12

NCERT SOLUTION

MATHEMATICS

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Chapter 1

Relations and Functions

Exercise 1.1

Q. 1A

Determine whether each of the following relations are reflexive, symmetric and transitive:

Relation R in the set A = {1, 2, 3, ..., 13, 14} defined as

R = {(x, y): 3x – y = 0}

Answer:

It is given that Relation R in the set A = {1, 2, 3, ..., 13, 14} defined as

R = {(x, y): 3x – y = 0}

Then, R = {(1,3), (2,6), (3,9), (4,12)}

REFLEXIVE

A relation is said to be reflexive if (x, x) ϵ R, where x is from domain.

R is not reflexive as (1,1), (2,2) …… (14,14) ∉ R

SYMMETRIC

A relation is said to be symmetric if (y, x) ϵ R whenever (x, y) ϵ R.

Also, R is not symmetric as (1,3) ϵ R, but (3,1) ∉ R

TRANSITIVE

A relation is said to be transitive if (x, z) ϵ R whenever (x, y) ϵ R and (y, z) ϵ R

And, also R is not transitive as (1,3), (3,9) ϵ R, but (1,9) ∉ R

Therefore, R is neither reflexive, nor symmetric, nor transitive.

Q. 1B

Relation R in the set N of natural numbers defined as

R = {(x, y): y = x + 5 and x < 4}

Determine if the given relation is reflexive, symmetric and transitive.

Answer:

It is given that Relation R in the set N of natural numbers defined as

R = {(x, y): y = x + 5 and x < 4}

Clearly,

R = {(1, 6), (2, 7), (3, 8)}

REFLEXIVE

A relation is said to be reflexive if (x, x) ϵ R, where x is from domain.
we can see that (1,1) ∉ R

⇒ R is not reflexive.

SYMMETRIC

A relation is said to be symmetric if (y, x) ϵ R whenever (x, y) ϵ R.

Now, (1,6) ϵ r but (6,1) ∉ R.

⇒ R is not symmetric.

TRANSITIVE

Now, since there is no pair in R such that (x, y) and (y, z) ∈R, then (x, z) cannot belong to R.

∴R is not transitive.

Therefore, R is neither reflexive, nor symmetric, nor transitive.

Q. 1C

Relation R in the set A = {1, 2, 3, 4, 5, 6} as

R = {(x, y): y is divisible by x}

Answer:

It is given that relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as

$R = \{(x, y): y \text{ is divisible by } x\}$

We know that any number (x) is divisible by itself.

$\Rightarrow (x, x) \in R$

$\Rightarrow$ R is reflexive.

Now, $(2,4) \in R$ but $(4,2) \notin R$.

$\Rightarrow$ R is not symmetric.

Let $(x, y), (y, z) \in R$. Then, y is divisible x and z is divisible by y.

$\Rightarrow$ z is divisible by x.

$\Rightarrow (x, z) \in R$

$\Rightarrow$ R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

Q. 1D

Relation R in the set Z of all integers defined as

$R = \{(x, y): x - y \text{ is an integer}\}$

Answer:

It is given that Relation R in the set Z of all integers defined as

$R = \{(x, y): x - y \text{ is an integer}\}$

Now, for every $x \in Z$, $(x, x) \in R$, as $x - x = 0$ is an integer.

$\Rightarrow$ R is reflexive.

Now, for every $x, y \in Z$ if $(x, y) \in R$, then $x - y$ is an integer.

$\Rightarrow -(x - y)$ is also an integer.

$\Rightarrow (y - x)$ is an integer.

$\Rightarrow (y, x) \in R$

$\Rightarrow$ R is symmetric.

Now, for every (x, y) and $(y, z) \in R$ where $x, y, z \in R$,

$\Rightarrow (x - y)$ and $(y - z)$ is an integer.

$\Rightarrow (x - z) = (x - y) + (y - x)$ is an integer.

$\Rightarrow (x, z) \in R$

$\Rightarrow$ R is transitive.

Therefore, R is reflexive, symmetric and transitive.

Q. 1E

Relation R in the set A of human beings in a town at a particular time given by

A. R = {(x, y): x and y work at the same place}

B. R = {(x, y): x and y live in the same locality}

C. R = {(x, y): x is exactly 7 cm taller than y}

D. R = {(x, y): x is wife of y}

E. R = {(x, y): x is father of y}

Answer:

(a) It is given that R = {(x, y): x and y work at the same place}

$\Rightarrow (x, x) \in R$

$\Rightarrow$ R is reflexive.

Now, if $(x, y) \in R$, then x and y work on the same place.

$\Rightarrow$ y and x work at the same place.

$\Rightarrow (y, x) \in R$

$\Rightarrow$ R is symmetric.

Now, let $(x, y), (y, z) \in R$

$\Rightarrow$ x and y work at the same place and y and z work at the same place.

$\Rightarrow$ x and z work at the same place

$\Rightarrow (x, z) \in R$

$\Rightarrow$ R is transitive.

Therefore, R is reflexive, symmetric and transitive.

(b) It is given that $R = \{(x, y)$: x and y live in the same locality$\}$

$\Rightarrow (x, x) \in R$ as x and x live in the same human being.

$\Rightarrow$ R is reflexive.

Now, if $(x, y) \in R$, then x and y live in the same locality.

$\Rightarrow$ y and x live in the same locality.

$\Rightarrow (y, x) \in R$

$\Rightarrow$ R is symmetric.

Now, let $(x, y), (y, z) \in R$

$\Rightarrow$ x and y live in the same locality and y and z live in the same locality.

$\Rightarrow$ x and z live in the same locality

$\Rightarrow (x, z) \in R$

$\Rightarrow$ R is transitive.

Therefore, R is reflexive, symmetric and transitive.

(c) It is given that $R = \{(x, y)$: x is exactly 7 cm taller than y$\}$

$\Rightarrow (x, x) \notin R$ as human being x cannot be taller than himself.

⇒ R is not reflexive.

Now, if $(x, y) \in R$, then x is exactly 7 cm taller than y.

⇒ But y is not taller than x.

⇒ $(y, x) \notin R$

⇒ R is not symmetric.

Now, let $(x, y), (y, z) \in R$

⇒ x is exactly 7 cm taller than y and y is exactly 7 cm taller than z.

⇒ x is exactly 14 cm taller than z.

⇒ $(x, z) \notin R$

⇒ R is not transitive.

Therefore, R is neither reflexive, nor symmetric, nor transitive.

(d) It is given that $R = \{(x, y)$: x is wife of y$\}$

⇒ $(x, x) \notin R$ as x cannot be the wife of herself.

⇒ R is not reflexive.

Now, if $(x, y) \in R$, then x is the wife of y.

⇒ But y is not wife of x.

⇒ $(y, x) \notin R$

⇒ R is not symmetric.

Now, let $(x, y), (y, z) \in R$

⇒ x is the wife of y and y is the wife of z.

⇒ This cannot be possible.

⇒ $(x, z) \notin R$

⇒ R is not transitive.

Therefore, R is neither reflexive, nor symmetric, nor transitive.

(e) It is given that R = {(x, y): x is father of y}

⇒ (x, x) ∉ R as x cannot be the father of himself.

⇒ R is not reflexive.

Now, if (x, y) ϵ R, then x is the father of y.

⇒ But y is not father of x.

⇒ (y, x) ∉ R

⇒ R is not symmetric.

Now, let (x, y), (y, z) ϵ R

⇒ x is the father of y and y is the father of z.

⇒ x is not the father of z.

⇒ Indeed x is the grandfather of z.

⇒ (x, z) ∉ R

⇒ R is not transitive.

Therefore, R is neither reflexive, nor symmetric, nor transitive.

Q. 2

Show that the relation R in the set R of real numbers, defined as

R = {(a, b): $a \leq b^2$} is neither reflexive nor symmetric nor transitive.

Answer:

It is given that R = {(a, b): $a \leq b^2$}

Check for reflexive:

We can see that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$, Since, $\frac{1}{2} > \left(\frac{1}{2}\right)^2$

Therefore, R is not reflexive.

Check for symmetric:

Now, $(1,4) \in R$ as $1 < 4^2$

But 4 is not less than 1^2.

Then, $(4,1) \notin R$

Therefore, R is not symmetric.

Check for transitive:

Now, $(3, 2), (2, 1.5) \in R$

But, $3 > (1.5)^2 = 2.25$.

Then, $(3, 1.5) \notin R$

Therefore, R is not transitive.

Therefore, R is neither reflexive, nor symmetric, nor transitive.

Q. 3

Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as

R = {(a, b): b = a + 1} is reflexive, symmetric or transitive.

Answer:

Let us take A = {1, 2, 3, 4, 5, 6}

A relation R is defined on set A as:

R = {(a, b): b = a + 1}

Then, R = {(1,2), (2,3), (3,4), (4,5), (5,6)}

Now, we will find $(a, a) \notin R$, where $a \in A$

For instance,

$(1,1), (2,2), (3,3), (4,4), (5,5), (6,6) \notin R$

Therefore, R is not reflexive.

We can see that $(1,2) \in R$, but $(2,1) \notin R$.

Therefore, R is not symmetric.

And now, $(1,2), (2,3) \in R$

But, $(1,3) \notin R$

Therefore, R is not transitive.

Therefore, R is neither reflexive, nor symmetric, nor transitive.

Q. 4

Show that the relation R in R defined as $R = \{(a, b): a \leq b\}$, is reflexive and transitive but not symmetric.

Answer:

It is given that $R = \{(a, b): a \leq b\}$,

It is clear that $(a, a) \in r$ as $a = a$.

Therefore, R is reflexive.

Now let us take $(2,4) \in R$ $(2 < 4)$

But, $(4,2) \notin R$ as 4 is greater than 2.

Therefore, R is not symmetric.

Now, let $(a, b), (b, c) \in R$

Then, $a \leq b$ and $b \leq c$

$\Rightarrow a \leq c \Rightarrow (a, c) \in R$

Therefore, R is a transitive.

Therefore, R is reflexive and transitive but not symmetric.

Q. 5

Check whether the relation R in R defined by R = {(a, b): a $\leq b^3$} is reflexive, symmetric or transitive.

Answer:

It is given that R = {(a, b): a $\leq b^3$}

Now, It can observed that as: $\left(\frac{1}{3}, \frac{1}{3}\right) \notin$ R as $\frac{1}{3} > \left(\frac{1}{3}\right)^3 = \frac{1}{3}$

Therefore, R is not reflexive.

Now, $\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in$ R as $3 < \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$

But, $\left(3, \frac{6}{5}\right) \notin$ R as $3 > \left(\frac{6}{5}\right)^3$

Therefore, R is not transitive.

Therefore, R is neither reflexive, nor symmetric, nor transitive.

Q. 6

Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

Answer:

Let us take A = {1, 2, 3}

A relation R is defined on set A as R = {(1,2), (2,1)}

It is seen that (1,1), (2,2), (3,3) $\notin$ R.

Therefore, R is not reflexive.

Now, we can see that (1,2) $\in$ R and (2,1) $\in$ R

Therefore, R is symmetric.

And now, $(1,2), (2,1) \in R$

But, $(1,1) \notin R$

Therefore, R is not transitive.

Therefore, R is symmetric but neither reflexive, nor transitive.

Q. 7

Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y): x and y have same number of pages} is an equivalence relation.

Answer:

It is given that the set A of all the books in a library of a college. Then,

R = {(x, y): x and y have same number of pages}

Now, R is reflexive

Since, $(x, x) \in R$ as x and x have same number of pages.

Let $(x, x) \in R$

$\Rightarrow$ x and y have the same number of pages

$\Rightarrow$ y and x have the same number of pages.

$\Rightarrow (y, x) \in R$

Therefore, R is symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow$ x and y have the same number of pages and y and z have the same number of pages.

$\Rightarrow$ x and z have the same number of pages.

$\Rightarrow (x, z) \in R$

Therefore, R is transitive.

Therefore, R is an equivalence relation.

