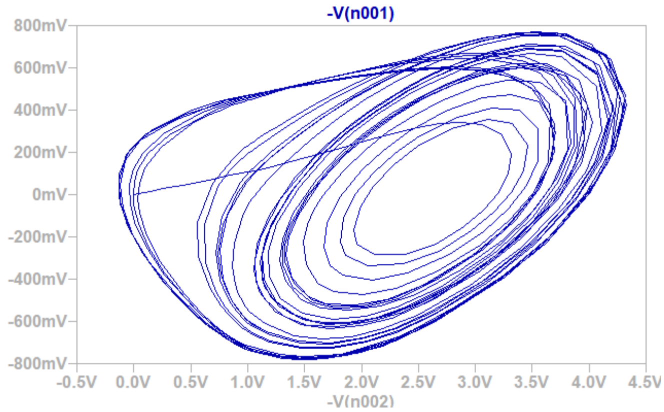

FIG. 21: Period-4 at $R = 1.79k\Omega$

III. Spiral scroll Chua attractor

This is the point where period doubling reaches infinity when resistance is gradually decreased towards zero. From here on, we have chaos. This appears at the resistance value of $1.76k\Omega$. Here, both outward and inward flow of orbits occurs in different planes around the center point.

IV. Double scroll Chua attractor

Two Chua attractors making a compound attractor are called a double scroll attractor. This occurs at the resistance values of $1.76k\Omega$ and $1.44k\Omega$ respectively.

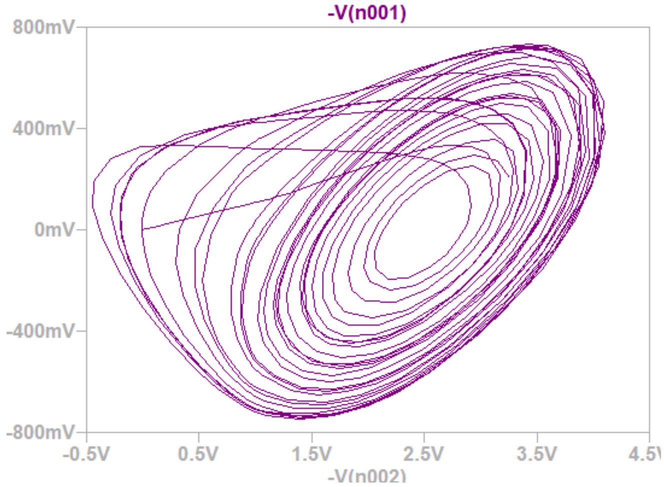

FIG. 22: Single scroll attractor at $R = 1.77k\Omega$

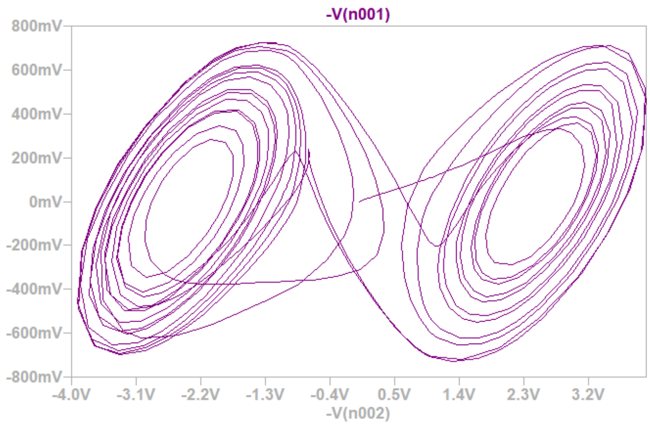

FIG. 23: Double scroll attractor at $R = 1.76k\Omega$

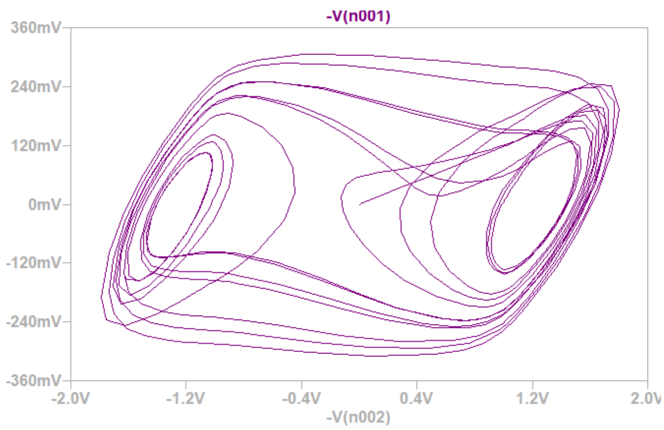

FIG. 24: Double scroll attractor at $R = 1.44k\Omega$

V. Large limit cycle

Finally, a large limit cycle occurs at the resistance value of $1.39k\Omega$. This corresponds to the outer segments of Chua's diode characteristics.

