ME 759: Nonlinear FEM
Assignment 4: 2-D Elasto-plasticity

Equations used for coding

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Assignment 4 – 2D Elastoplasticity 06/04/2022

— Variables passed in as information into UMAT by Abaqus.

STRAN (NTENS), DSTRA(NTENS)

— Solution dependent state variables used in code

ε^p ≡ epsilon-plas (NTENS) —— array with dimension NTENS

α ≡ alpha ⇐ Equivalent plastic strain

epsilon-plas (1) = STATEV(1)
epsilon-plas (2) = STATEV(2)
epsilon-plas (3) = STATEV(3)
epsilon-plas (4) = STATEV(4)
alpha = STATEV(5)

— In UMAT, it is necessary to rotate tensors during a finite-strain analysis. The matrix DROT that is passed into UMAT represents incremental rotation of the material basis system in which stress & strain are ~~not~~ stored.

— For elastic-plastic material that hardens isotropically, elastic & plastic strain tensors must be rotated to account for evolution of material direction.

— So, we have to use ROTSIG inbuilt function in code

CALL ROTSIG(STATEV(1), DROT, epsilon-plas, 2, NDI, NSHR)

Algorithm steps :

1) Take material properties from PROPS(..) array.

2) $\underset{\sim}{I} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$ $\{\underset{\sim}{1}\} = \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{Bmatrix}$

3) Compute DDSDDE(,) for elastic step.

4) Compute trial stress as

$\sigma^t_{n+1}(i) = \sigma_n(i) + \text{DDSDDE}(i,j) * \text{DSTRAN}(j)$

$\underset{\sim}{\sigma}^t_{n+1} = \underset{\sim}{\sigma}_n + [\underset{\sim}{C}]\{\Delta\varepsilon_n\}$

5) Compute hydrostatic stress : ~~$\sigma_{Hydro} = \underset{\sim}{\sigma}^t_{n+1}$~~

$\sigma_H = tr(\underset{\sim}{\sigma}^t_{n+1}) = \dfrac{\sigma^t_{n+1}(1) + \sigma^t_{n+1}(2) + \sigma^t_{n+1}(3)}{3}$

6) Compute deviatoric stress as

$\{S\}^t_{n+1} = \{\sigma^t_{n+1}\} - \sigma_H\{1\}$

7) $|\underset{\sim}{S}^t_{n+1}| = \left[(S^t_{11})^2 + (S^t_{22})^2 + (S^t_{33})^2 + 2*(S^t_{12})^2\right]^{1/2}$

8) Check yield condition. $f^t_{n+1} = \dfrac{|\underset{\sim}{S}^t_{n+1}|}{\sqrt{2}} - \dfrac{\sigma_y(\alpha_n)}{\sqrt{3}}$ --- $\sigma_y = \sigma_0 + H\alpha$ - for linear Hardening

9) If $f^t_{n+1} \leq 0$ then

$\{S_{n+1}\} = \{S^t_{n+1}\}$ & $\{\sigma_{n+1}\} = \{S^t_{n+1}\} + \sigma_H\{1\}$

$\{\varepsilon^p_{n+1}\} = \{\varepsilon^p_n\}$ & $\alpha_{n+1} = \alpha \rightarrow E$

else

i) $\Delta\gamma = \dfrac{f^t_{n+1}}{\mu + H/3}$ - - for linear hardening

ii) $\{v\}_{n+1} = \dfrac{1}{\sqrt{2}\,|\underset{\sim}{S}^t_{n+1}|} \begin{Bmatrix} S^t_{33} \\ S^t_{11} \\ S^t_{22} \\ 2*S^t_{12} \end{Bmatrix}$

iii) Update plastic strain, equivalent plastic strain & stress

$$\{\varepsilon\}^{p}_{n+1} = \{\varepsilon_n\}^{p} + \Delta\gamma \{\nu\}_{n+1}$$

$$\alpha_{n+1} = \alpha_n + \frac{\Delta\gamma}{\sqrt{3}}$$

$$\{\sigma\}_{n+1} = \{s\}^{t}_{n+1} - 2\mu \Delta\gamma [I] \{\nu\}_{n+1} + \sigma_H \{\mathbb{1}\}$$