Q. 8

Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b): |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

Answer:

It is given that the relation R in the set A = {1, 2, 3, 4, 5}

given by R = {(a, b): |a – b| is even},

It is clear that for any element a ϵ A, we have |a – a|= 0(which is even)

Therefore, R is reflexive.

Now, Let (a, b) ϵ R

⇒ |a – b| is even

⇒ |- (a – b) | = |b - a| is also even.

⇒ (b, a) ϵ R

Therefore, R is symmetric.

Now, Let (a, b) ϵ R and (b, c) ϵ R.

⇒ |a – b| is even and |b – c| is even

⇒ (a – b) is even and (b – c) is even

⇒ (a – c) = (a – b) + (b – c) is even

⇒ |a – c| is even

⇒ (a, c) ϵ R

Therefore, R is transitive.

Therefore, R is an equivalence relation.

Now, all elements of the set {1,3,5} are related to each other as all the elements of this subset are odd.

Therefore, the modules of the difference between any two elements will be even.

Similarly, all elements of the sets {2,4} are related each other as all the elements of this subset are even.

Also, no element of the subset {1, 3, 5} can be related to any element of {2, 4} as all elements of {1, 3, 5} are odd and all elements of {2,4} are even.

Therefore, the modulus of the difference between the two elements will not be even.

Q. 9A

Show that each of the relation R in the set $A = \{x \in Z: 0 \leq x \leq 12\}$, given by

$R = \{(a, b): |a - b| \text{ is a multiple of } 4\}$

is an equivalence relation. Find the set of all elements related to 1 in each case.

Answer:

It is given that the relation R in the set $A = \{x \in Z: 0 \leq x \leq 12\}$, given by

$R = \{(a, b): |a - b| \text{ is a multiple of } 4\}$

For any element $a \in A$, we have $(a, a) \in R$ as $|a-a|=0$ is a multiple of 4.

Therefore, R is reflexive.

Now, Let $(a, a) \in R$

$\Rightarrow |a - b|$ is a multiple of 4

⇒ $|b - a| = |a - b|$ is a multiple of 4

⇒ $(b, a) \in R$

Therefore, R is symmetric.

Now, Let $(a, b), (b, c) \in R$

⇒ $|a - b|$ is a multiple of 4 and $|b - c|$ is a multiple of 4

⇒ $|a - c| = |(a - b) + (b - c)|$ is a multiple of 4

⇒ $(a, c) \in R$

Therefore, R is transitive.

Therefore, R is an equivalence relation.

The set of elements related to 1 is $\{1,5,9\}$

$|1-1| = 0$ is multiple of 4

$|5-1| = 4$ is multiple of 4

$|9-1| = 8$ is multiple of 4.

Q. 9B

Show that each of the relation R in the set $A = \{x \in Z: 0 \leq x \leq 12\}$, given by

$R = \{(a, b): a = b\}$

is an equivalence relation. Find the set of all elements related to 1 in each case.

Answer:

It is given that the relation R in the set $A = \{x \in Z: 0 \leq x \leq 12\}$, given by

$R = \{(a, b): a = b\}$

For any element $a \in A$, we have $(a, a) \in R$ as $a = a$.

Therefore, R is reflexive.

Now, Let (a, a) $\in$ R

$\Rightarrow$ a = b

$\Rightarrow$ b = a

$\Rightarrow$ (b, a) $\in$ R

Therefore, R is symmetric.

Now, Let (a, b), (b, c) $\in$ R

$\Rightarrow$ a = b and b = c

$\Rightarrow$ a = c

$\Rightarrow$ (a, c) $\in$ R

Therefore, R is transitive.

Therefore, R is an equivalence relation.

The set of elements related to 1 will be those elements from set A which are equal to 1.

Therefore, the set of elements related to 1 is {1}.

Q. 10A

Give an example of a relation. Which is

Symmetric but neither reflexive nor transitive.

Answer:

Let A = {3,4,5}

Define a relation R on A as R = {(3,4), (4,3)}

Relation R is not reflexive as (3,3), (4,4) and (5,5) $\notin$ R.

Now, as (3,4) $\in$ R and also (4,3) $\in$ R, R is symmetric.

$\Rightarrow (3,4), (4,3) \in R$, but $(3,3) \notin R$

$\Rightarrow$ R is not transitive.

Therefore, relation R is symmetric but not reflexive or transitive.

Q. 10B

Give an example of a relation. Which is

Transitive but neither reflexive nor symmetric.

Answer:

Let a relation R in R defined as:

$R = \{(a, b): a<b\}$

For any $a \in R$, we have $(a, a)) \notin R$ as a cannot be strictly less than a itself.

In fact, $a = a$,

Therefore, R is not reflexive.

Now, $(1,2) \in R$ but $2 > 1$

$\Rightarrow (2,1)) \notin R$.

$\Rightarrow$ R is not symmetric.

Now, let $(a, b), (b, c) \in R$

$\Rightarrow a < b$ and $b < c$

$\Rightarrow a < c$

$\Rightarrow (a, c) \in R$

$\Rightarrow$ R is transitive.

Therefore, relation R is transitive but not reflexive and a symmetric.

Q. 10C

Give an example of a relation. Which is

Reflexive and symmetric but not transitive.

Answer:

Let us take A = {2,4,6}

Define a relation R on A as:

A = {(2,2), (4,4), (6,6), (2,4), (4,2), (4,6), (6,4)}

Relation of R is reflexive as for every a ϵ A,

(a, a) ϵ R

⇒ (2,2), (4,4), (6,6) ϵ R,

Relation R is symmetric as (a, b) ϵ R

⇒ (b, a) ϵ R for all a, b ϵ R

And Relation R is not transitive as (2,4), (4,6) ϵ R,

but (2,6) ∉ R

Therefore, relation R is reflexive and symmetric but not transitive.

Q. 10D

Give an example of a relation. Which is Reflexive and transitive but not symmetric.

Answer:

Let us define a relation R in R as

R = {(a, b): $a^3 \geq b^3$}

It is clear that (a, a) ϵ R as $a^3 = a^3$

⇒ R is reflexive.

Now, $(2,1) \in R$

But $(1,2) \notin R$

$\Rightarrow$ R is not symmetric.

Now, let $(a, b)\ (b, c) \in R$

$\Rightarrow a^3 \geq b^3$ and $b^3 \geq c^3$

$\Rightarrow a3 \geq c3$

$\Rightarrow (a, c) \in R$

$\Rightarrow$ R is transitive.

Therefore, relation R is reflexive and transitive but not symmetric.

Q. 10E

Give an example of a relation. Which is Symmetric and transitive but not reflexive.

Answer:

Let $A = \{-7, -8\}$

Define a relation R on A as:

$R = \{(-7, -8), (-8, -7), (-7, -7)\}$

Relation R is not reflexive as $(-8, -8) \notin R$

Relation R is symmetric as $(-7, -8) \in R$ and $(-8, -7) \in R$

But it is seen that $(-7, -8), (-8, -7) \in R$.

Also, $(-7, -7) \in R$.

$\Rightarrow$ R is transitive.

Therefore, relation R is symmetric and transitive but not reflexive.

Q. 11

Show that the relation R in the set A of points in a plane given by

R = {(P, Q): distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point P ≠ (0, 0) is the circle passing through P with origin as center.

Answer:

It is given that

R = {(P, Q): distance of the point P from the origin is same as the distance of the point Q from the origin},

Now, it is clear that

(P, P) ϵ R since the distance of point P from origin is always the same as the distance of the same point P from the origin.

Therefore, R is reflexive.

Now, let us take (P, Q) ϵ R,

⇒ The distance of point P from origin is always the same as the distance of the same point Q from the origin.

⇒ The distance of point Q from origin is always the same as the distance of the same point P from the origin.

⇒ (Q, P) ϵ R

Therefore, R is symmetric.

Now, Let (P, Q), (Q, S) ϵ R

⇒ The distance of point P and Q from origin is always the same as the distance of the same point Q and S from the origin.

⇒ The distance of points P and S from the origin is the same.

⇒ (P, S) ϵ R

Therefore, R is transitive.

Therefore, R is equivalence relation.

The set of all points related to $P \neq (0,0)$ will be those points whose distance from the origin is the same as the distance of point P from the origin.

So, if O (0,0) is the origin and OP = k, then the set of all points related to P is at a distance of k from the origin.

Therefore, this set of points forms a circle with the center as the origin and this circle passes through point P.

Q. 12

Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2)$: T_1 is similar to $T_2\}$, is equivalence relation. Consider three right angle triangles T1 with sides 3, 4, 5, T_2 with sides 5, T_2, 1^3 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Answer:

It is given that the relation R defined in the set A of all triangles as

$R = \{(T_1, T_2)$: T_1 is similar to $T_2\}$,

Now, R is reflexive as every triangle is similar to itself.

Now, if $(T_1, T_2) \in R$, then T_1 is similar to T_2.

$\Rightarrow T_2$ is similar to T_1.

$\Rightarrow (T_1, T_2) \in R$

Therefore, R is symmetric.

Now, if $(T_1, T_2), (T_2, T_3) \in R$,

$\Rightarrow T_1$ is similar to T_2 and T_2 is similar to T_3.

$\Rightarrow T_1$ is similar to T_3.

⇒ $(T_1, T_3) \in R$

Therefore, R is transitive.

Therefore, R is equivalence relation.

Now, we can see that,

$\frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$

Therefore, the corresponding sides of triangles T_1 and T_3 are in the same ratio.

Thus, triangle T_1 is similar to triangle T_3.

Therefore, T_1 is related to T_3.

Q. 13

Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2)$: P_1 and P_2 have same number of sides$\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Answer:

It is given that the relation R defined in the set A of all polygons as

$R = \{(P_1, P_2)$: P_1 and P_2 have same number of sides$\}$,

Then, R is reflexive since $(P_1, P_2) \in R$ as the same polygon has the same number of sides with itself.

Let $(P_1, P_2) \in R$

⇒ P_1 and P_2 have the same number of sides.

⇒ P_2 and P_1 have the same number of sides.

⇒ $(P_2, P_1) \in R$

Therefore, R is symmetric.

Now, let $(P_1, P_2), (P_2, P_3) \in R$

$\Rightarrow P_1$ and P_2 have the same number of sides. Also, P_2 and P_3 have the same number of sides.

$\Rightarrow P_1$ and P_3 have the same number of sides.

$\Rightarrow (P_1, P_3) \in R$

Therefore, R is transitive.

Thus, R is an equivalence relation.

The elements in A related to the right-angled triangle (T) with sides 3, 4 and 5 are those polygons which have 3 sides.

Therefore, the set of all elements in A related to triangle T is the set of all triangles.

Q. 14

Let L be the set of all lines in XY plane and R be the relation in L defined as

$R = \{(L_1, L_2): L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

Answer:

It is given that the relation in L defined as

$R = \{(L_1, L_2): L_1$ is parallel to $L_2\}$

R is reflexive as any line L_1 is parallel to itself

$\Rightarrow (L_1, L_2) \in R$

Now, Let $(L_1, L_2) \in R$

$\Rightarrow L_1$ is parallel to L_2.

$\Rightarrow L_2$ is parallel to L_1.

$\Rightarrow (L_2, L_1) \in R$

Therefore, R is symmetric.

Now, Let $(L_1, L_2), (L_2, L_3) \in R$

$\Rightarrow L_1$ is parallel to L_2. Also, L_2 is parallel to L_3.

$\Rightarrow L_1$ is parallel to L_3.

$\Rightarrow (L_1, L_3) \in R$

Therefore, R is transitive.

Therefore, R is an equivalence relation.

The set of all lines related to the line $y = 2x + 4$ is the set of all lines that are parallel to the line

$y = 2x + 4$

Slope of line $y = 2x + 4$ is $m = 2$

We know that parallel lines have the same slopes.

The line parallel to the given line is of the form $y = 2x + c$ where, $c \in R$.

Therefore, the set of all lines related to the given line by $y = 2x + c$, where $c \in R$.