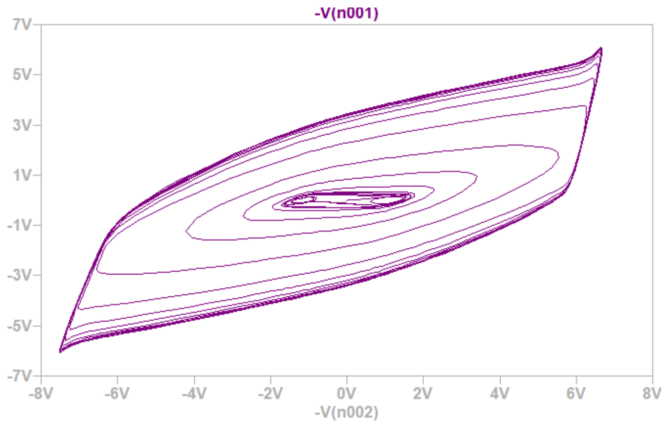

FIG. 25: Large limit cycle at $R = 1.42k\Omega$

CONSTRUCTION OF INDUCTORLESS CHUA CIRCUIT

Chua circuit is basically an oscillator connected to a non-linear resistor. An inductor is used in the oscillator portion of the Chua's Circuit. Inductors with any substantial amount of inductance tend to be physically large, heavy, and expensive. If there is more than one inductor in a circuit, their magnetic fields can interact with each other.We can replace an inductor by a synthetic inductor in Circuit (aka "impedance converter").

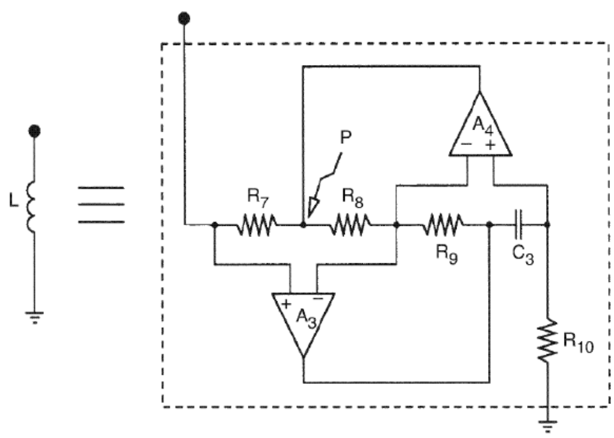

FIG. 26: Gyrator circuit

The schematic diagram and the component list for the inductorless Chua's Circuit we will discuss are shown below. As you can see in this schematic, the inductor in the original Chua's Circuit we discussed previously has been replaced by the inductance simulation circuit located in the box to the left of C_2. The value of the simulated inductance is given by

$$L_{eq} = \frac{R_7 R_9 R_{10} C_3}{R_8} \tag{7}$$

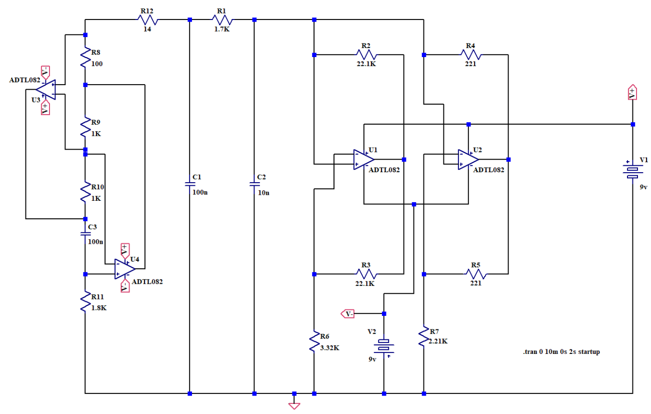

FIG. 27: Inductorless Chua circuit

The inductorless implementation of Chua's circuit facilitates the measurement of the system state variables and extends the flexibility of this already very popular circuit.

CHARACTERISTICS OF CHAOS

The $LTspice$ schematic diagram for the inductorless Chua Circuit is shown below. The strange attractor for this inductorless Chua Circuit is virtually identical with that for the Chua Circuit which used a real inductor and $TL082$ op-amps.