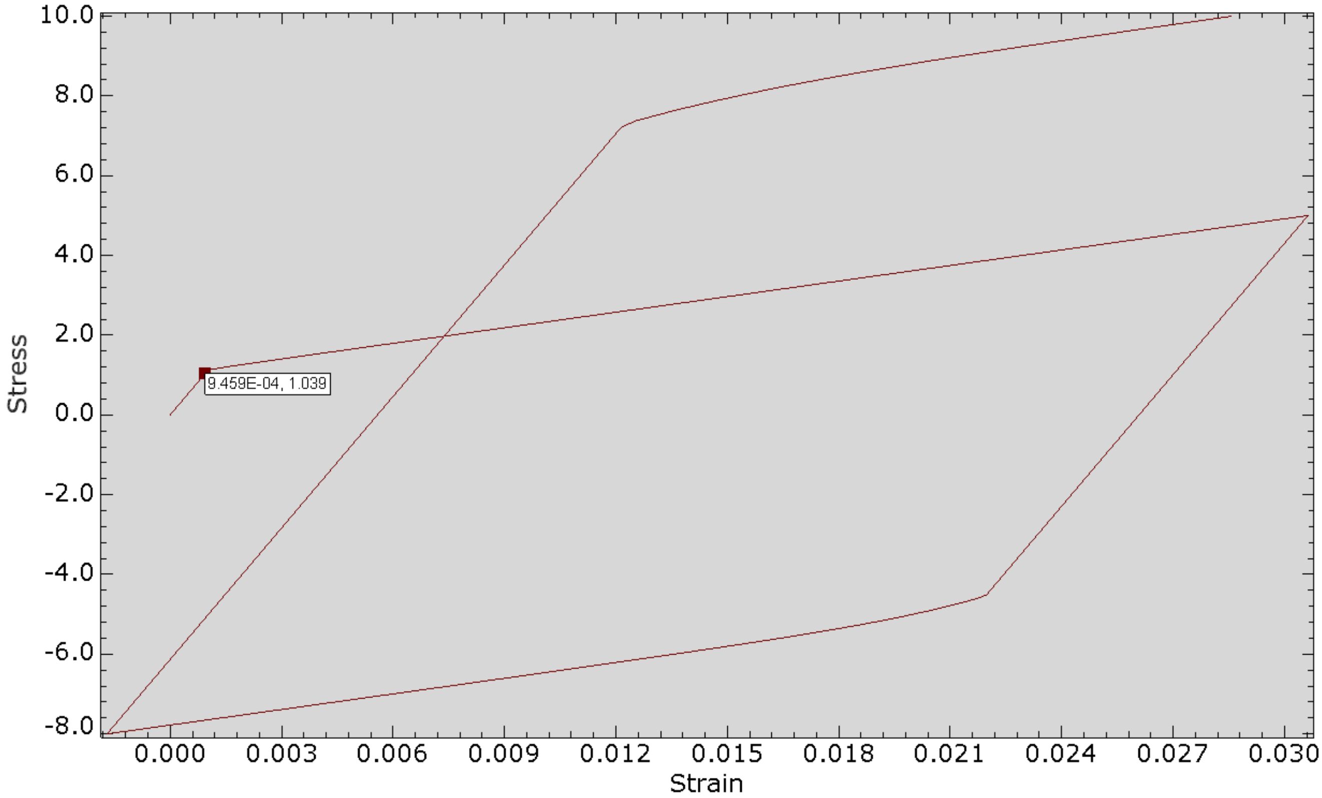
iv) Get consistent elastic plastic tangent modulus

$$\xi_{n+1} = 1 - \frac{\sqrt{2}\, \mu \Delta\gamma}{|\xi^{t}_{n+1}|} \; ; \; \tilde{\xi}_{n+1} = -(1 - \xi_{n+1}) + \frac{1}{1 + \frac{H}{3\mu}}$$

$$[C]_{n+1} = K \{\mathbb{1}\}\{\mathbb{1}\}^{T} + 2\mu\, \xi_{n+1} \left([I] - \frac{1}{3} \{\mathbb{1}\}\{\mathbb{1}\}^{t} \right) - 4\mu\, \tilde{\xi}_{n+1} \left([I]\{\nu\}_{n+1} \right) \left([I]\{\nu\}_{n+1} \right)^{T}$$

