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Introduction to Solid Mechanics

By

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CONTENTS

KCE 402 : Introduction to Solid Mechanics

1 UNIT

Simple and Compound Stress and Strains

CONTENTS

PART-1

Concept of Stress and Strain, Types of Stresses and Strains.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. Define stress and also classify it.

Answer

A. **Stress :**

1. The force of resistance per unit area, offered by a body against deformation is known as stress. The external force acting on the body is called the load or force. Mathematically, stress

$$\sigma = \frac{P}{A}$$

Where, P = External force or load, A = Cross-sectional area.

2. It is expressed in N/m².

B. **Types :** There are two types of stresses :

1. **Normal Stress :**

i. It is the stress, which acts in a direction perpendicular to the area. It is represented by σ.

ii. The normal stress is further divided into tensile stress and compressive stress.

Tensile Stress :

a. The stress induced in a body, when subjected to two equal and opposite pulls as a result of which there is an increase in length, is known as tensile stress (Fig. 1.1.1(*a*)).

b. The tensile stress acts normal to the area and it pulls on the area.

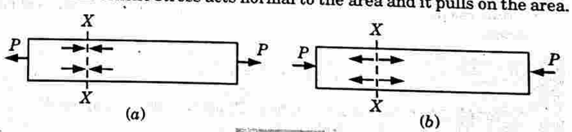


Fig. 1.1.1.

Compressive Stress :

a. The stress induced in a body, when subjected to two equal and opposite pushes as a result of which there is a decrease in length of the body, is known as compressive stress (Fig. 1.1.1(*b*)).

b. The compressive stress acts normal to the area and it pushes on the area.

2. **Shear Stress :**
i. The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as a result of which the body tends to shear off across the section, is known as shear stress (Fig. 1.1.2).
ii. It is given by, $\tau = \dfrac{\text{Shear resistance}}{\text{Shear area}} = \dfrac{P}{A}$

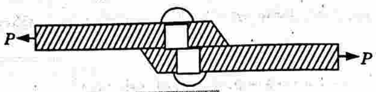


Fig. 1.1.2.

Que 1.2. **Define strain. What are different types of strain ?**

Answer

A. **Strain :**
1. The ratio of change in dimension of the body to the original dimension is known as strain.
2. These changes in the dimension of the body occur due to external load subjected on the body

$$\varepsilon = \frac{\delta l}{l}$$

B. **Types of Strain :** Following are the types of strain :
1. **Tensile Strain :** If there is some increase in length of a body due to external force, then the ratio of increase in length to the original length of the body is known as tensile strain.
2. **Compressive Strain :** If there is some decrease in length of the body due to external force, then the ratio of decrease in length of the body to the original length is known as compressive strain.
3. **Volumetric Strain :** The ratio of change in volume of the body to the original volume is known as volumetric strain.
4. **Shear Strain :** The strain produced by shear stress is known as shear strain.

PART-2

Hook's Law, Stress and Strain Diagram for Ductile and Brittle Material.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.3. **Define Hook's Law. Draw a typical stress-strain curve for mild steel and explain the salient point on it.**

OR

Draw the stress-strain diagram for mild steel under tensile load.

AKTU 2015-16, Marks 05

Answer

A. **Hook's Law :**
1. It states, "When a material is loaded, within its elastic limit, the stress is proportional to the strain." Mathematically,

$$\frac{\text{Stress}}{\text{Strain}} = E = \text{Constant}$$

2. It may be noted that Hook's Law equally holds good for tension as well as compression.

B. **Stress-Strain Diagram for Mild Steel :**
Fig. 1.3.1 shows stress *vs* strain diagram for the typical mild steel specimen. The following salient points are observed on stress-strain curve :

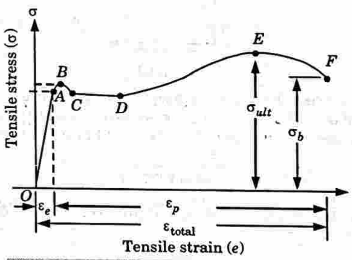


Fig. 1.3.1. Stress-Strain diagram for a typical structural steel in tension.

1. **Limit of Proportionality (A) :** It is the limiting value of the stress up to which stress is proportional to strain.
2. **Elastic Limit :** This is the limiting value of stress up to which if the material is stressed and then released (unloaded) strain disappears completely and the original length is regained. This point is slightly beyond the limit of proportionality.
3. **Upper Yield Point (B) :** This is the stress at which, the load starts reducing and the extension increases. This phenomenon is called yielding of material. At this stage strain is about 0.125 per cent and stress is about 250 N/mm².

4. **Lower Yield Point (C) :** At this stage the stress remains same but strain increases for same time.
5. **Ultimate Stress (E) :** This is the maximum stress the material can resist. This stress is about 370 – 400 N/mm^2. At this stage cross-sectional area at a particular section starts reducing very fast. This is called neck formation. After the stage load and hence the stress developed starts reducing.
6. **Breaking Point (F) :** The stress at which finally the specimen fails is called breaking point. At this point strain is 20 to 25 per cent.

Que 1.4. **Explain the stress-strain relationship for brittle material.**

Answer

Stress-Strain Relation in Brittle Material :

1. The typical stress-strain relation in a brittle material like cast iron is shown in Fig. 1.4.1.

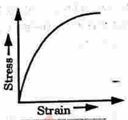


Fig. 1.4.1.

2. In this material, there is no appreciable change in rate of strain. There is no yield point and to necking takes place. Ultimate point and breaking point are one and the same. The strain at failure is very small.

PART-3

Lateral Strain, Poisson's Ratio, Volumetric Strain.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.5. **Write short notes on :**

i. **Lateral strain and longitudinal strain.**
ii. **Poisson's ratio.**

Answer

1. **Longitudinal Strain :**
The ratio of axial deformation to the original length of the body is known as longitudinal (or linear) strain. The longitudinal strain in also defined

as the deformation of the body per unit length in the direction of the applied load.

Let L = Length of the body,
P = Tensile force acting on the body.
δL = Increase in the length of the body in the direction of P.

