ME 759: Nonlinear FEM
Assignment 3: 1-D Elasto-viscoplasticity

Equations used for coding

23/03/2022 Assignment 3 – 1D viscoplasticity 203010005 Prafull Bhosale

Variables passed in as a information into UMAT from Abaqus

$\varepsilon_n = \text{STATEV(1)}$

$\varepsilon_n^{vp} = \text{STATEV(2)}$

$\beta_n = \text{STATEV(3)}$

$\alpha_n = \text{STATEV(4)}$

$\Delta\varepsilon_n = \text{DSTRAN(1)}$

~~Algorithm:~~

Equations used in Fortran code:

1) Trial stress: $\sigma_{n+1}^{t} = \sigma_n + E\,\Delta\varepsilon_n \rightarrow \text{STRESS(1)} = \text{STRESS(1)} + E * \text{DSTAN(1)}$

2) $\varepsilon_{n+1} = \varepsilon_n + \Delta\varepsilon_n$

3) Relative trial stress

$\xi_{n+1}^{t} = \sigma_{n+1}^{t} - \beta_n$

4) Back stress trial: $\beta_{n+1}^{t} = \beta_n$

5) Equivalent viscoplastic strain trial: $\alpha_{n+1}^{t} = \alpha_n$

6) Viscoplastic trial strain: $\varepsilon_{n+1}^{vp\,t} = \varepsilon_n^{vp}$

7) Loading function trial: $f_{n+1}^{t} = |\xi_{n+1}^{t}| - \sigma_y(\alpha_n)$

$= |\xi_{n+1}^{t}| - [\sigma_0 + H * \theta * \alpha_n]$ -- for linear hardening

if $f_{n+1}^{t} <= 0$:

DDSDDE = E

$\varepsilon_{n+1}^{vp} = \varepsilon_n^{vp}$, $\alpha_{n+1} = \alpha_n$, $\beta_{n+1} = \beta_n$

$\sigma_{n+1} = \sigma_{n+1}^{t}$

else: a) $\Delta\gamma = \dfrac{f_{n+1}^{t}}{E+H} * \dfrac{\Delta t/\tau}{1+\Delta t/\tau}$

b) $\sigma_{n+1} = \sigma_{n+1}^{t} - \Delta\gamma * E * \text{sign}(\xi_{n+1}^{t})$

c) $\varepsilon_{n+1}^{vp} = \varepsilon_{n+1}^{vp\,t} + \Delta\gamma * \text{sign}(\xi_{n+1}^{t})$

d) $\beta_{n+1} = \beta_{n+1}^{t} + \Delta\gamma * (1-\theta) * H * \text{sign}(\xi_{n+1}^{t})$

e) $\text{DDSDDE}(1,1) = \dfrac{E}{1+\frac{\Delta t}{\tau}} + \dfrac{\Delta t/\tau}{1+\frac{\Delta t}{\tau}} \left(\dfrac{EH}{E+H}\right)$

where $\tau \rightarrow$ Relaxation time $= \dfrac{\eta}{E+H}$

$\eta \rightarrow$ viscosity parameter

$\theta \rightarrow$ Parameter for combined isotropic-kinematic hardening

For given problem, $\theta = 1$

E_t = tangat modulus

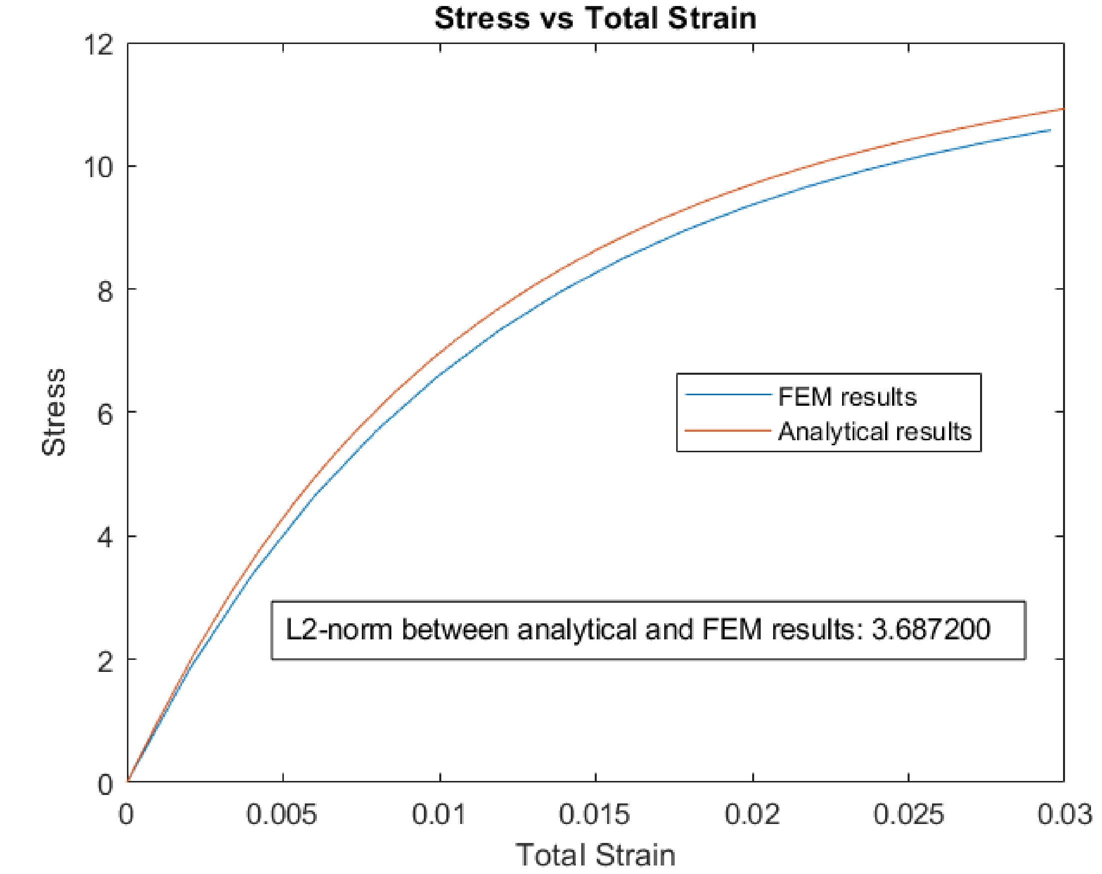
$$H = \frac{E * E_t}{E - E_t}$$

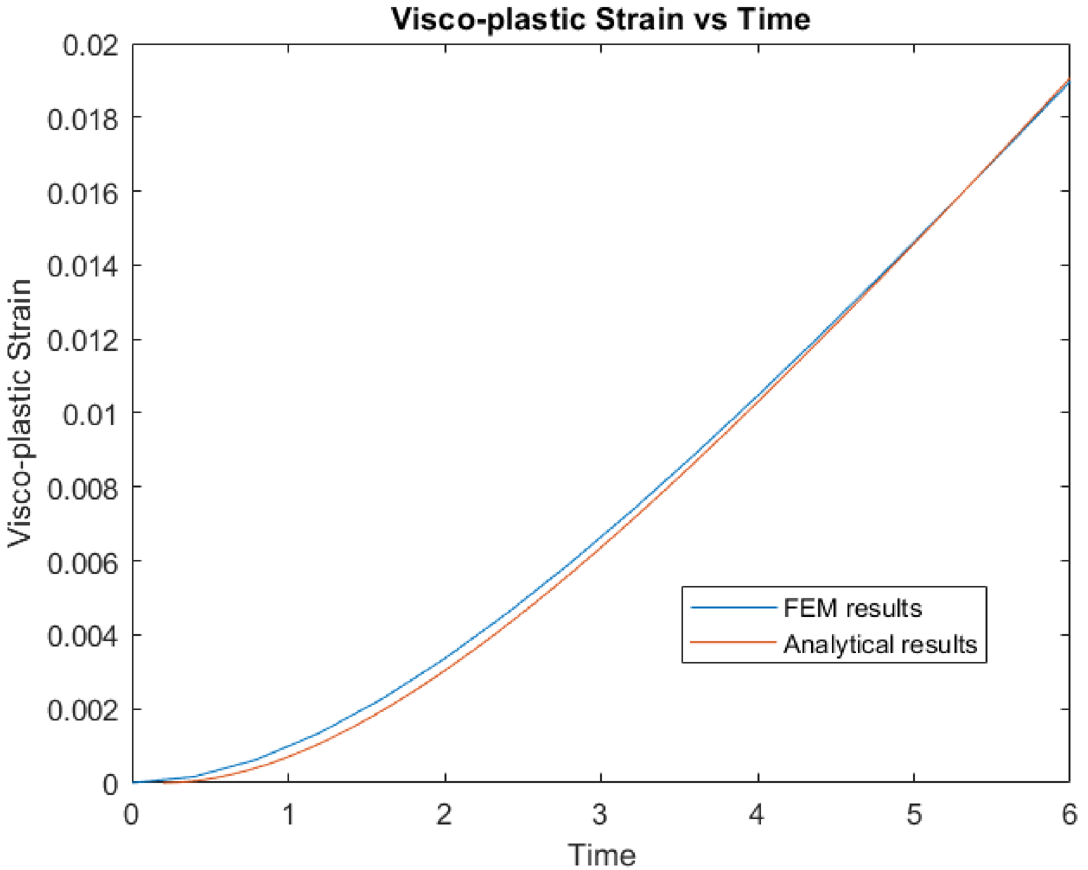
sigma_0 = σ_0 = Initial yield stress.

