Questions 1-25 carry 1 mark each

1. Consider a system of linear equations:

$$x - 2y + 3z = -1$$

$$x - 3y + 4z = 1$$

$$-2x + 4y - 6z = k$$

The value of k for which the system has infinitely many solutions is _____.

2. A function $f(x) = 1 - x^2 + x^3$is defined in the closed interval $[-1, 1]$. The value of x , in the open interval $(-1, 1)$ for which the mean value theorem is satisfied, is
(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

3. Suppose A and B are two independent events with probabilities $P(A) \neq 0$ and $P(B) \neq 0$. Let $\overline{A}$ and $\overline{B}$ be their complements. Which one of the following statements is FALSE?
(a) $P(A \cap B) = P(A)P(B)$ (b) $P(A \cup B) = P(A) + P(B)$
(c) $P(A|B) = P(A)$ (d) $P(\overline{A} \cap \overline{B}) = P(\overline{A})P(\overline{B})$

4. Let $z = x+iy$ be a complex variable. Consider that contour integration is performed along the unit circle in anticlockwise direction. Which one of the following statements is **NOT TRUE**?

(a) The residue of $\frac{z}{z^2-1}$ at $z = 1$ is$\frac{1}{2}$

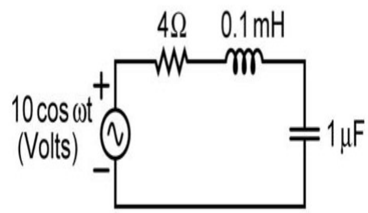
(b) $\oint_C z^2\, dz = 0$

(c) $\frac{1}{2\pi i} \oint_C \frac{1}{z}\, dz$

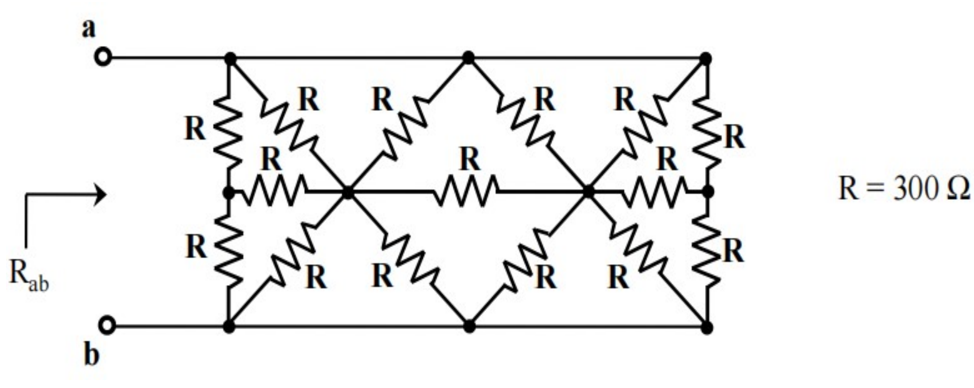
(d) $\overline{z}\,(complex conjugate of z)$is an analytical function

5. The value of p such that the vector $\begin{bmatrix}1\\2\\3\end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix}4 & 1 & 2\\ p & 2 & 1\\ 14 & -4 & 10\end{bmatrix}$ is _____.

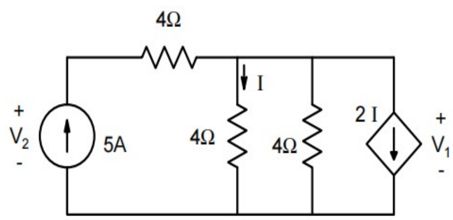
6. In the circuit shown, at resonance, the amplitude of the sinusoidal voltage (in Volts) across the capacitor is _____.



7. In the network shown in the figure, all resistors are identical with $R = 300\Omega$. The resistance $R_ab\,(in\Omega)$ of the network is _____.