Q. 15

Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

A. R is reflexive and symmetric but not transitive.

B. R is reflexive and transitive but not symmetric.

C. R is symmetric and transitive but not reflexive.

D. R is an equivalence relation.

A. R is reflexive and symmetric but not transitive.

B. R is reflexive and transitive but not symmetric.

C. R is symmetric and transitive but not reflexive.

D. R is an equivalence relation.

Answer:

It is given that the relation in the set {1, 2, 3, 4} given by

R = {(1, 2), (2, 2), (1, 1), (4,4), (1, 3), (3, 3), (3, 2)}

Check symmetric:

As (1,1), (2,2), (3,3), (4,4) ϵ R

It is seen that (a, a) ϵ R, for every a ϵ {1,2,3,4}

Therefore, R is reflexive.

Check symmetric:

We can see that (1, 2) ϵ R, but (2, 1) ∉R.

Therefore, R is not symmetric.

Check transitive:

(a, b), (b, c) ϵ R

⇒ (a, c) ϵ R

here (1,3) ϵ R, (3,2) ϵ R and (1,2) ϵ R

Therefore, R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

Q. 16

Let R be the relation in the set N given by R = {(a, b): a = b – 2, b > 6}.
Choose the correct answer.

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $(8, 7) \in R$

Answer:

It is given that the relation in the set N given by

$R = \{(a, b): a = b - 2, b > 6\}$

Now, $b > 6$, $(2, 4) \notin R$

Also, as $3 \neq 8 - 2$, $(3, 8) \notin R$

And $8 \neq 7 - 2$

Therefore, $(8,7) \notin R$

Now, consider $(6,8)$

We have $8 > 6$ and also $6 = 8 - 2$

Therefore, $(6, 8) \in R$.

Exercise 1.2

Q. 1

Show that the function f: R* → R* defined by f (x) = $\frac{1}{2}$ is one-one and onto, where R* is the set of all non-zero real numbers. Is the result true, if the domain R* is replaced by N with co-domain being same as R*?

Answer:

It is given that f: R* → R* defined by f (x) = $\frac{1}{2}$

check for one-one:

For a function to be one-one, if f(x) = f(y) then x = y. f(x) = f(y)

$= \frac{1}{x} = \frac{1}{y}$

⇒Therefore, f is one – one.

We can see that y ϵ R, there exists x = $\frac{1}{y}$ ∈ R, such that

$= f(x) = \frac{1}{\left(\frac{1}{y}\right)} = y$

⇒ f is onto.

Therefore, function f is one-one and onto.

Now, let us consider g: N → R* defined by

$= g(x) = \frac{1}{x}$

Then, we get

$\frac{1}{x_1} = \frac{1}{x_2}$

⇒ $x_1 = x_2$

$\Rightarrow$ g is one–one.

It can be observed that g is not onto as for 1.2 ϵ R there does not exist any x in N such that

$= g(x) = \frac{1}{1..2}$

Therefore, function g is one –one but not onto.

Q. 2 A

Check the infectivity and surjectivity of the following functions:

f: N → N given by $f(x) = x^2$

Answer:

It is given that f: N → N given by $f(x) = x^2$

We can see that for x, y ϵ N,

$f(x) = f(y)$

$\Rightarrow x^2 = y^2$

$\Rightarrow x = y$

$\Rightarrow$ f is injective.

Now, let 2 ϵ N. But, we can see that there does not exists any x in N such that

$f(x) = x^2 = 2$

$\Rightarrow$ f is not surjective.

Therefore, function f is injective but not surjective

Q. 2 B

Check the injectivity and surjectivity of the following functions:

f: Z → Z given by f (x) = x2

Answer:

It is given that f: Z → Z given by $f(x) = x^2$

We can see that f (-1) = f (1) = 1, but -1 ≠ 1

⇒ f is not injective.

Now, let -2 ϵ Z. But, we can see that there does not exists any x in Z such that

$f(x) = x^2 = -2$

⇒ f is not surjective.

Therefore, function f is neither injective nor surjective.

Q. 2 C

Check the injectivity and surjectivity of the following functions:

f: R → R given by $f(x) = x^2$

Answer:

It is given that f: R → R given by $f(x) = x^2$

We can see that f (-1) = f (1) = 1, but -1 ≠ 1

⇒ f is not injective.

Now, let -2 ϵ R. But, we can see that there does not exists any x in R such that

$f(x) = x^2 = -2$

⇒ f is not surjective.

Therefore, function f is neither injective nor surjective.

Q. 2 D

Check the injectivity and surjectivity of the following functions:

f: N → N given by f (x) = x^3

Answer:

It is given that f: N → N given by f (x) = x^3

We can see that for x, y ϵ N,

f(x) = f(y)

⇒ $x^3 = y^3$

⇒ x = y

⇒ f is injective.

Now, let 2 ϵ N. But, we can see that there does not exists any x in N such that

f(x) = x^3 = 2

⇒ f is not surjective.

Therefore, function f is injective but not surjective.

Q. 2 E

Check the injectivity and surjectivity of the following functions:

f: Z → Z given by f (x) = x^3

Answer:

It is given that f: Z → Z given by f (x) = x^3

We can see that for x, y ϵ N,

f(x) = f(y)

⇒ $x^3 = y^3$

⇒ x = y

⇒ f is injective.

Now, let 2 ϵ Z. But, we can see that there does not exists any x in Z such that

$f(x) = x^3 = 2$

⇒ f is not surjective.

Therefore, function f is injective but not surjective.

Q. 3

Prove that the Greatest Integer Function f: R → R, given by f (x) = [x], is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

Answer:

It is given f: R → R, given by f (x) = [x]

We can see that f (1.2) = [1.2] = 1

f (1.9) = [1.9] = 1

⇒ f (1.2) = f (1.9), but 1.2 ≠ 1.9.

⇒ f is not one- one.

Now, let us consider 0.6 ϵ R.

We know that f(x) = [x] is always an integer.

⇒ there does not exist any element x ϵ R such that f(x) = 0.6

⇒ f is not onto.

Therefore, the greatest integer function is neither one-one nor onto.

Q. 4

Show that the Modulus Function f: R → R, given by f (x) = |x|, is neither one-one nor onto, where |x| is x, if x is positive or 0 and |x| is – x, if x is negative.

Answer:

It is given that f: R → R, given by f (x) = | x|

We can see that f (-1) = |-1| = 1, f (1) = |1| = 1

⇒ f (-1) = f (1), but -1 ≠ 1.

⇒ f is not one-one.

Now, we consider -1 ϵ R.

We know that f(x) = |x| is always positive

Therefore, there doesn't exist any element x in domain R such that f(x) = |x| = -1

⇒ f is not onto.

Therefore, modulus function is neither one-one nor onto.

Q. 5

Show that the Signum Function f: R → R, given by

$$f(x) = \begin{cases} 1, & i\,fx > 0 \\ 0, & i\,fx = 0 \\ -1, & i\,fx < 0 \end{cases}$$

is neither one-one nor onto.

Answer:

It is given that f: R → R, given by

$$f(x) = \begin{cases} 1, & i\,fx > 0 \\ 0, & i\,fx = 0 \\ -1, & i\,fx < 0 \end{cases}$$

We can have observed that f (1) = f (2) = but 1 ≠ 2.

Thus, f is not one – one.

Now, as f (x) takes only 3 values (1, 0, -1) for the element -2 in co-domain R, there exists any x in domain R such that f(x) = -2

Thus, f is not onto.

Therefore, the Signum function is neither one-one nor onto.

Q. 6

Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.

Answer:

It is given that A = {1, 2, 3}, B = {4, 5, 6, 7}

f: A → B is defined as f = {(1, 4), (2, 5), (3, 6)}

Therefore, f (1) = 4, f (2) = 5, f (3) = 6We can see that the images of distinct elements of A under f are distinct.

Therefore, function f is one- one.

Q. 7 A

In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

f: R → R defined by f (x) = 3 – 4x

Answer:

It is given that f: R → R defined by f (x) = 3 – 4x

Let $x_1, x_2 \epsilon$ R such that $f(x_1) = f(x_2)$

$\Rightarrow 3 - 4x_1 = 3 - 4x_2$

$\Rightarrow -4x_1 = -4x_2$

$\Rightarrow x_1 = x_2$

⇒ f is one- one

For any real number (y) in R, there exist $\frac{3-y}{4}$ in R such that

$f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right)$

⇒ f is onto.

Therefore, function f is bijective.

Q. 7 B

In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

f: R → R defined by f (x) = 1 + x_2

Answer:

It is given that f: R → R defined by f (x) = 1 + x_2

Let x_1, $x_2 \epsilon$ R such that $f(x_1) = f(x_2)$

$= 1 + x_1^2 = 1 + x_2^2$

$= x_1^2 = x_1^2$

$= x_1 = \pm x_2$

Now, f (1) = f (-1) = 2

⇒ f (x1) = f (x2) which does means that x1 = x2

⇒ f is not one – one

Now consider an element -2 in co- domain R.

We can see that f(x) = 1 + x_2 is always positive.

⇒ there does not exist any x in domain R such that f(x) = -2

⇒ F is not onto.

Therefore, function f is neither one-one nor onto.

Q. 8 Let A and B be sets. Show that f: A × B → B × A such that f (a, b) = (b, a) is bijective function.

Answer:

It is given that f: A × B → B × A is defined as f (a, b) = (b, a)

Now let us consider $(a_1, b_1), (a_2, b_2) \in A \times B$

Such that $f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (b_1, a_1) = (b_2, a_2)$

$\Rightarrow b_1 = b_2$ and $a_1 = a_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

⇒ f is one-one.

Now, let (b, a) ϵ B × A be any element.

Then, there exists (a, b) ϵ A × B such that f (a, b) = (b, a)

⇒ f is onto.

Therefore, f is bijective.

Q. 9

Let f: N → N be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, i\ fn\ i\ sodd \\ \qquad for\ al\ ln\ \in N \\ \frac{n}{2}, i\ fn\ i\ seven \end{cases}$$

State whether the function f is bijective. Justify your answer.

Answer:

It is given that

f: N → N be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, i\ fn\ i\ sodd \\ for\ al\ ln\ \in N \\ \frac{n}{2}, i\ fn\ i\ seven \end{cases}$$

We can have observed that:

$f(1) = \frac{1+1}{2} = 1$ and $f(2) = \frac{2}{2} = 1$ (by using the definition of f)

Thus, f (1) = f (2), where $1 \neq 2$.

Therefore, f is not one-one.

Now, let us consider a natural number (n) in co domain N.

Case I: When n is odd.

Then, n = 2r +1 for some r ϵ N.

⇒ there exist 4r + 1 ϵ N such that $f(4r+1) = \frac{4r+1+1}{2} = 2r + 1$

Case II: When n is even.

Then, n = 2r for some r ϵ N.

⇒ there exist 4r ϵ N such that $f(4r) = \frac{4r}{2} = 2r$

Therefore, f is onto.

⇒ Function f is not one-one but it is onto.

Thus, Function f is not bijective function.

Q. 10

Let A = R – {3} and B = R – {1}. Consider the function f: A → B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Answer:

It is given that A = R – {3} and B = R – {1} f: A → B defined by

f (x) = $\left(\frac{x-2}{x-3}\right)$

Now, let x, y ϵ A such that f(x) = f(y)

⇒ $\frac{x-2}{x-3} = \frac{y-2}{y-3}$

⇒ (x - 2) (y - 3) = (y - 2) (x - 3)

⇒ xy – 3x - 2y + 6 = xy -3y - 2x +6

⇒ -3x - 2y = -3y - 2x

⇒ x = y

⇒ f is one –one.

Let y ϵ B = R – {1}

Then, y ≠ 1.

