Total No. Printed :03

KAS103/ KAS103T

Roll no. to be filled by candidate

2	1	0	0	4	30	2	0	00	2	2

B.Tech.

FIRST SEMESTER THEORY EXAMINATION, 2022-2023

KAS-103/KAS-103T

MATHEMATICS-I / ENGINEERING MATHEMATICS-I

Time: 3 Hours **Max. Mark: 100**

Note:-Attempt all questions:

1.) Attempt any four parts of the following **[4x5]**

a) Evaluate $A^2-3A+9I$, if I is the unit matrix of order 3 and [CO1]

$$A=\begin{bmatrix}1 & -2 & 3\\ 2 & 3 & -1\\ -3 & 1 & 2\end{bmatrix}$$

b) Find the rank of the matrix A [CO1]

if $A=\begin{bmatrix}2 & 3 & 4 & -1\\ 5 & 2 & 0 & -1\\ -4 & 5 & 12 & -1\end{bmatrix}$

c) Investigate for what values of λ and μ, do the system of equations [CO1]

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2\lambda+\lambda z=\mu$$

have (i) no solution (ii) unique solution.

d) Find the eigen value & eigen vector of [CO1]

$$\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$$

e) If $N=\begin{bmatrix}0 & 1+2i\\ -1+2i & 0\end{bmatrix}$, obtain the matrix $(I-N)(I+N)^{-1}$ and show that it is Unitary [CO1]

f) State Cayley Hamilton theorem and verify for the matrix [CO1]

$$A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

2.) Attempt any four parts of the following **[4x5]**

a) Trace the curve $y^2(2a-x)=x^3$ [CO2]

b) State and prove Cauchy's mean value theorem. [CO2]

c) If $y^{1/m}+y^{-1/m}=2x$, then prove that [CO2]

$$(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$$

d) Find the envelope of the family of straight lines $y=mx+\frac{a}{m}$, the parameter being m. [CO2]

OR

Find the evaluate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ [CO2]

e) Show that the function f(x) is defined by

$f(x)=\begin{cases} x^2, & x\neq 1 \\ 2, & x=1 \end{cases}$ is discontinuous at $x=1$ [CO2]

f) Verify Rolle's theorem for the following function $f(x)=2x^3+x^2-4x-2$ [CO2]

3.) Attempt any four parts of the following **[4x5]**

a) If $x^x y^y z^z=c$ show that $\frac{\partial^2 z}{\partial x\partial y}=-(x\log ex)^{-1}$ [CO3]

b) Verify Euler's theorem for the following function [CO3]

$$f(x)=\sin^{-1}\left(x/y\right)+\tan^{-1}\left(y/x\right)$$

c) Expand $e^{ax\sin by}$ in power of x and y as for as the terms of third degree. [CO3]

d) If u,v,w are the roots of the cubic $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ in λ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ [CO3]

e) If the kinetic energy T is given by $T=\frac{1}{2}mv^2$, find approximately, the change in T as the mass m changes from 49 to 49.5 and the velocity changes from 1600 to 1590. [CO3]

f) In a plane triangle ABC. Find the maximum values of $\cos A \cos B \cos C$ [CO3]

4.) Attempt any two parts of the following **[2x10]**

a) (i) Evaluate $\int_0^2\int_1^{e^x} dxdy$ by changing the order of integration [CO4]

(ii) Evaluate $\iiint \frac{dxdydz}{(x+y+z)^2}$, the integral being taken throughout the volume bounded by the planes $x=0, y=0, z=0$ and $x+y+z=1$ [CO4]

b) Find the double integration, the area lying inside the circle $r=a\sin\theta$ and outside the cardioids $r=a(1-\cos\theta)$. [CO4]

c) Find the mass of a lamina in the form of the cardiod $r=a(1+\cos\theta)$ whose density at any point varies as the square of its distance from the initial line. [CO4]

5.) Attempt any two parts of the following **[2x10]**

a) (i) Find the directional derivative of $\phi=\left(x^2+y^2+z^2\right)^{-1/2}$ at the point $p(3,1,2)$ in the direction of the vector $yzi+zxj+xyk$. [CO5]

