Measuring Mean Lifetime of Muon using Orctech Nim

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In this experiment the mean lifetime of cosmic ray muons in their rest frame has measured. We have obtained an exponential time distribution by measuring the time interval, from the point where the muon enters the detector to the point it decays, for a large number of muon decay events. By calculating the gradient, using curve fitting, of this distribution the mean lifetime of the muon in its rest frame will be determined. Another purpose of this experiment is to understand the workings of Photo-multiplier tubes, scintillators and high speed electronic modules such as discriminators, logic gates, time-to-amplitude converter and multi-channel analyzer.

AIM

- To understand how photomultiplier tubes and scintillators combined work as a particle detector.
- To study the formation of muons from primary cosmic rays in the atmosphere.
- To analyze the working of high speed electronic modules so that they can be used to obtain the time difference to calculate the lifetime.
- To understand the workings of the multi-channel analyzer and realize how it is a crucial part of counting different voltage pulses in this experiment.
- To calculate mean life of Muon by fitting our data for different discriminator voltage.

THEORY

Muons are elementary or fundamental particles belonging to the second generation of the lepton family. Generation in particle physics is a classification of particles according to their masses. Electrons are the lightest leptons and hence belong to the first generation. Muons being heavier than them belong to the second. They are fermions of spin $1/2$ and have one unit of electronic charge. They have a mass of about $105.7 MeV/c^2$. The anti-particle of the muon is the antimuon(μ^+), also called positive muon. Muons are unstable and decay into electrons and neutrinos. Their decay reaction is as shown below

$$\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$$
$$\mu^+ \longrightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

Muons take part in both strong and electromagnetic interactions. Muons being heavier than electrons do not get accelerated by electromagnetic fields as easily, in matter. Therefore they emit much less bremsstrahlung radiation. They are more penetrating than electrons. While passing through matter they can also be captured by nuclei in the following way

$$\mu^+ + p \longrightarrow n + \nu_\mu$$

Sources of Muon

Energy of about $105.7MeV$ (rest mass energy) is required for a muon to be created in the COM frame. Normal radioactive decays do not possess such energies and neither do fission and fusion reactions occurring in nuclear reactors and weapons. Although single atom nuclear fission has the energy of this order, due to conservation laws, muons are not produced. The only known natural source of muons is the cosmic rays. Muons can be produced in high energy accelerators. In cosmic rays coming from space, about 89% of the nuclei are hydrogen (protons), 10% helium, and about 1% heavier elements. When the cosmic rays reach the upper atmosphere the protons interact with the atoms in the atmosphere and produce pions. Pions subsequently decay into muons. π^0 decays into two gamma photons and does not produce muons. The schematic diagram is as shown below.

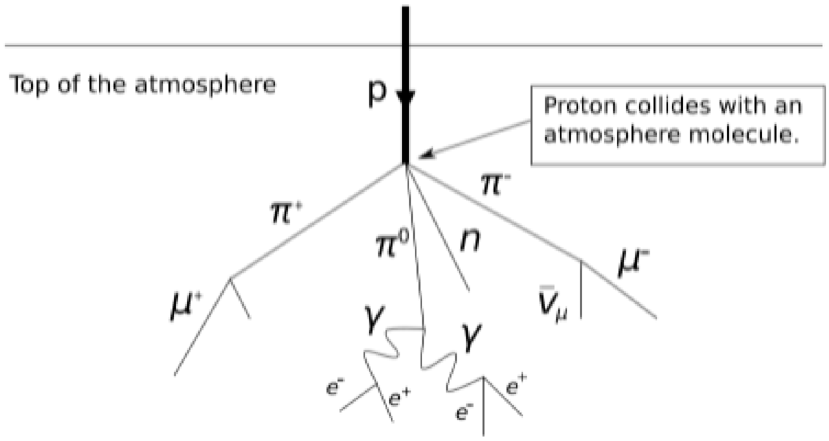


FIG. 1: Muon creation at upper atmosphere

Pions decay into muons as

$$\pi^- \longrightarrow \mu^- + \bar{\nu}_\mu$$
$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

Muon flux

The incident muon flux at sea level has the following relationship with θ (the zenith angle or the angle measured from the vertical)

$$I = I_\circ \cos^2 \theta$$

For low energy incident muons the muon flux decreases in general as the angle increases. This is because at larger angles the muons have to penetrate a larger distance through the atmospheric layer of air particles to reach the ground. Therefore increasing angle increases the probability of the muons interacting with an air particle or decaying before reaching the ground. The exception to this general trend are the high energy muons which are mostly not affected and their flux steadily increases with increasing angle. Therefore one of the main parameter which concerns ones when performing such calculations is the amount of matter above any atmospheric layer through which these particles had to pass. This is atmospheric depth measured in g/cm^2 . Temperature and density variations affect the interaction of cosmic ray particles and air molecules in the atmosphere. Theoretically one can also calculate the muon lifetime in its rest frame using the Fermi's Golden rule. The Golden Rule provides ones with the decay rate for the muons which is given by

$$\Gamma = \frac{G^2(m_\mu c)^5}{192\pi^3}$$

Where G is the Fermi constant and m_μ is the mass of the muon which is roughly equal to $106m_e$. The value of G is $1.136 \times 10^{-5} GeV^{-2}$. The lifetime is then given by $\tau = h/\Gamma$.