8. In the given circuit, the values of V_1 and V_2 respectively are



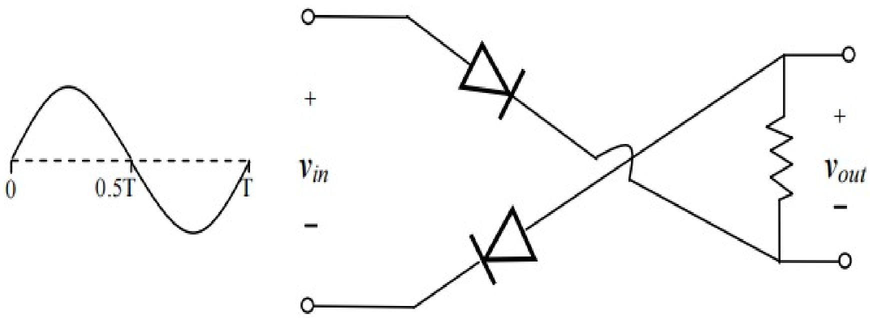
(a) $5V, 25V$ (b) $10V, 30V$ (c) $15V, 35V$ (d) $0V, 20V$

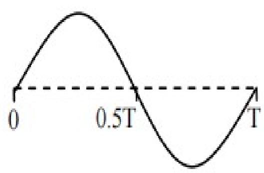
9. A region of negative differential resistance is observed in the current voltage characteristics of a silicon PN junction if

   (a) both the P-region and the N-region are heavily doped
   (b) the N-region is heavily doped compared to the P-region
   (c) the P-region is heavily doped compared to the N-region
   (d) an intrinsic silicon region is inserted between the P-region and the N-region

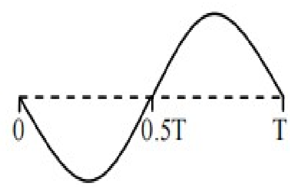
10. A silicon sample is uniformly doped with donor type impurities with a concentration of $10^16/cm^3$.The electron and hole mobilities in the sample are $1200cm^2/V-s$ and $400cm^2/V-s$ respectively. Assume complete ionization of impurities. The charge of an electron is 1.610^-19C. The resistivity of the sample $(in\Omega - cm)$ is __________.

11. For the circuit with ideal diodes shown in the figure, the shape of the output (v_{out}) for the given sine wave input (v_{in}) will be

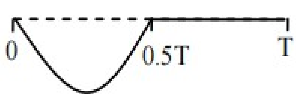




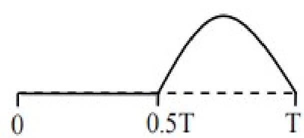
a)



b)

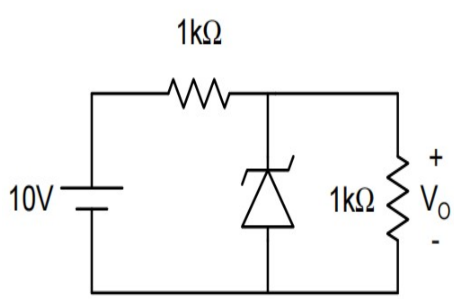


c)

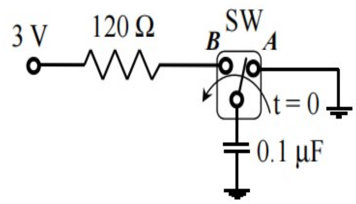


d)

12. In the circuit shown below, the Zener diode is ideal and the Zener voltage is $6V$. The output voltage V_o(in volts) is _____.



13. In the circuit shown, the switch SW is thrown from position A to position B at time $t = 0$. The energy $(in \mu J)$ taken from the $3V$ source to charge the $0.1\mu F$ capacitor from $0V$ to $3V$ is



(a) 0.3 (b) 0.45 (c) 0.9 (d) 3

14. In an 8085 microprocessor, the shift registers which store the result of an addition and the overflow bit are, respectively

(a) B and F
(b) A and F
(c) H and F
(d) A and C

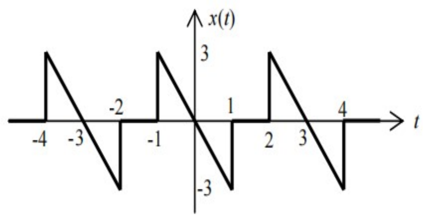
15. A $16Kb$ (=16,384 bit) memory array is designed as a square with an aspect ratio of one (number of rows is equal to the number of columns). The minimum number of address lines needed for the row decoder is _____.

