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4-noded quadrilateral element

shape functions.

$$N_1^{4Q}(\xi,\eta) = \frac{1}{4}(1-\xi)(1-\eta)$$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)$$

$$[x^e \; y^e] = \underset{4\times 2}{\begin{bmatrix} x_1^e & y_1^e \\ x_2^e & y_2^e \\ x_3^e & y_3^e \\ x_4^e & y_4^e \end{bmatrix}},$$

$$[J^e] = \frac{1}{4}\begin{bmatrix} \eta-1 & 1-\eta & 1+\eta & -\eta-1 \\ \xi-1 & -\xi-1 & 1+\xi & 1-\xi \end{bmatrix} [x^e \; y^e]$$

$$|J^e| =$$

$$(J^e)^{-1} =$$

$$[B^e] =$$

$$[K^e] = \int \sum_{i=1}^{2}\sum_{j=1}^{2} W_i W_j \, |J^e(\xi_i,\eta_j)| \; B^{eT}(\xi_i,\eta_j) D^e B^e(\xi_i,\eta_j)$$

$$\begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} & \frac{\partial N_4}{\partial y} \end{bmatrix} = \underset{(\xi_i,\eta_j)}{(J^e)^{-1}} [GN^{4Q}](\xi_i,\eta_j)$$

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$$[GN^{4\varphi}] = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \frac{\partial N_3}{\partial \xi} & \frac{\partial N_4}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \frac{\partial N_3}{\partial \eta} & \frac{\partial N_4}{\partial \eta} \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} \eta-1 & 1-\eta & 1+\eta & -\eta-1 \\ \xi-1 & -\xi-1 & 1+\xi & 1-\xi \end{bmatrix}_{2\times 4}$$

$$[J^e]_{2\times 2} = [GN^{4\varphi}]_{2\times 4} \begin{bmatrix} x^e & y^e \end{bmatrix}_{4\times 2}$$

$$[B^e] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} & 0 & \frac{\partial N_4}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial y} & \frac{\partial N_4}{\partial x} \end{bmatrix}$$

$$[D^e] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

for 2D plain &

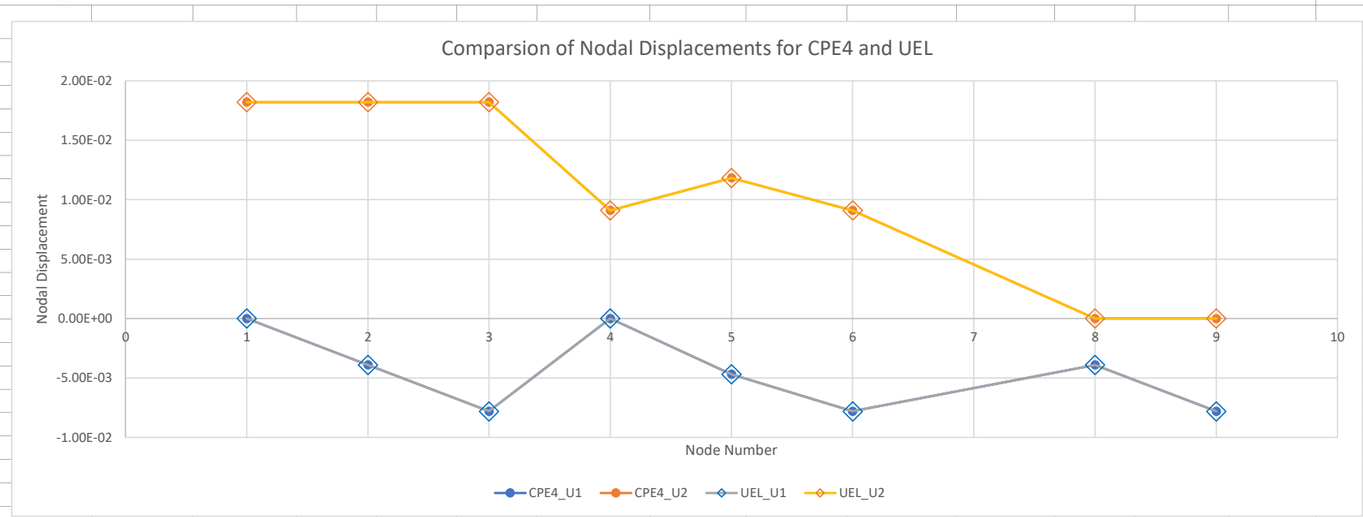
$$\begin{Bmatrix} \sigma_{33} \\ \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = [D_{-e}] \begin{Bmatrix} \varepsilon_{33} \\ \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix}$$

ME759 Assignment 1 (Roll No.203010005, Prafull Bhosale)

Part a

Nodal Displacements

CPE4			UEL		
Nodes	U1	U2	Nodes	U1	U2
1	0.00E+00	1.82E-02	1	0.00E+00	1.82E-02
2	-3.90E-03	1.82E-02	2	-3.90E-03	1.82E-02
3	-7.80E-03	1.82E-02	3	-7.80E-03	1.82E-02
4	0.00E+00	9.10E-03	4	0.00E+00	9.10E-03
5	-4.68E-03	1.18E-02	5	-4.68E-03	1.18E-02
6	-7.80E-03	9.10E-03	6	-7.80E-03	9.10E-03
8	-3.90E-03	0.00E+00	8	-3.90E-03	0.00E+00
9	-7.80E-03	0.00E+00	9	-7.80E-03	0.00E+00