I. Period doubling or Bifurcation

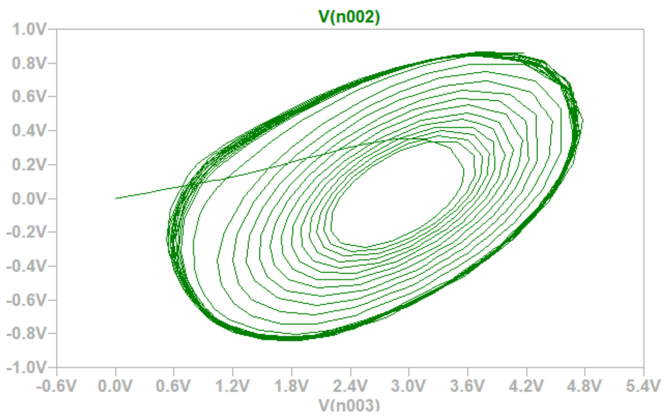

FIG. 28: Period-1 at $R = 1.84k\Omega$

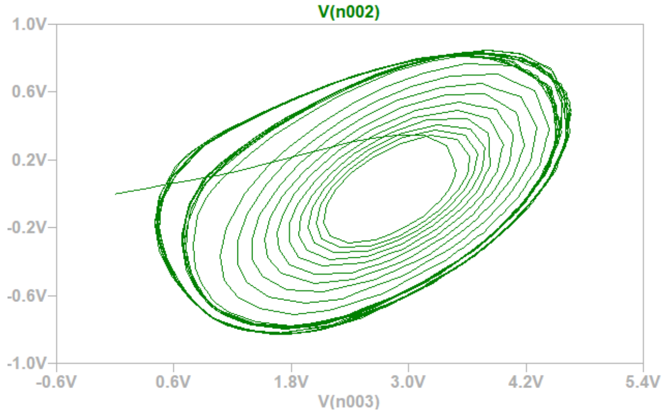

FIG. 29: Period-2 at $R = 1.83k\Omega$

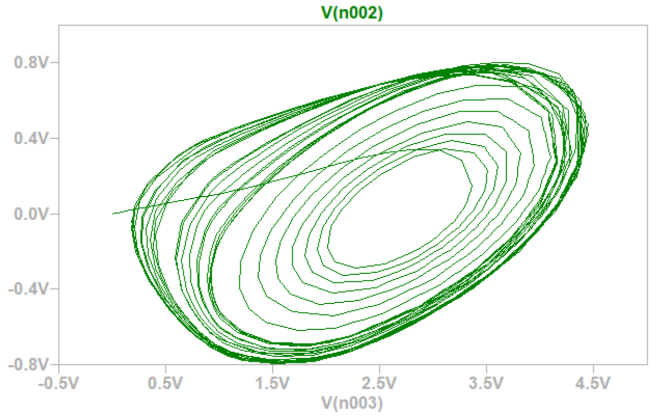

FIG. 30: Period-3 at $R = 1.81k\Omega$

II. Periodic windows

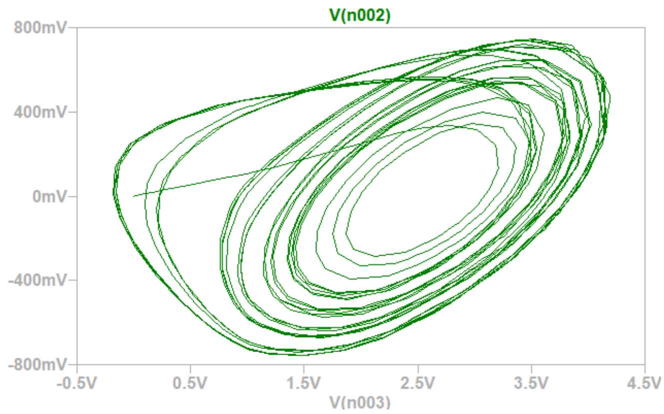

FIG. 31: Period-4 at $R = 1.79k\Omega$

III. Spiral scroll Chua attractor

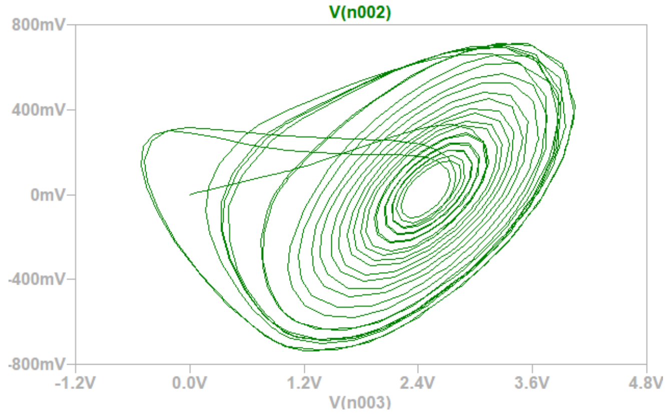

FIG. 32: Single scroll attractor at $R = 1.77k\Omega$

IV. Double scroll Chua attractor

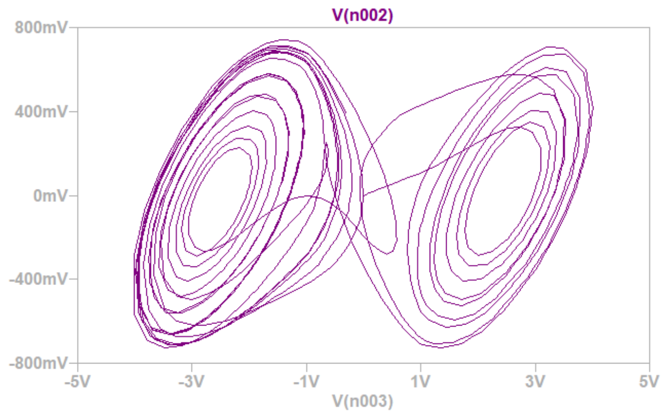

FIG. 33: Double scroll attractor at $R = 1.765k\Omega$

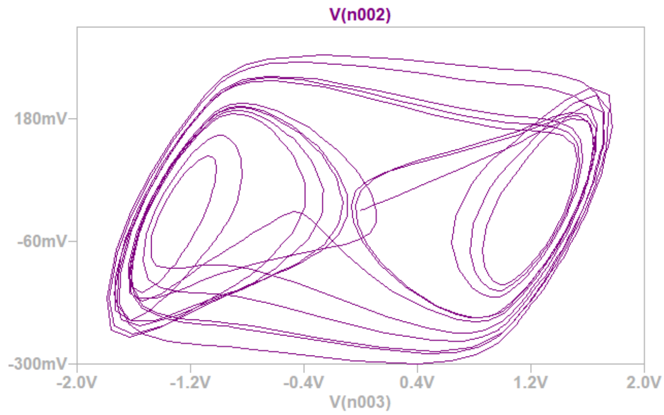

FIG. 34: Double scroll attractor at $R = 1.44k\Omega$

V. Large limit cycle

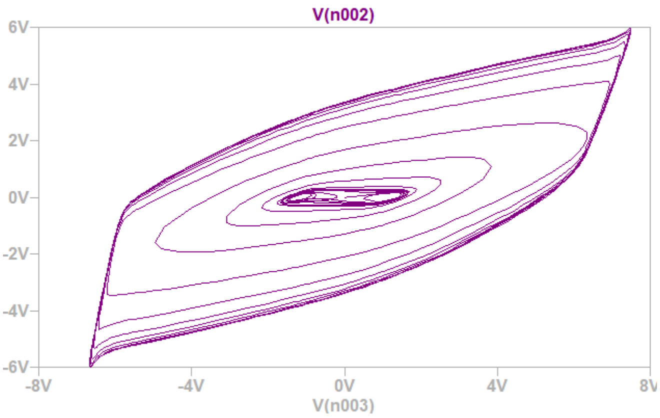

FIG. 35: Large limit cycle at $R = 1.42k\Omega$

CONCLUSION

Chua circuit is constructed with Kennedy's two op-amp realization of Chua's diode using the *LTspice* simulation software. Then all the characteristics of chaotic systems such as periodic doubling, periodic windows, spiral Chua attractor, double scroll attractor, and large limit cycle are observed with the help of Chua's circuit by precise control of resistance parameter in a very easy and inexpensive manner. We have also presented an implementation