t = t1

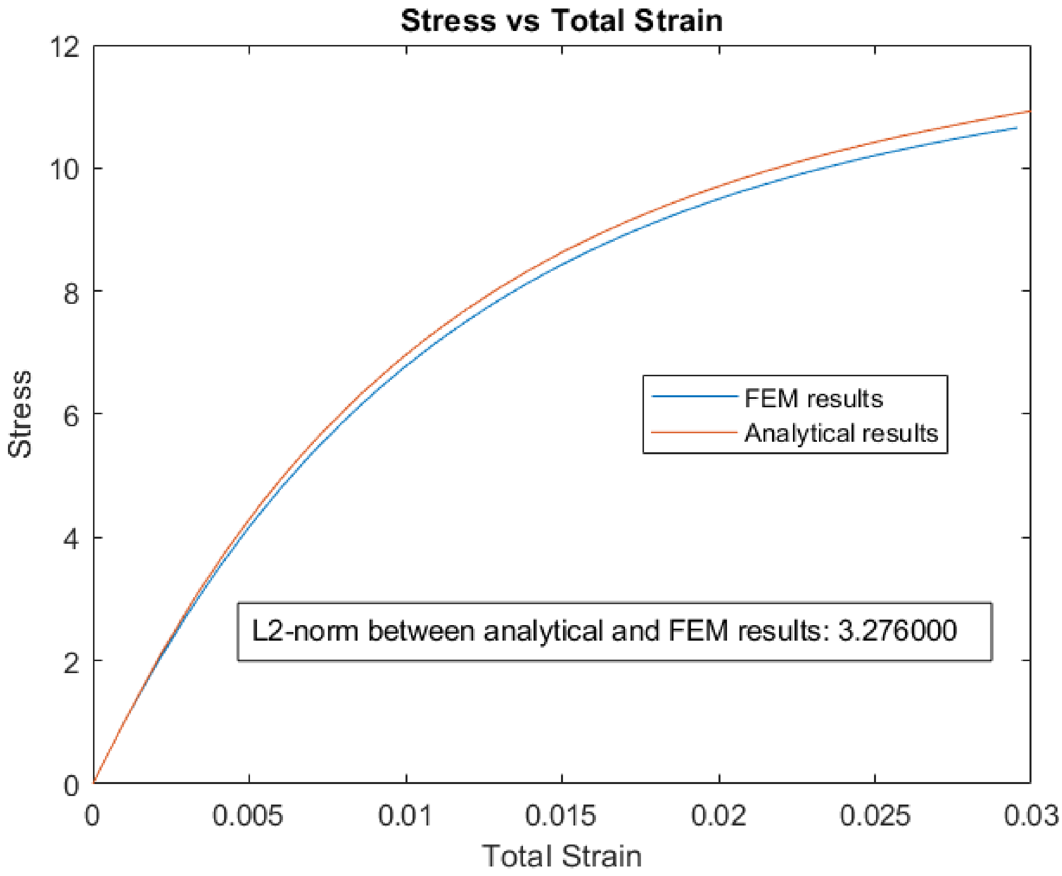
t1 = 6

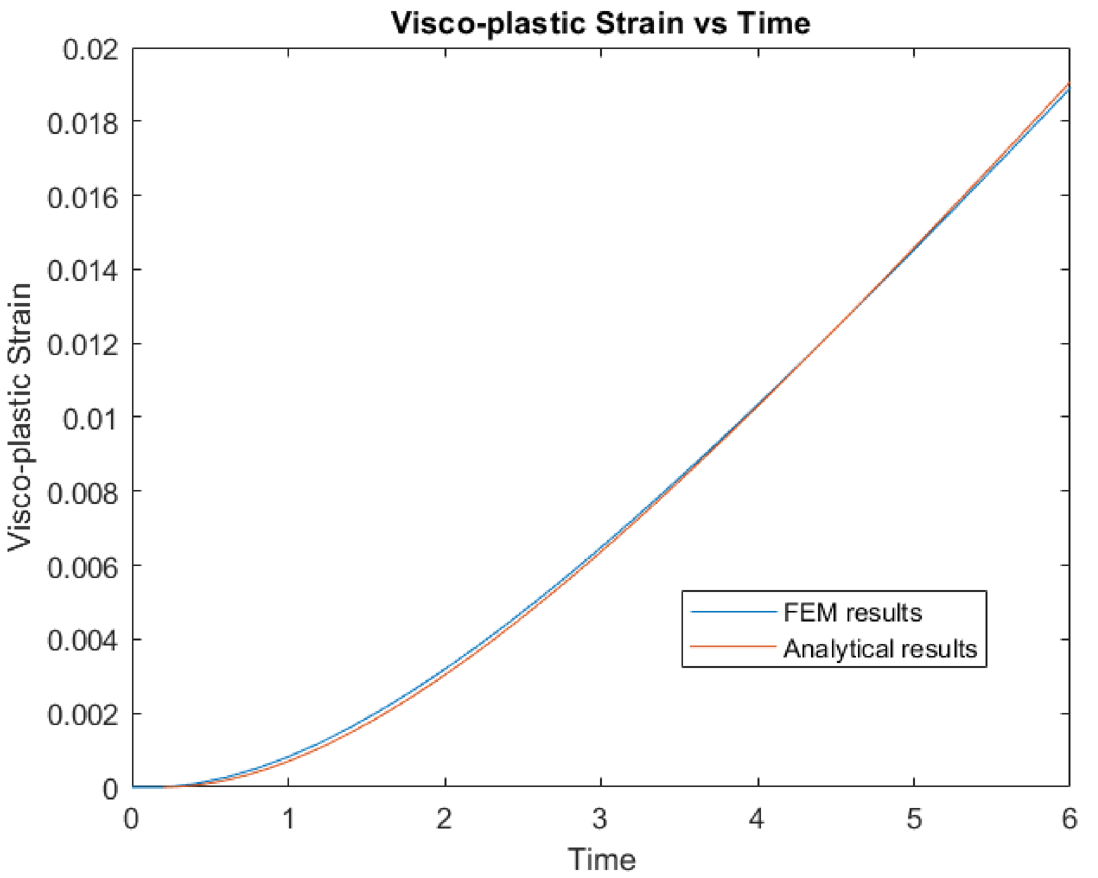
c = 0.005, $\Delta t = 2 * \frac{\epsilon_0}{c} = 0.4$