16. Consider a four bit D to A converter. The analog value corresponding to digital signals of values 0000 and 0001 are $0V$ and $0.0625V$ respectively. The analog value (in Volts) corresponding to the digital signal 1111 is _____.

17. The result of the convolution $x(-t) * \delta(-t - t_0)$ is

(a) $x(t + t_0)$ (b) $x(t - t_0)$ (c) $x(-t + t_0)$ (d) $x(-t - t_0)$

18. The waveform of a periodic signal $x(t)$ is shown in the figure



A signal $g(t)$ is defined by $g(t) = x\left(\dfrac{x-1}{2}\right)$.The average power of $g(t)$ is _____.

19. Negative feedback in a closed-loop control system **DOES NOT**

    (a) reduce the overall gain
    (b) reduce bandwidth
    (c) improve disturbance rejection
    (d) reduce sensitivity to parameter variation

20. A unity negative feedback system has the open-loop transfer function $G(t) = \dfrac{K}{s(s+1)(s+3)}$. The value of the gain $K\,(>0)$ at which the root locus crosses the imaginary axis is _____.

21. The polar plot of the transfer function

$$G(s) = \frac{10(s+1)}{s(s+10)} \quad \text{for} \quad 0 \leq \omega < \infty$$

    will be in the

    (a) first quadrant
    (b) second quadrant
    (c) third quadrant
    (d) fourth quadrant

22. A sinusoidal signal of $2kHz$ frequency is applied to a delta modulator. The sampling rate and step-size Δ of the delta modulator are $20,000$

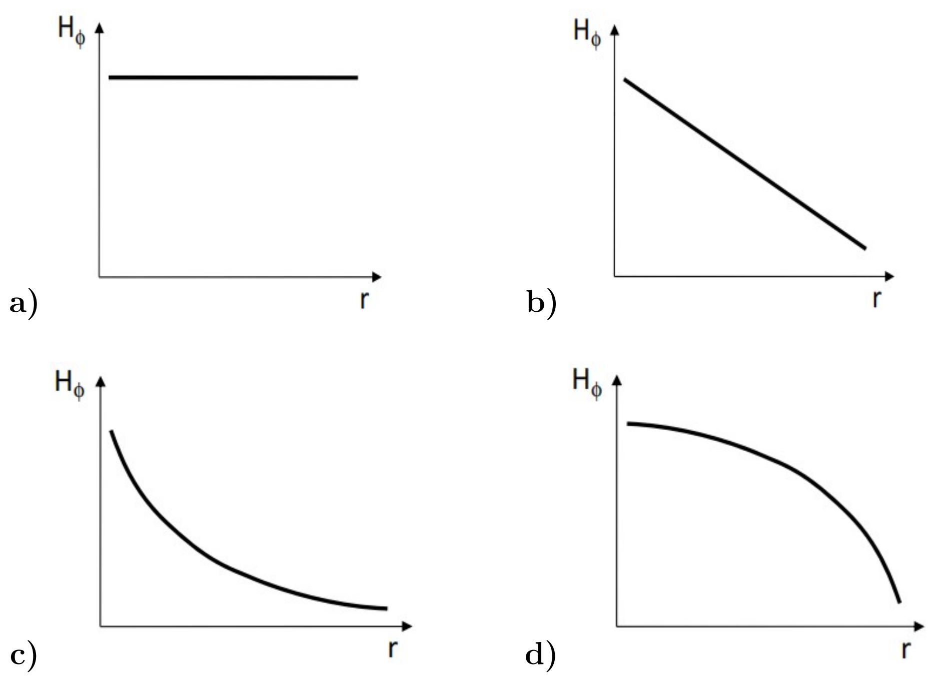
samples per second and $0.1V$, respectively. To prevent slope overload, the maximum amplitude of the sinusoidal signal (in Volts) is