Part a

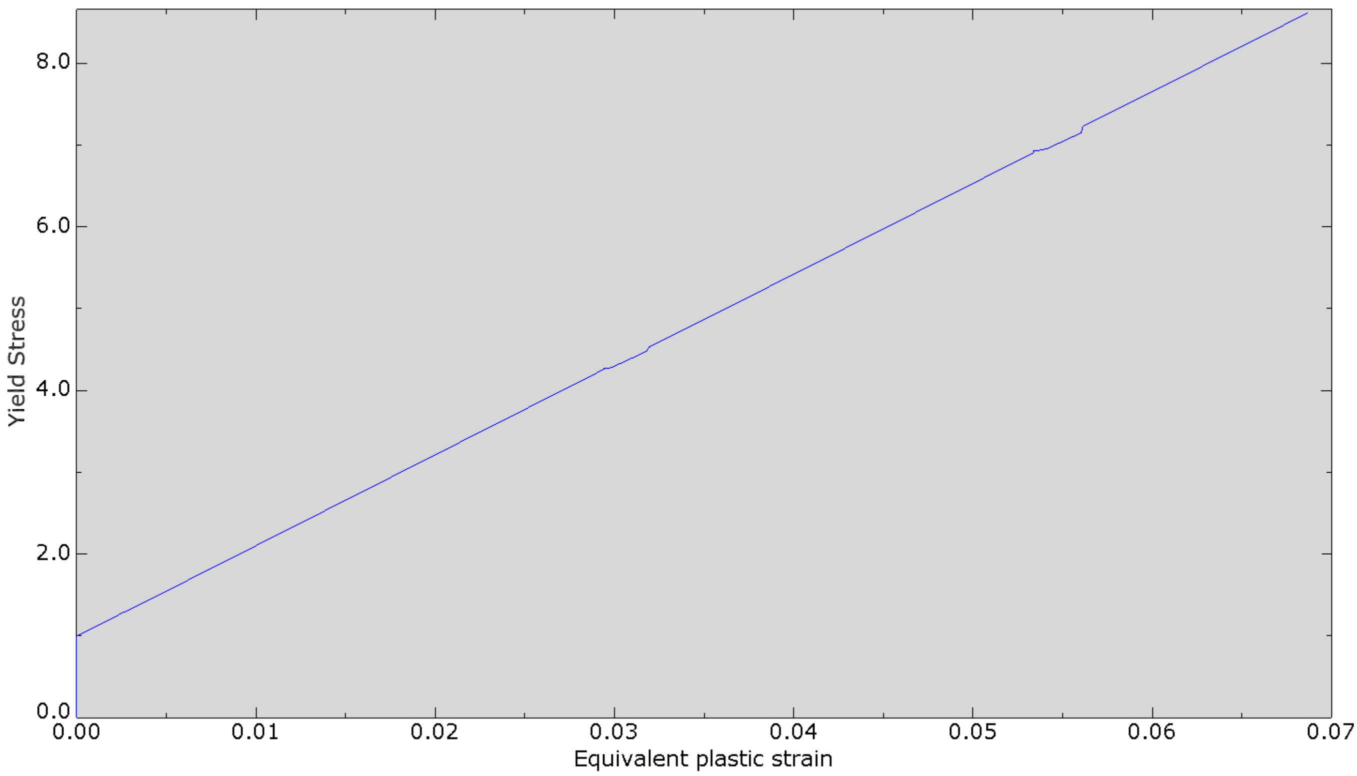
Plot of σ_{22} vs ϵ_{22}



Observations:

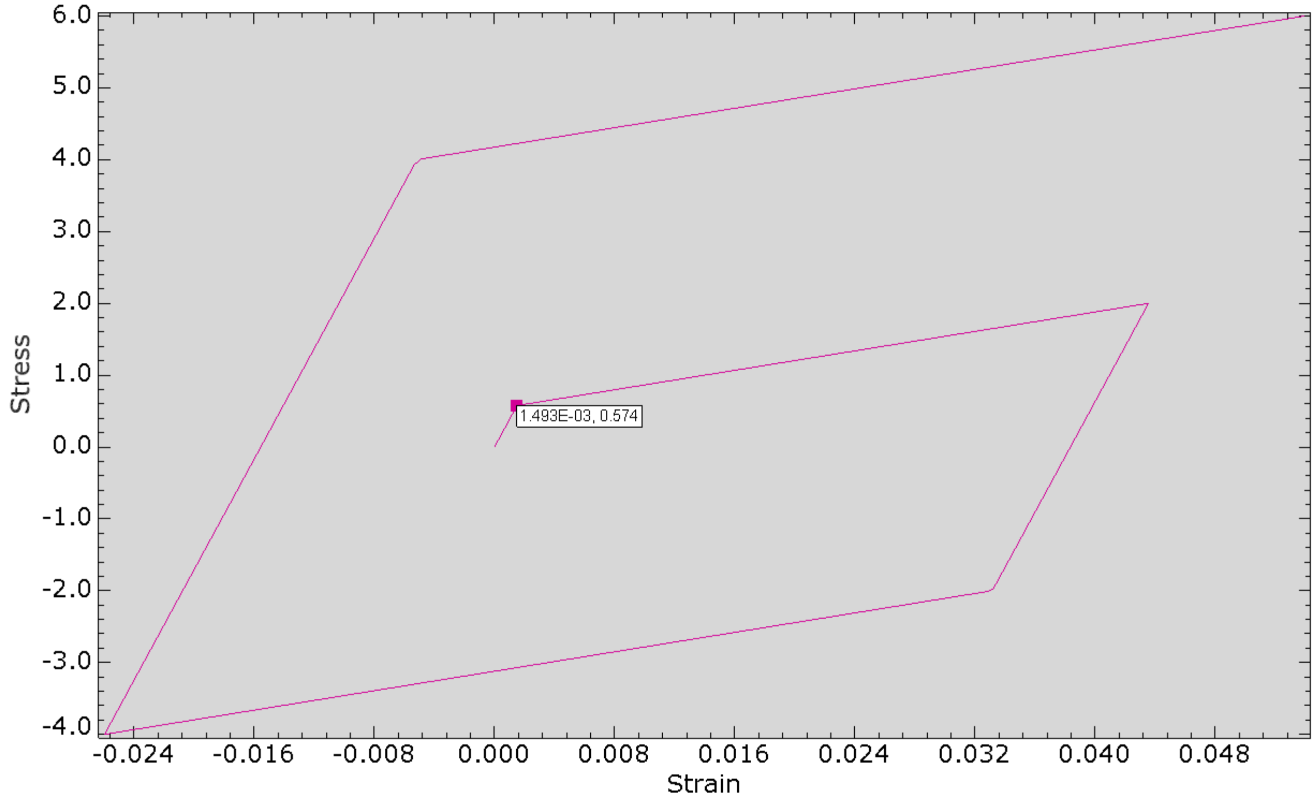
Since we are using von-mises stress criteria, we can see that the yielding starts at σ_0 for when we apply normal traction on top surface.