Then, longitudinal strain = $\dfrac{\delta L}{L}$

2. **Lateral Strain :**
i. The strain at right angles to the direction of applied load is known as lateral strain.
ii. Let a rectangular bar of length, L, breadth, b and depth, d is subjected to an axial tensile load, P as shown in Fig. 1.5.1. The length of the bar will increase while the breadth and depth will decrease.

Let, δL = Increase in length.
δb = Decrease in breadth.
δd = Decrease in depth.

$$\text{Lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}$$

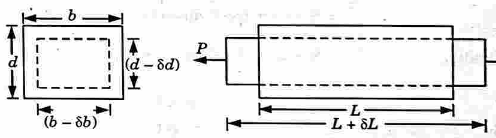


Fig. 1.5.1.

3. **Poisson's Ratio :** The ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poisson's ratio and it is generally denoted by μ.

$$\text{Poisson's ratio, } \mu = -\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Que 1.6. **Derive a relation for the volumetric strain of a body.**

Answer

Volumetric Strain of a Rectangular Bar Subjected to Three Forces which are Mutually Perpendicular :

1. Consider a rectangular block of dimensions x, y and z subjected to three direct tensile stresses along three mutually perpendicular axis as shown in Fig. 1.6.1.

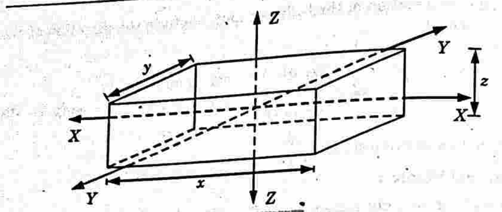


Fig. 1.6.1.

2. Then volume of block, $V = xyz$
3. Taking logarithm to both sides, we have
$$\log V = \log x + \log y + \log z$$
4. Differentiating the above equation, we get
$$\frac{dV}{V} = \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} \quad ...(1.6.1)$$

Volumetric strain $= \frac{dV}{V} = \frac{\text{Change of volume}}{\text{Original volume}}$

$$\frac{dx}{x} = \text{Strain in the } X\text{-direction} = \varepsilon_x$$

Similarly, $\frac{dy}{y}$ = Strain in the Y-direction $= \varepsilon_y$

$$\frac{dz}{z} = \text{Strain in the } Z\text{-direction} = \varepsilon_z$$

Substituting these values in eq. (1.6.1), we get
$$\frac{dV}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z \quad ...(1.6.2)$$

Now,
σ_x = Tensile stress in X-X direction.
σ_y = Tensile stress in Y-Y direction.
σ_z = Tensile stress in Z-Z direction.
E = Young's modulus.
μ = Poisson's ratio.

5. Now σ_x will produce, a tensile strain $= \sigma_x/E$, in the direction of X a compressive strain $= (\mu \times \sigma_x)/E$, in the direction of Y and Z.
6. Similarly, σ_y will produce, a tensile strain $= \sigma_y/E$, in the direction of Y and a compressive strain $(\mu \times \sigma_y)/E$, in the direction of X and Z.
σ_z will produce, a tensile strain $= \sigma_z/E$, in the direction of Z and a compressive strain $(\mu \times \sigma_z)/E$ in the direction of X and Y.
7. Hence, σ_y and σ_z will produce compressive strains equal to $\mu \times \sigma_y / E$ and $\mu \times \sigma_z / E$ in the direction of x. So, Net tensile strain along X-direction is given by,
$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu \times \sigma_y}{E} - \frac{\mu \times \sigma_z}{E} = \frac{\sigma_x}{E} - \mu\left(\frac{\sigma_y + \sigma_z}{E}\right)$$

Similarly, $\varepsilon_y = \frac{\sigma_y}{E} - \mu\left(\frac{\sigma_x + \sigma_z}{E}\right)$; $\varepsilon_z = \frac{\sigma_z}{E} - \mu\left(\frac{\sigma_x + \sigma_z}{E}\right)$

8. Putting the values of e_x, e_y and e_z in eq. (1.6.2), we get
$$\frac{dV}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$
$$= \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z) - \frac{2\mu}{E}(\sigma_x + \sigma_y + \sigma_z)$$
$$\therefore \quad \frac{dV}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu) \quad ...(1.6.3)$$
9. Equation (1.6.3) gives the volumetric strain. In this equation the stresses σ_x, σ_y and σ_z are all tensile. If any of the stresses is compressive, it may be regarded as negative, and the above equation will hold good. If the value of dV/V is positive, it represents increase in volume whereas the negative value of dV/V represents a decrease in volume.

Que 1.7. **Prove that the volumetric strain of a cylindrical rod which is subjected to an axial tensile load is equal to strain in the length minus twice the strain of diameter.**

Answer

1. Consider a cylindrical rod which is subjected to an axial tensile load P.
Let d = Diameter of the rod.
L = Length of the rod.
2. Due to tensile load P, there will be increase in the length of the rod, but the diameter of the rod will decrease as shows in Fig. 1.7.1.

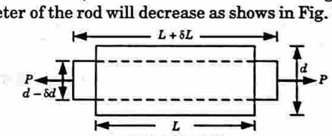


Fig. 1.7.1.

$\therefore$ Final length $= L + \delta L$
$\therefore$ Final diameter $= d - \delta d$

3. Now original volume of the rod, $V = \frac{\pi}{4} d^2 \times L$

Final volume, $V = \frac{\pi}{4}(d - \delta d)^2 (L + \delta L)$
$$= \frac{\pi}{4}(d^2 + \delta d^2 - 2d \times \delta d)(L + \delta L)$$
$$= \frac{\pi}{4}(d^2 \times L + \delta d^2 \times L - 2d \times L \times \delta d + d^2 \times \delta L + \delta d^2 \times \delta L - 2d \times \delta d \times \delta L)$$

$$= \frac{\pi}{4}(d^2 \times L - 2d \times L \times \delta d + d^2 \times \delta L)$$

Neglecting the products and higher powers of small quantities.