The function f is onto if there exist x ϵ A such that f(x) = y

Now, f(x) = y

$\frac{x-2}{x-3}$ = y

⇒ x – 2 = xy - 3y

⇒ x(1-y) = -3y + 2

⇒ x = $\frac{2-3y}{1-y} \in$ A

⇒ y ϵ B, there exists $\frac{2-3y}{1-y}$ ϵ A such that

$$\Rightarrow \left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)}{\left(\frac{2-3y}{1-y}\right)} = \frac{2-3y-3+2y}{2-3y-3+3y} = \frac{-y}{-1}$$

⇒ f is onto.

Therefore, function f is one- one and onto.

Q. 11

Let f: R → R be defined as $f(x) = x^4$. Choose the correct answer.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto.

Answer:

f: R → R be defined as $f(x) = x^4$.

Let x, y ϵ R such that f(x) = f(y)

$\Rightarrow x^4 = y^4$

$\Rightarrow x = y$

Therefore, $f(x_1) = f(x_2)$ which does not implies $x_1 = x_2$.

For instance, f (1) = f (-1) = 1

Therefore, f is not one-one.

Now, an element 2 in co-domain R.

We can see that there does not exist any x in domain R such that

f (x) = 2

Therefore, f is not onto.

Therefore, function f is neither one-one nor onto.

Q. 12

Let f: R → R be defined as f (x) = 3x. Choose the correct answer.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto.

Answer:

It is given that f: R → R be defined as f (x) = 3x.

Let x, y ϵ R such that f(x) = f(y).

⇒ 3x = 3y

⇒ x = y

⇒ f is one–one.

Also, for any real number (y) in co–domain R, there exists $\frac{y}{3}$ in R such that

$f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right)$

Therefore, f is onto.

Therefore, function f is one-one and onto.

Exercise 1.3

Q. 1

Let f: $\{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and g: $\{1, 2, 5\} \rightarrow \{1, 3\}$ be given by

$f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down g of.

Answer:

It is given that f: $\{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and g: $\{1, 2, 5\} \rightarrow \{1, 3\}$ be given by

$f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Then, g of (1) = g (f (1)) = g (2) = 3 g of (3)

= g (f (3)) = g (5) = 1 gof (4) = g (f (4)) = g (1) = 3

Therefore, gof = $\{(1,3), (3,1), (4,3)\}$

Q. 2

Let f, g and h be functions from R to R. Show that

(f + g) oh = foh + goh

(f. g) oh = (foh). (goh)

Answer:

(i) (f + g) oh = foh + goh

Let us consider ((f + g) oh) (x) = (f + g) (h(x))

= f(h(x)) + g(h(x))

= (foh)(x) + (goh)(x)

= {(foh) + (goh)} (x)

Then, ((f + g) oh) (x) = {(foh) +(goh)} (x) $\forall$ x $\in$ R

Therefore, (f + g) oh = foh + goh.

(ii) (f. g) oh = (foh). (goh)

Let us consider $((f. g) oh) (x) = (f. g) (h(x))$

$= f(h(x)). g(h(x))$

$= f (h(x)). g(h(x))$

$= (fog)(x). (goh)(x)$

$= \{(fog). (goh)\} (x)$

Then, $((f. g) oh) (x) = \{(fog). (goh)\} (x) \forall x \in R$

Therefore, $(f. g) oh = (fog). (goh)$

Q. 3 A

Find gof and fog, if

(i) $f (x) = | x |$ and $g(x) = | 5x – 2 |$

Answer:

(i) $f(x) = |x|$ and $g(x) = |5x -2|$

Then, $(gof)(x) = g (f(x)) = g(|x|) = |5|x|-2|$

$(fog)(x) = f(g(x)) = f (|5x -2 |) = ||5x -2 || = |5x -2 |$

Q. 3 B

Find gof and fog, if

$f (x) = 8x^3$ and $g (x) = x^{\frac{1}{3}}$

Answer:

(ii) $f (x) = 8x^3$ and $g (x) = x^{\frac{1}{3}}$

Then, $(gof) (x) = g (f(x)) = g(8x3) = (8x^3)^{\frac{1}{3}} = 2$

$(fog)(x) = f(g(x)) = f\left(x^{\frac{1}{3}}\right) = 8\left(x^{\frac{1}{3}}\right)^3 = 8x$

Q. 4

If f (x) = $\frac{(4x+3)}{(6x-4)}$, $x \neq \frac{2}{3}$, show that fof(x) = x, for all x $\neq \frac{2}{3}$. What is the inverse of f?

Answer:

It is given that f (x) = $\frac{(4x+3)}{(6x-4)}$,x $\neq \frac{2}{3}$,

(fof) (x) = f (f (x)) = f $\frac{(4x+3)}{(6x-4)} = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = \frac{16x+12+18x-12}{24x-18-24x+16} = \frac{34x}{34}$ = x

Therefore, fof(x) = x, for all x $\neq \frac{2}{3}$

= > fof = 1

Therefore, the given function f is invertible and the inverse of f itself.

Q. 5 A

State with reason whether following functions have inverse

f: {1, 2, 3, 4} → {10} with

f = {(1, 10), (2, 10), (3, 10), (4, 10)}

Answer:

It is given that f: {1, 2, 3, 4} → {10} with

f = {(1, 10), (2, 10), (3, 10), (4, 10)}

From the given definition of f,

We can see that f is a many one function as:

F (1) = f (2) = f (3) = f (4) = 10

therefore, f is not one-one.

Therefore, function f does not have an inverse.

Q. 5 B

State with reason whether following functions have inverse

g: {5, 6, 7, 8} → {1, 2, 3, 4} with

g = {(5, 4), (6, 3), (7, 4), (8, 2)}

Answer:

It is given that: {5, 6, 7, 8} → {1, 2, 3, 4} with

g = {(5, 4), (6, 3), (7, 4), (8, 2)}

Necessary Condition for a function to have inverse: Function should be one-one and onto.

From the given definition,

We can see that f is a many one function as:

G (5) = g (7) = 4

[As at two points function have same values, the function is not one-one]

i.e. g is not one- one.

Therefore, function g does not have an inverse.

Q. 5 C

State with reason whether following functions have inverse

h: {2, 3, 4, 5} → {7, 9, 11, 13} with

h = {(2, 7), (3, 9), (4, 11), (5, 13)}

Answer:

It is given that h: {2, 3, 4, 5} → {7, 9, 11, 13} with

h = {(2, 7), (3, 9), (4, 11), (5, 13)}

We can see that all distinct elements of the set (2, 3, 4, 5) have distinct images under h.

⇒ h is one–one.

Also, h is onto since for every element of the set {7, 9, 11, 13}, there exists an element x in the set {2, 3, 4, 5} such that h(x) = y.

Therefore, h is a one–one and onto function.

Therefore, h has an inverse.

Q. 6

Show that f: [–1, 1] → R, given by f (x) = $\frac{x}{x+2}$ is one-one. Find the inverse of the function f: [–1, 1] → Range f.

(Hint: For y ∈ Range f, y = f (x) = $\frac{x}{x+2}$, for some x in [–1, 1], i.e., x = $\frac{2y}{(1-y)}$)

Answer:

It is given that f: [–1, 1] → R, given by f (x) = $\frac{x}{x+2}$

Now, Let f(x) = f(y)

$\Rightarrow \frac{x}{x+2} = \frac{y}{y+2}$

⇒ xy + 2x = xy +2y

⇒ 2x = 2y

⇒ x = y

⇒ f is a one- one function.

Now, Let y = $\frac{x}{x+2}$, xy = x + 2y so x = $\frac{y}{1-y}$

So, for every y in the range there exists x in the domain such that f(x) = y

⇒ f is onto function.

⇒ f: [-1,1] → Range f is one-one and onto

⇒ the inverse of the function: f: [-1, 1] → Range f exists.

Let g: Range f → [-1, 1]be the inverse of range f.

Let y be an arbitrary element of range f.

Since, f: [-1, 1] → Range f is onto, we get:

y = f(x) for same x ϵ [-1, 1]

⇒ y = $\frac{x}{x+2}$

⇒ xy +2y = x

⇒ x (1 - y) = 2y

⇒ x = $\frac{2y}{1-y}$, y ≠ 1

Now, let us define g: Range f → [-1, 1]

g(y) = $\frac{2y}{1-y}$, y ≠ 1

Now, (gof)(x) = g(f(x)) = g $\left(\frac{x}{x+2}\right) = \frac{2\left(\frac{x}{x+2}\right)}{1-\frac{x}{x+2}} = \frac{2x}{x+2-x} = \frac{2x}{2}$

(fog)(y) = f(g(y)) = f $\left(\frac{2y}{1-y}\right) = \frac{\frac{2y}{1-y}}{\frac{2y}{1-y}+2} = \frac{2y}{2y+2-2y} = \frac{2y}{2}$

Thus, gof = I [-1,1] and fog = $I_{\text{Range f}}$

⇒ f-1 = g

Therefore, f-1(y) = $\frac{2y}{1-y}$, y ≠ 1

Q. 7

Consider f: R → R given by f (x) = 4x + 3. Show that f is invertible. Find the inverse of f.

Answer:

It is given that f: R → R given by f (x) = 4x + 3

Let f(x) = f(y)

⇒ 4x +3 = 4y +3

⇒ 4x = 4y

⇒ x = y

⇒ f is one- one function.

Now, for y ϵ R, Let y = 4x +3

= x = $\frac{y-3}{4}$ ∈ R

⇒ for any y ϵ R, there exists x = $\frac{Y-3}{4}$ ϵ R

such that, f(x) = f $\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3$ = y

⇒ F is onto function.

Since, f is one –one and onto

⇒ f-1 exists.

Let us define g: R → R by g(x) = $\frac{x-3}{4}$

Now, (gof)(x) = g(f(x)) = g (4x + 3) = $\frac{(4x+3)-3}{4}$ = x

(fog)(y) = f(g(xy)) = f $\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3$ = y – 3 + 3 = y

Therefore, gof = fog = IR

Therefore, f is invertible and the inverse of f is given by

f^{-1} (y) = g(y) = $\frac{y-3}{4}$.

Q. 8

Consider f: R+→ [4, ∞) given by f (x) = x2 + 4. Show that f is invertible with the inverse f^{-1} of f given by f-1 = $\sqrt{y-4}$, where R+ is the set of all non-negative real numbers.

Answer:

It is given that f: R+→ [4, ∞) given by f (x) = $x^2 + 4$.

Now, Let f(x) = f(y)

⇒ $x^2 + 4 = y^2 + 4$

⇒ $x^2 = y^2$

⇒ x = y

⇒ f is one-one function.

Now, for y ϵ [4, ∞), let y = $x^2 + 4$.

⇒ $x^2 = y - 4 \geq 0$

= x = $\sqrt{y-4} \geq 0$

⇒ for any y ϵ R, there exists x = $\sqrt{y-4}$ ϵ R such that

= f (x) = $f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2$ y -4 + 4 = y.

⇒ f is onto function.

Therefore, f is one–one and onto function, so f-1 exists.

Now, let us define g: [4, ∞) → R+ by,

g(y) = $\sqrt{y-4}$

Now, gof(x) = g(f(x)) = g ($x^2 + 4$) = $\sqrt{(x^2+4)+4} = \sqrt{x^2}$ = x

And, fog(y) = f(g(y)) = $f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4$ = (y – 4) + 4 = y

Therefore, gof = gof = IR.

Therefore, f is invertible and the inverse of f is given by

f-1(y) = g(y) = $\sqrt{y-4}$

Q. 9

Consider f: R+ → [–5, ∞) given by f(x) = 9x^2 + 6x – 5. Show that f is invertible with f^{-1} (y) = $\left\{\frac{(\sqrt{y+6})-1}{3}\right\}$

Answer:

It is given that f: R+ → [–5, ∞) given by f(x) = 9x^2 + 6x – 5.