Plot of yield stress σ_y vs equivalent plastic strain α



Part b

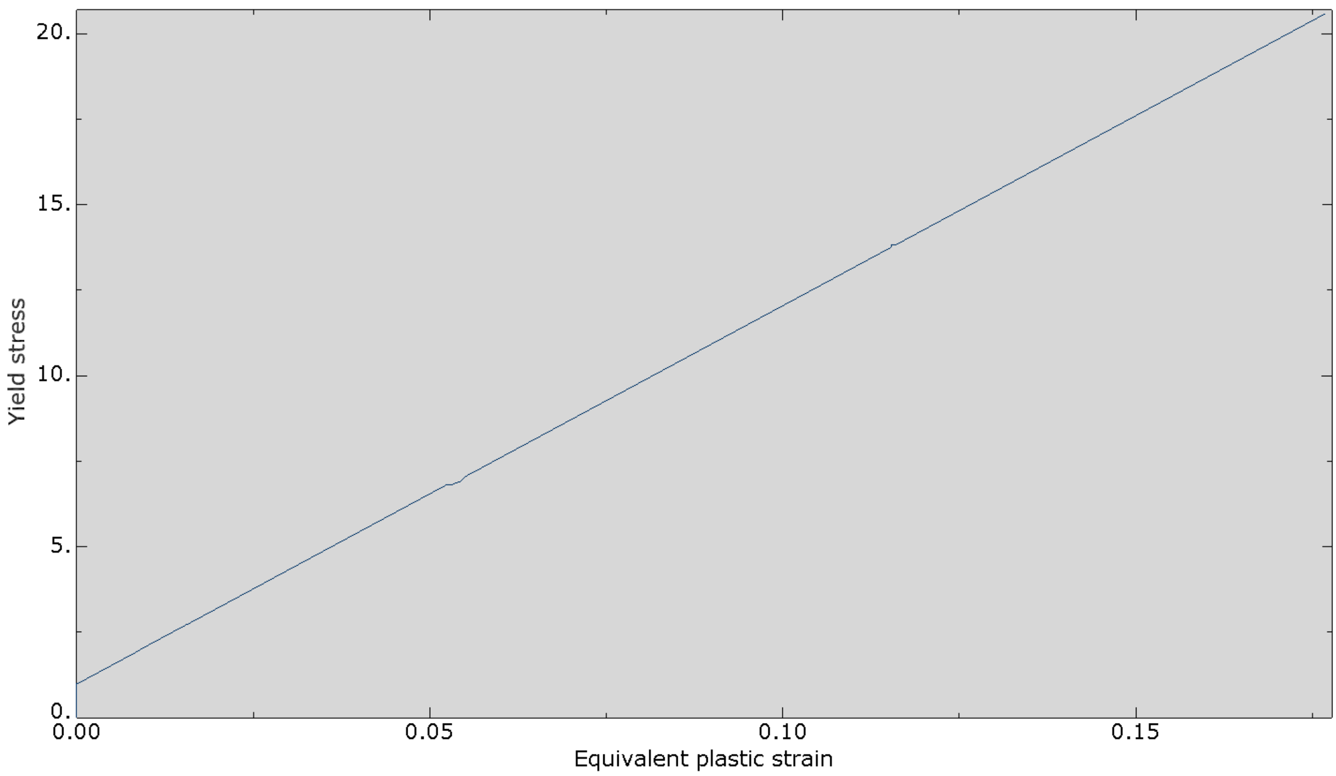
Plot of σ_{12} vs ϵ_{12}



Observations:

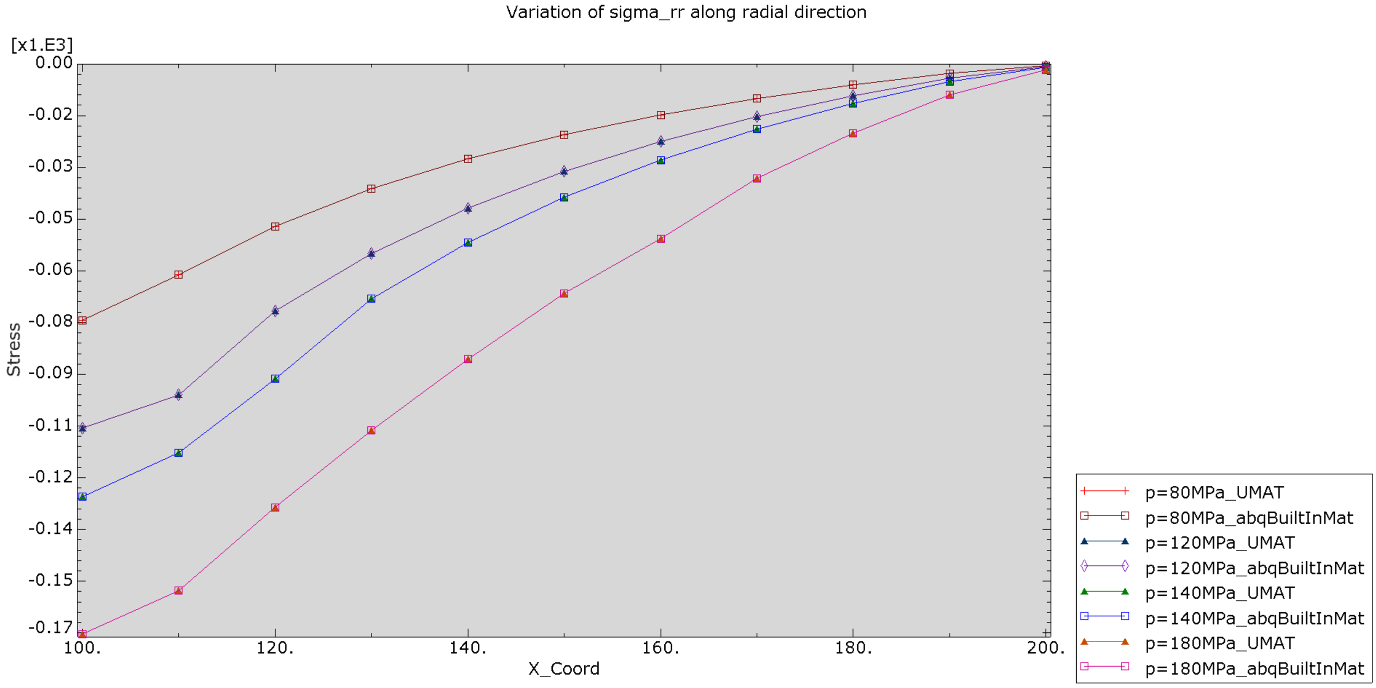
Since we are using von-mises stress criteria, we can see that the yielding starts at 0.577*σ_0 for pure shear.