4. Change in volume, δV = Final volume – Original volume

$$= \frac{\pi}{4}(d^2 \times L - 2d \times L \times \delta d + d^2 \times \delta L) - \frac{\pi}{4} d^2 \times L$$

$$= \frac{\pi}{4}(d^2 \times \delta L - 2d \times L \times \delta d)$$

5. Volumetric strain, $e_V = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\delta V}{V}$

$$= \frac{\pi/4(d^2 \times \delta L - 2d \times L \times \delta d)}{\frac{\pi}{4} d^2 \times L} = \frac{\delta L}{L} - \frac{2\delta d}{d}$$

where, $\frac{\delta L}{L}$ is the strain of length and $\frac{\delta d}{d}$ is the strain of diameter.

6. Volumetric strain = Strain of length – Twice the strain of diameter.

PART-4

Elastic Moduli and Relation Between Them.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.8. **Explain the various types of elastic constants.**

Answer

1. **Young's Modulus :** It is the ratio between tensile stress and tensile strain or compressive stress and compressive strain. It is denoted by E. It is the same as modulus of elasticity. Hence, mathematically,

   Young's Modulus, $E = \frac{\sigma}{\varepsilon}$

2. **Modulus of Rigidity :** It is defined as the ratio of shear stress, τ to shear strain and is denoted by G. It is also called shear modulus of elasticity. It is given by, $G = \frac{\tau}{\varepsilon_s}$

3. **Bulk or Volume Modulus of Elasticity :** It may be defined as the ratio of normal stress (on each face of a solid cube) to volumetric strain and is denoted by K. Hence, mathematically,

$$K = \frac{\sigma_n}{\varepsilon_v}$$

Que 1.9. **Derive the following expression for the elastic constants :**

$$K = \frac{E}{3(1-2\mu)}$$

Answer

Relation between Young's Modulus (E) and Bulk Modulus (K) :

1. Bulk Modulus, $K = \frac{\text{Normal stress}}{\text{Volumetric strain}} = \frac{\sigma}{\varepsilon_v}$

   Consider a cubical element subjected to stress σ along X, Y and Z directions (as shown in Fig. 1.9.1).

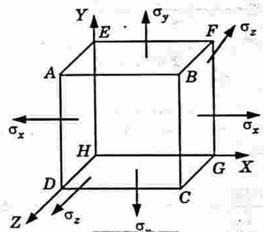


Fig. 1.9.1.

[Forces are same along each face $P_x = P_y = P_z = P$]

2. Strain in X-direction, $\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$

   $\because \quad \sigma_x = \sigma_y = \sigma_z = \sigma$

   $\therefore \quad \varepsilon_x = \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E} = \frac{\sigma}{E}(1-2\mu)$

   Similarly, $\varepsilon_y = \varepsilon_z = \frac{\sigma}{E}(1-2\mu)$

3. Volumetric strain, $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = 3\frac{\sigma}{E}(1-2\mu)$

$$\varepsilon_v = 3.\frac{\sigma}{E}(1-2\mu)$$

$$E = 3\frac{\sigma}{e_v}(1-2\mu) = 3K(1-2\mu)$$

$$K = \frac{E}{3(1-2\mu)}$$

Que 1.10. **Establish the relation between modulus of elasticity and shear modulus.**

Answer

Relation between E and G :

1. Consider a cubic element $ABCD$ with fixed bottom (BC) and top face AD subjected to force P (tangential).

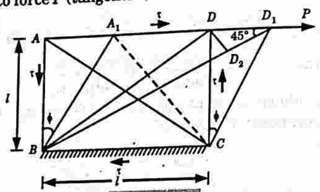


Fig. 1.10.1.

2. **Due to load P :**
   i. Shear stress τ is induced at AD and BC.
   ii. Complimentary shear stresses are set up at AB and DC due to which cubic element $ABCD$ distorts to BA_1D_1C.

$$AA_1 = DD_1$$

3. From ΔDCD_1, $\dfrac{DD_1}{l} = \tan\phi = \phi$ (Since ϕ is very small)

$$\therefore \quad DD_1 = l\phi$$

Diagonal BD elongates to BD_1.
Diagonal AC shortens to A_1C.

4. Longitudinal strain in $BD = \dfrac{BD_1 - BD}{BD} = \dfrac{BD_1 - BD_2}{BD}$

[DD_2 is perpendicular from D to BD_1]

5. DD_1 is small, therefore $\angle BDC \approx \angle BD_1C \approx 45°$

$$\therefore \quad \angle DD_1D_2 \approx 45°$$

6. Longitudinal strain in $BD = \dfrac{D_1D_2}{BD} = \dfrac{DD_1 \cos 45°}{BD} = \dfrac{l\phi(1/\sqrt{2})}{l\sqrt{2}}$

7. Longitudinal strain in $BD = \dfrac{\phi}{2} = \dfrac{\tau}{2G}$ $[\because \tau = G\phi]$ ...(1.10.1)

8. Strain in diagonal BD is also given by :
(Strain due to tensile stress in diagonal BD) – (Strain due to complimentary stress in diagonal AC)

$$= \frac{\tau}{E} - \left(-\mu\frac{\tau}{E}\right) = \frac{\tau}{E}(1+\mu) \quad ...(1.10.2)$$

9. From equation (1.10.1) and (1.10.2), we get

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\mu) \Rightarrow E = 2G(1+\mu)$$

Que 1.11. **Show that E, G and K are related by the following expression :**

$$E = \frac{9KG}{3K+G}$$

Answer

Relation between E, G and K :

1. We know that, $E = 2G(1+\mu)$ ...(1.11.1)
and $E = 3K(1-2\mu)$ ...(1.11.2)

2. From eq. (1.11.1), $\mu = \dfrac{E}{2G} - 1$ and put it in eq. (1.11.2).

$$E = 3K\left[1 - 2\left(\frac{E}{2G} - 1\right)\right] = 3K\left[1 - \frac{E}{G} + 2\right]$$

$$E = 3K\left[3 - \frac{E}{G}\right] = 3K\left[\frac{3G - E}{G}\right] = \left[\frac{9KG - 3KE}{G}\right]$$

$$GE = 9KG - 3KE$$

$$GE + 3KE = 9KG \Rightarrow E(3K+G) = 9KG$$

$$E = \frac{9KG}{3K+G}$$

Que 1.12. **A steel bar 2 m long 20 mm wide, 10 mm thick is subjected to a pull of 20 kN in the direction of length. Find the changes in length, breadth, thickness of bar. Take $E = 2 \times 10^5$ N/mm² and Poisson's ratio 0.3.**

Answer

Given : Length of steel bar, L = 2 m, b = 20 mm, t = 10 mm load, P = 20 kN (in the direction of length) $E = 2 \times 10^5$ N/mm², $\mu = 0.3$.
To Find : Change in length breadth and thickness of bar.