Possibility of detection and measurement

Pions have a lifetime of the order of $10^{-8}s$. Hence they are not observed near sea level. The muons produced subsequently after their decay, decay as shown before. Muon lifetime (average) from many other experiments is found out to be about $2.2\mu s$. If we use the formula distance = velocity x time, we see that it (muon) can only travel about $\approx 700m$. But we are able to observe muons at sea level even though they are formed at heights of several kilometers. This is because of relativistic time dilation. Muons from cosmic rays possess energies in the order of GeV. Assuming the energy of the muon to be about $1GeV$, we find that it will have a velocity of about $0.99c$, where $'c'$ is the speed of light. Using the formula $t = t'\sqrt{1-(v^2/c^2)}$(t is the time in the rest frame and t' is the time as measured in the muon frame.) and taking $t' = 2.2\mu s$, we see that the muon's life increases to $16\mu s$. By using the factor γ which is about 7.1 in this case, we see that, a distance of $\approx 5000m$ would be seen as $\approx 700m$ by the muon.

EXPERIMENTAL SETUP

The components used in the setup are as follows

- Plastic Scintillation Detector
- High Voltage Power Supply
- Discriminator
- Time to Amplitude Converter
- Multi Channel Analyzer
- Computer

Plastic Scintillation detector

This scintillator is very well suited for fast counting and has a very sharp lifetime. It produces scintillation's in the blue and low UV region of the electromagnetic spectrum. The scintillator has been wrapped by two types of papers one above the other. First, it is wrapped by 'Tyvec' paper which has a reflective property. Scintillation produced are always spherically distributed. It helps in refocusing them. The second wrapping is of 'Tedlar', which prevents ambient light from entering into the scintillator. The scintillator which we are using is a Bicron BC-404. It is an Organic Fluor polyvinyl toluene plastic scintillator. The dimensions are $24cm \times 24cm \times 14.5cm$. Plastic scintillators can be shaped and fabricated into rods, cylinders and flat sheets. They are

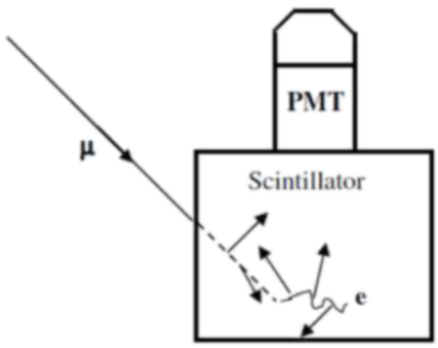


FIG. 2: Plastic Scintillation detector

relatively inexpensive and can be made in large forms. Also they find very good application in fast counting. They can also sometimes recover from radiation damage over periods of time.

Photo multiplier tube

Photo multiplier tube is used to convert the light output of the scintillator to an electric signal. The PM tube we are using has a semitransparent bialkali photo-cathode. When particle hit the scintillation material in the detector, electrons in the material go to higher orbital then come back to lower orbital by emitting photons. When these are absorbed by photo cathode, electrons are emitted from it. These electrons again attracted by the $+ve$ charged plates in the PMT. PMT is connected to high voltage power supply. Using voltage divider, different plates of PMT are set at different voltage.Because of varying voltage of the plates in the PMT a high current beam is produced.

The gain or current amplification, G, of a PMT is the ratio of the anode current to the photo-cathode current. It varies as a power of the supply voltage (usually >5) and:

$$G = \frac{G_2}{G_1} = \left(\frac{V_2}{V_1}\right)^{\alpha N}$$

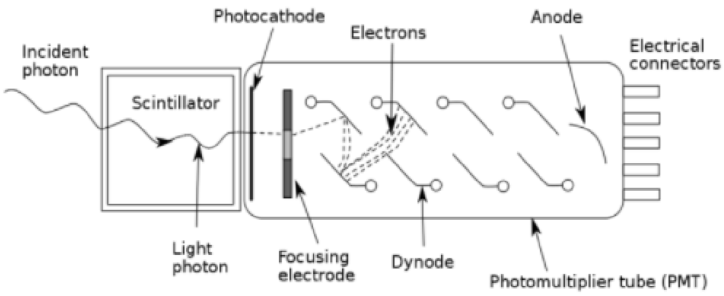


FIG. 3: Photo multiplier tube

where $G2$ and $G1$ are the gains at supply voltages $V2$ and $V1$ respectively, α is a coefficient $(0.6\ \ to\ \ 0.8)$ set by the dynode material and geometry, and N is the number of dynodes.

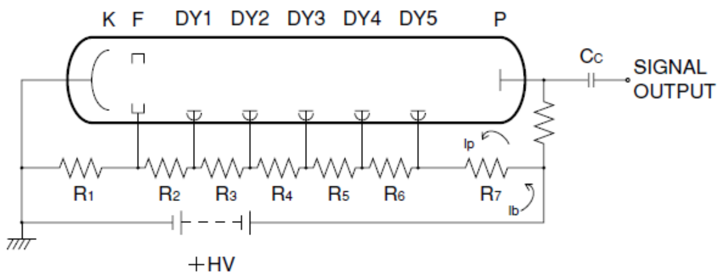


FIG. 4: PMT connections

Discriminator

Pulses from the PMT's are shown in $fig(5)$. They are negative but not square. However the electronic logic units only work with negative square pulses which are also known as logic pulses. The function of the discriminators is to convert the negative pulse of the PMT into a negative square pulse so that they can be then used by the electronic logic units.