Plot of yield stress σ_y vs equivalent plastic strain α

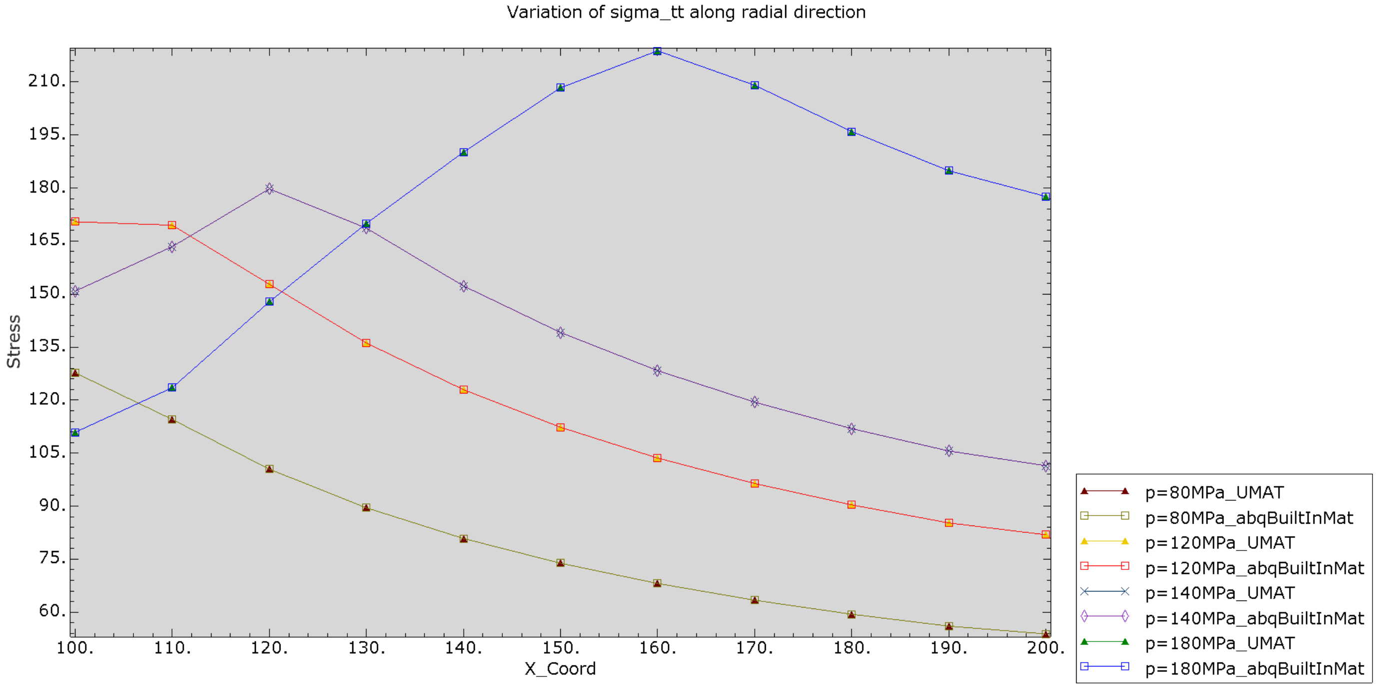


Part c

Variation of σ_{rr} through the thickness of the cylinder



Variation of $\sigma_{\theta\theta}$ through the thickness of the cylinder



Plot of radial displacement of inner surface of the cylinder vs pressure