1. Longitudinal strain,

$$\frac{\delta l}{l} = \frac{\text{Stress}}{\text{Modulus of elasticity}} = \frac{P/A}{E} = \frac{P}{AE} = \frac{20 \times 10^3}{(20 \times 10) \times 2 \times 10^5}$$

$$= 0.5 \times 10^{-3}$$

2. Change is length, δl = Longitudinal strain × Original length
$= (0.5 \times 10^{-3}) \times (2 \times 10^3) = 1.0$ mm (increase)

3. Lateral strain = Poisson's ratio × Longitudinal strain
$= 0.3 \times (0.5 \times 10^{-3}) = 0.15 \times 10^{-3}$

4. Change in breadth, $\delta b = b \times$ Lateral strain $= 20 \times (0.15 \times 10^{-3})$

$= 3 \times 10^{-3}$ mm (decrease)

5. Change in thickness, $\delta t = t \times$ Lateral strain $= 10 \times (0.15 \times 10^{-3})$
$= 1.5 \times 10^{-3}$ mm (decrease)

Que 1.13. **Determine the Poisson's ratio and bulk modulus of a material, for which Young's modulus is 120 GPa and modulus of rigidity 48 GPa.**

AKTU 2016-17, Marks 05

Answer

Given : Young's modulus, $E = 120$ GPa $= 1.2 \times 10^5$ N/mm^2
Modulus of rigidity, $G = 48$ GPa $= 4.8 \times 10^4$ N/mm^2
To Find : Poisson's ratio and bulk modulus.

1. Elastic modulus is given by, $E = 2G(1 + \mu)$

$$1.2 \times 10^5 = 2 \times 4.8 \times 10^4 (1 + \mu)$$
$$\mu = 1.25 - 1.0 = 0.25.$$

2. Bulk modulus is given by,

$$K = \frac{E}{3(1-2\mu)} = \frac{1.2 \times 10^5}{3(1 - 0.25 \times 2)} \quad (\because \mu = 0.25)$$
$$= 8 \times 10^4 \text{ N/mm}^2.$$

Que 1.14. **The modulus of rigidity of material is 39 GPa. A 10 mm diameter rod of the material is subjected to an axial tensile force of 5 kN and the change in its diameter is 0.002 mm. Calculate the Poisson's ratio of the material.**

Answer

Given : Modulus of rigidity, $G = 39$ GPa, Diameter, $d = 10$ mm
Tensile force, $F = 5$ kN, Change in diameter, $\delta d = 0.002$ mm
To Find : Poisson's ratio of the material.

1. The stress induced in the rod by tensile force,

$$\sigma = \frac{F}{A} = \frac{5 \times 10^3}{\frac{\pi}{4} \times (10)^2} = 63.66 \text{ N/mm}^2$$

2. Longitudinal strain, $\varepsilon = \dfrac{\sigma}{E} = \dfrac{\sigma}{2G(1+\mu)}$ $\quad [\because E = 2G(1+\mu)]$

$$\varepsilon = \frac{63.66}{2 \times 39 \times 10^3 (1 + \mu)} = \frac{8.16 \times 10^{-4}}{(1 + \mu)} \text{ mm/mm}$$

4. Poisson's ratio,

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{(\delta d / d)}{(8.16 \times 10^{-4}) / (1 + \mu)}$$

$$\mu = \frac{(0.002/10)}{(8.16 \times 10^{-4})/(1+\mu)} = 0.324$$

Que 1.15. **Derive the expression for elongation of a uniform bar due to its self-weight.**

AKTU 2016-17, Marks 05

OR

Drive the expression for extension in the vertically suspended bar due to self weight.

AKTU 2014-15, Marks 05

Answer

1. Fig. 1.15.1 shows a bar AB fixed at end A and hanging freely under its own weight.

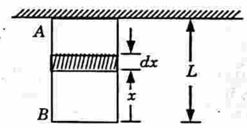


Fig. 1.15.1.

2. Let L = Length of bar, A = Area of cross-section, E = Young's modulus for the bar material, w = Weight per unit volume of the bar material.
3. Consider a small strip of thickness dx at a distance x from the lower end. Weight of the bar for a length of x is given by,

$$W_x = \text{Specific weight} \times \text{volume of bar upto length } x$$
$$= w \times A \times x$$

This means that on the strip, a weight of $w \times A \times x$ is acting in the downward direction. Due to this weight, there will be some increase in the length of element. But length of the element is dx.

4. Now stress on the element

$$= \frac{\text{Weight acting on element}}{\text{Area of cross section}} = \frac{w \times A \times x}{A} = w \times x \quad ...(1.15.1)$$

5. The eq. (1.15.1) shows that stress due to self weight in a bar is not uniform. It depends on x. The stress increases with the increase of x.
6. Therefore, elongation of the element = Strain × Length of element

$$= \frac{w \times x}{E} \times dx \quad \left[\because \text{Strain} = \frac{\text{Stress}}{E} = \frac{w \times x}{E}\right]$$

7. Total elongation of the bar is obtained by integrating the above equation between limits zero and L.

$$\delta L = \int_0^L \frac{w \times x}{E} dx = \frac{w}{E} \int_0^L x\, dx$$
$$= \frac{w}{E}\left[\frac{x^2}{2}\right]_0^L = \frac{w}{E} \times \frac{L^2}{2} = \frac{WL}{2AE} \quad (\because W = w \times A \times L)$$

Que 1.16. **Derive the expression for elongation of a conical bar due to its self-weight.**

AKTU 2016-17, Marks 05

Answer

1. Consider a small length dx of the bar at a distance x from the free end (Fig. 1.16.1).

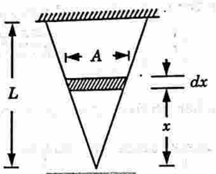


Fig. 1.16.1.