(a) $\frac{1}{2\pi}$ (b) $\frac{1}{\pi}$ (c) $\frac{2}{\pi}$ (d) π

23. Consider the signal $s(t) = m(t)\,cos(2\pi f_c t) + \hat{m}(t)\,sin(2\pi f_c t)$ where $\hat{m}(t)$ denotes the Hilbert transform of m(t) and the bandwidth of m(t) is very small compared to f_c . The signal s(t) is a

   (a) high-pass signal
   (b) low-pass signal
   (c) band-pass signal
   (d) double sideband suppressed carrier signal

24. Consider a straight, infinitely long, current carrying conductor lying on the z-axis. Which one of the following plots (in linear scale) qualitatively represents the dependence of H_ϕ on r, where H_ϕ is the magnitude of the azimuthal component of magnetic field outside the conductor and r is the radial distance from the conductor?



25. The electric field component of a plane wave traveling in a lossless dielectric medium is given by $\overrightarrow{E}(z,t) = \hat{a}_y 2cos\left(10^8 t - \frac{z}{sqrt2}\right) V/m$.The wavelength (in m) for the wave is _____.

Questions 26-55 carry 2 marks each

26. The solution of the differential equation $\frac{d^2y}{dy^2} + 2\frac{dy}{dt} + y = 0$ with $y(0) = y'(0) = 1$ is

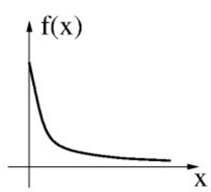
(a) $(2-t)\,e^{t}$ (b) $(1+2t)\,e^{-t}$

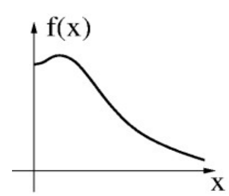
(c) $(2+t)\,e^{-t}$ (d) $(1-2t)\,e^{t}$

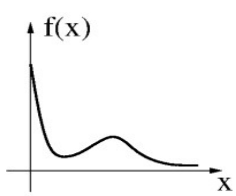
27. A vector $\overrightarrow{P}$ is given by $\overrightarrow{P} = x^3 y \overrightarrow{a_x} - x^2 y^2 \overrightarrow{a_y} - x^2 y x \overrightarrow{a_z}$. Which one of the following statements is **TRUE**?

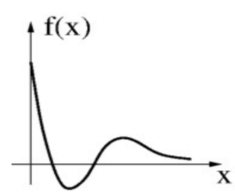
(a) $\overrightarrow{P}$ is solenoidal, but not irrotational
(b) $\overrightarrow{P}$ is irrotational, but not solenoidal
(c) $\overrightarrow{P}$ is neither solenoidal nor irrotational
(d) $\overrightarrow{P}$ is both solenoidal and irrotational

28. Which one of the following graphs describes the function $f(x) = e^{-x}(x^2 + x + 1)$?

a) 

b) 

c) 

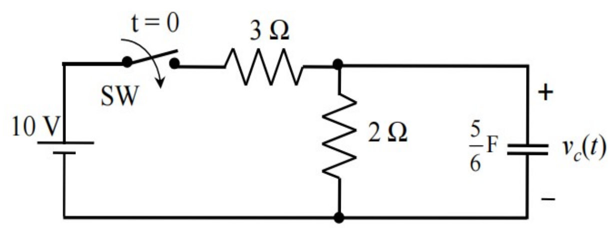
d) 