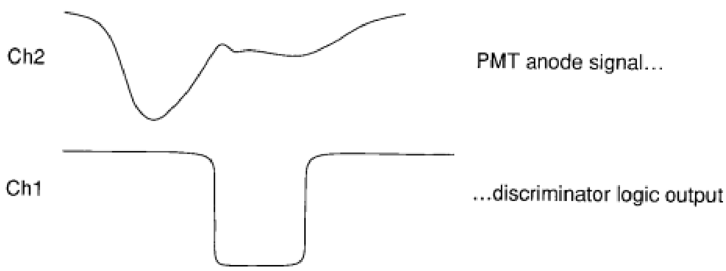


FIG. 5: Signal from PMT and Discriminator respectively

Discriminators take PMT pulses and produce negative logic pulses ONLY IF the PMT pulses exceed a certain threshold voltage which is variable. It also helps to eliminate the electronic noise which becomes significant at very high voltages. This makes sure that a logic pulse would only be generated if the detector has actually ‘seen’ a particle, because a particle would give off a pulse with a large amplitude, much larger than the amplitude of the noise signal.

Time to Amplitude Converter(*TAC*)

For measuring the time period between the coming of the muon and it’s decay, we use a *TAC* or a time to amplitude converter. Measures the time difference between the signals coming to the start and the stop port, then convert it into amplitude of pulse. In *TAC* a time window is set when there is a logic 1 signal at the start port the *TAC* starts measuring. If a 1 signal pulse comes in the stop port before the time window is crossed then that data is passed on and the clock resets to wait for another start signal. If there is no signal in the range the clock resets without sending any pulse. This range is important because once the muon passes through the detector there will be no decay signal. This basically consists of three parts.

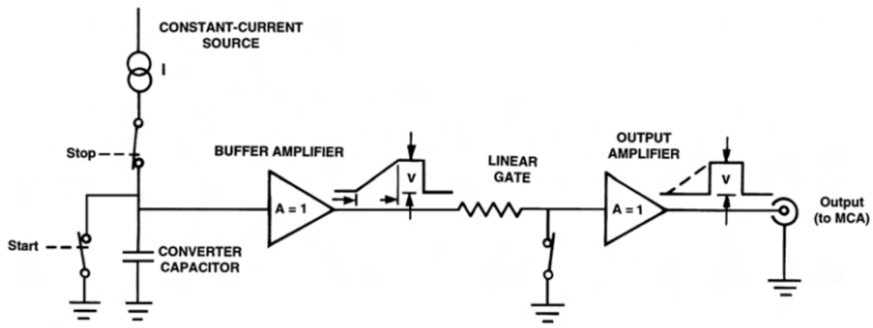


FIG. 6: A function diagram of *TAC*

- **Start Circuitry:**
  The start circuitry is used to generate a logic signal which begins the time-to-amplitude conversion in the TAC. When the first signal from the discriminator comes to the input of start circuitry it sets the D1 input in U2 to 1 (HIGH). This MC10131 is a type of master-slave D-flip flop. By setting this D1 to 1, Q2 becomes 1 and $\overline{Q2}$ becomes 0. There is another transistor connected to a capacitor (C38). These two signals go thorough level-shifting transistors. When Q and $\overline{Q2}$ reach the transistor they turn off the transistor (Q17) so C38 connected to it starts discharging. The discharge rate of the C38 is controlled by some other circuitry.
- **Stop circuitry:**
  The stop circuitry works identical to the start circuitry one. There is U4 (MC10131) present in this circuitry. When stop signal signal reaches the circuit, the D1 input is set to 1 the output at Q2 is 1 and at $\overline{Q2}$ is 0. These two signals turn off another transistor to turn off another transistor to terminate the current flow out of the capacitor and also end the time-to-amplitude conversion cycle.

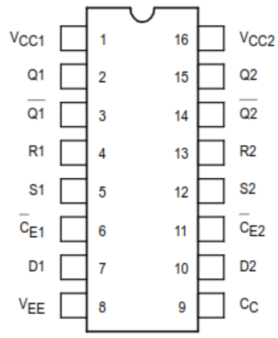


FIG. 7: MC10131 chip

- **Gated baseline restorer:**
  The voltage on the capacitor is buffered by Q21-Q23 transistors. The emitter voltage of Q23 is held at 0V. This BLR(Base Line Restorer) acts as high-gain feed back in this inactive state. In inactive state this BLR sets the output of Q23 to be 0V. When there is a start signal the feedback path is opened and proper dc voltage is maintained throughout the loop.
- **Strobe circuitry :**
  This part of TAC is responsible for the time window. i.e. if the second signal does not come until certain time of start signal (in our case it is $25\mu s$) it resets the the circuit and does not send output to MCA.

