Advanced Quantum Computing, Assignment 1

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Q1.

QMA is the quantum analogue of **NP**. It represents the class of decision problems for which a quantum proof can be verified by a quantum polynomial-time algorithm.
BQP is the class of problems solvable in polynomial time by a quantum computer with bounded error. If a BQP algorithm exists for a QMA-complete problem, this implies:

$$\text{QMA} \subseteq \text{BQP} \Rightarrow \text{QMA} = \text{BQP}$$

Since NP $\subseteq$ QMA, it follows that:

$$\text{NP} \subseteq \text{QMA} \implies \text{NP} \subseteq \text{BQP}$$

Q2.

NP problems are those for which a solution can be verified in polynomial time.
NP-hard problems are at least as hard as the hardest problems in NP. Solving an NP-hard problem in polynomial time implies all NP problems can also be solved in polynomial time. However, NP-hard problems may not be verifiable in polynomial time.

- **Example of NP problem:** SAT (Boolean satisfiability problem)
- **Example of NP-hard problem:** Traveling Salesman Problem (optimization checking)

Q3.

Probability of error for a single run: $P_{\text{error}} = \frac{1}{3}$.
The experiment is repeated twice, and we take a majority vote to determine the final result.

- Probability of both correct: $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$
- Probability of one correct, one incorrect (i.e tie): $2 \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$
- Probability of both incorrect: $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

Since majority voting only fails when both runs are incorrect, the effective probability of error is:

$$P_{\text{effective error}} = \frac{1}{9}.$$

Q4.

<u>$(W = VU)$</u>

Time Complexity:

$$\text{Complexity}(U) = O\left(\frac{N}{\epsilon}\right), \quad \text{Complexity}(V) = O\left(\frac{N^2}{\epsilon^2}\right)$$

The total complexity is additive:

$$\text{Complexity}(W) = O\left(\frac{N}{\epsilon} + \frac{N^2}{\epsilon^2}\right)$$

Error Probability:

Let $\epsilon_U = \epsilon$ and $\epsilon_V = \epsilon$. The worst-case error is:

$$\epsilon_W \leq \epsilon_U + \epsilon_V = 2\epsilon.$$

If errors are independent, the probability that both succeed is $(1-\epsilon)^2$, so:

$$\epsilon_W = 1 - (1-\epsilon)^2 = 2\epsilon - \epsilon^2 \approx 2\epsilon \quad \text{(for small } \epsilon\text{)}.$$

<u>**(U and V Act in Parallel)**</u>

Time Complexity:

The runtime is dominated by the slower unitary:

$$\text{Complexity}(W) = \max\left(O\left(\frac{N}{\epsilon}\right), O\left(\frac{N^2}{\epsilon^2}\right)\right) = O\left(\frac{N^2}{\epsilon^2}\right).$$

Error Probability:

The system fails if either U or V fails:

$$\epsilon_W \leq \epsilon_U + \epsilon_V = 2\epsilon.$$

For independent errors:

$$\epsilon_W = 2\epsilon - \epsilon^2 \approx 2\epsilon.$$