29. The maximum area (in square unit) of a rectangle whose vertices lie on the ellipse $x^2 + 4y^2 = 1$ is _____.

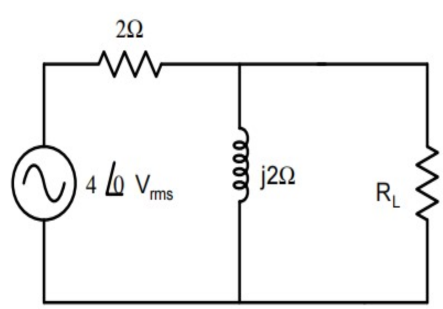
30. The damping ratio of a series RLC circuit can be expressed as

(a) $\dfrac{R^2C}{2L}$ (b) $\dfrac{2L}{R^2C}$ (c) $\dfrac{R}{2}\sqrt{\dfrac{C}{L}}$ (d) $\dfrac{2}{R}\sqrt{\dfrac{L}{C}}$

31. In the circuit shown, switch SW is closed at $t = 0$. Assuming zero initial conditions, the value of $v_c(t)$ (in Volts) at $t = 1$ sec is _____.



32. In the given circuit, the maximum power (in Watts) that can be transferred to the load R_L is (C_J)



33. The built-in potential of an abrupt p-n junction is $0.75V$. If its junction capacitance (C_J) at a reverse bias (V_R) of $1.25V$ is $5pF$, the value of C_J (in pF) when $V_R = 7.25V$ is _____.

34. A MOSFET in saturation has a drain current of $1mA$ for $V_DS = 0.5V$. If the channel length modulation coefficient is $0.05V^{-1}$, the output resistance (in kΩ) of the MOSFET is _____.

35. For a silicon diode with long P and N regions, the accepter and donor impurity concentrations are $1 \times 10^17\ cm^-3$ and $1 \times 10^15\ cm^-3$, respectively. The lifetimes of electrons in P region and holes in N region are both 100 μs. The electron and hole diffusion coefficients are 49 cm^2/s and 36 cm^2/s, respectively. Assume $kT/q = 26\ mV$, the intrinsic carrier concentration is $1 \times 10^10\ cm^-3$, and $q = 1.6 \times 10^-19$ C. When a forward voltage of 208 mV is applied across the diode, the hole current density (in nA/cm^2) injected from P region to N region is ________.

36. The Boolean expression $F(X, Y, Z) = \overline{X}Y\overline{Z} + X\overline{Y}\overline{Z} + XY\overline{Z} + XYZ$ converted into the canonical product of sum (POS) form is

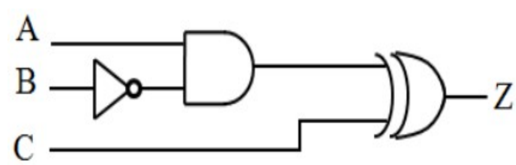
(a) $(X + Y + Z)(X + Y + \overline{Z})(X + \overline{Y} + \overline{Z})(\overline{X} + Y + \overline{Z})$

(b) $(X + \overline{Y} + Z)(\overline{X} + Y + \overline{Z})(\overline{X} + \overline{Y} + Z)(\overline{X} + \overline{Y} + \overline{Z})$

(c) $(X + Y + Z)(\overline{X} + Y + \overline{Z})(X + \overline{Y} + Z)(\overline{X} + \overline{Y} + \overline{Z})$

(d) $(X + \overline{Y} + \overline{Z})(\overline{X} + Y + Z)(\overline{X} + \overline{Y} + Z)(X + Y + Z)$

37. All the logic gates shown in the figure have a propagation delay of 20 ns. Let $A = C = 0$ and $B = 1$ until time $t = 0$. At $t = 0$, all the inputs flip (i.e., $A = C = 1$ and $B = 0$) and remain in that state. For $t > 0$, output $Z = 1$ for a duration (in ns) of ________.



38. A 3-input majority gate is defined by the logic function $M(a, b, c) = ab + bc + ca$. Which one of the following gates is represented by the function $M\left(\overline{M(a, b, c)}, M(a, b, \overline{c}), c\right)$?