A *TAC* can achieve such exceptional precision because it uses an analog technique to convert small time intervals to pulse amplitudes. Before a time measurement starts, all the switches in $fig(6)$ are closed. The arrival of the leading edge of the "start" signal opens the "start" switch, and the converter capacitor begins to charge at a rate set by the constant-current source. The leading edge of the "stop" signal opens the "stop" switch and prevents any further charging of the capacitor. Because the charging current I is constant, the voltage developed on the capacitor is given by

$$V = \frac{It}{C}$$

where t is the time interval between start and stop pulses and C is the capacitance of the converter capacitor. Consequently, the voltage is proportional to the time interval. This voltage pulse is passed through the buffer amplifier to the linear gate. A short time after the stop pulse arrives, the linear gate switch opens to pass the voltage pulse through the output amplifier to the *TAC* output. Calibration of *TAC* is performed to show the linear dependence of the *TAC* output to its input. In order to check this, we have to generate two successive signals (start/stop) with a known time difference and record the output amplitude of *TAC*. If this is done for several time differences the results can be plotted to see the behaviour.

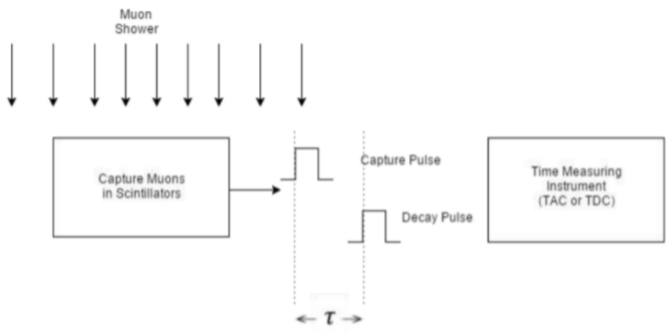


FIG. 8: Time difference of muon capture pulse and decay pulse

Multi Channel Analyzer(*MCA*)

The *MCA* is a device which takes only positive square pulses and distinguishes and counts them according to their amplitude (voltage). It contains channels and each channel corresponds to a specific voltage. Whenever the *MCA* receives a pulse of a specific amplitude it makes a count of that pulse on the corresponding amplitude channel. So each channel tells us that how many pulses the *MCA* received of that particular amplitude.Then the histogram is then fitted with a exponential decay curve and using that the muon lifetime is calculated.

LEMO cable

In our setup we have connected all the devices through *LEMO* cable(Coaxial cable) *i.e* from *PMT* to discriminator, from discriminator to *TAC* and from *TAC* to *MCA* for impedance matching. Coaxial cable is a type of electrical cable consisting of an inner conductor surrounded by a concentric conducting shield, with the two separated by a dielectric (insulating material) many coaxial cables also have a protective outer sheath or jacket. The term coaxial refers to the inner conductor and the outer shield sharing a geometric axis.

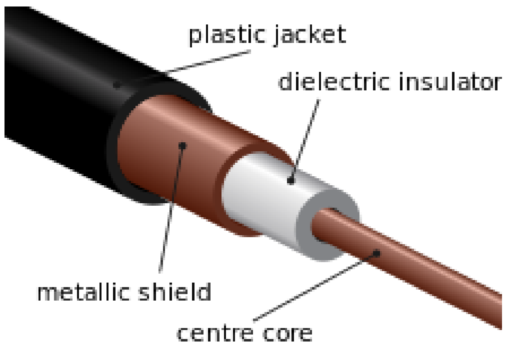


FIG. 9: Crossection of LEMO cable

Coaxial cable is a type of transmission line, used to carry high-frequency electrical signals with low losses. It differs from other shielded cables because the dimensions of the cable and connectors are controlled to give a precise, constant conductor spacing, which is needed for it to function efficiently as a transmission line. For the coaxial cable the characteristics impedance is given by

$$z_{\circ} = \frac{1}{2\pi} ln\left(\frac{b}{a}\right)\sqrt{\frac{\mu}{\varepsilon}}$$

Where a and b are the radii of the inner and outer conductor respectively. μ and ε are permeability and permittivity of dielectric used in between the inner and outer conductors. In our experiment impedance of $LEMO$ cable $z_{\circ} \approx 50\Omega$. So to minimize the loss of signal a uniform cable characteristic impedance is important. The source and load impedance are chosen to match the impedance of the cable to ensure maximum power transfer and to avoid reflection of signal.

APPROACH

In order to measure the mean lifetime of muons in the lab, we have stopped cosmic-ray muons in a plastic scintillator block, as shown in $Fig(10)$ interactions with the dense plastic material caused the muons to deposit energy into the scintillator. When enough energy was deposited within a short period of time, the scintillator flashed, and the photomultiplier tubes (PMTs) amplified this signal (henceforth called the arrival signal). The flux rate of muons was sufficiently small that we expected only a single muon to be present (and stopped) by the detector at any particular time.