Let y be any element of [-5, ∞)

Now, let y = 9x^2 + 6x – 5

⇒ y = (3x+1)^2 -1-5 = (3x+1)^2 – 6

⇒ 3x + 1 = $\sqrt{y+6}$

= x = $\frac{\sqrt{y+6}-1}{3}$

⇒ f is onto and its range is f = [-5, ∞)

Now, let us define g: [-5, ∞) → R+ as g(y) = $\frac{\sqrt{y+6}-1}{3}$

Now, we have:

(gof)(x) = g(f(x)) = g (9x^2 + 6x – 5)

= g((3x+1)2 – 6)

= $\frac{\sqrt{(3x+1)^2-6+6}-1}{3}$

= $\frac{3x+1-1}{3}$ = x

And, (fog)(y) = f(g(y)) = f $\left(\frac{\sqrt{y+6}-1}{3}\right)$

$= \left[3\left(\frac{\sqrt{y+6}-1}{3}\right)+1\right]^2 - 6$

$= \left(\sqrt{y+6}\right)^2 - 6 = y + 6 - 6 = y$

Thus, gof = IR and fog = I (-5, ∞)

Therefore, f is invertible and the inverse of f is given by

$f^{-1}(y) = g(y) = \frac{\sqrt{y+6}-1}{3}$

Q. 10

Let f: X → Y be an invertible function. Show that f has unique inverse.

(Hint: suppose g_1 and g_2 are two inverses of f. Then for all y ∈ Y,

$fog_1(y) = 1_Y(y) = fog_2(y)$. Use one-one ness of f).

Answer:

It is given that f: X → Y be an invertible function.

Also, suppose f has two inverse

Then, for all y ϵ Y, we get:

$fog_1(y) = I_1(y) = fog_2(y)$

$\Rightarrow f(g_1(y)) = f(g_2(y))$

$\Rightarrow g_1(y) = g_2(y)$

$\Rightarrow g_1 = g_2$

Therefore, f has a unique inverse.

Q. 11

Consider f: {1, 2, 3} → {a, b, c} given by f (1) = a, f (2) = b and f (3) = c. Find f^{-1} and show that $(f^{-1})^{-1} = f$.

Answer:

It is given that f: $\{1, 2, 3\} \to \{a, b, c\}$ given by

$f(1) = a$, $f(2) = b$ and $f(3) = c$

So, if we define g:

$\{a, b, c\} \to \{1, 2, 3\}$ as

$g(a) = 1$, $g(b) = 2$, $g(c) = 3$, then we get:

$(fog)(a) = f(g(a)) = f(1) = a$

$(fog)(b) = f(g(b)) = f(2) = b$

$(fog)(c) = f(g(c)) = f(3) = c$

And

$(gof)(1) = g((1)) = g(a) = 1$

$(gof)(2) = g(f(2)) = g(b) = 2$

$(gof)(3) = g(f(3)) = g(c) = 3$

Therefore, gof = IX and fog = IY, where $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$

Thus, the inverse of f exists and f-1 = g.

Then, f^{-1}: $\{a, b, c\} \to \{1, 2, 3\}$ is given by

$f^{-1}(a) = 1$, $f^{-1}(b) = 2$, $f^{-1}(c) = 3$

Let us now find the inverse of f^{-1},

So, if we define h: $\{1, 2, 3\} \to \{a, b, c\}$ as

$h(1) = a$, $h(2) = b$, $h(3) = c$, then we get:

$(goh)(1) = g(h(1)) = g(a) = 1$

$(goh)(2) = g(h(2)) = g(b) = 2$

$(goh)(3) = g(h(3)) = g(c) = 3$

And,

$(hog)(a) = h(g(a)) = h(1) = a$

$(hog)(b) = h(g(b)) = h(2) = b$

$(hog)(c) = h(g(c)) = h(3) = c$

$\Rightarrow goh = I_X$ and $hog = I_Y$, where $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$.

$\Rightarrow$ The inverse of g exists and $g^{-1} = h$

$\Rightarrow (f^{-1})^{-1} = h$

$\Rightarrow h = f$

$\Rightarrow (f^{-1})^{-1} = f$

Q. 12

Let f: X → Y be an invertible function. Show that the inverse of f^{-1} is f, i.e., $(f^{-1})^{-1} = f$.

Answer:

It is given that f: X → Y be an invertible function.

Then, there exists a function g: Y → X such that gof = Ix and fog = Iy.

Then, $f^{-1} = g$.

Now, gof = Ix and fog = Iy

$\Rightarrow f^{-1}of = Ix$ and fof-1 = Iy

Thus, f^{-1}: Y→X is invertible and f is the inverse of f^{-1}.

Therefore, $(f^{-1})^{-1} = f$.

Q. 13

If f: R → R be given by f (x) = $(3 - x^3)^{\frac{1}{3}}$, then fof (x) is

A. $x^{\frac{1}{3}}$

B. x3

C. x

D. $(3 - x^3)$

Answer:

It is given that f: R → R be given by f (x) = $(3-x^3)^{\frac{1}{3}}$,

Then, fof(x) = f(f(x)) = f $\left((3-x^3)^{\frac{1}{3}}\right)$ = $\left[3-\left((3-x^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{3}}$

= 3 $[3-(3-x^3)]^{\frac{1}{3}} = (x^3)^{\frac{1}{3}}$ = x

⇒ fof(x) = x

Q. 14

Let f: R $-\left\{-\frac{4}{3}\right\}$ →R be a function defined as f (x) = $\frac{4x}{3x+4}$. The inverse of f is the map g: Range f → R $-\left\{-\frac{4}{3}\right\}$given by

A. g (y) = $\frac{3y}{3-4y}$

B. g (y) = $\frac{4y}{4-3y}$

C. g (y) = $\frac{3y}{4-3y}$

D. g (y) = $\frac{4y}{3-4y}$

Answer:

It is given that f: R $-\left\{-\frac{4}{3}\right\}$ → R be a function defined as f (x) = $\frac{4x}{3x+4}$.

Let y be any element of Range f.

Then, there exists x ϵ R -$\left\{-\frac{4}{3}\right\}$ such that y = f(x)

$= y = \frac{4x}{3x+4}$

$\Longrightarrow 3xy + 4y = 4x$

$\Longrightarrow x\,(4 - 3y) = 4y$

$\Longrightarrow x = \frac{4y}{4-3y}$

Let us define g: Range $f \rightarrow R - \left\{-\frac{4}{3}\right\}$ as $g(y) = \frac{4y}{4-3y}$

Now, $(gof)(x) = g(f(x)) = g\left(\frac{4y}{4-3y}\right) = \frac{4\left(\frac{4x}{3x+4}\right)}{4-3\left(\frac{4y}{4-3y}\right)} = \frac{16x}{12x+16-12x} = \frac{16x}{16}$

And, $(fog)(y) = f(g(y)) = f\left(\frac{4y}{4-3y}\right) = \frac{4\left(\frac{4y}{4-3y}\right)}{3\left(\frac{4y}{4-3y}\right)+4} = \frac{16y}{12y+16-12y} = \frac{16y}{16} = y$

Therefore, $gof = {}^{I}\,R - \left\{-\frac{4}{3}\right\}$ and $fog = I_{\text{Range } f}$

Thus, g is the inverse of f

Therefore, the inverse of f is the map

: Range $f \rightarrow R - \left\{-\frac{4}{3}\right\}$, which is given by $g(y) = \frac{4y}{4-3y}$

Exercise 1.4

Q. 1 A

Determine whether or not each of the definition of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this.

On Z^+, define $*$ by $a * b = a – b$

Answer:

It is given that On Z+, define $*$ by $a * b = a – b$

It is not a binary operation as the image of (1, 2) under * is

$1 * 2 = 1 – 2 = -1 \notin Z^+$.

Q. 1 B

Determine whether or not each of the definition of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this.

On Z^+, define $*$ by $a * b = ab$

Answer:

It is given that on Z^+, define $*$ by $a * b = ab$

We can see that for each $a, b \in Z^+$, there is a unique element ab in Z^+.

$\Rightarrow$ * carries each pair (a, b) to a unique element $a * b = ab$ in Z^+.

Therefore, * is a binary operation.

Q. 1 C

Determine whether or not each of the definition of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this.

On R, define $*$ by $a * b = ab^2$

Answer:

It is given that On R, define $*$ by $a * b = ab^2$

We can see that for each $a, b \in R$, there is a unique element ab^2 in R.

$\Rightarrow$ $*$ carries each pair (a, b) to a unique element $a * b = ab^2$ in R.

Therefore, $*$ is a binary operation.

Q. 1 D

Determine whether or not each of the definition of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this.

On Z^+, define $*$ by $a * b = |a - b|$

Answer:

It is given On Z+, define $*$ by $a * b = | a - b|$

We can see that for each $a, b \in Z^+$, there is a unique element $|a - b|$ in Z^+.

$\Rightarrow$ $*$ carries each pair (a, b) to a unique element $a * b = |a - b|$ in Z^+.

Therefore, $*$ is a binary operation.

Q. 1 E

Determine whether or not each of the definition of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this.

On Z^+ define $*$ by $a * b = a$

Answer:

It is given that On Z^+ define $*$ by $a * b = a$

We can see that for each $a, b \in Z^+$, there is a unique element a in Z^+.

⇒ * carries each pair (a, b) to a unique element a * b = a in Z^+.

Therefore, * is a binary operation.

Q. 2 A

For each operation ∗ defined below, determine whether ∗ is binary, commutative or associative.

On Z, define a ∗ b = a – b

Answer:

It is given that On Z, define a ∗ b = a – b

a –b ϵ Z. so the operation * is binary.

We can see that 1 * 2 = 1-2 = -1 and 2 * 1 = 2-1 = 1.

⇒ 1 *2 ≠ 2 * 1, where 1,2 ϵ Z.

⇒ the operation * is not commutative.

Q. 2 B

For each operation ∗ defined below, determine whether ∗ is binary, commutative or associative.

On Q, define a ∗ b = ab + 1

Answer:

It is given that On Q, define a ∗ b = ab + 1

ab + 1 ϵ Q, so operation * is binary

We know that ab = ba for a, b ϵ Q

⇒ ab + 1 = ba + 1for a, b ϵ Q

⇒ a * b = a * b for a, b ϵ Q

⇒ 1 *2 ≠ 2 * 1, where 1,2 ϵ Z.

⇒ The operation * is commutative.

Also, we get,

$(1 * 2) * 3 = (1 \times 2) * 3 = 3 * 3 = 3 \times 3 + 1 = 10$

$1 * (2 * 3) = 1 * (2 \times 3) = 1 * 7 = 1 \times 7 + 1 = 8$

$\Rightarrow (1 * 2) * 3 \neq 1 * (2 * 3)$

⇒ the operation * is not associative.

Q. 2 C

For each operation $*$ defined below, determine whether $*$ is binary, commutative or associative.

On Z^+, define $a * b = 2ab$

Answer:

It is given that on Z^+, define $a * b = 2ab$

$2ab \in Z^+$, so operation * is binary

We know that $ab = ba$ for $a, b \in Z^+$

$\Rightarrow 2ab = 2ba$ for $a, b \in Z^+$

$\Rightarrow a * b = a * b$ for $a, b \in Z^+$

⇒ The operation * is commutative.

Also, we get,

$(1 * 2) * 3 = 2(1\times2) *3 = 4 * 3 = 2(4\times3) = 212$

$1 * (2 * 3) = 1 * 2(2 \times 3) = 1 * 26 = 1 \times 64 =264$

$\Rightarrow (1 * 2) * 3 \neq 1 * (2 * 3)$

⇒ The operation * is not associative.

Q. 2 D

For each operation ∗ defined below, determine whether ∗ is binary, commutative or associative.

On Z^+, define a ∗ b = a^b

Answer:

It is given that On Z^+, define a ∗ b = a^b

abϵ Z^+, so operation * is binary

We know that ab = ba for a, b ϵ Z^+

⇒ 1 * 2 =1^2 and 2 * 1 = 2^1 =2

⇒ 1 * 2 ≠ 2 *1, where 1,2 ϵ Z^+

⇒ The operation * is not commutative.