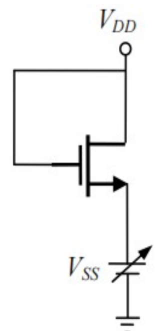
(a) 3-input NAND gate

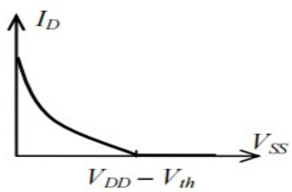
(b) 3-input XOR gate

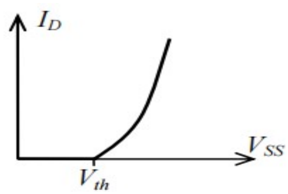
(c) 3-input NOR gate

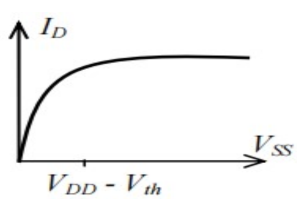
(d) 3-input XNOR gate

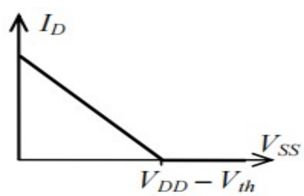
39. For the NMOSFET in the circuit shown, the threshold voltage isV_{th}, where $V_{th} > 0$. The source voltage V_{SS} is varied from 0 to V_{DD}. Neglecting the channel length modulation, the drain current I_d as a function of V_{SS} is represented by



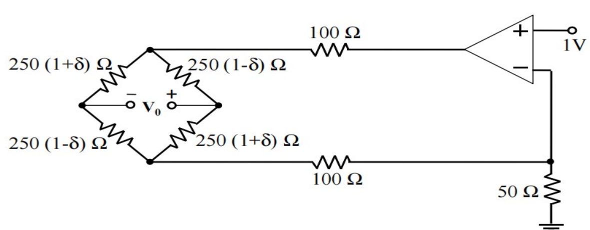
a) 

b) 

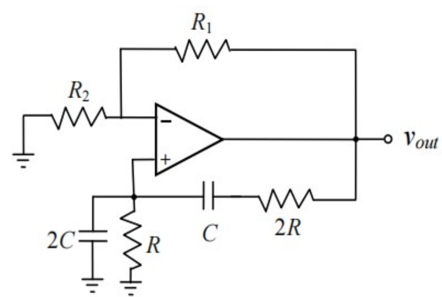
c) 

d) 

40. In the circuit shown, assume that the opamp is ideal. The bridge output voltage V_0 (in mV) for $\delta = 0.05$ is _____.



41. The circuit shown in the figure has an ideal opamp. The oscillation frequency and the condition to sustain the oscillations, respectively, are



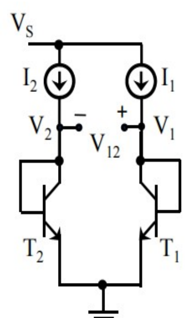
(a) $\frac{1}{CR}$ and $R_1 = R_2$

(b) $\frac{1}{CR}$ and $R_1 = 4R_2$

(c) $\frac{1}{2CR}$ and $R_1 = R_2$

(d) $\frac{1}{2CR}$ and $R_1 = 4R_2$

42. In the circuit shown, $I_1 = 80$mA and $I_2 = 4$mA. Transistors T_1 and T_2 are identical. Assume that the thermal voltage V_T is 26mV at $27°C$. At $50°C$, the value of the voltage V_12 (in mV) is _________.