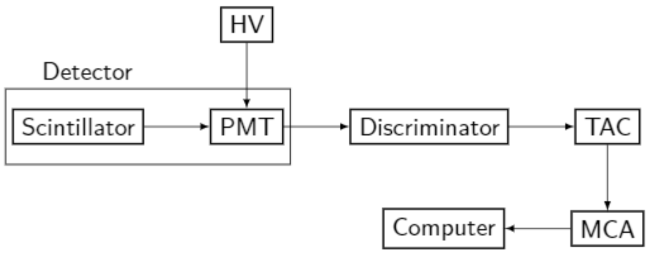


FIG. 10: Setup for time measuring unit

When enough energy was deposited by a particular muon, it came to a halt in the scintillator. Such muons decayed some time later, and the PMT amplified this signal, too (henceforth called the decay signal). In order to reduce the noise (false positives) of the PMT, we piped the signal first through discriminator and then into a coincidence circuit. Unfortunately, we neglected to measure the individual count rates of the PMT, making a theoretical prediction of remaining noise impossible.

Because the arrival signal and the decay signal were expected to be extremely close together, relative to the expected spacing between signals from different muons, we were able to determine the time between arrival and decay using a time to amplitude converter (TAC) which measured

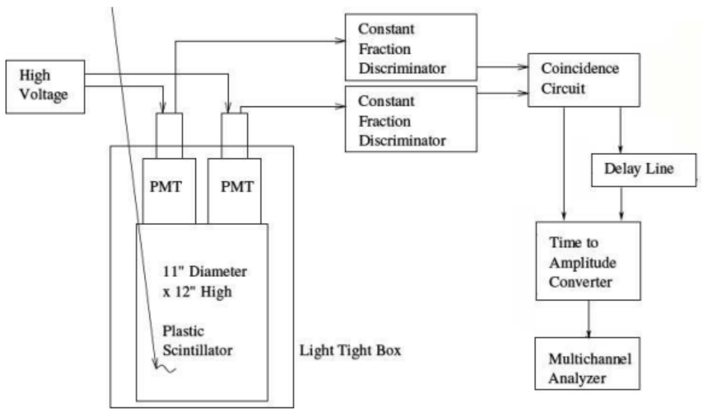


FIG. 11: Schematic diagram of time measuring circuit

the time interval between a time starting shortly after any signal and ending at the next signal. The delay between the incoming signal and the start time (on the order of a few nanoseconds) was judged to be insignificant relative to the decay time (on the order of a few microseconds). A multi channel analyzer (MCA) was then used to record the decay times. Because the probability distribution for decay is time translation invariant (the probability that a muon will decay at a time t from now is always $\tau^{-1}e^{-t/\tau}$,(assuming that it has not yet decayed), we may infer the mean lifetime from the decay-time curve, despite the fact that we didn't measure a creation event.

PROCEDURE

The lifetime of a muon as seen by us is not a constant quantity. The decay of the muon is a statistical process. Therefore the aim of this experiment is to determine the average lifetime of the muon. We do it by stopping muons in a plastic scintillator and detecting its decay. The time information is collected from the time measuring circuit on a PC, with the use of a parallel port. A computer program is written, which takes a note of the time interval between the muons arrival and its decay on a text file. The experiment is run till sufficient number of counts is obtained. The recorded time intervals follow an exponential distribution. The distribution of the events is governed by the relation

$$N = N_0 e^{-t/\tau}$$

where, N_0 is the initial population of muons, N is the population muons at time t and T is the lifetime of the muon. However background events are also present in the readings. They might be due to natural radioactivity, PMT after pulses, and other through going muons. The background pulses arising due to entry of one muon and due to the entry of another muon within the time interval of $25.5\mu s$ is quite negligible as the avg. muon flux at the sea level is about $1 muon/sq.cm/min$. Assuming that the background is constant over a period of time, the decay spectrum could be written as

$$N = N_0 e^{-t/\tau} + b$$

where b is the number of background events. The values are binned and plotted on a logarithmic scale. The inverse of the slope of the curve gives the muon lifetime.

ANALYSIS

Basically we are taking the time difference t_d between entering and decaying of muon at detector. Let the total number of muons which got decayed in detector = N_0. Also we can say that entering time $t = 0$ and decayed at some $t = t_d$. Hence $\frac{dN}{dt}$=$-kN_0 \exp(-kt)$, where N is number of muons at time t. So number of particle decayed in $[t, t+\Delta t]$ is $-\Delta N = kN_0 \exp(-kt)\Delta t$. So we have to fit our histogram data with the function $f(t) = a \times \exp(-\frac{t}{b}) + c$. So that we could get mean lifetime $t_{mean} = b$.

OBSERVATION

First we have calibrated the *TAC* and *MCA* using the function generator. For this purpose the following data table has been taken, varying the period of function generator and noting the value of the channel that shows readings at that setting. The maximum range of *TAC* had been set as $20\mu s$.

Form the fitting between channel number and time period of function generator as shown in $fig12$ we have got the relation

$$\text{Time}(\mu\text{sec.}) = \frac{\text{Channel No.} + 11.37}{407.9}$$

Time(μs)	Channel No.	Time(μs)	Channel No.
1	395	11	4461
2	804	12	4879
3	1213	13	5288
4	1617	14	5700
5	2031	15	6106
6	2435	16	6515
7	2844	17	6924
8	3258	18	7333
9	3666	19	7737
10	4071		

(a) Calibration data with channel

Time(μs)	$V_{ac}(20\mu s)$	$V_{ac}(10\mu s)$
1	0.008	0.014
2	0.014	0.136
3	0.049	0.402
4	0.135	0.692
5	0.253	0.906
6	0.394	1.237
7	0.503	1.404
8	0.588	1.543
9	0.657	1.683
10	0.711	1.822