2. Let, A = Area of cross-section at the small length, w = Weight per unit volume of the bar, W = Weight of the whole bar = wAL, W_x = Weight of the bar below the section = $wAx/3$.
3. The extension of a small length = $\dfrac{W_x dx}{AE} = \dfrac{wAxdx}{3AE}$
4. Extension of the whole rod = $\displaystyle\int_0^L \frac{wx}{3E}dx = \frac{w}{3E}\int_0^L x dx = \frac{w}{3E}\left(\frac{x^2}{2}\right)_0^L$

$$= \frac{wL^2}{6E} = \frac{(wL)AL}{6AE} = \frac{WL}{6AE}$$

PART-5

Bar of Varying Cross-Section.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.17. **Three sections of a bar are having different lengths and different diameters. The bar is subjected to an axial load P. Determine the total change in length of the bar. Take Young's modulus of different section same.**

Answer

1. A bar of different lengths and of different diameters (and hence of different cross-sectional areas) is shown in Fig. 1.17.1.

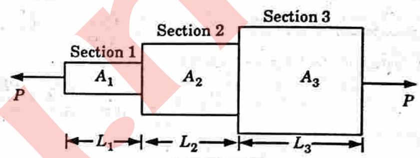


Fig. 1.17.1.

2. Let this bar is subjected to an axial load P.
3. Though each section is subjected to the same axial load P, yet the stresses, strains and change in lengths will be different.
4. The total change in length will be obtained by adding the changes in length of individual section.
5. Let,
   P = Axial load acting on the bar.
   L_1 and A_1 = Length and Cross-sectional area of section-1.
   L_2 and A_2 = Length and cross-sectional area of section-2.
   L_3 and A_3 = Length and cross-sectional area of section-3.
   E = Young's modulus for the bar.
6. Then stress for the section-1, $\sigma_1 = \dfrac{\text{Load}}{\text{Area of section}-1} = \dfrac{P}{A_1}$
7. Similarly stresses for the section-2 and section-3 are given as,
$$\sigma_2 = \frac{P}{A_2} \text{ and } \sigma_3 = \frac{P}{A_3}$$
8. Using equation $E = \dfrac{\sigma}{\varepsilon}$, the strains in different sections are obtained.

$\therefore$ Strain of section-1, $\varepsilon_1 = \dfrac{\sigma_1}{E} = \dfrac{P}{A_1 E}$ $\left(\because \sigma_1 = \dfrac{P}{A_1}\right)$

Similarly the strains of section-2 and section-3 are,
$$\varepsilon_2 = \frac{\sigma_2}{E} = \frac{P}{A_2 E} \text{ and } \varepsilon_3 = \frac{\sigma_3}{E} = \frac{P}{A_3 E}$$
9. Strain in section-1, $\varepsilon_1 = \dfrac{\text{Change in length of section}-1}{\text{Length of section}-1} = \dfrac{dL_1}{L_1}$

where, dL_1 = Change in length of section-1.
10. Change in length of section-1, $dL_1 = \varepsilon_1 L_1 = \dfrac{PL_1}{A_1 E}$ $\left(\because \varepsilon_1 = \dfrac{P}{A_1 E}\right)$
11. Similarly, Change in length of section-2, $dL_2 = \varepsilon_2 L_2 = \dfrac{PL_2}{A_2 E}$

$\left(\because \varepsilon_2 = \dfrac{P}{A_2 E}\right)$

change in length of section-3, $dL_3 = \varepsilon_3 L_3 = \frac{PL_3}{A_3 E}$ $\left(\because \ \varepsilon_3 = \frac{P}{A_3 E}\right)$

12. Total change in the length of the bar,

$$dL = dL_1 + dL_2 + dL_3 = \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E}$$

$$dL = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right] \quad ...(1.17.1)$$

13. If the Young's modulus of different sections is different, then total change in length of the bar is given by,

$$dL = P\left[\frac{L_1}{E_1 A_1} + \frac{L_2}{E_1 A_2} + \frac{L_3}{E_1 A_3}\right]$$

Que 1.18. **Explain the principle of superposition.**

Answer

Principle of Superposition :

1. When a number of loads are acting on a body, the resulting strain, according to principle of superposition, will be the algebraic sum of strains caused by individual loads.
2. While using this principle for an elastic body which is subjected to a number of direct forces (tensile or compressive) at different sections along the length of the body, first the free body diagram of individual section is drawn.
3. Then the deformation of the each section is obtained.
4. The total deformation of the body will be then equal to the algebraic sum of deformations of the individual sections.

Que 1.19. **A steel bar is subjected to loads as shown in Fig. 1.19.1. Determine the change in length of bar. The bar is 200 mm in diameter. Take E = 200 GPa.**

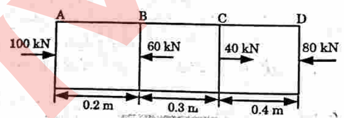
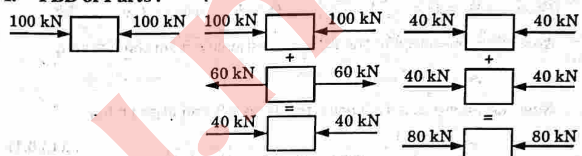


Fig. 1.19.1.

Answer

Given : Bar subjected to load as shown in Fig. 1.20.1, Diameter of bar, d = 200 mm, E = 200 GPa.
To Find : Change in length of bar.

1. **FBD of Parts :**



(a) FBD of part AB (b) FBD of part BC (c) FBD of part CD

Fig. 1.19.2.

2. All the three segments are subjected to compressive loads, hence total change in length of the bar,

$$\delta l = \frac{\Sigma P_i l_i}{AE} = \frac{(100 \times 10^3 \times 200 + (40 \times 10^3 \times 300) + (80 \times 10^3 \times 400)}{\pi / 4 \times (200)^2 \times 200 \times 10^3}$$

$$= 0.0102 \text{ mm}$$

This change in length is decrease in length because all loads are of compressive type.