Also, we get,

(1 * 2) * 3 = 2^3 *3 = 8 * 3 = 2^4

1 * (2 * 3) = 1 * 3^2 = 1 * 9 = 9

⇒ (1 * 2) * 3 ≠ 1 * (2 * 3), where 1,2,3 ϵ Z^+

⇒ The operation * is not associative.

Q. 2 E

For each operation ∗ defined below, determine whether ∗ is binary, commutative or associative.

On R – {–1}, define a ∗ b =

Answer:

It is given that On R, * – {– 1}, define a ∗ b = $\frac{a}{b+1}$

$\frac{a}{b+1}$ ϵ R for b ≠ -1, so the operation * is binary.

We can see that 1 * 2 = $\frac{1}{2+1} = \frac{1}{3}$ and 2 * 1= $\frac{2}{1+1} = \frac{2}{2}$ = 1

⇒ 1 * 2 ≠ 2 * 1; where 1,2 ϵ R – {-1}

⇒ the operation * is not commutative.

Now, we can have observed that

(1 * 2) * 3 = $\frac{1}{3} * 3 = \frac{\frac{1}{3}}{3+1} = \frac{1}{12}$

1 * (2 * 3) = 1 * $\frac{2}{3+1} = 1 * \frac{2}{4} = 1 * \frac{1}{2} = \frac{1}{\frac{1}{2}+1} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

⇒ (1 * 2) * 3 ≠ 1 * (2 * 3), where 1,2,3 ϵ R * – {– 1}

⇒ The operation * is not associative.

Q. 3

consider the binary operation ∧ on the set {1, 2, 3, 4, 5} defined by a ∧ b = min {a, b}. Write the operation table of the operation ∧.

Answer:

The binary operation ^ on the set {1, 2, 3, 4, 5} is defined as a ^ b = min {a, b} for a, b ϵ {1, 2, 3, 4, 5}

Therefore, the operation table of the given operation ^ can be given as:

^	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3

4	1	2	3	4	4
5	1	2	3	4	5

Q. 4

Consider a binary operation $*$ on the set $\{1, 2, 3, 4, 5\}$ given by the following multiplication table (Table 1.2).

(i) Compute $(2 * 3) * 4$ and $2 * (3 * 4)$

(ii) Is $*$ commutative?

(iii) Compute $(2 * 3) * (4 * 5)$.

(Hint: use the following table)

Table 1.2

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

Answer:

(i) $(2 * 3) * 4 = 1 * 4 = 1$

$2 * (3 * 4) = 2 * 1 = 1$

(ii) For every a, b ϵ {1, 2, 3, 4, 5},

We have, a * b = b * a

$\Rightarrow$ the operation * is commutative.

(iii) $(2 * 3) = 1$

$\Rightarrow (2 * 3) * (4 * 5) = 1 * 1 = 1$

Q. 5

Let $*'$ be the binary operation on the set {1, 2, 3, 4, 5} defined by a $*'$ b = H.C.F. of a and b. Is the operation $*'$ same as the operation $*$ defined in Exercise 4 above? Justify your answer.

Answer:

The binary operation *' on the set {1, 2, 3, 4, 5} is defined as

a *' b = HCF of a and b.

The operation table for the operation *' is given by:

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

From the above table we can see that the operation tables for the operation * and *' are the same.

Therefore, the operation *' is same as the operation *

Q. 6

Let $*$ be the binary operation on N given by $a * b$ = L.C.M. of a and b. Find

(i) $5 * 7$, $20 * 16$

(ii) Is $*$ commutative?

(iii) Is $*$ associative?

(iv) Find the identity of $*$ in N

(v) Which elements of N are invertible for the operation $*$?

Answer:

(i) It is given that the binary operation on N given by $a * b$ = L.C.M. of a and b.

Then, $5 * 7$ = LCM of 5 and 7 = 35

$20 * 16$ = LCM of 20 and 16 = 80.

(ii) It is given that the binary operation on N given by $a * b$ = L.C.M. of a and b.

We know that LCM of a and b = LCM of b and a, $a, b \in N$.

$\Rightarrow a * b = b * a$

Therefore, the operation $*$ is commutative.

(iii) It is given that the binary operation on N given by $a * b$ = L.C.M. of a and b.

For $a, b, c \in N$

$(a * b) * c$ = (LCM of a and b) $* c$ = LCM of a, b and c

$a * (b * c) = a *$ (LCM of b and c) =LCM of a, b and c

$\Rightarrow (a * b) * c = a * (b * c)$

Therefore, the operation * is associative.

(iv) It is given that the binary operation on N given by a ∗ b = L.C.M. of a and b.

We know that LCM of a and 1 = a = LCM of 1 and 1, a ϵ N

⇒ a * 1 = a = 1 * a, a ϵ N

Therefore, 1 is the identity of * in N.

(v) It is given that the binary operation on N given by a ∗ b = L.C.M. of a and b.

An element a in N is invertible w.r.t. the operation * if there exists an element b in N,

Such that a * b = e =b * a

Now, if e = 1

⇒ LCM of a and b = 1= LCM of b and a

⇒ This is only possible when a = b = 1

Therefore, 1 is the only invertible element of N w.r.t. the operation *.

Q. 7

Is ∗ defined on the set {1, 2, 3, 4, 5} by a ∗ b = L.C.M. of a and b a binary operation? Justify your answer.

Answer:

The operation * on the set A = {1, 2, 3, 4, 5} is defined as

a * b = LCM of a and b.

2 * 3 = LCM of 2 and 3 = 6.

But 6 does not belong to the given set.

Therefore, the given operation * is not a binary operation.

Q. 8

Let $*$ be the binary operation on N defined by $a * b$ = H.C.F. of a and b. Is $*$ commutative? Is $*$ associative? Does there exist identity for this binary operation on N?

Answer:

It is given that the binary operation on N defined by $a * b$ = H.C.F. of a and b.

We know that HCF of a and b = HCF of b and a, $a, b \in N$.

$\Rightarrow a * b = b * a$

$\Rightarrow$ The operation * is commutative.

For $a, b\ c \in N$, we get,

$(a * b) * c$ = (HCF of a and b) $* c$ = HCF of a, b and c

$a * (b * c) = a *$ (HCF of b and c) = HCF of a, b and c

$\Rightarrow (a * b) * c = a * (b * c)$

$\Rightarrow$ The operation * is associative.

Now, an element $e \in N$ will be the identity for the operation.

Now, if $a * e = a = e * a$, $a \in N$.

But, this is not true for any $a \in N$.

Therefore, the operation * does not have any identity in N.

Q. 9 A

Let $*$ be a binary operation on the set Q of rational numbers as follows:

$a * b = a - b$

Find which of the binary operations are commutative and which are associative.

Answer:

It is given that $*$ be a binary operation on the set Q of rational numbers as

$a * b = a – b$

We can have observed that for $2,3,4 \in Q$

$2 * 3 = 2-3 = -1$ and $3 * 2 = 3-2 =1$

$2 * 3 \neq 3 * 2$

$\Rightarrow$ the operation * is not commutative.

Also, $(2 * 3) * 4 = (-1) * 4 = -1 -4 = -5$ and

$2 * (3 * 4) = 2 * (-1) = 2 – (-1) = 3.$

$\Rightarrow (2 * 3) * 4 \neq 2 * (3 * 4)$

Therefore, the operation * is not associative.

Q. 9 B

Let $*$ be a binary operation on the set Q of rational numbers as follows:

$a * b = a^2 + b^2$

Find which of the binary operations are commutative and which are associative.

Answer:

It is given that $*$ be a binary operation on the set Q of rational numbers is defined as

$a * b = a^2 + b^2$

For $a, b \in Q$, we get,

$a * b = a^2 + b^2 = b^2 + a^2 = b * a$

$\Rightarrow$ the operation is commutative.

We can see that $(1 * 2) * 3 \neq 1 *(2 * 3)$, where $1,2,3 \in Q$

Therefore, the operation * is not associative.

Q. 9 C

Let $*$ be a binary operation on the set Q of rational numbers as follows:

$a * b = a + ab$

Find which of the binary operations are commutative and which are associative.

Answer:

It is given that $*$ be a binary operation on the set Q of rational numbers is defined as

$a * b = a + ab$

We can see that $1 * 2 = 1 + 1 \times 2 = 1 + 2 = 3$

and $2 * 1 = 2 + 2 \times 1 = 2+2 = 4$

$\Rightarrow 1 *2 \neq 2 * 1$: where $1,2 \in Q$.

$\Rightarrow$ the operation * is not commutative.

Also, we can see that $(1 * 2) * 3 \neq 1 *(2 * 3)$, where $1,2,3 \in Q$

Therefore, the operation * is not associative.

Q. 9 D

Let $*$ be a binary operation on the set Q of rational numbers as follows:

$a * b = (a - b)^2$

Find which of the binary operations are commutative and which are associative.

Answer:

It is given that $*$ be a binary operation on the set Q of rational numbers is defined as

$a * b = (a - b)^2$

For a, b $\in$ Q, we have,

$a * b = (a - b)^2$

$b * a = (b - a)^2 = [-(a - b)]^2 = (a - b)^2$

$\Rightarrow a * b = b * a$

$\Rightarrow$ the operation * is commutative.

Also, we can see that $(1 * 2) * 3 \neq 1 *(2 * 3)$, where $1,2,3 \in Q$

Therefore, the operation * is not associative.

Q. 9 E

Let $*$ be a binary operation on the set Q of rational numbers as follows:

$a * b = \frac{ab}{4}$

Find which of the binary operations are commutative and which are associative.

Answer:

It is given that $*$ be a binary operation on the set Q of rational numbers is defined as

$a * b = \frac{ab}{4}$

For a, b $\in$ Q, we get,

$a * b = \frac{ab}{4} = \frac{ba}{4} = b * a$

$\Rightarrow$ the operation is commutative.

For, a, b, c ϵ Q, we get,

$(a * b) * c = \frac{ab}{4} * c = \frac{\frac{ab}{4}.c}{4} = \frac{abc}{16}$

$a * (b * c) = a * \frac{bc}{4} = \frac{a.\frac{bc}{4}}{4} = \frac{abc}{16}$

⇒ (a * b) * c = a * (b * c)

Therefore, the operation * is associative.

Q. 9 F

Let ∗ be a binary operation on the set Q of rational numbers as follows:

$a * b = ab^2$

Find which of the binary operations are commutative and which are associative.

Answer:

It is given that ∗ be a binary operation on the set Q of rational numbers is defined as

$a * b = ab^2$

We can see that for 2, 3 ϵ Q

$2 * 3 = 2.\ 3^2 = 18$ and $3 * 2 = 3.\ 2^2 = 12$

⇒ 2 * 3 ≠ 3 * 2

Therefore, the operation * is not commutative.

Also, we can see that (1 * 2) * 3 ≠ 1 *(2 * 3), where 1,2,3 ϵ Q

Therefore, the operation * is not associative.

Q. 10

Find which of the operations given above has identity.

Answer:

(i) An element $e \in Q$ will be the identity element for the operation * if

$a * e = a = e * a, \forall a \in Q.$

$a * b = a - b$

This operation is not commutative,

Therefore, it does not have identity element.

(ii) An element $e \in Q$ will be the identity element for the operation * if

$a * e = a = e * a, \forall a \in Q.$

$a * b = a^2 + b^2$

If $a * e = a$, then $a^2 + e^2 = a$.

For $a = -2$, $(-2)^4 + e^2 \neq -2$.

Therefore, there is no identity element.

(iii) An element $e \in Q$ will be the identity element for the operation * if

$a * e = a = e * a, \forall a \in Q.$

Now, $a * b = a + ab$

This is not commutative.

Therefore, there is no identity element.

(iv) An element $e \in Q$ will be the identity element for the operation * if

$a * e = a = e * a, \forall a \in Q.$

$a * b = (a - b)^2$

If $a * e = a$, then $(a - e)^2 = a$.

A square is always positive, thus for $a = -2$, $(-2 - e)^2 \neq -2$.

Therefore, there is no identity element.