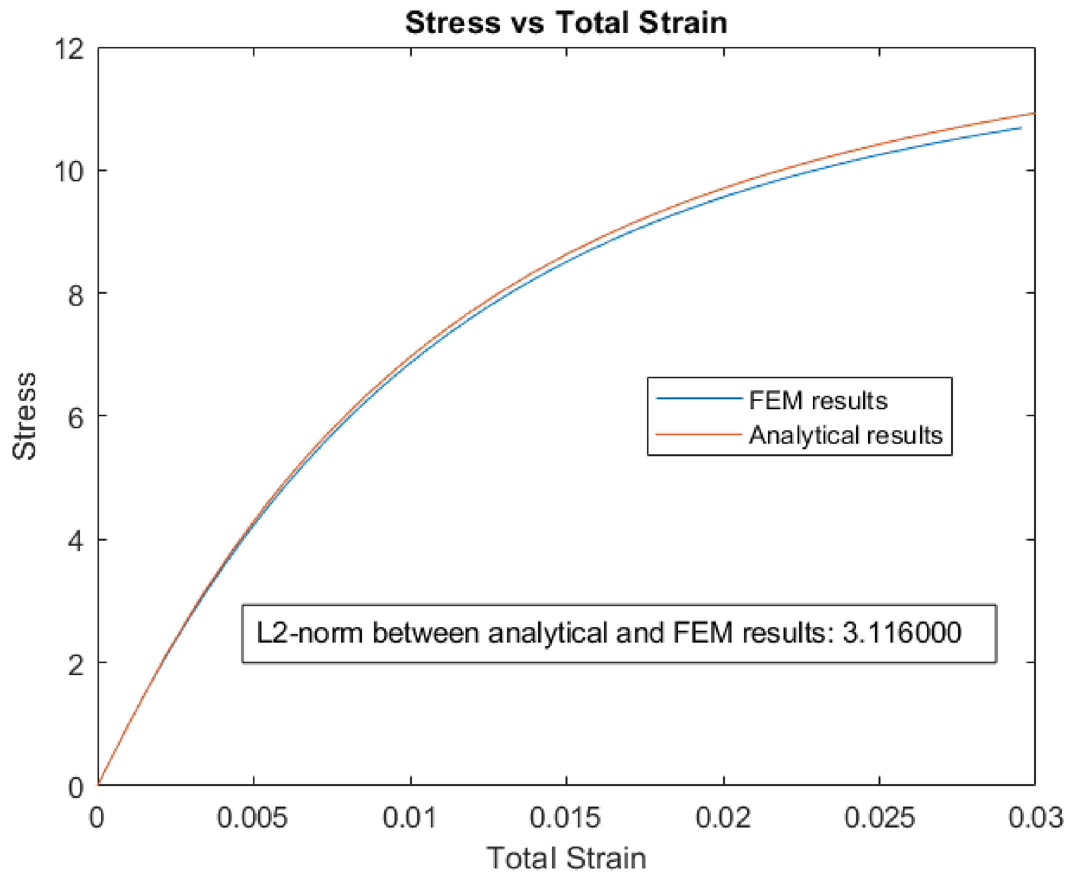


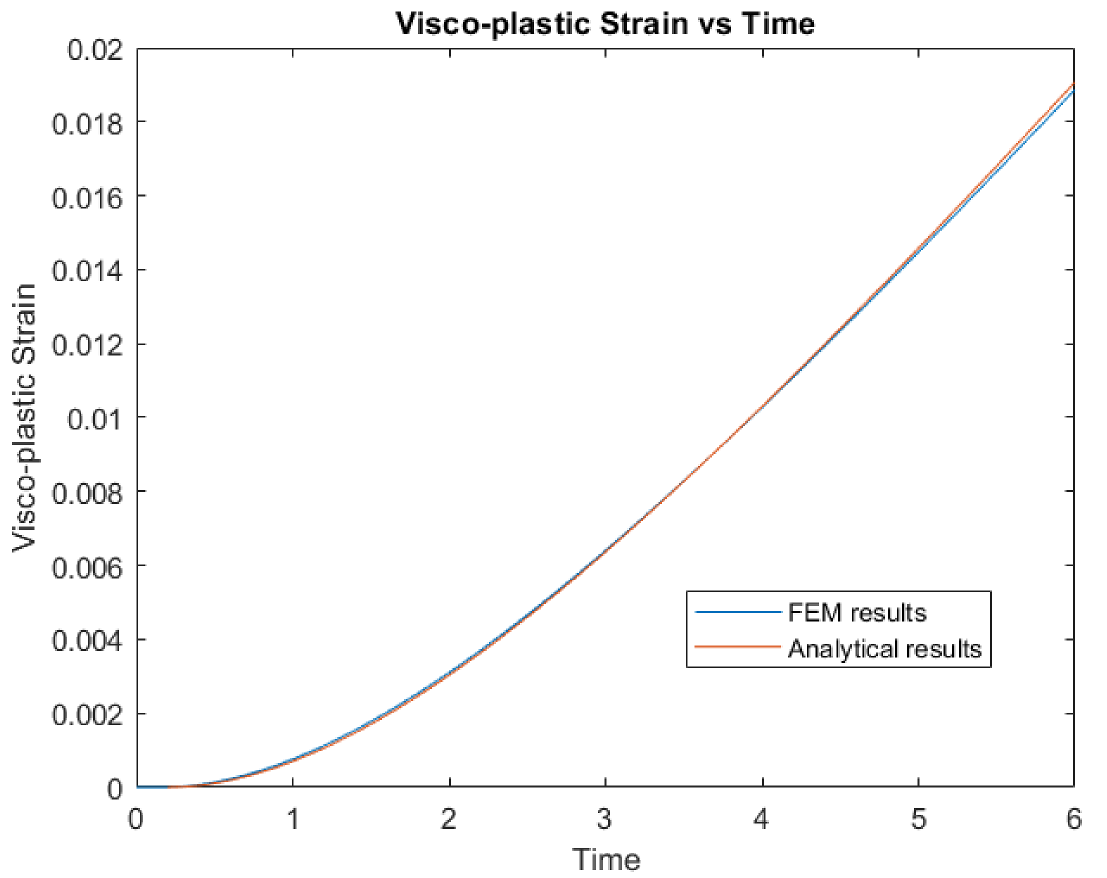
c = 0.005, $\Delta t = \frac{\epsilon_0}{c} = 0.2$





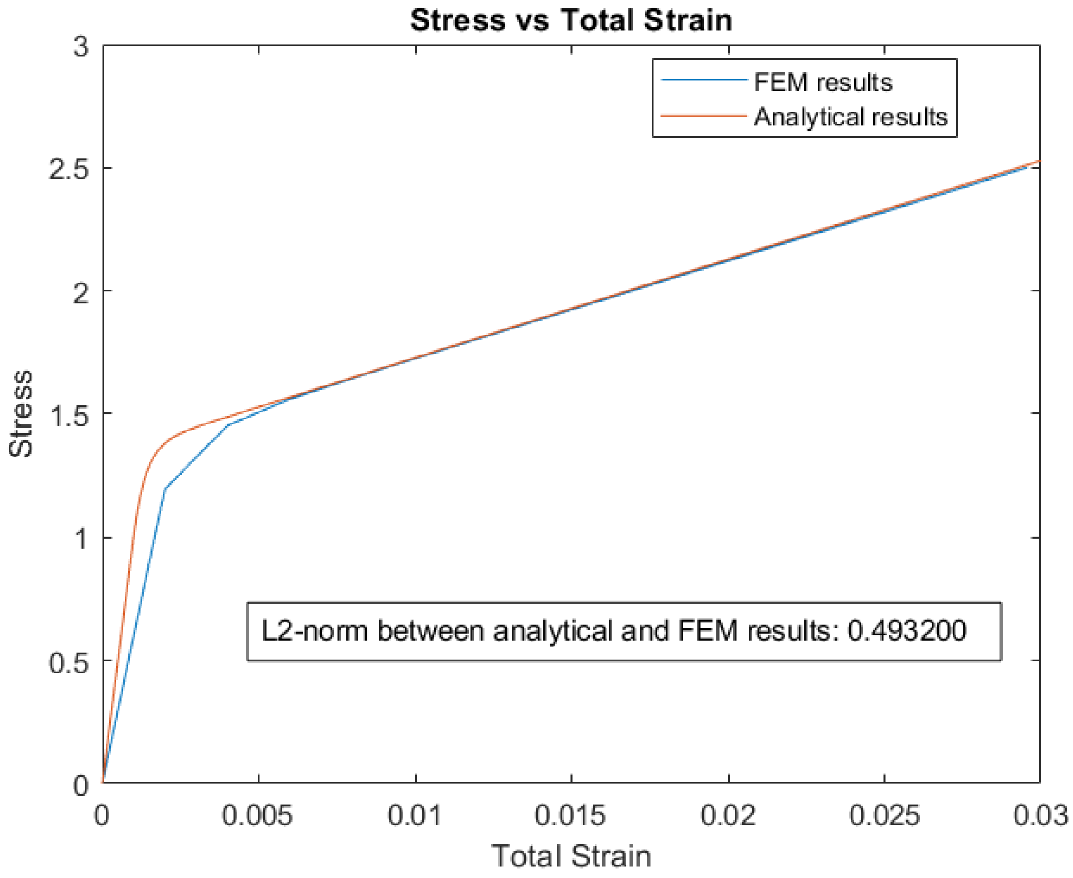
c = 0.005, $\Delta t = 0.5 * \frac{\epsilon_0}{c} = 0.1$

