1 Efficient Clifford+T Decomposition

Efficient decomposition of quantum circuits into the Clifford+T gate set is a central problem in fault-tolerant quantum computing. While Clifford gates can be implemented with relatively low overhead, the T gate represents a costly non-Clifford resource. Consequently, the primary goal of compilation is to minimize the T-count while preserving unitary fidelity.

1.1 Gate Set and Universality

The Clifford+T gate set

$$\{H,\ S,\ T,\ \text{CNOT}\}$$

is universal for quantum computation. Any unitary can be approximated to arbitrary precision using this set. However, naive numerical synthesis methods often result in prohibitively large T-counts, motivating more structured decomposition techniques.

1.2 Separation of Clifford and Non-Clifford Structure

An effective strategy is to decompose a target circuit into:

1. A purely Clifford subcircuit, and
2. A minimal non-Clifford component expressed using phase rotations.

Since Clifford operations normalize the Pauli group, they can be algebraically commuted, conjugated, and simplified without affecting the T-count. This allows large portions of a circuit to be reduced before introducing any T gates.

1.3 Phase Gadgets and Pauli Rotations

Many non-Clifford operations can be written as exponentials of Pauli operators,

$$U = e^{i\theta P}, \quad P \in \{I, X, Y, Z\}^{\otimes n}.$$

Such operators are implemented using *phase gadgets*, which consist of:

- Clifford basis changes mapping P to a Z-type operator,
- A single-axis $R_Z(\theta)$ rotation, and
- The inverse basis transformation.

This representation isolates all non-Clifford behavior into a single operation, enabling targeted optimization.

1.4 Clifford Simplification and Cancellation

Symbolic compilation enables exact simplifications when non-Clifford rotations appear in symmetric or conjugated forms. In such cases, rotations may cancel or reduce to Clifford gates, completely eliminating the need for T gates while preserving exact unitary equivalence.

1.5 Phase Polynomial Optimization for Diagonal Unitaries

Diagonal unitaries with phases restricted to multiples of $\pi/4$ can be represented as phase polynomials over $\mathbb{Z}_8$. Each odd coefficient corresponds to a T or $T^\dagger$ gate. Reed–Muller based optimization minimizes the number of odd coefficients, thereby directly minimizing the T-count.

1.6 Avoiding Numerical Approximation

Rather than relying on numerical approximation methods such as Solovay–Kitaev, modern compilers favor algebraic and symbolic decompositions. Numerical synthesis is used only when algebraic simplification is impossible, allowing explicit control over approximation error and resource overhead.

1.7 Summary

An efficient Clifford+T compilation workflow consists of rewriting circuits into Pauli rotations, simplifying Clifford structure, isolating non-Clifford phases, optimizing phase polynomials, and synthesizing the final circuit using Clifford+T primitives. This approach yields low T-counts, high fidelity, and scalability for fault-tolerant quantum computation.