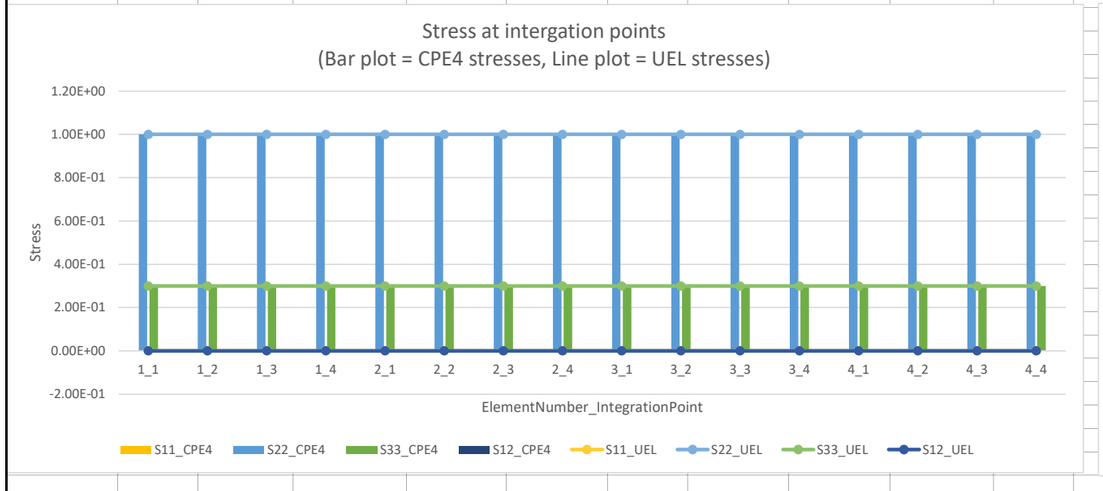
Stresses

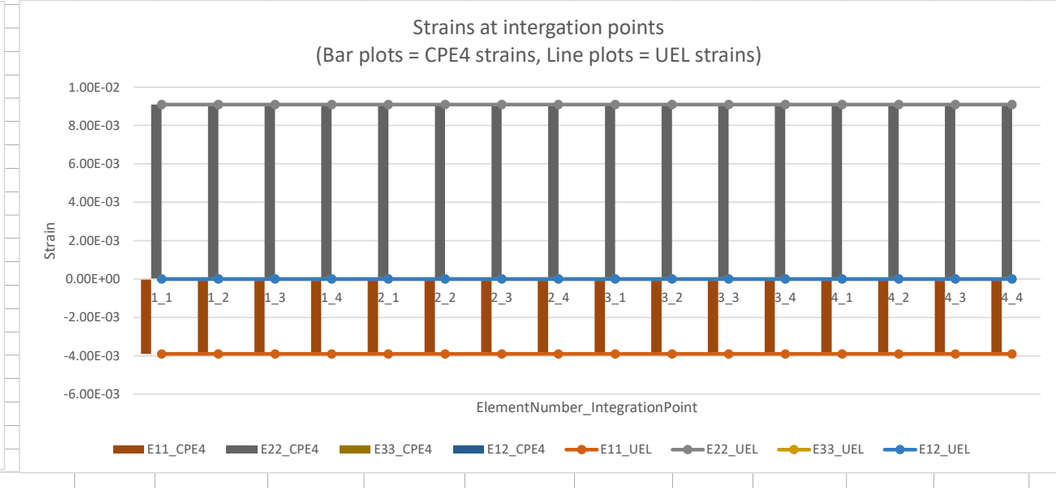
Stress and strains at intergration points for plain strain element available in abaqus (CPE4)

ELEMENT_IntPt	PT	Integration points COORDS COORD1	COORD2	S11_CPE4	S22_CPE4	S33_CPE4	S12_CPE4	E11_CPE4	E22_CPE4	E33_CPE4	E12_CPE4
1_1	1	0.2447	1.261	5.55E-16	1.00E+00	3.00E-01	-1.67E-16	-3.90E-03	9.10E-03	0.00E+00	-4.34E-18
1_2	2	0.9131	1.398	-1.11E-16	1.00E+00	3.00E-01	-1.00E-16	-3.90E-03	9.10E-03	0.00E+00	-2.60E-18
1_3	3	0.2203	1.802	4.44E-16	1.00E+00	3.00E-01	6.67E-17	-3.90E-03	9.10E-03	0.00E+00	1.73E-18
1_4	4	0.822	1.839	0.00E+00	1.00E+00	3.00E-01	2.67E-16	-3.90E-03	9.10E-03	0.00E+00	6.94E-18
2_1	1	1.336	1.398	-1.11E-16	1.00E+00	3.00E-01	2.50E-17	-3.90E-03	9.10E-03	0.00E+00	6.51E-19
2_2	2	1.822	1.261	1.11E-16	1.00E+00	3.00E-01	-1.33E-16	-3.90E-03	9.10E-03	0.00E+00	-3.47E-18
2_3	3	1.245	1.839	-1.11E-16	1.00E+00	3.00E-01	-1.33E-16	-3.90E-03	9.10E-03	0.00E+00	-3.47E-18
2_4	4	1.798	1.802	1.11E-16	1.00E+00	3.00E-01	-6.67E-17	-3.90E-03	9.10E-03	0.00E+00	-1.73E-18
3_1	1	0.2203	0.2247	2.22E-16	1.00E+00	3.00E-01	2.50E-16	-3.90E-03	9.10E-03	0.00E+00	6.51E-18
3_2	2	0.822	0.2613	2.22E-16	1.00E+00	3.00E-01	5.00E-17	-3.90E-03	9.10E-03	0.00E+00	1.30E-18
3_3	3	0.2447	0.8387	0.00E+00	1.00E+00	3.00E-01	5.00E-16	-3.90E-03	9.10E-03	0.00E+00	1.30E-17
3_4	4	0.9131	0.9753	-2.22E-16	1.00E+00	3.00E-01	2.34E-16	-3.90E-03	9.10E-03	0.00E+00	6.07E-18
4_1	1	1.245	0.2613	-2.22E-16	1.00E+00	3.00E-01	-3.34E-17	-3.90E-03	9.10E-03	0.00E+00	-8.67E-19
4_2	2	1.798	0.2247	-3.33E-16	1.00E+00	3.00E-01	1.00E-16	-3.90E-03	9.10E-03	0.00E+00	2.60E-18
4_3	3	1.336	0.9753	0.00E+00	1.00E+00	3.00E-01	6.67E-17	-3.90E-03	9.10E-03	0.00E+00	1.73E-18
4_4	4	1.822	0.8387	1.11E-16	1.00E+00	3.00E-01	1.33E-16	-3.90E-03	9.10E-03	0.00E+00	3.47E-18