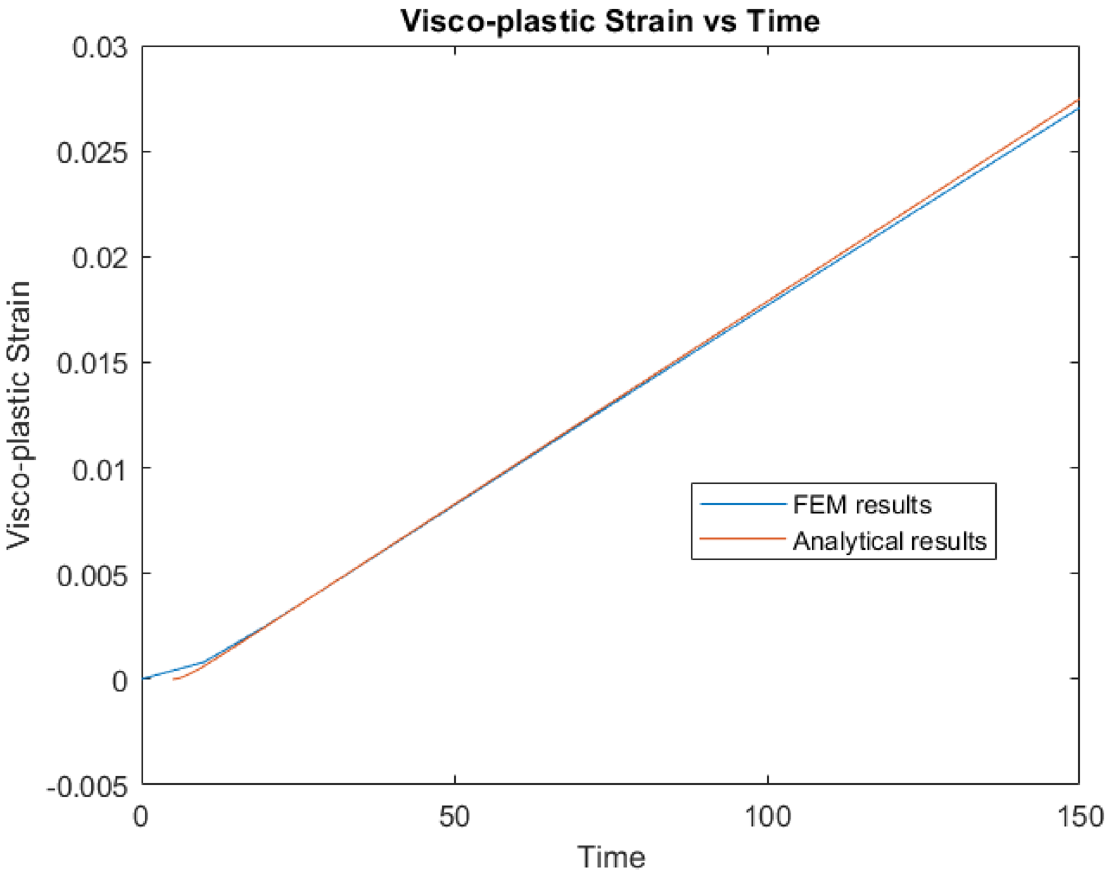




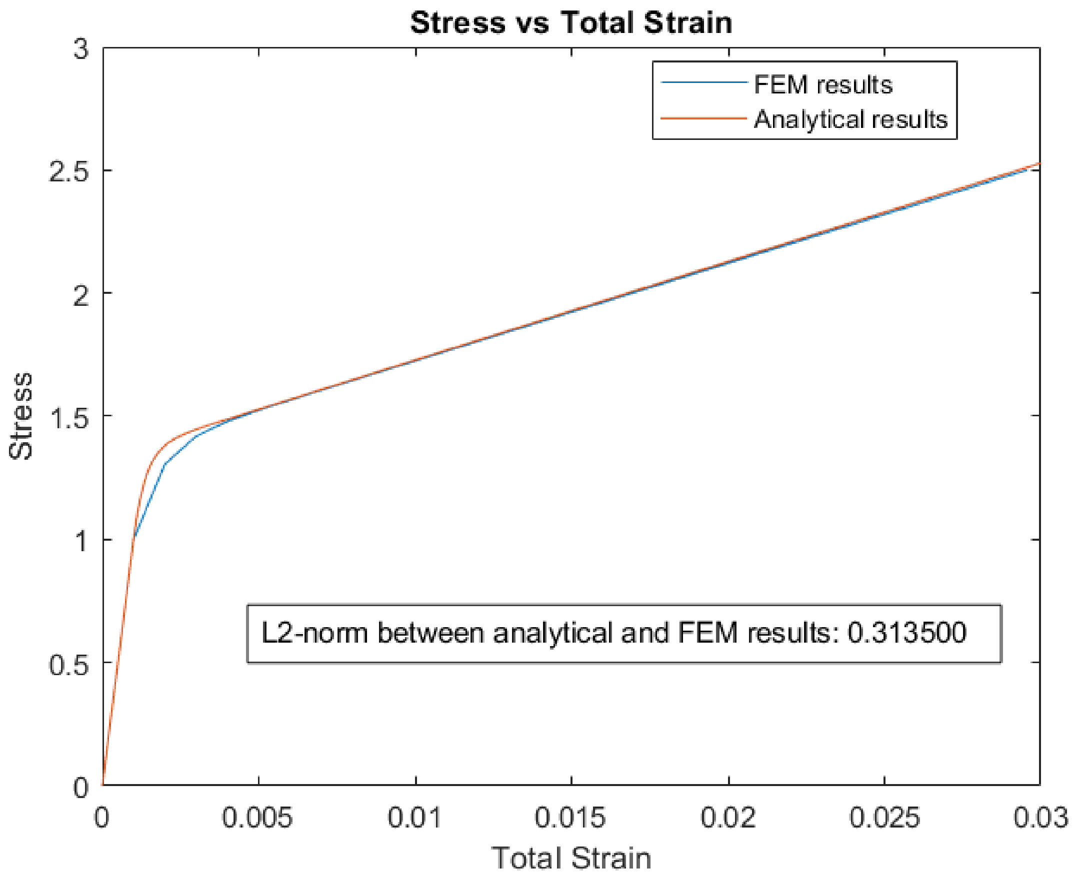
t1 = 150

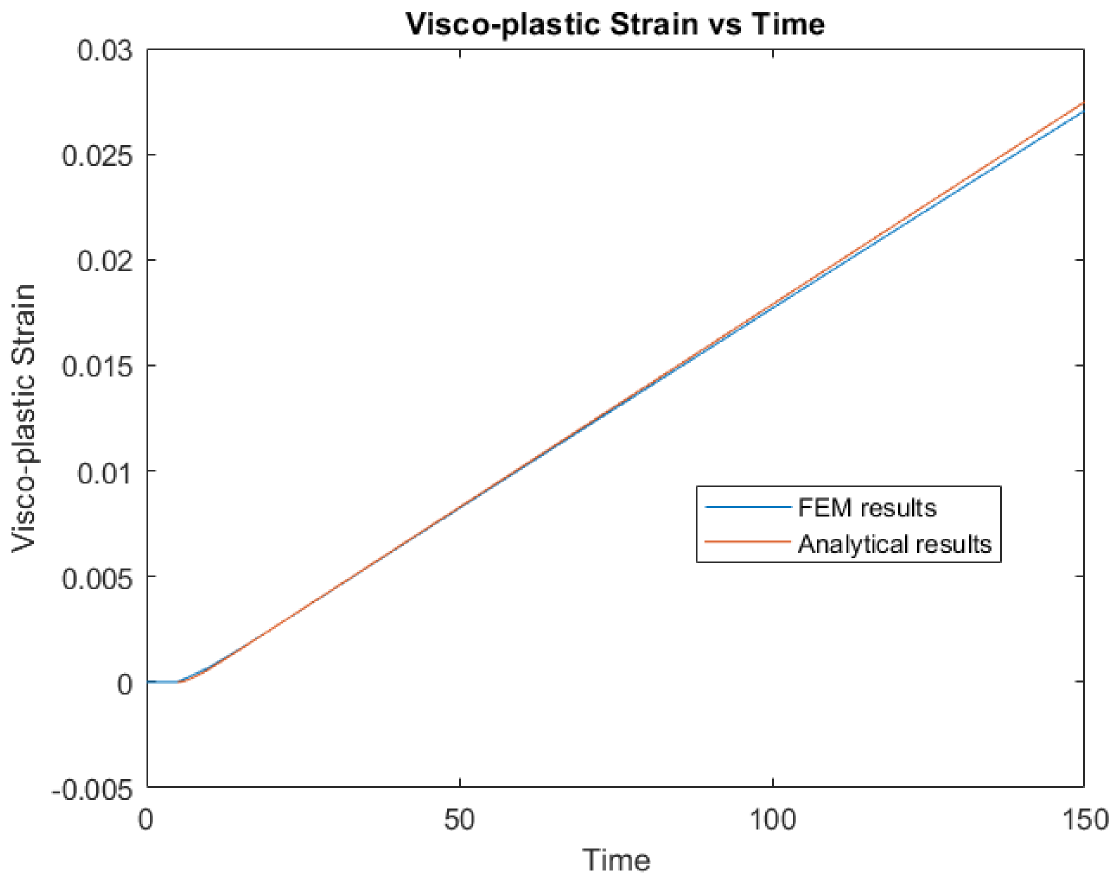
c = 0.0002, $\Delta t = 2 * \frac{\epsilon_0}{c} = 10$