of Chua circuit that does not rely on the use of inductors. Only capacitors, resistors and operational amplifiers are employed in this circuit. Since this circuit has more parameters which could be used for chaos tuning, it may be useful for various practical applications.

ACKNOWLEDGMENT

I would like to thank Dr. Pratap kumar sahoo and Dr. Gunda santosh babu for their guidance in this project, which helped me in fulfilment of this project.

[1] `http://www.linear.com/LTspice`
[2] LTspice Getting Started Guide `https://www.analog.com/media/en/simulation-models/spice-models/LTspiceGettingStartedGuide.pdf?modelType=spice-models`
[3] Michael Peter Kennedy, Three steps to chaos-part II: A Chua's circuit primer, *IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications, Vol.40, No.10, pp.657–674, 1993.*
[4] Michael Peter Kennedy, Robust op-amp realization of Chua's circuit, *Frequenz, Vol.46, No.3–4, pp.66–80,1992.*
[5] `https://www.chaotic-circuits.com/8-simulating-chaus-circuit-with-ltspice/`
[6] Torres, Leonardo & Aguirre, Luis. (2000). Inductorless Chua's circuit. *Electronics Letters. 36. 1915 - 1916. 10.1049/el:20001363.*
[7] `https://www.chaotic-circuits.com/11-an-inductorless-chaus-circuit/`

Appendixes: Circuit Simulation

A. Chua circuit SPICE Netlist

B. Inductor less Chua circuit SPICE Netlist