Stress and strains at intergration points for UEL

ELEMENT_IntPt	PT	Integration points COORDS COORD1	COORD2	S11_UEL	S22_UEL	S33_UEL	S12_UEL	E11_UEL	E22_UEL	E33_UEL	E12_UEL
1_1	1	0.24466	1.2613	0.00E+00	1.00E+00	3.00E-01	3.34E-17	-3.90E-03	9.10E-03	0.00E+00	8.67E-19
1_3	3	0.22026	1.8021	3.33E-16	1.00E+00	3.00E-01	6.67E-17	-3.90E-03	9.10E-03	0.00E+00	1.73E-18
1_2	2	0.91308	1.3979	1.11E-16	1.00E+00	3.00E-01	1.00E-16	-3.90E-03	9.10E-03	0.00E+00	2.60E-18
1_4	4	0.82201	1.8387	1.11E-16	1.00E+00	3.00E-01	6.67E-17	-3.90E-03	9.10E-03	0.00E+00	1.73E-18
2_1	1	1.3357	1.3979	-5.55E-16	1.00E+00	3.00E-01	-2.38E-16	-3.90E-03	9.10E-03	0.00E+00	-6.18E-18
2_3	3	1.2447	1.8387	-1.11E-16	1.00E+00	3.00E-01	6.67E-17	-3.90E-03	9.10E-03	0.00E+00	1.73E-18
2_2	2	1.822	1.2613	-3.33E-16	1.00E+00	3.00E-01	0.00E+00	-3.90E-03	9.10E-03	0.00E+00	0.00E+00
2_4	4	1.7976	1.8021	1.11E-16	1.00E+00	3.00E-01	0.00E+00	-3.90E-03	9.10E-03	0.00E+00	0.00E+00
3_1	1	0.22026	0.22472	-1.11E-16	1.00E+00	3.00E-01	6.67E-17	-3.90E-03	9.10E-03	0.00E+00	1.73E-18
3_3	3	0.24466	0.83868	-2.22E-16	1.00E+00	3.00E-01	1.67E-16	-3.90E-03	9.10E-03	0.00E+00	4.34E-18
3_2	2	0.82201	0.26132	0.00E+00	1.00E+00	3.00E-01	6.67E-17	-3.90E-03	9.10E-03	0.00E+00	1.73E-18
3_4	4	0.91308	0.97528	1.11E-16	1.00E+00	3.00E-01	1.33E-16	-3.90E-03	9.10E-03	0.00E+00	3.47E-18
4_1	1	1.2447	0.26132	-1.11E-16	1.00E+00	3.00E-01	1.67E-17	-3.90E-03	9.10E-03	0.00E+00	4.34E-19
4_3	3	1.3357	0.97528	-3.33E-16	1.00E+00	3.00E-01	0.00E+00	-3.90E-03	9.10E-03	0.00E+00	0.00E+00
4_2	2	1.7976	0.22472	-1.11E-16	1.00E+00	3.00E-01	-3.34E-17	-3.90E-03	9.10E-03	0.00E+00	-8.67E-19
4_4	4	1.822	0.83868	-3.33E-16	1.00E+00	3.00E-01	-6.67E-17	-3.90E-03	9.10E-03	0.00E+00	-1.73E-18





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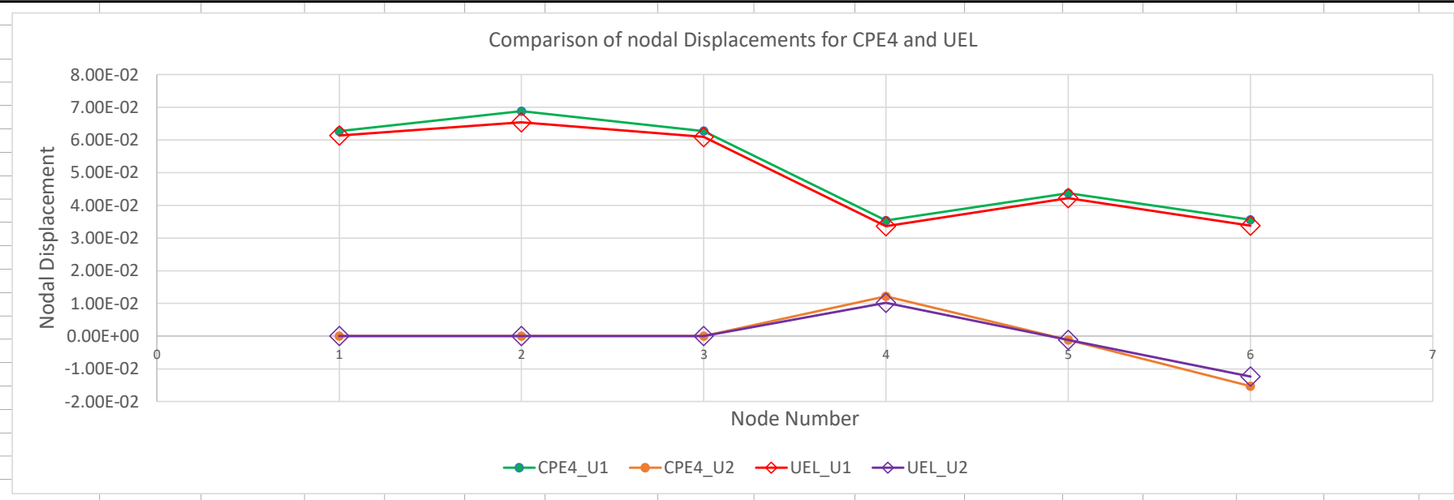
Part b

Nodal Displacements

CPE4			UEL		
Nodes	U1	U2	Nodes	U1	U2
1	6.27E-02	0.00E+00	1	6.14E-02	0.00E+00
2	6.88E-02	0.00E+00	2	6.54E-02	0.00E+00
3	6.27E-02	0.00E+00	3	6.09E-02	0.00E+00
4	3.54E-02	1.22E-02	4	3.36E-02	1.02E-02
5	4.37E-02	-1.16E-03	5	4.23E-02	-1.18E-03
6	3.56E-02	-1.53E-02	6	3.38E-02	-1.24E-02