Que 1.20. **Prove that the total extension of a uniformly tapering rod of diameters D_1 and D_2, when the rod is subjected to an axial load P is given by,**

$$dL = \frac{4PL}{\pi E D_1 D_2}$$

where, L = Total length of the rod.

Answer

1. A bar uniformly tapering from a diameter D_1 at one end to a diameter D_2 at the other end is shown in Fig. 1.20.1.
Let, P = Axial tensile load on the bar.
L = Total length of the bar.
E = Young's modulus.
2. Consider a small element of length dx of the bar at a distance x from the left end. Let the diameter of the bar be D_x at a distance x from the left end.

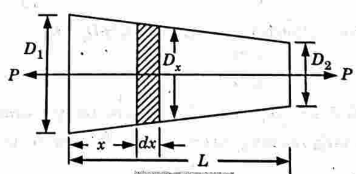


Fig. 1.20.1.

Then, $D_x = D_1 - \left(\frac{D_1 - D_2}{L}\right)x = D_1 - kx$, where, $k = \frac{D_1 - D_2}{L}$

3. Area of cross-section of the bar at a distance x from the left end,

$$A_x = \frac{\pi}{4} D_x^2 = \frac{\pi}{4}(D_1\, k.x)^2$$

4. Now, the stress at a distance x from the left end is given by,

$$\sigma_x = \frac{\text{Load}}{A_x} = \frac{P}{\frac{\pi}{4}(D_1 - kx)^2} = \frac{4P}{\pi(D_1 - kx)^2} \qquad ...(1.20.1)$$

5. The strain, ε_x in the small element of diameter d_x is obtained by using eq. (1.20.1), $\varepsilon_x = \frac{\sigma_x}{E} = \frac{4P}{\pi E(D_1 - kx)^2}$

6. Extension of the small elemental length, $dx = \varepsilon_x\, dx = \frac{4P}{\pi E(D_1 - k.x)^2}\, dx$

7. Total extension of the bar is obtained by integrating the above equation between the limits 0 and L.

Total extension, $dL = \int_0^L \frac{4P}{\pi E(D_1 - kx)^2}\, dx = \frac{4P}{\pi E}\int_0^L (D_1 - kx)^{-2}\, dx$

$$= \frac{4P}{\pi E}\int_0^L \frac{(D_1 - kx)^{-2} \times (-k)}{(-k)}\, dx$$

[Multiplying and dividing by $(-k)$]

$$= \frac{4P}{\pi E}\left[\frac{(D_1 - k.x)^{-1}}{(-1)\times(-k)}\right]_0^L = \frac{4P}{\pi E k}\left[\frac{1}{D_1 - kL} - \frac{1}{D_1}\right]$$

8. Substituting the value of $k = \frac{D_1 - D_2}{L}$ in the above equation, we get

Total extension,

$$dL = \frac{4P}{\pi E\left(\frac{D_1 - D_2}{L}\right)}\left[\frac{1}{D_1 - \left(\frac{D_1 - D_2}{L}\right)L} - \frac{1}{D_1}\right] = \frac{4PL}{\pi E D_1 D_2}$$

9. If the rod is of uniform diameter, then $D_1 = D_2 = D$

Total extension, $dL = \frac{4PL}{\pi E D^2}$

Que 1.21. **Find an expression for the total elongation of a uniformly tapering rectangular bar when it is subjected to an axial load *P*.**

Answer

Expression for Elongation of a Rectangular Tapered Bar :

1. Consider a rectangular tapered bar of width b_1 as bigger end and b_2 as smaller end.

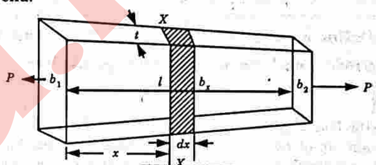


Fig. 1.21.1.

2. Take an element at distance x from bigger end of length dx being subjected to load 'P'.

3. Width of the element section at X–X, $b_x = b_1 - \frac{b_1 - b_2}{l}\, x$

$$b_x = b_1 - kx \qquad \left(\text{Where, } k = \frac{b_1 - b_2}{l}\right)$$

4. Cross-sectional area at the section X–X, $A_x = b_x t = (b_1 - kx)t$

5. Stress at the element, $\sigma_x = \frac{P}{A_x} = \frac{P}{(b_1 - kx)t}$

6. Strain at the element, $\varepsilon_x = \frac{\sigma_x}{E} = \frac{P}{Et(b_1 - kx)}$

7. Elongation of element, $dl_x = \varepsilon_x dx = \frac{P}{Et(b_1 - kx)}\, dx$

8. For total elongation of the bar, integrate the above expression between limits, $x = 0$ to $x = l$

$$dl = \int_0^l \frac{P}{Et(b_1 - kx)}\, dx = \frac{P}{-Etk}[\ln(b_1 - kx)]_0^l$$

$$= \frac{P}{-Etk}\left[\ln\frac{b_1 - kl}{b_1}\right] = -\frac{P}{Etk}\left[\ln\frac{b_2}{b_1}\right] \qquad \{\because b_2 = b_1 - kl\}$$

$$dl = -\frac{P}{Et\left(\frac{b_1 - b_2}{l}\right)}\left(\ln\frac{b_2}{b_1}\right) = \frac{Pl}{Et(b_1 - b_2)}\ln\frac{b_1}{b_2}$$

PART-6

Composite Bar and Temperature Stress.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.22. **Define a composite bar. How will you find the stresses and load carried by each member of a composite bar ?**

Answer

A. Composite Bar :

1. A bar, made up of two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for extension or compression when subjected to an axial tensile or compressive load is called a composite bar.
2. For the composite bar the following two points are important :

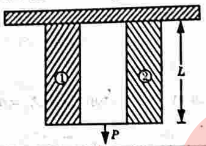


Fig. 1.22.1. Composite bar made up of two different materials.

i. The extension or compression in each bar is equal. Hence deformation per unit length *i.e.*, strain in each bar is equal.

ii. The total external load on the composite bar is equal to the sum of the loads carried by each different material.