(v) An element $e \in Q$ will be the identity element for the operation * if

$a * e = a = e * a, \forall a \in Q$.

$a * b = \frac{ab}{4}$

If $a * e = a$, then $\frac{ae}{4} = a$

Therefore, $e = 4$ is the identity element.

$a * 4 = 4 * a = \frac{4a}{4} = a$.

(vi) An element $e \in Q$ will be the identity element for the operation * if

$a * e = a = e * a, \forall a \in Q$.

Now, $a * b = ab^2$

This operation is not commutative,

Therefore, there is not have identity element.

Q. 11

Let $A = N \times N$ and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$

Show that $*$ is commutative and associative. Find the identity element for $*$ on A, if any.

Answer:

It is given that $A = N \times N$ and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$

Let $(a, b), (c, d) \in A$

Then, $a, b, c, d \in N$

Now, we have,

$(a, b) * (c, d) = (a + c, b + d)$

$(c, d) * (a, d) = (c + a, d + b) = (a + c, b + d)$

$\Rightarrow (a, b) * (c, d) = (c, d) * (a, b)$

$\Rightarrow$ The operation * is commutative.

Now, $(a, b), (c, d), (e, f) \in A$

Then, $a, b, c, d, e, f \in N$

Now, we have,

$((a, b) * (c, d)) * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)$

$(a, b) * ((c, d) * (e, f)) = (a, b) * (c + e, d + f) = (a + c + e, b + d + f)$

Then, $((a, b) * (c, d)) * (e, f) = (a, b) * ((c, d) * (e, f))$

Therefore, the operation * is associative.

Now, an element $e = (e_1, e_2) \in A$ will be an identity for the operation *

if $a * e = a = e * a\ \forall\ a = (a_1, a_2) \in A$,

$(a_1 + e_1, a_2 + e_2) = (a_1, a_2) = (e_1 + a_1, e_2 + a_2)$

which is not true for any element in A.?

Therefore, the operation * does not have any identity element.

Q. 12 A

State whether the following statements are true or false. Justify. For an arbitrary binary operation $*$ on a set N, $a * a = a\ \forall a \in N$.

Answer:

For an arbitrary binary operation $*$ on a set N, $a * a = a\ \forall a \in N$.

The above statement is false.

Explanation: It is given that an operation $*$ on a set N, $a * a = a\ \forall a \in N$

Then, in particular, for $b = a = 3$, we get,

$3 * 3 = 3 + 3 = 6 \neq 3$

Q. 12 B

State whether the following statements are true or false. Justify. If $*$ is a commutative binary operation on N, then $a * (b * c) = (c * b) * a$

Answer:

If $*$ is a commutative binary operation on N, then $a * (b * c) = (c * b) * a$

The above statement if true.

Explanation: RHS = $(c * b) * a$

$= (b * c) * a$ (* is commutative)

$= a * (b * c)$ (as * is commutative)

= LHS.

Therefore, $a * (b * c) = (c * b) * a$.

Hence Proved

Q. 13

Consider a binary operation $*$ on N defined as $a * b = a^3 + b^3$. Choose the correct answer.

A. Is $*$ both associative and commutative?

B. Is $*$ commutative but not associative?

C. Is $*$ associative but not commutative?

D. Is $*$ neither commutative nor associative?

Answer:

On N, the operation * is defined as $a * b = a^3 + b^3$

For, a, b $\in$ N, we get,

$a * b = a^3 + b^3 = b^3 + a^3 = b * a$ [Addition is commutative in N]

⇒ the operation * is commutative.

We can have observed that (1*2) *3 = (1^3+2^3) *3 = 9 * 3 = $9^3 + 3^3$ = 729 + 27 = 756

Also, 1*(2*3) = 1*$(2^3 + 3^3)$ = 1*(8 +27) = 1 × 35 = $1^3 + 35^3$ = 1 + $(35)^3$ = 1 + 42875 = 42876.

Therefore, (1 * 2) *3 ≠ 1*(2*3); where 1,2,3 ϵ N

Therefore, the operation * is not associative.

Therefore, the operation * is commutative, but not associative.

Miscellaneous Exercise

Q. 1

Let f: R → R be defined as f (x) = 10x + 7. Find the function g: R → R such that g o f = f o g = 1R.

Answer:

It is given that f: R → R be defined as f (x) = 10x + 7

Let f(x) = f(y), where x, y ϵ R.

⇒ 10x + 7 = 10y + 7

⇒ x = y

⇒ f is a one – one function.

For y ϵ R, let y = 10x + 7.

⇒ x = $\frac{y-7}{10}$ ∈ R

Therefore, for any y ϵ R, there exists x = $\frac{y-7}{10}$ ∈ R such that

f (x) = f $\left(\frac{y-7}{10}\right)$ = 10 $\left(\frac{y-7}{10}\right)$ + 7 = y – 7 + 7 = y

⇒ f is onto.

⇒ f is an invertible function.

Let us define g: R → R as g (y) = $\frac{y-7}{10}$

Now, we get:

gof(x) = g(f(x)) = g (10x + 7)

= $\frac{(10x+7)-7}{10} = \frac{10x}{10}$ = 10

And,

fog(y) = f (g(y)) = f $\left(\frac{y-7}{10}\right)$ = 10 $\left(\frac{y-7}{10}\right)$ + 7 = y – 7 + 7 = y

⇒ gof = IR and gof = IR

Therefore, the required function g : R → R is defined as g(y) = $\frac{y-7}{10}$.

Q. 2

Let f: W → W be defined as f (n) = n – 1, if n is odd and f (n) = n + 1, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

Answer:

It is given that f: W → W be defined as

f(n) = $\begin{cases} n-1, if\ n\ is\ odd \\ n+1, if\ n\ is\ even \end{cases}$

Let f(n) = f(m)

We can see that if n is odd and m is even, then we will have n -1 = m +1.

⇒ n – m = 2

⇒ this is impossible

Similarly, the possibility of n being even and m being odd can also be ignored under a similar argument.

Therefore, both n and m must be either odd or even.

Now, if both n and m are odd, then we get:

f(n) = f(m)

⇒ n -1 = m -1

⇒ n = m

Again, if both n and m are even, the we get:

f(n) = f(m)

⇒ n +1 = m + 1

⇒ n = m

⇒ f is one – one.

Now, it is clear that any odd number 2r + 1 in co-domain N is the image of 2r in domain N and any even number 2r in co – domain N is the image of 2r +1 in domain N.

⇒ f is onto.

⇒ f is an invertible function.

Now, let us define g: W → W be defined as

$$g\ (m) = \begin{cases} m+1, if\ m\ is\ even \\ m-1, if\ m\ is\ odd \end{cases}$$

Now, when n is odd:

gof(n) = g(f(n)) = g (n-1) = n -1 +1 = n (when n is odd, then n-1 is even)

And when n is even:

gof(n) = g(f(n)) = g (n+1) = n +1 -1 = n (when n is even, then n+1 is odd)

Similarly, when m is odd:

fog(m) = f(g(m)) = f (m-1) = m -1 +1 = m

And when n is even:

fog(m) = f(g(m)) = f (m+1) = m +1 -1 = m

Therefore, gof = IW and fog = IW

Therefore, f is invertible and the inverse of f is given by f-1 = g, which is the same as f.

Thus, the inverse of f is f itself.

Q. 3

If f: R → R is defined by $f(x) = x^2 - 3x + 2$, find f (f (x)).

Answer:

It is given that f: R → R is defined by $f(x) = x^2 - 3x + 2$.

$f(f(x)) = f(x^2 - 3x + 2)$

$= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$

$= x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2$

$= x^4 - 6x^3 + 10x^2 - 3x$.

Q. 4

Show that the function f: R → {x ∈ R: – 1 < x < 1} defined by f (x) = $\frac{X}{1+|X|}$, X ∈ R is one-one and onto function.

Answer:

It is given that f: R → {x ∈ R: – 1 < x < 1} defined by f(x) = $\frac{X}{1+|X|}$, x ∈ R

Now, suppose that f(x) = f(y), where x, y ϵ R.

$= \frac{X}{1+|X|} = \frac{Y}{1+|y|}$

We can see that if x is positive and y is negative, then we get:

$\frac{X}{1+X} = \frac{y}{1+y} = 2xy = x - y$

Since, x is positive, and y is negative.

Then, $2xy \neq x - y$.

Thus, the case of x being positive and y being negative can be ruled out.

Similarly, x being negative and y being positive can also be ruled out.

Therefore, x and y have to be either positive or negative.

When x and y are both positive, we get:

f(x) = f(y)

$= \frac{X}{1+X} = \frac{y}{1+y} = x + \ xy = y + xy = x = y$

And when x and y are both negative, we get:

f(x) = f(y)

$= \frac{X}{1-X} = \frac{y}{1-y} = x - xy = y - xy = x = y$

⇒ f is one- one.

Now, let y ϵ R such that -1 < y < 1.

It y is negative, then there exists $X = \frac{y}{1+y} \in R$, such that

$f\,(x) = f\left(\frac{y}{1-y}\right) = \frac{\left(\frac{y}{1-y}\right)}{1+\left(\frac{y}{1-y}\right)} = \frac{\frac{y}{1-y}}{1+\frac{y}{1-y}} = \frac{y}{1-y+y}$

⇒ f is onto.

Therefore, f is one – one and onto.

Q. 5

Show that the function f: R → R given by f (x) = x^3 is injective.

Answer:

Let f: R → R given by f (x) = x^3.

Suppose f(x) = f(y), where x, y ϵ R.

⇒ $x^3 = y^3$ … (1)

Now, we need to show that x = y.

Suppose x ≠ y, their cubes will also not be equal.

$\Rightarrow x^3 \neq y^3$

However, this will be contraction to (1).

Thus, x = y

Therefore, f is injective.

Q. 6

Give examples of two functions f: N → Z and g: Z → Z such that g o f is injective but g is not injective.

(Hint: Consider f (x) = x and g(x) = |x|).

Answer:

Define f: N → Z as f(x) = x and g: Z → Z as g(x) = |x|

Now, we can see that

G (-1) = |-1| = 1

g (1) = |1| = 1

⇒ g (-1) = g (1), but -1 ≠ 1

⇒ g is not injective.

Now, gof: N → Z is defined as gof(x) = g(f(x)) = g(x) = |x|

Let x, y ϵ N such that gof(x) = gof(y).

⇒ |x| = |y|

⇒ x = y

Therefore, gof is injective.

Q. 7 Give examples of two functions f: N → N and g: N → N such that g o f is onto but f is not onto.

(Hint: Consider f (x) = x + 1 and g (x) = $\begin{cases} x - 1 \ if\ x > 1 \\ 1 \ if\ x = 1 \end{cases}$

Answer:

It is given that f: N → N by, f(x) = x + 1

And, g: N → N by,

$$g(x) = \begin{cases} x - 1 & \text{if } x > 1 \\ 1 & \text{if } x = 1 \end{cases}$$

Now, consider element 1 in co-domain N. So, it is clear that this element is not an image of any of the elements in domain N.

⇒ f is onto.

Now, gof: N → N is defined as:

gof(x) = g(f(x)) = g(x+1) = (x + 1) – 1 [x ϵ N = > (x+1) > 1]

Then we can see that for y ϵ N, there exists x = y ϵ N such that gof(x) = y.

Therefore, gof is onto.

Q. 8

Given a non-empty set X, consider P(X) which is the set of all subsets of X.

Define the relation R in P(X) as follows:

For subsets A, B in P(X), ARB if and only if A ⊂ B. Is R an equivalence relation on P(X)? Justify your answer.

Answer:

We know that every set is a subset of itself, ARA for all A ϵ P(X).

⇒ R is reflexive.

This cannot be implied to B ⊂ A.

So, if A = {1, 2} and B = {1, 2, 3}, then it cannot be implied that B is related to A.

⇒ R is not symmetric.

So, if ARB and BRC, then $A \subset B$ and $B \subset C$.