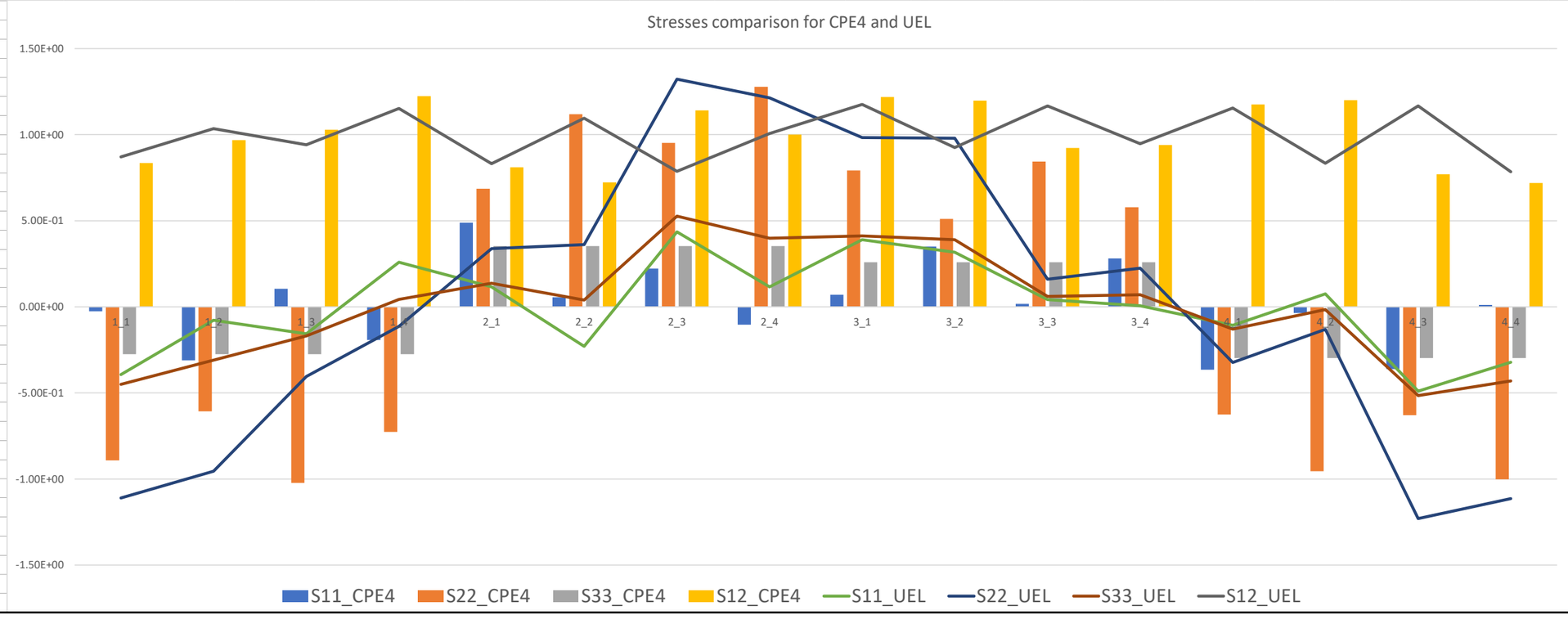
Stresses and strains at integration points

Stress and strains at intergration points for plain strain element available in abaqus (CPE4)

ELEMENT_Int Pt	PT	S11_CPE4	S22_CPE4	S33_CPE4	S12_CPE4	E11_CPE4	E22_CPE4	E33_CPE4	E12_CPE4
1_1	1	-2.63E-02	-0.892	-0.2755	0.8348	3.24E-03	-8.01E-03	0	2.17E-02
1_2	2	-0.3112	-0.6071	-0.2755	0.9681	-4.65E-04	-4.31E-03	0	2.52E-02
1_3	3	0.1051	-1.023	-0.2755	1.029	4.95E-03	-9.72E-03	0	2.68E-02
1_4	4	-0.1924	-0.7259	-0.2755	1.224	1.08E-03	-5.86E-03	0	3.18E-02
2_1	1	0.489	0.6862	0.3526	0.8101	1.77E-03	4.34E-03	0	2.11E-02
2_2	2	5.59E-02	1.119	0.3526	0.7223	-3.86E-03	9.97E-03	0	1.88E-02
2_3	3	0.2223	0.9529	0.3526	1.141	-1.69E-03	7.80E-03	0	2.97E-02
2_4	4	-0.1032	1.278	0.3526	1.001	-5.93E-03	1.20E-02	0	2.60E-02
3_1	1	6.97E-02	0.7919	0.2585	1.219	-2.45E-03	6.93E-03	0	3.17E-02
3_2	2	0.3506	0.511	0.2585	1.198	1.20E-03	3.28E-03	0	3.12E-02
3_3	3	1.78E-02	0.8437	0.2585	0.9226	-3.13E-03	7.61E-03	0	2.40E-02
3_4	4	0.2819	0.5796	0.2585	0.9405	3.05E-04	4.18E-03	0	2.45E-02
4_1	1	-0.365	-0.6253	-0.2971	1.176	-8.83E-04	-4.27E-03	0	3.06E-02
4_2	2	-3.55E-02	-0.9548	-0.2971	1.2	3.40E-03	-8.55E-03	0	3.12E-02
4_3	3	-0.361	-0.6293	-0.2971	0.7703	-8.30E-04	-4.32E-03	0	2.00E-02
4_4	4	1.13E-02	-1.002	-0.2971	0.7189	4.01E-03	-9.16E-03	0	1.87E-02

