Assignment set 2 for QCA course (March 2025)

Some quick points:

-In general, please explain each solution in a step by step and in a systematic and clear way.
-If you think that a question has implicitly made some assumption(s), please feel free to state them.
-If you want to make any assumptions while answering a question, please state them and justify why you made such an assumption or assumptions.
-Very important: the questions need **not** be easy, so I will grade you also on the basis of your efforts taken to solve the problem, and not necessarily just the final solution itself. Therefore, please make sure to also write down your different thoughts and approaches towards solving these problems (even if you think it makes no sense!).

Deadline: 20th of March, 5 pm.

Assignment questions:

Question 1: Prove $(A \otimes B)(C \otimes D) = AC \otimes BD$, where $A,\ B,\ C, \text{ and } D$ are $(N \times N)$ square matrices. (1 mark)
Question 2: Begin with the matrix representation of the CX gate in the Z basis, and arrive at its matrix form in the X basis. (1 mark)
Question 3: What is the computational cost of finding tensor product of two $(N \times N)$ matrices? Please explain how you arrived at the estimate. (1 mark)
Suggest two methods to reduce the cost, and explain how they help in detail, perhaps with an example? (2 marks)
Write a python code that can find the tensor product of any two $(N \times N)$ matrices (without using libraries like kron from numpy, etc). (2 marks)
Question 4: Given two unitaries $U \text{ and } V$, is $(U + V)$ also a unitary? If yes, why? If no, why? If no, what are the conditions under which the sum is unitary? (2 marks)
Question 5: Please write a python code to reduce the depth of a quantum circuit (pick a 6-qubit circuit with 60 one-qubit gates and 30 two-qubit gates arranged at random; use only the gates H, X, Y, Z, RZ(θ), and CX), given a set of circuit identities (pick at least 10 of your favourite circuit identities for this purpose). List the final number of one- and two-qubit gates in the optimized circuit, and comment on the effectiveness of your procedures. Make sure to explain the logic behind your code clearly. (5 marks)
Question 6: Can you prepare the state $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$ with fewer than two one-qubit gates and one two-qubit gate? If yes, why? If no, why? (1 mark)
Question 7: Do the X gate on the first qubit (with identity on the second) and the CX gate always commute? If yes, why? If no, why? (1 mark)
Question 8: Pick your favourite quantum computer (one example could be Forte-I from IonQ, another could be IBM Brisbane from IBMQ, etc), and list its specifications. Comment in detail on the size of the circuits that you think such computers can handle, including the circuit you chose for question 5 before and after optimization. (4 marks)
Question 9: Find the quantum circuit that implements the normalized version of the state $|00> - 3|11> + |10>$. (5 marks)