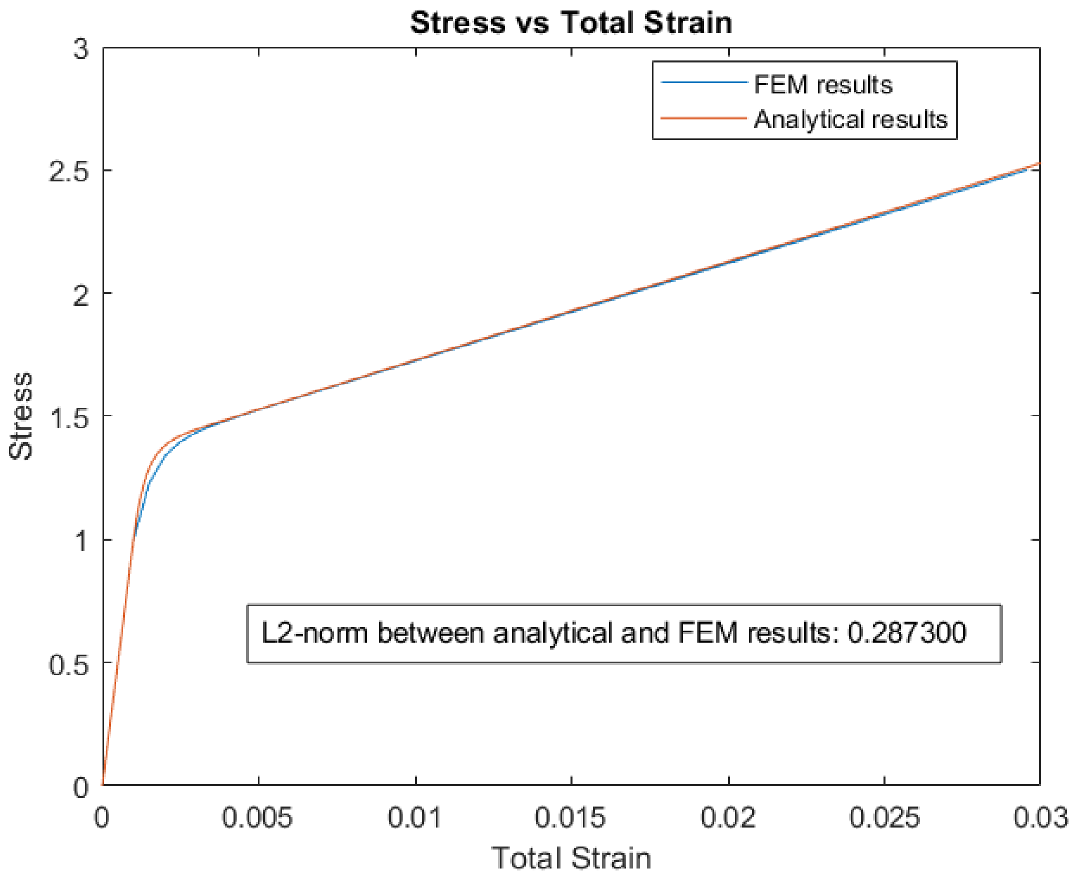


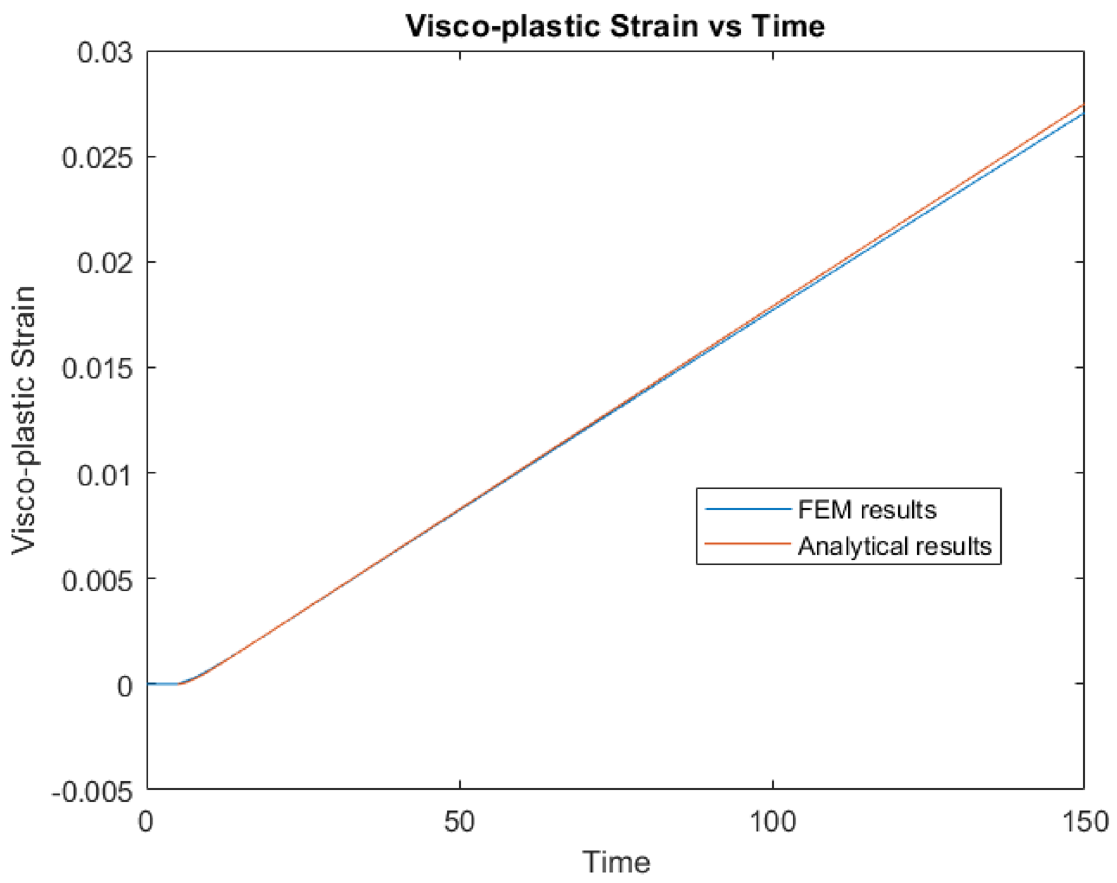
c = 0.0002, $\Delta t = \frac{\epsilon_0}{c} = 5$





c = 0.0002, $\Delta t = 0.5 * \frac{\epsilon_0}{c} = 2.5$





Observations:

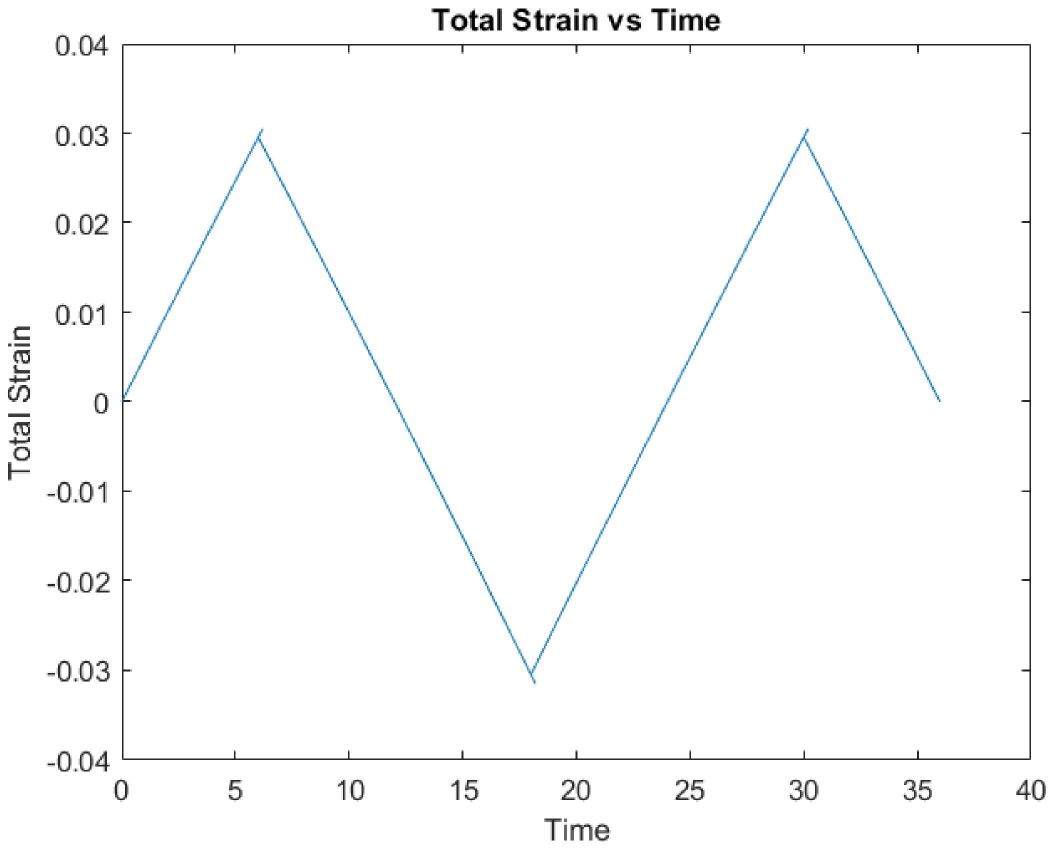
1. The L2-norm between analytical and FEM results was observed to decerease as we reduce the time step Δt.

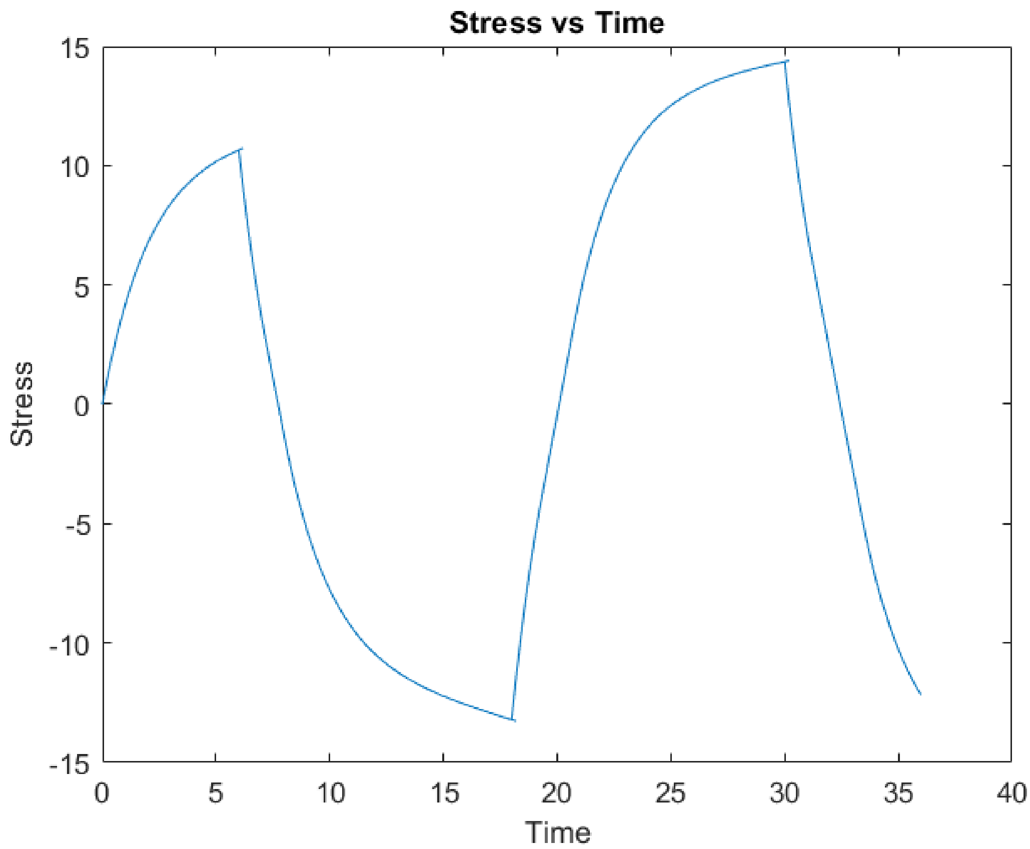
Δt	$L_2 - norm$
0.4	0.4932
0.2	0.3135
0.1	0.2873

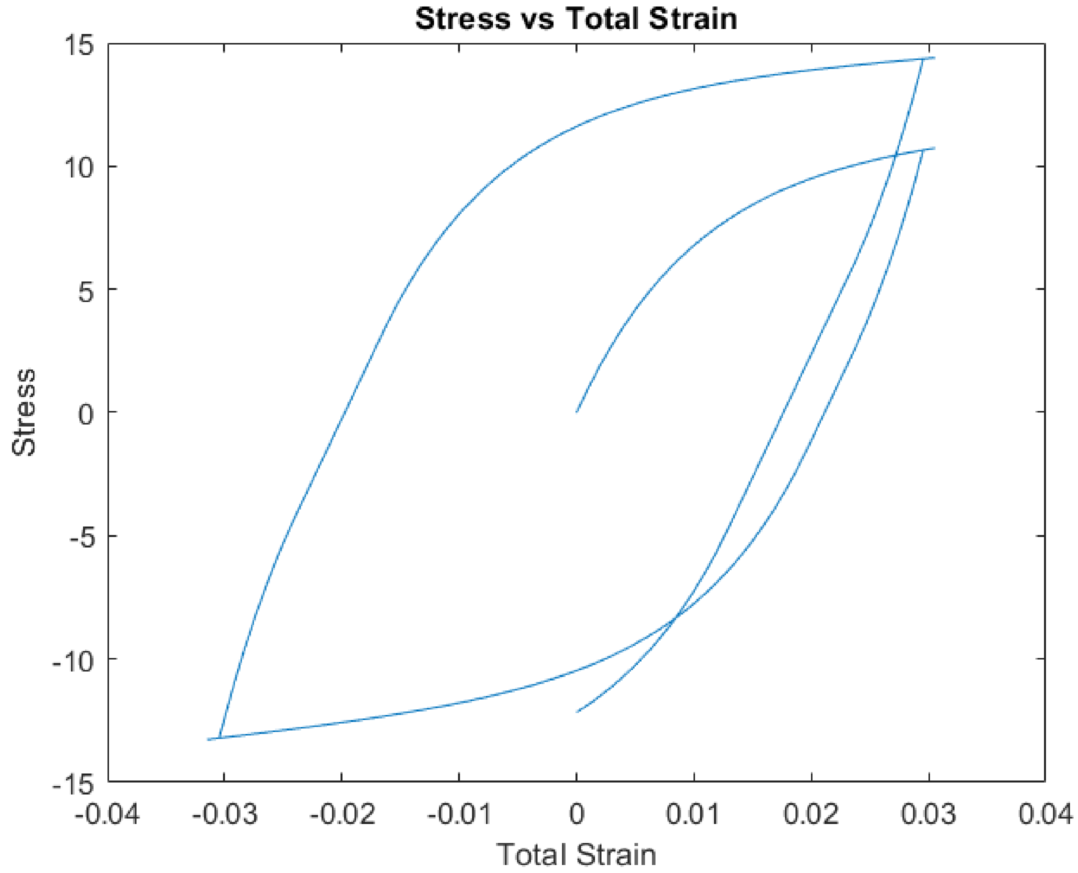
2. We can also see that as we reduce the time step, the stress-strain plot for FEM comes closer to analytical plot.