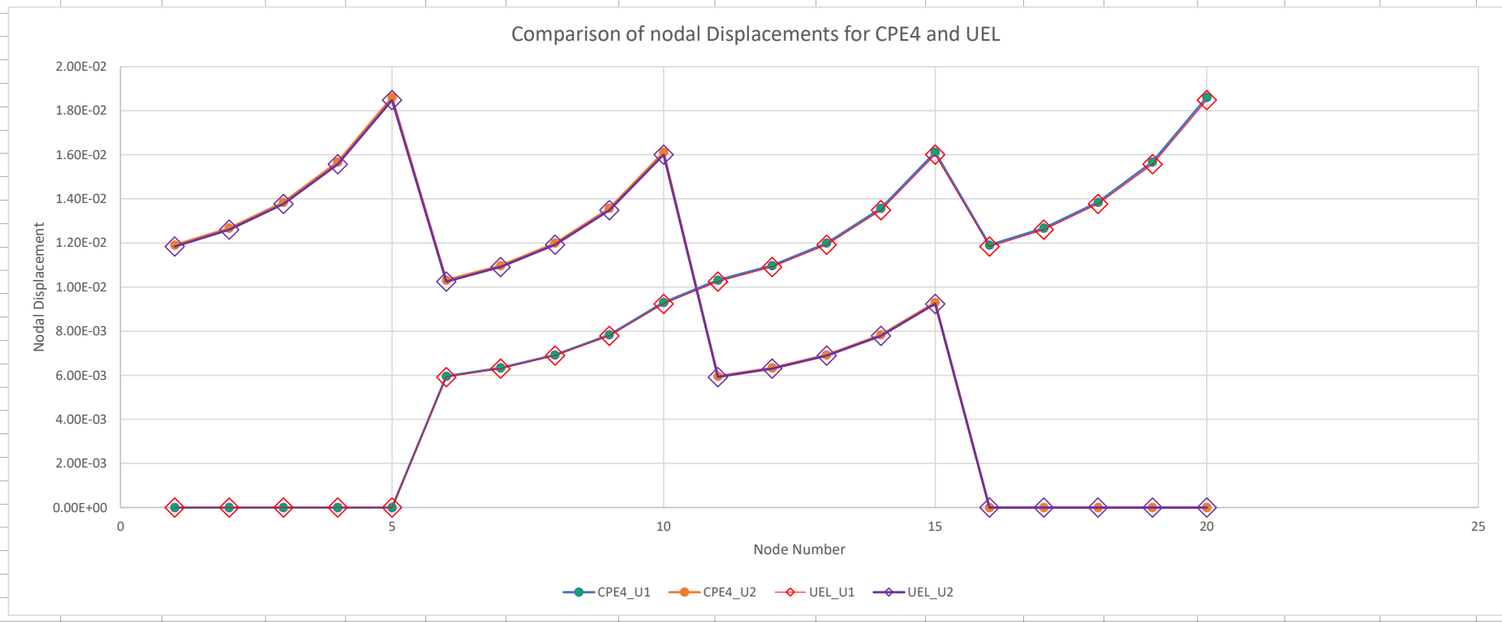
Stress and strains at intergration points for UEL

ELEMENT_IntPt	COORD1	COORD2	S11_UEL	S22_UEL	S33_UEL	S12_UEL	E11_UEL	E22_UEL	E33_UEL	E12_UEL
1_1	2.45E-01	1.26E+00	-3.93E-01	-1.11E+00	-4.51E-01	8.71E-01	7.57E-04	-8.57E-03	0.00E+00	2.27E-02
1_3	2.20E-01	1.80E+00	-7.76E-02	-9.54E-01	-3.10E-01	1.04E+00	3.02E-03	-8.38E-03	0.00E+00	2.69E-02
1_2	9.13E-01	1.40E+00	-1.57E-01	-4.04E-01	-1.68E-01	9.42E-01	1.50E-04	-3.07E-03	0.00E+00	2.45E-02
1_4	8.22E-01	1.84E+00	2.60E-01	-1.14E-01	4.38E-02	1.15E+00	2.81E-03	-2.05E-03	0.00E+00	3.00E-02
2_1	1.34E+00	1.40E+00	1.16E-01	3.39E-01	1.36E-01	8.31E-01	-2.62E-04	2.63E-03	0.00E+00	2.16E-02
2_3	1.24E+00	1.84E+00	-2.30E-01	3.62E-01	3.97E-02	1.10E+00	-3.50E-03	4.19E-03	0.00E+00	2.85E-02
2_2	1.82E+00	1.26E+00	4.36E-01	1.32E+00	5.27E-01	7.88E-01	-1.19E-03	1.03E-02	0.00E+00	2.05E-02
2_4	1.80E+00	1.80E+00	1.16E-01	1.21E+00	3.99E-01	1.01E+00	-3.68E-03	1.06E-02	0.00E+00	2.62E-02
3_1	2.20E-01	2.25E-01	3.91E-01	9.83E-01	4.12E-01	1.18E+00	-2.78E-04	7.42E-03	0.00E+00	3.06E-02
3_3	2.45E-01	8.39E-01	3.17E-01	9.80E-01	3.89E-01	9.26E-01	-9.38E-04	7.68E-03	0.00E+00	2.41E-02
3_2	8.22E-01	2.61E-01	4.22E-02	1.60E-01	6.07E-02	1.17E+00	-2.40E-04	1.29E-03	0.00E+00	3.03E-02
3_4	9.13E-01	9.75E-01	6.32E-03	2.25E-01	6.95E-02	9.48E-01	-8.22E-04	2.03E-03	0.00E+00	2.47E-02
4_1	1.24E+00	2.61E-01	-1.07E-01	-3.24E-01	-1.29E-01	1.16E+00	2.89E-04	-2.53E-03	0.00E+00	3.00E-02
4_3	1.34E+00	9.75E-01	7.54E-02	-1.30E-01	-1.63E-02	8.34E-01	1.19E-03	-1.47E-03	0.00E+00	2.17E-02
4_2	1.80E+00	2.25E-01	-4.90E-01	-1.23E+00	-5.16E-01	1.17E+00	3.38E-04	-9.28E-03	0.00E+00	3.03E-02
4_4	1.82E+00	8.39E-01	-3.22E-01	-1.11E+00	-4.31E-01	7.85E-01	1.42E-03	-8.89E-03	0.00E+00	2.04E-02