(b) Voltage of *TAC* with different time for $10\mu s$ and $20\mu s$ window

For measuring mean life of muon we had two parameters to vary, the voltage applied to the photo-multiplier tube and the voltage set in the discriminator. In our experiment we have set

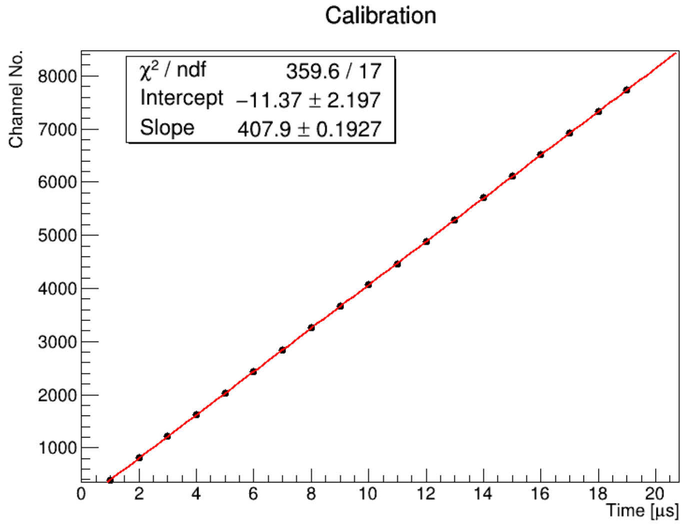
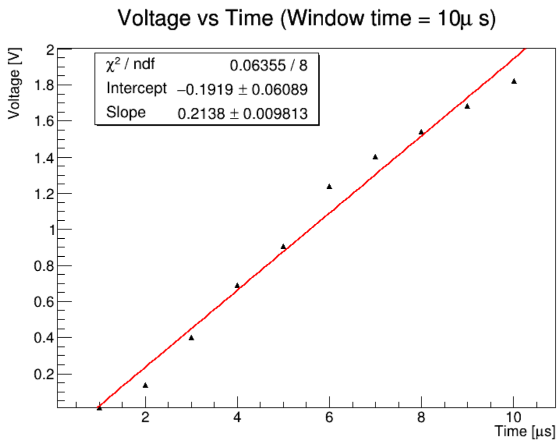
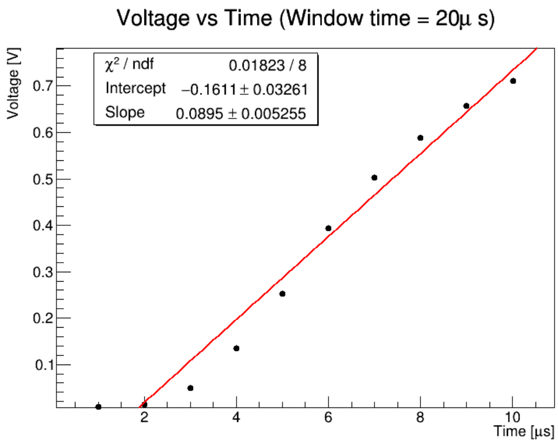


FIG. 12: Calibration of *TAC* and *MCA*



(a) Window time $10\mu sec$.



(b) Window time $20\mu sec$.

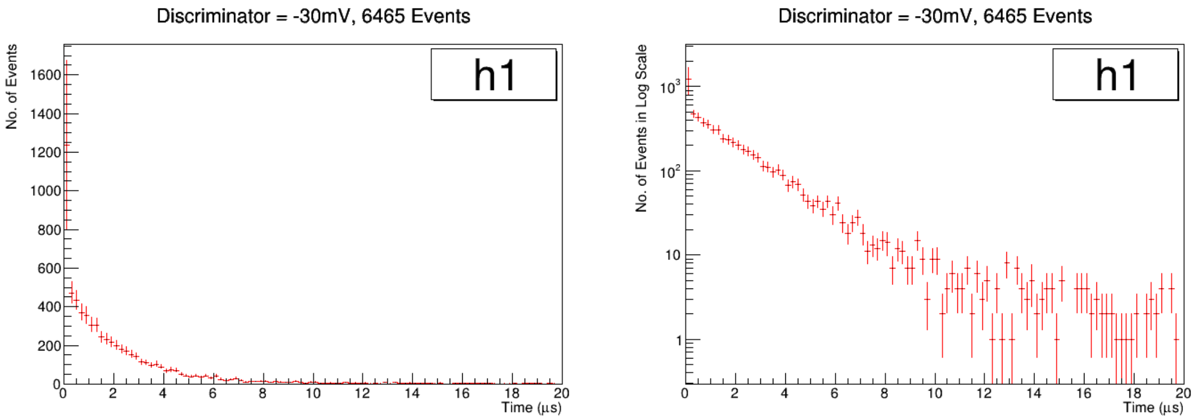
FIG. 13: TAC voltage vs. Time

our *PMT* voltage to $700volt$ for different discriminator voltage. These plots are then fitted using ROOT with the equation

$$y = a \times exp^{-t/\tau} + b$$

The values of a, $1/\tau$ and b are given below each plot. First number is the *PMT* voltage in V and second the discriminator voltage in mV. Here x-axis is time in μsec and y-axis is just the count for all graphs that follows.

