Advanced Quantum Computing, Assignment 1

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February 3, 2025

1 Questions

Vector Space: A set V with two operations (addition and scalar multiplication) satisfying certain axioms. In quantum mechanics, the state space of a quantum system is a complex vector space, often referred to as a Hilbert space.
Example: The set of all possible quantum states of a spin-1/2 particle forms a 2-dimensional complex vector space.

Subspace: A subset $W \subseteq V$ that is itself a vector space under the operations of V. In quantum mechanics, subspaces correspond to sets of quantum states that share specific properties, like energy levels.
Example: The set of all eigenstates corresponding to a specific eigenvalue of an observable forms a subspace.

Span: The set of all linear combinations of a given set of vectors $\{v_1, v_2, \ldots, v_k\}$. In quantum mechanics, any quantum state can be expressed as a linear combination (superposition) of basis states.
Example: The span of $|0\rangle$ and $|1\rangle$ forms the state space of a qubit.

Basis: A set of linearly independent vectors that span the vector space. In quantum mechanics, basis vectors represent orthonormal states, such as the computational basis $|0\rangle$ and $|1\rangle$ for qubits.
Example: The set $\{|0\rangle, |1\rangle\}$ is a basis of the state space of a qubit.

Norm: A function $\|\cdot\| : V \to \mathbb{R}$ that satisfies positivity, scalar multiplication, triangle inequality, and is zero only for the zero vector. In quantum mechanics, the norm of a state vector represents its probability amplitude.
Example: For a quantum state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, the norm is $\||\psi\rangle\| = \sqrt{|\alpha|^2 + |\beta|^2}$.

Hilbert Space: A complete inner product space where the norm is derived from the inner product. Hilbert space structure is fundamental for describing quantum states and observables.
Example: $L^2(\mathbb{R})$, the space of square-integrable wave functions, is a Hilbert space.

Linear Operator: A mapping $T : V \to V$ that satisfies linearity: $T(av+bw) = aT(v)+bT(w)$ for all $v, w \in V$, and scalars a, b. In quantum mechanics, observables and evolution operators are linear operators.
Example: The Hamiltonian operator $\hat{H}$ governs the time evolution of a quantum system.

Matrix Representation: A linear operator represented as a matrix relative to a basis. Operators like the Pauli matrices represent spin operators in matrix form.
Example: The Pauli-X operator is represented as $\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Observable: In quantum mechanics, an observable is a Hermitian operator representing a measurable quantity. The eigenvalues correspond to possible measurement outcomes.
Example: The spin operator $\hat{S}_z$ represents the measurement of spin along the z-axis.

Measurement: The process of obtaining the eigenvalues of an observable, which correspond to possible outcomes. The system collapses to the corresponding eigenstate post-measurement.
Example: Measuring the spin of an electron along the z-axis yields $+\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$.

Direct Sum: The combination of two vector spaces V and W such that each element is an ordered pair (v, w). This describes composite systems with distinguishable subsystems.
Example: The state space of a particle with spin and spatial degrees of freedom can be written as a direct sum of spatial and spin state spaces.

Tensor Product: A construction that combines two vector spaces $V \otimes W$ to form a new space. This represents composite systems of multiple particles.
Example: The combined state space of two qubits is $\mathbb{C}^2 \otimes \mathbb{C}^2 \cong \mathbb{C}^4$.

2 Essay

Quantum computing has the potential to change the world in ways we're just beginning to imagine. Its incredible processing power can revolutionize drug discovery by simulating complex molecules, speeding up the search for new treatments. In material science, it can help us design advanced materials with properties we've never seen before. When it comes to optimization whether improving supply chains or financial strategies quantum algorithms can find solutions faster and more efficiently than ever. Plus, with quantum cryptography, our data could be more secure than ever. As this technology evolves, it's set to transform industries and solve real-world challenges.