⇒ $A \subset C$

⇒ R is transitive.

Therefore, R is not an equivalence relation since it is not symmetric.

Q. 9

Given a non-empty set X, consider the binary operation $*: P(X) \times P(X) \to P(X)$ given by $A * B = A \cap B \ \forall\ A, B$ in $P(X)$, where $P(X)$ is the power set of X. Show that X is the identity element for this operation and X is the only invertible element in $P(X)$ with respect to the operation $*$.

Answer:

It is given that $*: P(X) \times P(X) \to P(X)$ given by

$A * B = A \cap B \ \forall\ A, B \in P(X)$.

As we know that,

⇒ $A * X = A = x * A \ \forall\ A \in P(X)$.

Thus, X is the identity element for the given binary operation *.

Now, an element $A \in P(X)$ is invertible if there exists $B \in P(X)$ such that

$A * B = X = B * A$ (As X is the identity element)

$A \cap B = X = B \cap A$

This can be possible only when $A = X = B$.

Therefore, X is the only invertible element in $P(X)$ w.r.t. given operation *.

Hence Proved.

Q. 10

Find the number of all onto functions from the set $\{1, 2, 3, ..., n\}$ to itself.

Answer:

Onto function from the set $\{1, 2, 3, ..., n\}$ to itself is simply a permutation on n symbols $1, 2, 3, \ldots, n$.

Therefore, the total number of onto maps from $\{1, 2, 3, \ldots, n\}$ to itself is the same as the total number of permutations on n symbols $1, 2, 3, \ldots, n$, which is n!

Q. 11 A

Let $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$. Find F^{-1} of the following functions F from S to T, if it exists.

$F = \{(a, 3), (b, 2), (c, 1)\}$

Answer:

(i) It is given that $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$

$F: S \rightarrow T$ is defined as:

$F = \{(a, 3), (b, 2), (c, 1)\}$

$=> F(a) = 3, F(b) = 2, F(c) = 1$

Therefore, $F^{-1}: T \rightarrow S$ is given by:

$F^{-1} = \{(3, a), (2, b), (1, c)\}$.

Q. 11 B

Let $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$. Find F^{-1} of the following functions F from S to T, if it exists.

$F = \{(a, 2), (b, 1), (c, 1)\}$

Answer:

It is given that $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$

$F: S \to T$ is defined as:

$F = \{(a, 2), (b, 1), (c, 1)\}$

$=> F(b) = 1, F(c) = 1$, F is not one-one.

Therefore, F is not invertible

$\Rightarrow F^{-1}$ does not exist.

Q. 12

Consider the binary operations $*: R \times R \to R$ and $o: R \times R \to R$ defined as $a * b = |a - b|$ and $a \text{ o } b = a, \forall a, b \in R$. Show that $*$ is commutative but not associative, o is associative but not commutative. Further, show that $\forall a, b, c \in R, a * (b \text{ o } c) = (a * b) \text{ o } (a * c)$. [If it is so, we say that the operation $*$ distributes over the operation o]. Does o distribute over $*$? Justify your answer.

Answer:

It is given that $*: R \times R \to R$ and $o: R \times R \to R$ defined as

$a * b = |a - b|$ and $a \text{ o } b = a, \forall a, b \in R$.

For $a, b \in R$, we get:

$a*b = |a-b|$

$b*a = |b-a| = |-(a-b)| = |a-b|$

Therefore, $a*b = b*a$

$\Rightarrow$ the operation * is commutative.

We can see that

$(1*2)*3 = (|1-2|)*3 = |1-3| = 2$

$1*(2*3) = 1*(|2-3|) = 1*1 = 1$

Therefore, the operation * is not associative.

Now, consider the operation o:

We can have observed that 1o2 = 1 and 2o1 = 2.

$\Rightarrow$ 1o2 $\neq$ 2o1(where 1, 2 ϵ R)

$\Rightarrow$ the operation o is not commutative.

Let a, b, c ϵ R. Then, we get:

(a o b) o c = a o c = a

a o (b o c) = a o b = a

$\Rightarrow$ (a o b) o c = a o (b o c)

$\Rightarrow$ the operation o is associative.

Now, let a, b, c ϵ R, then we have:

a * (b o c) = a * b = |a –b|

(a * b) o (a * c) = (|a –b|) o (|a –c| = |a –b|

Thus, a * (b o c) = (a * b) o (a * c)

Now,

1 o (2 * 3) = 1 o (|2 –3|) = 1 o 1 = 1

(1 o 2) * (1o3) = 1 * 1 = |1 –1| = 0

Therefore, 1 o (2 * 3) $\neq$ (1 o 2) * (1o3) (where 1, 2, 3 ϵ R)

Therefore, the operation o does not distribute over *.

Q. 13 Given a non-empty set X, let $*$: P(X) × P(X) → P(X) be defined as A * B = (A – B) ∪ (B – A), ∀A, B ∈ P(X). Show that the empty set φ is the identity for the operation $*$ and all the elements A of P(X) are invertible with

A^{-1} = A.

(Hint: (A – φ) ∪ (φ – A) = A and (A – A) ∪ (A – A) = A ∗ A = φ).

Answer:

It is given that ∗: P(X) × P(X) → P(X) be defined as

A * B = (A – B) ∪ (B – A), A, B ∈ P(X).

Now, let A ϵ P(X). Then, we get,

A * ∅ = (A –∅) ∪ (∅ – A) = A ∪ ∅ = A

∅ * A = (∅ - A) ∪ (A - ∅) = ∅ ∪ A = A

⇒ A * ∅ = A = ∅ * A, A ϵ P(X)

Therefore, ∅ is the identity element for the given operation *.

Now, an element A ϵ P(X) will be invertible if there exists B ϵ P(X) such that

A * B = ∅ = B * A. (as ϕ is an identity element.)

Now, we can see that A * A = (A –A) ∪ (A – A) = ∅ ∪ ϕ = ∅ A ϵ P(X).

Therefore, all the element A of P(X) are invertible with $A^{-1} = A$.

Q. 14

Define a binary operation ∗ on the set {0, 1, 2, 3, 4, 5} as a * b =
$$\begin{cases} a+b, if\, a+b < 6 \\ a+b-6, if\, a+b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element a ≠ 0 of the set is invertible with 6 – a being the inverse of a.

Answer:

Let X = {0, 1, 2, 3, 4, 5}

The operation * on X is defined as

a * b = $\begin{cases} a+b, if\, a+b < 6 \\ a+b-6, if\, a+b \geq 6 \end{cases}$

An element e ϵ X is the identity element for the operation *,

If a * e = a = e * a ∀ a ϵ X.

For a ϵ X, we can see that:

a * 0 = a + 0 = a [a ϵ X = > a + 0 < 6]

0 * A = 0 + a = a [a ϵ X = > a + 0 < 6]

⇒ a * 0 = a = 0 * a ∀ a ϵ X.

Therefore, o is the identity element for the given operation *.

An element a ϵ X is invertible if there exists b ϵ X such that

a * b = 0 = b * a.

= $\begin{cases} a+b = 0 = b+a, if\, a+b < 6 \\ a+b-6 = 0 = b+a-6, if\, a+b \geq 6 \end{cases}$

a = -b or b = 6 – a

But, X = {0, 1, 2, 3, 4, 5} and a, b ϵ X. Then, a ≠ -b.

Therefore, b = 6 – a is the inverse of a ϵ X.

Thus, the inverse of an element a ϵ X, a ≠ 0 is 6 – a, a-1 = 6 – a.

Q. 15

Let A = {–1, 0, 1, 2}, B = {–4, –2, 0, 2} and f, g: A → B be functions defined by f (x) = x2 – x, x ∈ A and g (x) = 2 $\left|x-\frac{1}{2}\right|-1$, x ∈ A. Are f and g equal?

Justify your answer. (Hint: One may note that two functions f: A → B and g: A → B such that f (a) = g(a) ∀ a ∈ A, are called equal functions).

Answer:

It is given that A = {–1, 0, 1, 2}, B = {– 4, – 2, 0, 2}

And also, it is given that f, g: A → B be functions defined by f (x) = x^2 – x, x ∈ A and g (x) = 2 $\left|x-\frac{1}{2}\right|-1$, x ∈ A

We can see that

f (-1) = $(-1)^2$ – (-1) = 1 + 1 = 2

g (-1) = 2 $\left|(-1)-\frac{1}{2}\right|$ – 1 = 2 $\left(\frac{3}{2}\right)$ – 1 = 3 – 1 = 2

⇒ f (-1) = g (-1)

F (0) = (0)2 – 0 = 0 + 0 = 0

g (-1) = 2 $\left|0-\frac{1}{2}\right|$ – 1 = 2 $\left(\frac{1}{2}\right)$ – 1 = 1 – 1 = 0

⇒ f (0) = g (0)

f (1) = $(1)^2$ – 1 = 1- 1 = 0

g (1) = 2 $\left|1-\frac{1}{2}\right|$ – 1 = 2 $\left(\frac{1}{2}\right)$ – 1 = 1 – 1 = 0

⇒ f (1) = g (1)

F (2) = $(2)^2$ – 2 = 4- 2 = 2

g (2) = 2 $\left|2-\frac{1}{2}\right|$ – 1 = 2 $\left(\frac{3}{2}\right)$ – 1 = 3 – 1 = 2

f (2) = g (2)

Thus, f(a) = g(a) ∀. a ϵ A

Therefore, the functions f and g are equal.

Q. 16

Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is

A. 1

B. 2

C. 3

D. 4

Answer:

This is because relation R is reflexive as (1, 1), (2, 2), (3, 3) ϵ R.

Relation R is symmetric as (1, 2), (2, 1) ϵ R and (1, 3), (3, 1) ϵ R.

But relation R is not transitive as (3, 1), (1, 2) ϵ R but (3, 2) $\in$ R.

Now, if we add any one of the two pairs (3, 2) and (2, 3) (or both) to relation R,

Then, relation R will become transitive.

Therefore, the total number of desired relations is one.

Q. 17

Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is

A. 1

B. 2

C. 3

D. 4

Answer:

It is given that A = {1, 2, 3}.

An equivalence relation is reflexive, symmetric and transitive.

The smallest equivalence relations containing (1, 2) is equal to

R1 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}

Now, only four pairs are left (2, 3), (3, 2), (1, 3) and (3, 1).

So, if we add one pair to R, then for symmetry we must add (3, 2).

Also, for transitivity we required to add (1, 3) and (3, 1).

Thus, the only equivalence relation is the universal relation.

Therefore, the total number of equivalence relations containing (1, 2) is 2.

Q. 18

Let f: R → R be the Signum Function defined as

$$f(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$$

and g: R → R be the Greatest Integer Function given by g(x) = [x], where [x] is greatest integer less than or equal to x. Then, does fog and gof coincide in (0, 1]?

Answer:

It is given that

f: R → R be the Signum Function defined as

$$f(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$$

Also, g: R → R is defined as g(x) = [x], where [x] is the greatest integer less than or equal to x.

Now, let x ϵ (0, 1]

Then, we get,

[x] = 1 if x = 1 and [x] = 0 if 0 < x < 1

Therefore, fog(x) = f(g(x)) = f([x])

$= \begin{cases} f\ (1), i\ fx = 1 \\ f\ (0), i\ fx\ \in (0,1) \end{cases} = \begin{cases} 1, i\ fx = 1 \\ 0, i\ fx\ \in (0,1) \end{cases}$

gof(x) = g(f(x))

= g (1) [x > 0]

= [1] = 1

Then, when x ϵ (0,1), we get fog(x) = 0 and gof(x) = 1

But fog (1) ≠ gof (1)

Therefore, fog and gof do not coincide in (0, 1].

Q. 19

Number of binary operations on the set {a, b} are

A. 10

B. 16

C. 20

D. 8

Answer:

A binary option * on {a, b} is a function from

{a, b} × {a, b} → {a, b}

i.e. y

Therefore, the total number of binary operations on the set {a, b} is 24 that is 16.