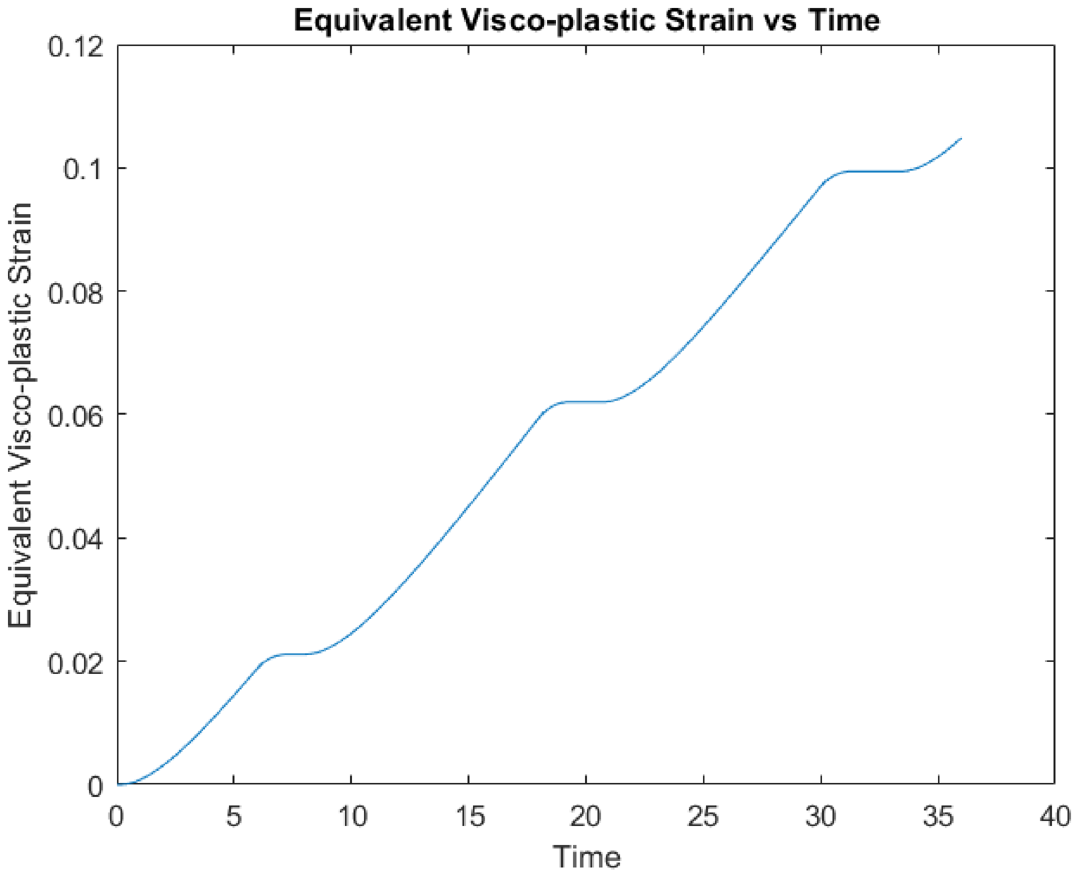
t = 6 * t1

$$\mathrm{t1} = 6.0, \mathrm{c} = 0.005, \Delta t = \frac{\epsilon_0}{c}$$

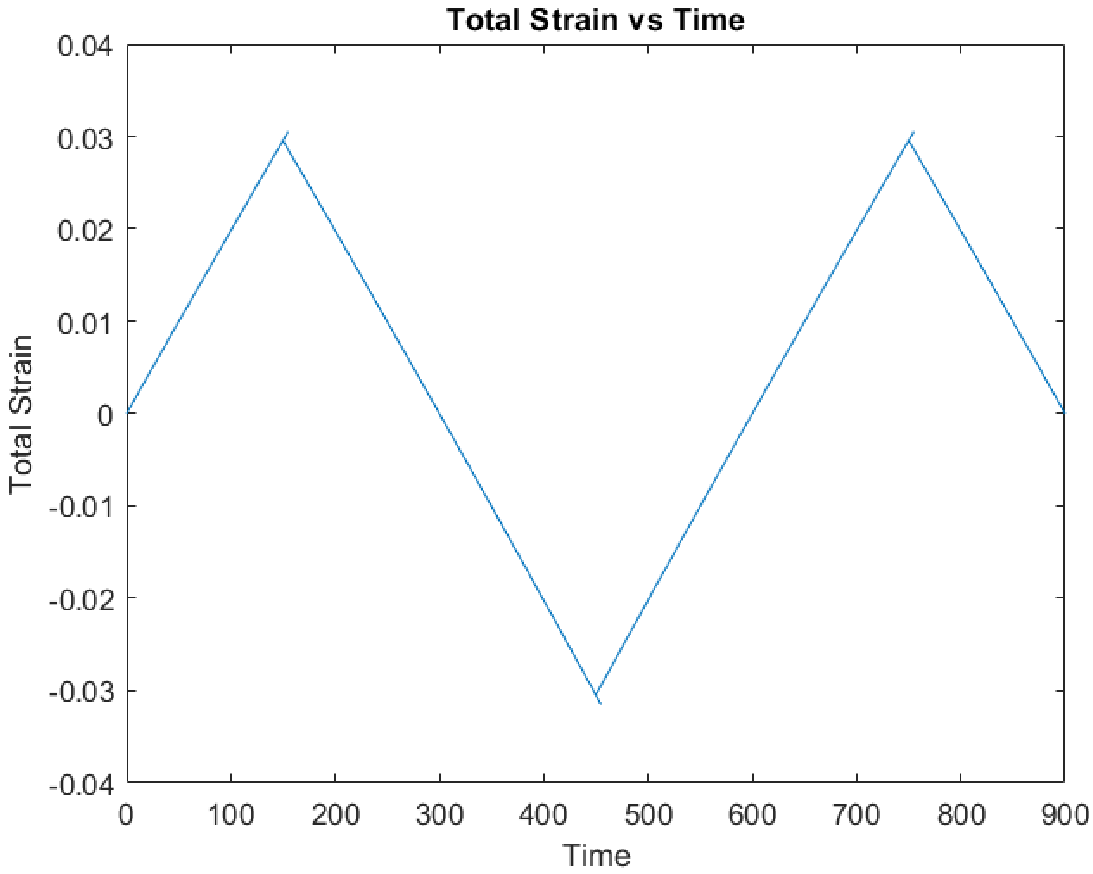


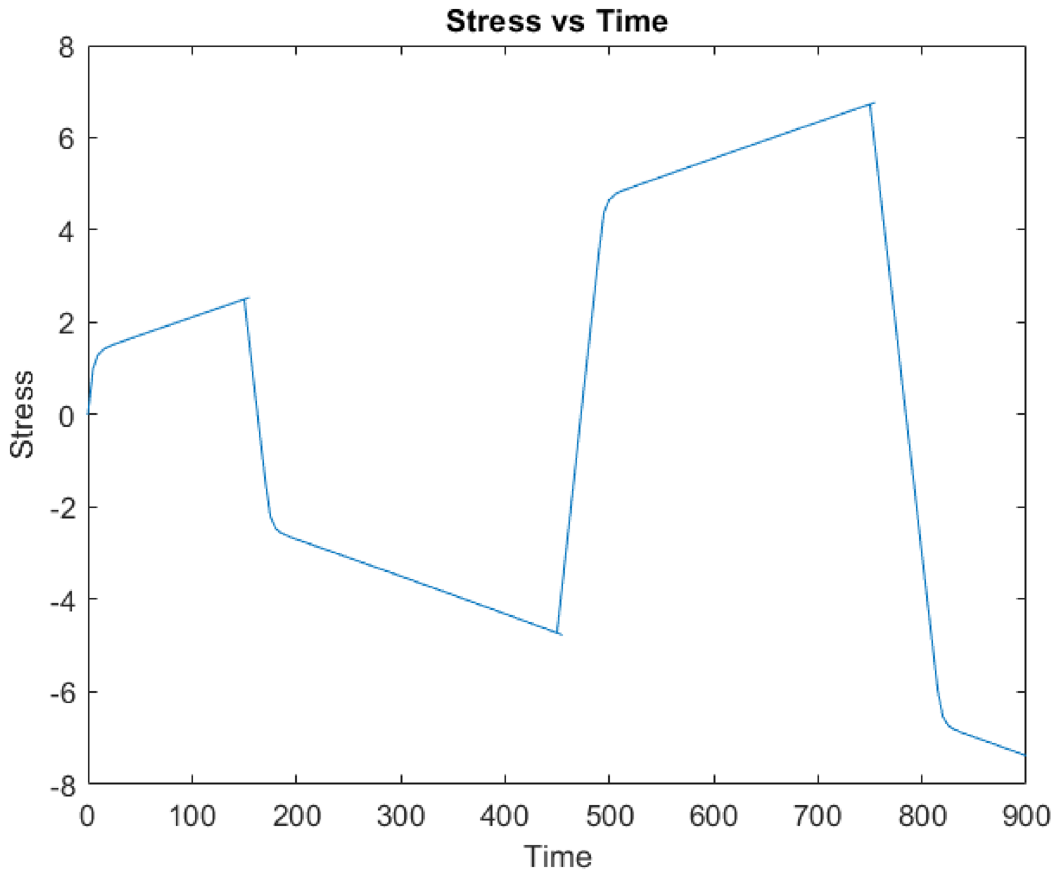


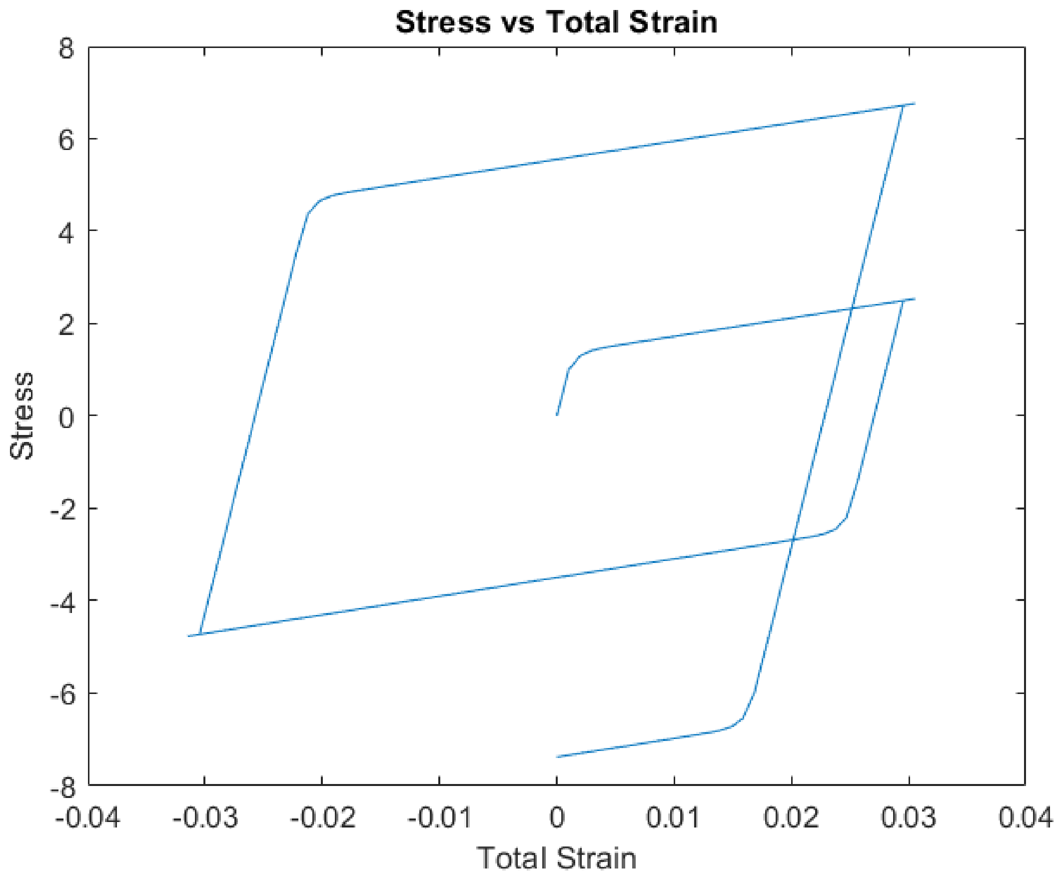

Stress vs Total Strain
Stress
Total Strain
15
10
5
0
-5
-10
-15
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04