(ii) Find $div\,\vec{F}$ and $curl\,\vec{F}$ of the vector $\vec{F}$, $\vec{F}=xyzi+3x^2yj+(xz^2-y^2z)k$ at the point $(2,-1,1)$. [CO5]

b) (i) Determine the work done by the forces

$$\vec{F}=(2y+3)\hat{\imath}+xz\hat{\jmath}+(yz-x)\hat{k}$$

when it moves a particle from the point (0,0,0) to the point(2,1,1) along the curve $x=2t^2, y=t, z=t^3$. [CO5]

(ii) Prove that $curl(\vec{a} \times \vec{b}) = (b.\nabla)a - bdiva - (a.\nabla)b + adivb$ [CO5]

OR

(ii) Evaluate $\iint \left(\nabla \times \vec{A}\right).\hat{n}\, ds$ where S is the surface of the cone $z = 2 - \sqrt{x^2 + y^2}$.

Above the xy-plane and $\vec{A} = (x-2)i + (x^2 + yz)j - 3xy^2 k$ [CO5]

c) Verify stokes theorem for $\vec{F} = (x^2 + y^2)i - 2xyj$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$ [CO5]

Total Number of Printed Pages: 04

KAS 103T

(Roll No. to be filled by candidate)

21	00	4	30	2	0	0	0	2	4

B. TECH.

FIRST SEMESTER THEORY EXAMINATION, 2021-22

KAS-103T

ENGINEERING MATHEMATICS-I

Time: 03 Hours **Max. Marks: 100**

Note:

- Attempt all questions. All questions carry equal marks.

1. Attempt any *TWO* parts of the following: 2×10 CO1

a. (i) If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ is a matrix, then prove that $(I-N)(I+N)^{-1}$ is unitary matrix, where I is identity matrix.

(ii) Reduce the matrix AB into Echelon form by using elementary transformations and hence find its rank, where

$$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & 7 \end{bmatrix}$$

b. (i) Apply elementary transformations to find the inverse of the following matrix:

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

(ii) Apply elementary transformations to test the consistency of the following system and solve them(if consistent)

$x+2y+z=3; 2x+3y+2z=5; 3x-5y+5z=2; 3x+9y-z=4$

c. Verify Cayley-Hamilton theorem for the matrix and hence find A^{-2}.

$$A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

2. Attempt any ***FOUR*** parts of the following: 4×5 CO2

a. Determine the values of a, b, c so that the following function is continuous for all values of x

$$f(x)=\begin{cases} \dfrac{\sin(a+1)x+\sin x}{x}, & x<0 \\ c, & x=0 \\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}}, & x>0 \end{cases}$$

b. Find y_n if $y=\sin^4 x\cos^4 x$.

c. If $y=\tan^{-1}\left(\dfrac{a+x}{a-x}\right)$, prove that $(x^2+a^2)y_{n+2}+2(n+1)xy_{n+1}+n(n+1)y_n=0$.

d. Determine the values of constants a, b if Rolle's theorem holds good for the function $f(x)=x^3+ax^2+bx,\ 1\le x\le 2$ at the point $x=\dfrac{4}{3}$.

e. Find the envelope of the circles described on the lines joining the centre of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ to any point on it as diameter.

f. Trace the curve $y^2(a-x)=x^3$.

3. Attempt any ***FOUR*** parts of the following: 4×5 CO3

a. Verify Euler's theorem for the following function

$$f(x)=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$$

Also evaluate $x^2\dfrac{\partial^2 f}{\partial x^2}+2xy\dfrac{\partial^2 f}{\partial x\partial y}+y^2\dfrac{\partial^2 f}{\partial y^2}$.