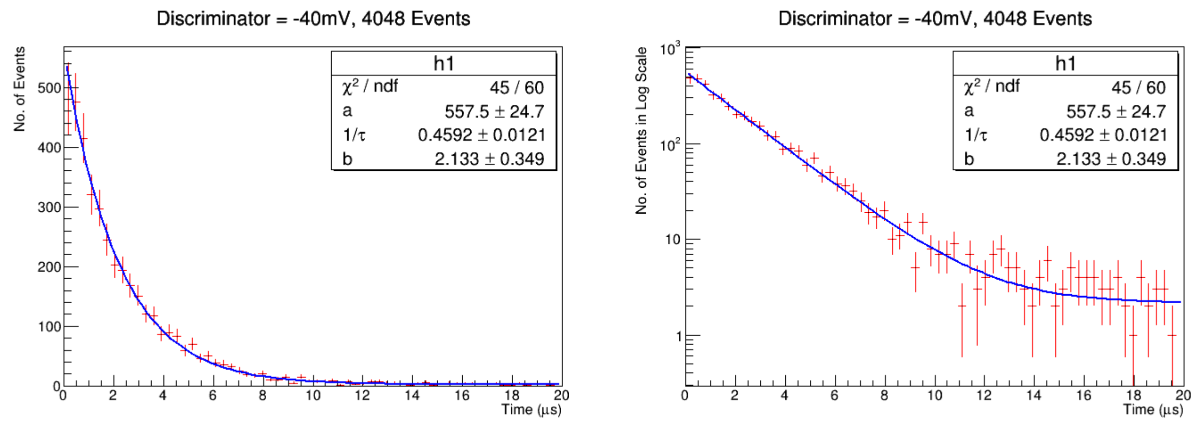
For discriminator level=$-30mV$, the exponential fit is very bad fit due to there are many noise in signal with muon signal. So the curve is not exponentially fitted. However we got exponential distribution for $-40mV$ and $-50mV$ discriminator level. For discriminator level=$-40mV$, from



(a) No. of events vs Time(μs)

(b) No. of events vs Time(μs) in log scale

FIG. 14: At discriminator voltage = $-30mV$ and $700V$ supply voltage



(a) No. of events vs Time(μs)

(b) No. of events vs Time(μs) in log scale

FIG. 15: At discriminator voltage = $-40mV$ and $700V$ supply voltage

exponential fit of no of events vs time we got the parameters which are

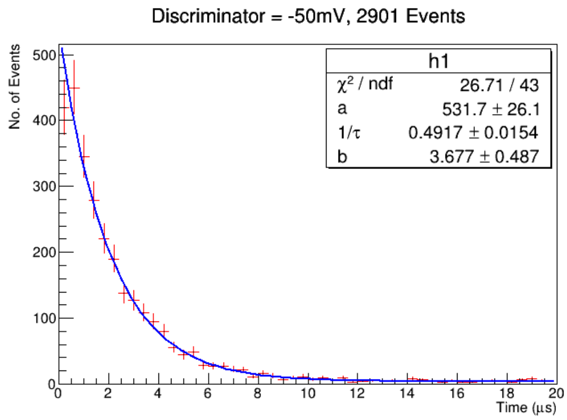
$$a = 557.5 \pm 24.7$$
$$1/\tau = 0.4592 \pm 0.0121$$
$$b = 2.133 \pm 0.349$$

So value of mean life (τ) at $-40mV$ discriminator level =

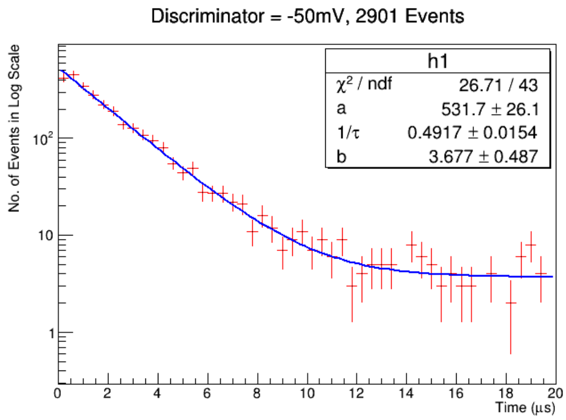
$$\frac{1}{0.4592} = 2.178 \mu sec$$

For discriminator level=$-50mV$, from exponential fit of no of events vs time we got the parameters which are

$$a = 531.7 \pm 26.1$$
$$1/\tau = 0.4917 \pm 0.0154$$
$$b = 3.677 \pm 0.487$$



(a) No. of events vs Time(μs)



(b) No. of events vs Time(μs) in log scale

FIG. 16: At discriminator voltage = $-50mV$ and 700V supply voltage

So value of mean life (τ) at $-50mV$ discriminator level =

$$\frac{1}{0.4917} = 2.034\mu sec$$

ERROR ANALYSIS

For discriminator level = $-40mV$, error in mean life is given by

$$\delta\tau = \frac{\delta\left(\frac{1}{\tau}\right)}{\left(\frac{1}{\tau}\right)} \times \tau = \frac{0.0121}{0.4592} \times 2.178 = 0.057\mu sec$$

For discriminator level = $-50mV$, error in mean life is given by

$$\delta\tau = \frac{\delta\left(\frac{1}{\tau}\right)}{\left(\frac{1}{\tau}\right)} \times \tau = \frac{0.0154}{0.4917} \times 2.034 = 0.064\mu sec$$

RESULT

- For discriminator voltage = $-40mV$, we got the mean life of muon = $2.178 \pm 0.057\mu sec$
- For discriminator voltage = $-50mV$, we got the mean life of muon = $2.034 \pm 0.064\mu sec$

SOURCES OF ERROR

- The random radiations which form a background in the data are the main source of error.
- The muon must enter the detector, or be created by a cosmic ray within the detector.
- The muon must decay within the detector.