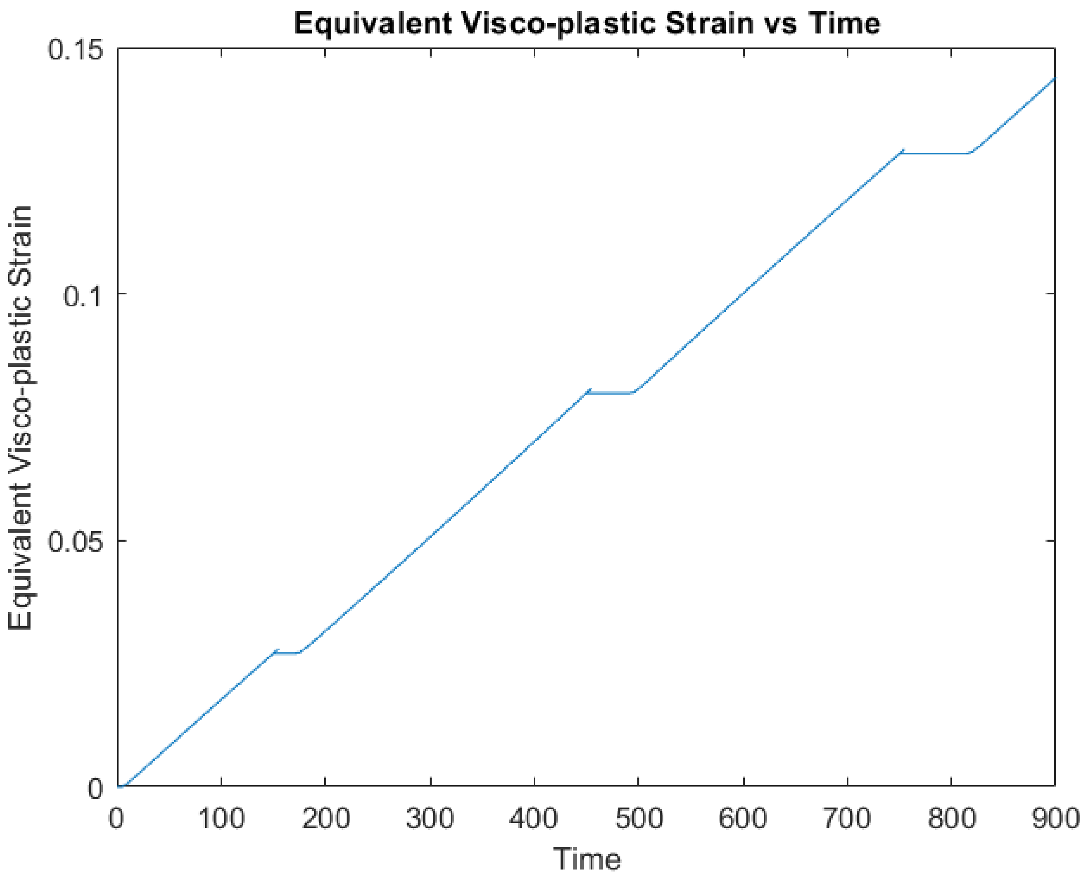

Equivalent Visco-plastic Strain vs Time
Equivalent Visco-plastic Strain
Time
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30
35
40

$$t1 = 150, c = 0.0002, \Delta t = \frac{\epsilon_0}{c}$$









Observations:

- We can see the strain hardening effect in stress-strain plots