4x5 (10)

b. Apply method of partial differentiation to find $\frac{du}{dy}$, if $u = x\log(xy)$, where $x^3 + y^3 + 3xy = 1$,

c. Find the percentage error in computing the parallel resistance r of three resistances r_1, r_2, r_3 when r_1, r_2, r_3 are in error by 1.5%.

d. If $u = xyz, v = x^2 + y^2 + z^2, w = x + y + z$, then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

e. Obtain the Taylor's series expansion of maximum order for the function $f(x,y) = 2x^2 - xy + y^2 + 3x - 4y + 1$ in powers of $(x-1)$ and $(y+1)$.

f. Apply Lagrange's multiplier method to determine the greatest value of $u = xyz$, where x, y and z are positive real numbers for which $4x + 2y + z = 12$.

4. Attempt any *TWO* parts of the following: 2×10 CO4

a. (i) Evaluate $\iint_D x^2 dxdy$, where D is the region bounded by the ellipse having length of major and minor axis as $2a$ and $2b$ respectively.

(ii) Apply double integration to find the mass of the triangular plate in the xy-plane which is bounded by the lines $x = 0$, $y = 0$ and $\frac{x}{a} + \frac{y}{b} = 1$ having mass density $\rho = x\sqrt{y}$ at any point (x, y).

b. Evaluate $\int_0^a \int_y^a \frac{x}{x^2 + y^2} dxdy$ by changing the order of integration. Also verify the result by evaluating the integral using polar coordinate.

c. Use method of triple integration to determine the volume enclosed between the two surfaces $z = 2x^2 + 3y^2$ and $z = 16 - 2x^2 - y^2$.

5. Attempt any *TWO* parts of the following: 2×10 CO5

a. (i) Find the directional derivative of $\nabla.\mathbf{u}$ at the point (2,2,1) in the direction of the outer normal to the sphere $x^2+y^2+z^2=9$ where

$$\mathbf{u} = x^2 z\mathbf{i} + y^2 x\mathbf{j} + z^2 y\mathbf{k}.$$

(ii) Show that the vector field **F** given by

$$\mathbf{F}=(x^2-yz)\,\mathbf{i}+(y^2-xz)\,\mathbf{j}+(z^2-xy)\,\mathbf{k}$$

is irrotational. Find its scalar potential.

b. (i) Find the work done in moving a particle once around a circle C in the xy-plane, if the circle has centre at the origin and radius a and if the force field is given by

$$\mathbf{F} = (2x - y + z)\,\mathbf{i} + (x + y - z^2)\mathbf{j} + (3x - 2y + 4z)\,\mathbf{k}.$$

(ii) Prove that $\nabla^2 f(r) = f''(r) + \frac{1}{r} f'(r)$. Find $f(r)$ such that $\nabla^2 f(r) = 0$.

c. Verify Gauss Divergence theorem for the function $\mathbf{F}=4xz\mathbf{i}-y^2\mathbf{j}+yz\mathbf{k}$ and S, the surface of the cube $x=0, x=1,\ y=0, y=1,\ z=0, z=1.$

$\frac{-1}{2} \times \frac{1}{2} = \frac{-1}{4}$

$\frac{-1}{2} - 1$

$= -1 - 2$

Bundelkhand Institute of Engineering & Technology, Jhansi
Second Class Test, Odd (I) Semester, 2021-22
Mathematics-I (KAS-103T) (For EC, EE & CH)

Max. Marks: 15 | **Time: 01 Hr**

1. Attempt *ALL* parts of this question. [CO3]

(a) Apply chain rule of differentiation to find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial \theta}$ for $f(x, y) = x^2 + y^2$ where $x = r\cos\theta, y = r\sin\theta$.

[1.5 Marks]

(b) Determine the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ where $u = \log\sqrt{x^2 + y^2 + z^2}$ [1.0 Mark]

(c) Apply Taylor's series expansion for the function $f(x, y) = e^{2x-y}$ about the point (0, 1) up to the quadratic terms to estimate the value of $f(x, y)$ at $(-0.1, 1.1)$. [2.5 Marks]

OR

Apply Lagrange's multiplier method to determine the greatest value of $u = xyz$, where x, y and z are positive real numbers for which $4x + 2y + z = 12$. [2.5 Marks]

(d) If $x + y + z = u, y + z = uv, z = uvw$, then determine $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (2.5)

(P.T.O)