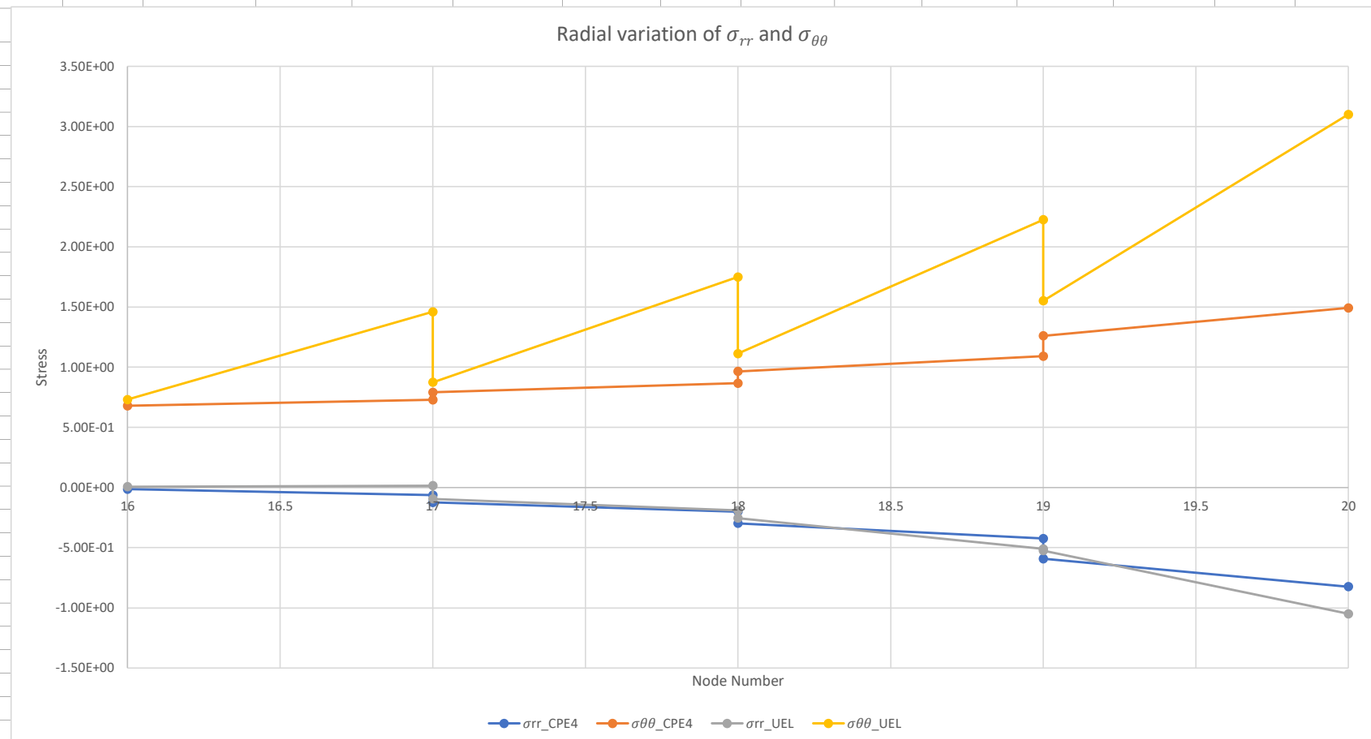
Part c

CPE4			UEL		
Nodes	U1	U2	Nodes	U1	U2
1	0.00E+00	1.19E-02	1	0.00E+00	1.18E-02
2	0.00E+00	1.27E-02	2	0.00E+00	1.26E-02
3	0.00E+00	1.39E-02	3	0.00E+00	1.38E-02
4	0.00E+00	1.57E-02	4	0.00E+00	1.56E-02
5	0.00E+00	1.86E-02	5	0.00E+00	1.85E-02
6	5.95E-03	1.03E-02	6	5.92E-03	1.03E-02
7	6.34E-03	1.10E-02	7	6.30E-03	1.09E-02
8	6.93E-03	1.20E-02	8	6.89E-03	1.19E-02
9	7.83E-03	1.36E-02	9	7.79E-03	1.35E-02
10	9.30E-03	1.61E-02	10	9.24E-03	1.60E-02
11	1.03E-02	5.95E-03	11	1.03E-02	5.92E-03
12	1.10E-02	6.34E-03	12	1.09E-02	6.30E-03
13	1.20E-02	6.93E-03	13	1.19E-02	6.89E-03
14	1.36E-02	7.83E-03	14	1.35E-02	7.79E-03
15	1.61E-02	9.30E-03	15	1.60E-02	9.24E-03
16	1.19E-02	0.00E+00	16	1.18E-02	0.00E+00
17	1.27E-02	0.00E+00	17	1.26E-02	0.00E+00
18	1.39E-02	0.00E+00	18	1.38E-02	0.00E+00
19	1.57E-02	0.00E+00	19	1.56E-02	0.00E+00
20	1.86E-02	0.00E+00	20	1.85E-02	0.00E+00