43. Two sequences $[a, b, c]$ and $[A, B, C]$ are related as

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{-1} & W_3^{-2} \\ 1 & W_3^{-2} & W_3^{-4} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ where } W_3 = e^{j\frac{2\pi}{3}}.$$

If another sequence $[p, q, r]$ is derived as,

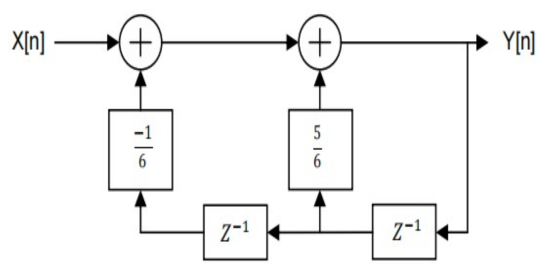
$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^1 & W_3^2 \\ 1 & W_3^2 & W_3^4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & W_3^2 & 0 \\ 0 & 0 & W_3^4 \end{bmatrix} \begin{bmatrix} A/3 \\ B/3 \\ C/3 \end{bmatrix}$$

then the reltionship between sequence $[p, q, r]$ and $[a, b, c]$ is

(a) $[p, q, r] = [b, a, c]$ (b) $[p, q, r] = [b, c, a]$

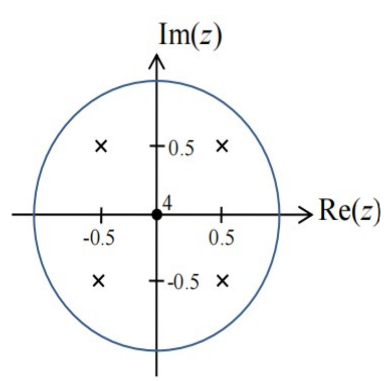
(c) $[p, q, r] = [c, a, b]$ (d) $[p, q, r] = [c, b, a]$

44. For the discrete-time system shown in the figure, the poles of the system transfer function are located at



(a) $2, 3$ (b) $\frac{1}{2}, 3$ (c) $\frac{1}{2}, \frac{1}{3}$ (d) $2, \frac{1}{3}$

45. The pole-zero diagram of a causal and stable discrete-time system is shown in the figure. The zero at the origin has multiplicity 4. The impulse response of the system is $h[n]$. If $h[0] = 1$, we can conclude,



(a) $h[n]$ is real for all n

(b) $h[n]$ is purely imaginary for all n

(c) $h[n]$ is real for only even

(d) $h[n]$ is purely imaginary for only odd

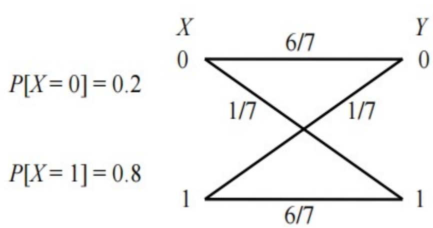
46. The open-loop transfer function of a plant in a unity feedback configuration is given as $G(s) = \dfrac{K(s+4)}{(s+8)(s^2-9)}$. The value of the gain $K(>0)$ for which $-1+j2$ lies on the root locus is _____.

47. A lead compensator network includes a parallel combination of R and C in the feed-forward path. If the transfer function of the compensator is $G_c(s) = \dfrac{s+2}{s+4}$, the value of RC is _____.

48. A plant transfer function is given as $G_c(s) = \left(K_P + \dfrac{K_I}{s}\right)\dfrac{1}{s(s+2)}$. When the plant operates in a unity feedback configuration, the condition for the stability of the closed loop system is

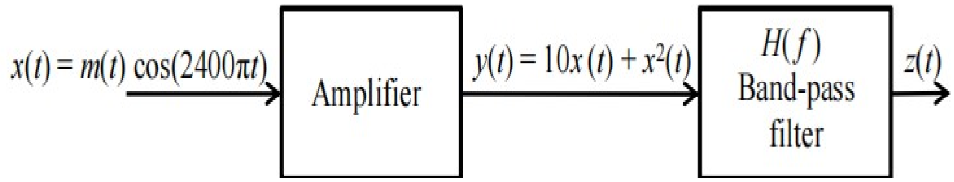
(a) $K_p > \dfrac{K_I}{2} > 0$ (b) $2K_I > K_p > 0$ (c) $2K_I < K_p$ (d) $2K_I > K_p$

49. The input X to the Binary Symmetric Channel(BSC) shown in the figure is '1′ with probability 0.8. The cross-over probability is 1/7. If the received bit $Y = 0$, the conditional probability that '1′ was transmitted is _____.