3. Let, P = Total load on the composite bar.
   L = Length of composite bar and also length of bars of different materials.
   A_1 and A_1 = Area of cross-section of bar-1 and bar-2.
   E_1 and E_2 = Young's Modulus of bar-1 and bar-2.
   P_1 and P_2 = Load shared by bar-1 and bar-2.
   σ_1 and σ_2 = Stress induced in bar-1 and bar-2.
4. Now, the total load on the composite bar is equal to the sum of the load carried by the two bars, $P = P_1 + P_2$ ...(1.22.1)
5. The stress in bar-1, $\sigma_1 = \dfrac{\text{Load carried by bar-1}}{\text{Area of cross-section of bar-1}}$

$\therefore \quad \sigma_1 = \dfrac{P_1}{A_1}$ or $P_1 = \sigma_1 A_1$ ...(1.22.2)

Similarly, stress in bar-2, $\sigma_2 = \dfrac{P_2}{A_2}$ or $P_2 = \sigma_2 A_2$ ...(1.22.3)

Substituting the values of P_1 and P_2 in equation (1.22.1), we get

$P = \sigma_1 A_1 + \sigma_2 A_2$ ...(1.22.4)

6. Since the ends of the two bars are rigidly connected, each bar will change in length by the same amount. Also the length of each bar is same and hence the ratio of change in length to the original length (*i.e.*, strain) will be same for each bar.

Strain in bar-1, $\varepsilon_1 = \dfrac{\text{Stress in bar}-1}{\text{Young's modulus of bar}-1} = \dfrac{\sigma_1}{E_1}$

Similarly, strain in bar-2, $\varepsilon_2 = \dfrac{\sigma_2}{E_2}$.

strain in bar-1 = Strain in bar-2

$\therefore \quad \dfrac{\sigma_1}{E_1} = \dfrac{\sigma_2}{E_2}$ ...(1.22.5)

7. From eq. (1.22.4) and (1.22.5), the stresses σ_1 and σ_2 can be determined. By substituting the values of σ_1 and σ_2 in eq. (1.22.2) and (1.22.3), the load carried by different materials may be computed.

Que 1.23. **A rod whose ends are fixed to rigid supports, is heated so that rise in temperature is T °C. Prove that the thermal strain and thermal stresses set up in the rod are given by,**

Thermal strain = αT and Thermal stress = αTE

where, α = Co-efficient of linear expansion.

Answer

1. The stresses induced in a body due to change in temperature are known as thermal stresses.
2. Consider a body which is heated to a certain temperature.
   Let, L = Original length of the body.
   T = Rise in temperature.
   E = Young's modulus.
   α = Co-efficient of linear expansion.
   dL = Extension of rod due to rise of temperature.
3. If the rod is free to expand, then extension of the rod is given by,
   $dL = \alpha TL$
4. If the ends of the body are fixed to rigid supports, so that its expansion is prevented, then compressive stress and strain will be set up in the rod. These stresses and strains are known as thermal stresses and thermal strain.
5. Thermal strain, $\phi = \dfrac{\text{Chane in length}}{\text{Original length}} = \dfrac{dL}{L} = \dfrac{\alpha TL}{L} = \alpha T$

6. Thermal stress, σ = Thermal strain $\times E = \phi \times E = \alpha TE$

Que 1.24. **A steel rod of 3 cm diameter is enclosed centrally in a hollow copper tube of external diameter 5 cm and internal diameter 4 cm. The composite bar is then subjected to an axial pull of 45000 N. If the length of each bar is equal to 15 cm.**
Determine :
i. The stresses in the rod and the tube.
ii. Load carried by each rod.
***E* for steel = 200 GPa and for Copper = 100 GPa.**

AKTU 2016-17, Marks 05

Answer

Given : Diameter of steel rod = 3 cm = 30 mm,
External diameter of copper tube = 5 cm = 50 mm,
Internal diameter of copper tube = 4 cm = 40 mm,
Axial pull on composite bar, P = 45000 N,
Length of each bar, L = 15 cm,
Young's modulus for steel, $E_s = 2 \times 10^5$ N/mm²
Young's modulus for copper, $E_c = 1 \times 10^5$ N/mm²
To Find : i. Stresses in the rod and the tube.
ii. Load carried by each rod.

1. Area of steel rod, $A_s = \frac{\pi}{4}(30)^2 = 706.86$ mm²
2. Area of copper tube, $A_c = \frac{\pi}{4}[50^2 - 40^2]$ mm² $= 706.86$ mm²
3. Let σ_s = Stress in steel, P_s = Load carried by steel rod, σ_c = Stress in copper, and P_c = Load carried by copper tube.
4. Now, strain in steel = Strain in copper

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c} \qquad \left(\because \ \frac{\sigma}{E} = \text{Strain}\right)$$

$$\therefore \quad \sigma_s = \frac{E_s}{E_c} \times \sigma_c = \frac{2 \times 10^5}{1 \times 10^5} \times \sigma_c = 2\,\sigma_c \qquad ...(1.24.1)$$

5. Now, $\text{Stress} = \frac{\text{Load}}{\text{Area}} \Rightarrow \text{Load} = \text{Stress} \times \text{Area}$
6. Load on steel + Load on copper = Total load

$$\sigma_s \times A_s + \sigma_c \times A_c = P \qquad (\because \ \text{Total load} = P)$$
$$2\,\sigma_c \times 706.86 + \sigma_c \times 706.86 = 45000$$
$$\sigma_c = 21.22 \text{ N/mm}^2$$

5. Substituting the value of σ_c in eq. (1.23.1), we get

$$\sigma_s = 2 \times 21.22 \text{ N/mm}^2 = 42.44 \text{ N/mm}^2$$

6. Load carried by steel rod = Stress × Area = 42.44 × 706.86 = 29999.13 N
7. Load carried by copper tube = 45000 – 29999.13 = 15000.87 N

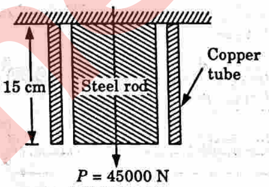


Fig. 1.24.1.