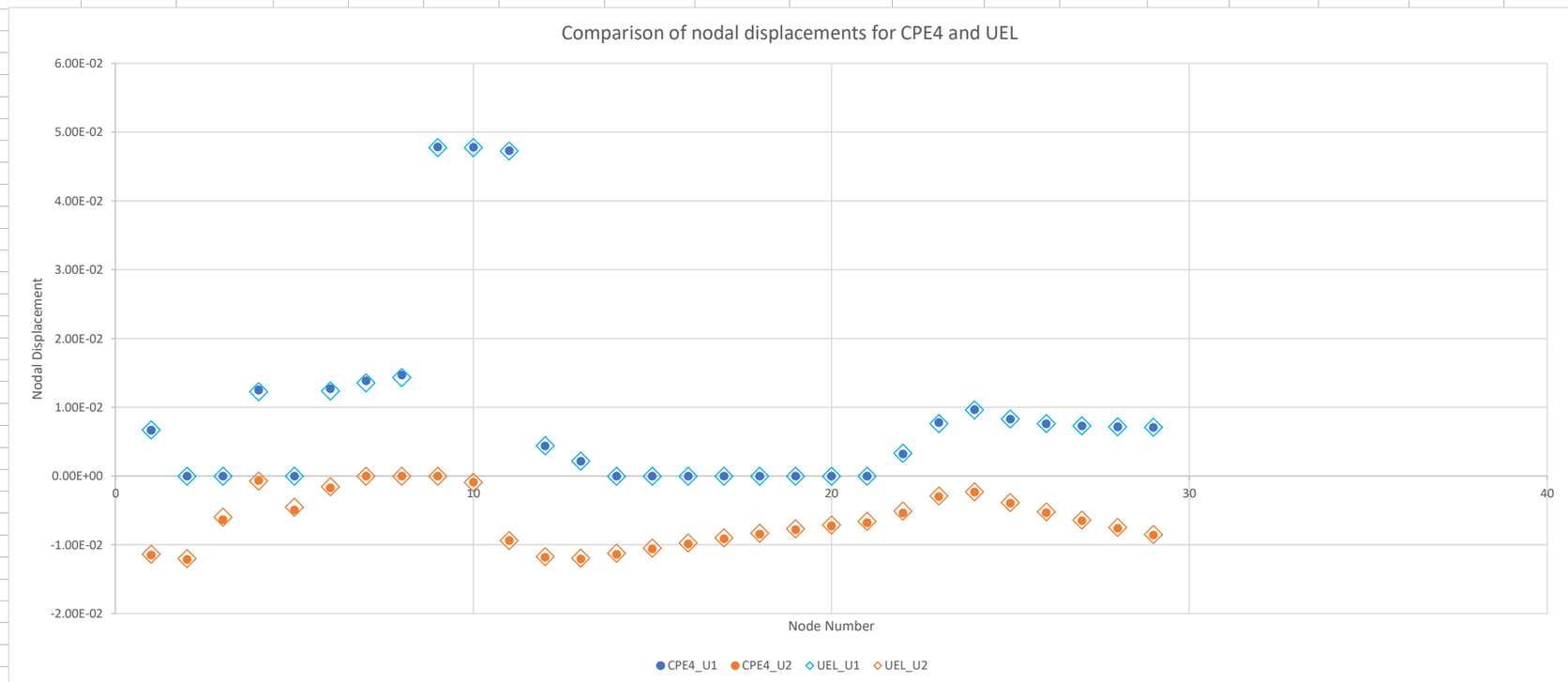
Radial variation of sigma_rr and sigma_tt for CPE4 and UEL

CPE4				UEL					
ELEMENT	ND	σrr_CPE4	$\sigma\theta\theta$_CPE4	ELEMENT	Nodes	σrr_UEL	$\sigma\theta\theta$_UEL	S33	S12
9	16	-1.39E-02	0.6798	9	16	7.17E-03	0.7306	0.2213	-0.1403
9	17	-6.34E-02	0.7293	9	17	1.43E-02	1.461	0.4427	-0.2807
10	17	-0.1244	0.7906	10	17	-9.54E-02	0.8746	0.2338	-0.1882
10	18	-0.2007	0.8669	10	18	-0.1909	1.749	0.4675	-0.3764
11	18	-0.2978	0.9646	11	18	-0.2555	1.113	0.2573	-0.2655
11	19	-0.424	1.091	11	19	-0.511	2.226	0.5146	-0.531
12	19	-0.5924	1.261	12	19	-0.5249	1.551	0.3077	-0.4027
12	20	-0.824	1.493	12	20	-1.05	3.101	0.6154	-0.8053



Part d

CPE4				UEL		
Nodes	U1	U2		Nodes	U1	U2
1	6.67E-03	-1.15E-02		1	6.69E-03	-1.14E-02
2	0.00E+00	-1.21E-02		2	0.00E+00	-1.20E-02
3	0.00E+00	-6.32E-03		3	0.00E+00	-5.98E-03
4	1.25E-02	-6.48E-04		4	1.23E-02	-7.06E-04
5	0.00E+00	-4.92E-03		5	0.00E+00	-4.51E-03
6	1.27E-02	-1.72E-03		6	1.24E-02	-1.55E-03
7	1.39E-02	0.00E+00		7	1.35E-02	0.00E+00
8	1.47E-02	0.00E+00		8	1.43E-02	0.00E+00
9	4.78E-02	0.00E+00		9	4.78E-02	0.00E+00
10	4.78E-02	-8.86E-04		10	4.78E-02	-8.90E-04
11	4.73E-02	-9.34E-03		11	4.73E-02	-9.36E-03
12	4.38E-03	-1.18E-02		12	4.40E-03	-1.17E-02
13	2.17E-03	-1.20E-02		13	2.18E-03	-1.19E-02
14	0.00E+00	-1.14E-02		14	0.00E+00	-1.12E-02
15	0.00E+00	-1.06E-02		15	0.00E+00	-1.05E-02
16	0.00E+00	-9.84E-03		16	0.00E+00	-9.71E-03
17	0.00E+00	-9.11E-03		17	0.00E+00	-8.98E-03
18	0.00E+00	-8.42E-03		18	0.00E+00	-8.29E-03
19	0.00E+00	-7.80E-03		19	0.00E+00	-7.65E-03
20	0.00E+00	-7.25E-03		20	0.00E+00	-7.08E-03
21	0.00E+00	-6.78E-03		21	0.00E+00	-6.57E-03
22	3.22E-03	-5.35E-03		22	3.30E-03	-5.06E-03
23	7.77E-03	-2.97E-03		23	7.66E-03	-2.88E-03
24	9.67E-03	-2.32E-03		24	9.60E-03	-2.30E-03
25	8.28E-03	-3.93E-03		25	8.29E-03	-3.84E-03
26	7.62E-03	-5.30E-03		26	7.64E-03	-5.18E-03
27	7.31E-03	-6.49E-03		27	7.32E-03	-6.37E-03
28	7.16E-03	-7.57E-03		28	7.17E-03	-7.46E-03
29	7.08E-03	-8.59E-03		29	7.08E-03	-8.48E-03