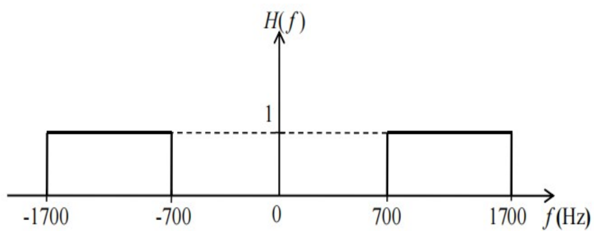


50. The transmitted signal in a GSM system is of 200 kHz bandwidth and 8 users share a common bandwidth using TDMA. If at a given time 12 users are talking in a cell, the total bandwidth of the signal received by the base station of the cell will be at least $(inkHz)$ _________.

51. In the system shown in Figure(a),$m(t)$ is a low-pass signal with bandwidth W Hz. The frequency response of the band-pass filter $H(f)$ is shown in Figure(b). If it is desired that the output signal $z(t) = 10x(t)$, the maximum value of W (Hz) should be strictly less than _____.



(a)



(b)

52. A source emits bit 0 with probability $\frac{1}{3}$ and bit 1 with probability $\frac{2}{3}$.The emitted bits are communicated to the receiver. The receiver decides for either 0 or 1 based on the received value R. It is given that the conditional density functions of R are as

$$f_{R|0}(r) = \begin{cases} \frac{1}{4}, & -3 \le r \le 1 \\ 0 & \text{otherwise.} \end{cases} \quad \text{and} \quad f_{R|1}(r) = \begin{cases} \frac{1}{6}, & -1 \le r \le 5 \\ 0 & \text{otherwise.} \end{cases}$$

The minimum decision error probability is

(a) 0 (b) 1/12 (c) 1/9 (d) 1/6

53. The longitudinal component of the magnetic field inside an air-filled rectangular waveguide made of a perfect electric conductor is given by

the following expression

$$H_z(x, y, z, t) = 0.1cos(25\pi x)\,cos(30.3\pi y)\,cos\left(12y \times 10^9 t - \beta z\right)(A/m)$$

The cross-sectional dimensions of the waveguide are given as $a = 0.08m$ and $b = 0.033m$. The mode of propagation inside the waveguide is

(a) TM_{12} (b) TM_{21}

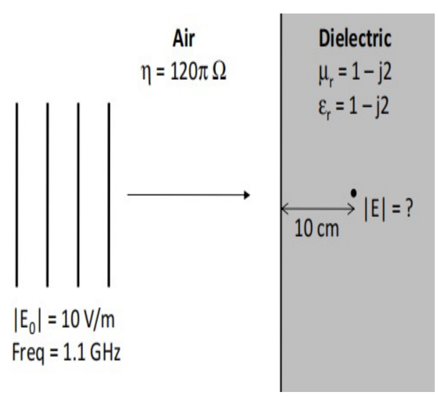
(c) TE_{21} (d) TE_{12}

54. The electric field intensity of a plane wave traveling in free space is given by the following expression

$$\mathbf{E}(x, t) = \mathbf{a}_y 24\pi cos(\omega t - k_0 x)(V/m)$$

In this field, consider a square area 10cm $\times$ 10cm on a plane $x + y = 1$. The total time-averaged power (in mW) passing through the square area is _____.

55. Consider a uniform plane wave with amplitude (E_0) of 10V/m and 1.1 GHz frequency travelling in air, and incident normally on a dielectric medium with complex relative permittivity (ε_r) and permeability (μ_r) as shown in the figure.



The magnitude of the transmitted electric field component $(in V/m)$ after it has travelled a distance of 10 cm inside the dielectric region is __________.