- Photons from both creation and decay must reach the *PMT*, which must detect them.
- The signals from the *PMT* must reach the *TAC* while it's ready to start/stop timing.
- positive muons can capture an electron from the atoms of the scintillator and for a muonic atom,
- Negative muons on the other hand react with the protons to give $\mu^- + p \longrightarrow n + \nu_\mu$ which decrease the measurement of mean life.

CONCLUSION

The purpose of this experiment was to measure the mean or average muon lifetime using a *PMT* coupled plastic scintillator, where, the incoming muons are stopped and electrons are produced by their decay. The decay times are counted in large numbers so that an exponential distribution is obtained. The inverse of the slope of the logarithmic plot gives the average lifetime of the muon. For $-40mV$ and $-50mV$ discriminator voltage level,mean life of muon is found to be $2.178 \pm 0.057\mu sec$ and $2.034 \pm 0.064\mu sec$ respectively. This experiment provides an opportunity to get acquainted with the realistic introduction to important concepts in experimental modern physics such as exponential distributions, nonlinear fitting, and useful equipment including scintillator detectors and photomultiplier tubes. This is a well-designed experiment, capable of very accurate and precise measurements of the muon lifetime.

ACKNOWLEDGMENT

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Appendixes: *Creating a discriminator using breadboard*

We have tried to create a discriminator using ICs. For this we have used IC741(OpAmp) and LM311(Fast Comparator).Here we have amplified the incoming signal by 100 times and then used the comparator to pass the signals which are greater than the threshold voltage. Here V_{ref} was

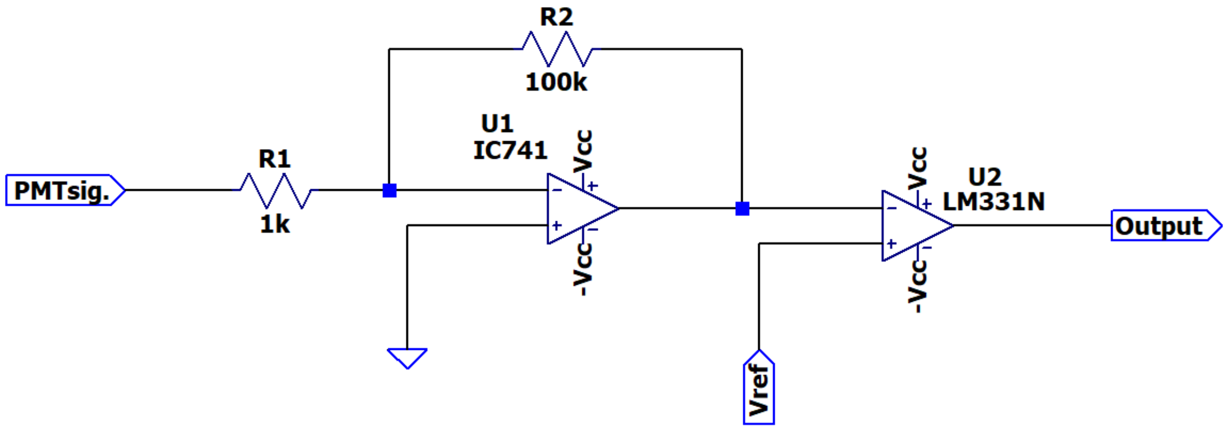

FIG. 17: Discriminator circuit

taken 4V or 5V. Since after amplification the signal was about 6V to 7V. The problem with V_{ref} was that the constant voltage supplier has was fluctuating about 2V around the given value. Which is not good. To make it more precise, we used voltage divider method as in figure below.

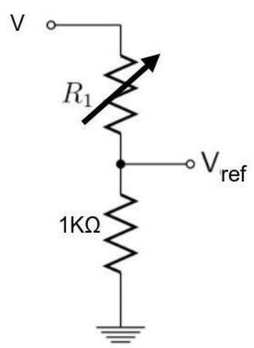

FIG. 18: Voltage divider method.

In the above figure we can adjust R_1 and V to get desired V_{ref}. The typical value of V is given around 30V. So for reducing fluctuation in constant voltage supplier or V_{ref} we have to use R_1 to be $6K\Omega$ to $10K\Omega$. The problem with the circuit in fig(17) is that the amplifier stage. The amplifier circuit we constructed does not amplify such high frequency signal as the PMT signal has frequency around $1MHz$. So this circuit failed to discriminate the signal. To improve this we have to use fast OpAmp instead of normal OpAmp or we could shape the signal before amplifying so that it would be easier to amplify.