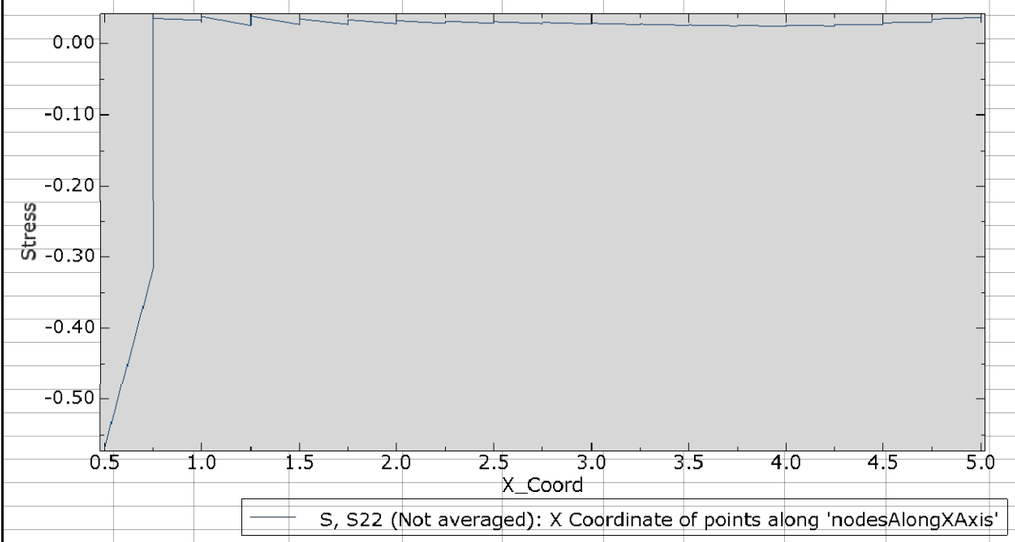


Course mesh(SIF = 2.57607)

Variation of sigma_yy along X axis

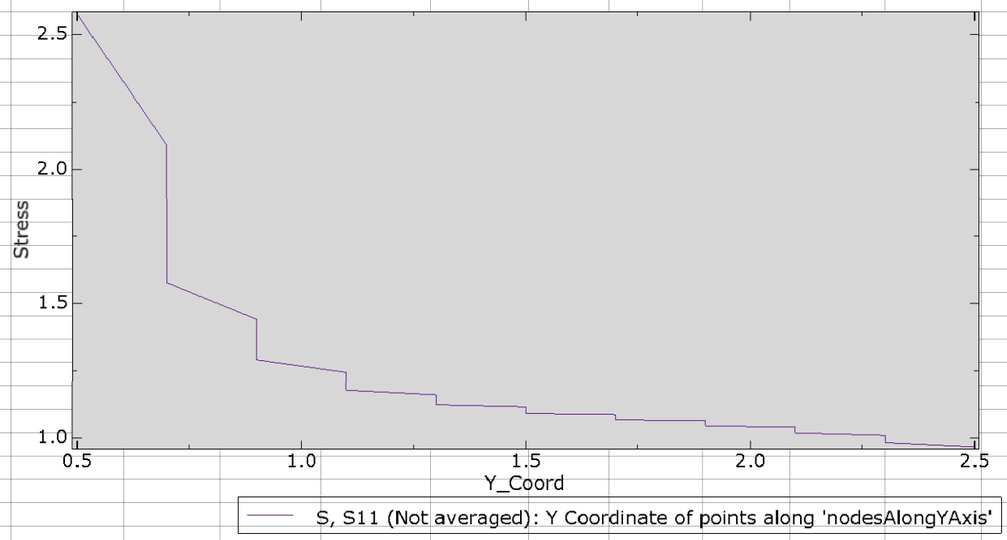
Values are determined at nodes

Stress field is discontinuous for linear element. Hence, we are seeing discontinuos lines in plot



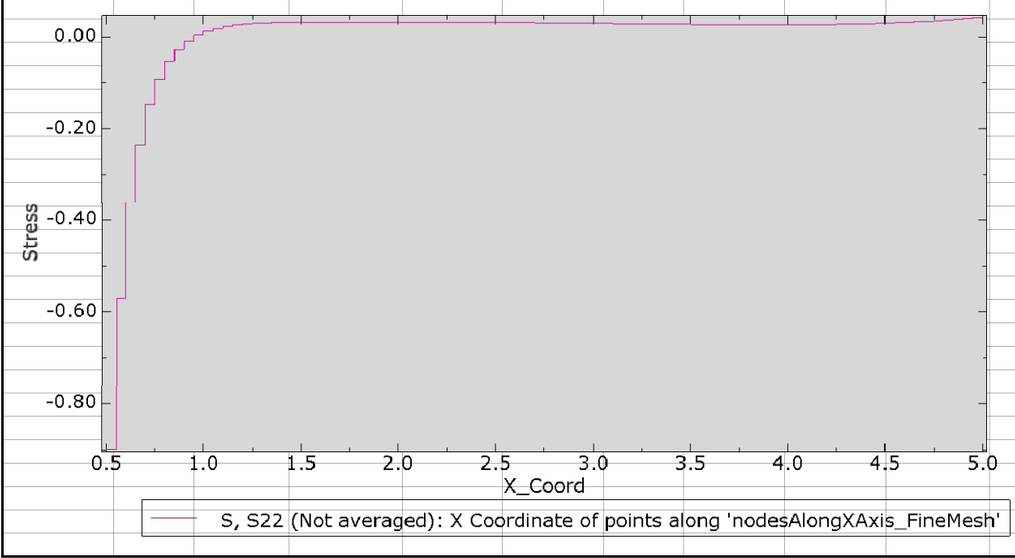
Variation of sigma_xx along Y axis

For course mesh, we get stress intensity factor = 2.57607



Fine mesh (SIF = 2.8410)

Variation of sigma_yy along X axis



Variation of sigma_xx along Y axis