Que 1.25. **A steel tube of 24 mm external diameter and 18 mm internal diameter encloses a copper rod 15 mm diameter to which it is rigidly attached at each end. If, at a temperature of 10 °C there is no longitudinal stress, calculate the stresses in the tube and rod when the temperature is raised to 200 °C.**

$$E_{steel} = 210 \text{ kN/mm}^2, \ E_{copper} = 210 \text{ kN/mm}^2$$

Coefficients of linear expansion :

$$\alpha_{steel} = 11 \times 10^{-6} /°C, \ \alpha_{copper} = 11 \times 10^{-6} /°C$$

AKTU 2018-19, Marks 07

Answer

Given : D_S = 24 mm, d_s = 18, D_C 15 mm, t_1 = 10 °C, t_2 = 200 °C, E_s = 210 kN/mm², $\alpha_s = 11 \times 10^{-6}$/°C
To Find : Stress in the tube and rod.

Note : Since the given value of E and α for steel and copper are same hence it is not possible to solve the numerical.
So assuming the value of E and α for copper as

$$E_C = 110 \text{ kN/mm}^2$$
$$\alpha_c = 18 \times 10^{-6} / °C$$

and solving the numerical

1. Rise in temperature, $t = t_2 - t_1 = 200 - 10 = 190$ °C
2. **Stresses in the Rod and the Tube, σ_c, σ_s :**
   i. Fig. 1.25.1 since $\alpha_c > \alpha_s$, elongation of copper will naturally be more than that of steel for the same rise of temperature but since they are rigidly jointed at each end, the copper rod will venture to pull the steel tube along with it; whereas the steel tube will struggle to bring the copper rod back.
   ii. Ultimately, they will compromise and become stable at certain common position.
3. Extension of copper rod when free to expand = $ab = l\,\alpha_c \cdot t$
   Extension of steel rod when free to expand = $ac = l\,\alpha_s\,t$
4. Being connected together, suppose they compromise at the position dd; which means that steel tube will be pulled from c to d, and the

copper rod pushed back from b to d. In this way steel is under tension and the copper is under compression.

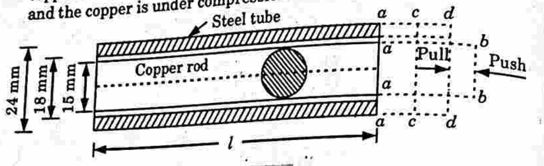


Fig. 1.25.1.

5. Compressive strain in copper rod,

$$\varepsilon_c = \frac{bd}{l} = \frac{ab - ad}{l} = \frac{ab}{l} - \frac{ad}{l}$$

$$= \alpha_c t - \frac{ad}{l} \qquad (\because ab = l\,\alpha_c t)$$

or, $$\varepsilon_c = \alpha_c t - \varepsilon \qquad \text{(where, } ad/l = \text{common strain} = e) \qquad ...(1.25.1)$$

6. Tensile strain in steel tube, $\varepsilon_s = \frac{cd}{l} = \frac{ad - ac}{l} = \frac{ad}{l} - \frac{ac}{l}$

$$= \frac{ad}{l} - \alpha_s t \qquad (\because ac = l\,\alpha_s t)$$

or, $$\varepsilon_s = \varepsilon - \alpha_s t \qquad ...(1.25.2)$$

7. Adding eq. (1.25.2) and (1.25.1), we get

$$\varepsilon_c + \varepsilon_s = \alpha_c t - \alpha_s t = t(\alpha_c - \alpha_s)$$

$$\frac{\sigma_c}{E_c} + \frac{\sigma_s}{E_s} = t(\alpha_c - \alpha_s) \qquad \left(\because \varepsilon_c = \frac{\sigma_c}{E_c} \text{ and } \varepsilon_s = \frac{\sigma_s}{E_s}\right)$$

$$\frac{\sigma_c}{1.0 \times 10^5} + \frac{\sigma_s}{2.1 \times 10^5} = 190\,(18 \times 10^{-6} - 11 \times 10^{-6})$$

$$2.1\,\sigma_c + \sigma_s = 279.3 \qquad ...(1.25.3)$$

8. But at the stabilized or common position dd,
Push on copper rod = Pull on steel tube

$$\sigma_c A_c = \sigma_s A_s$$

$$\sigma_c \times \frac{\pi}{4} \times 15^2 = \sigma_s \times \frac{\pi}{4} \times (24^2 - 18^2)$$

$$\sigma_c \times 225 = \sigma_s (576 - 324)$$

$$\sigma_s = \frac{225\,\sigma_c}{(576 - 324)} = 0.89\,\sigma_c \qquad ...(1.25.4)$$

9. Substituting for σ_s in eq. (1.25.3), we have

$$2.1\,\sigma_c + 0.89\,\sigma_c = 279.3$$

$$\therefore \quad \sigma_c = 279.3 / 2.99 = 93.41 \text{ N/mm}^2$$

10. From eq. (1.25.4), we get $\sigma_s = 0.89 \times 93.41 = 83.13 \text{ N/mm}^2$

PART-7

Strain Energy for gradual Sudden and Impact Loading.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.26. **Define resilience, proof resilience and modulus of resilience.**

Answer

1. **Resilience :** The total strain energy stored in a body is commonly known as resilience. Whenever the straining force is removed from the strained body, the body is capable of doing work. Hence the resilience is also defined as the capacity of a strained body for doing work on the removal of the straining force.
2. **Proof Resilience :** The maximum strain energy stored in a body is known as proof resilience. The strain energy stored in the body will be maximum when the body is stressed upto elastic limit. Hence the proof resilience is the quantity of strain energy stored in a body when strained upto elastic limit.
3. **Modulus of Resilience :** It is defined as the proof resilience of a material per unit volume. It is an important property of a material. Mathematically,

$$\text{Modulus of resilience} = \frac{\text{Proof resilience}}{\text{Volume of the body}}$$

Que 1.27. **Derive the expression for strain energy stored in a body when the load is applied gradually. Also write the expression for sudden load applied.**

Answer

Strain Energy Stored in a Body when the Load is applied gradually :

1. Fig. 1.27.1 shows load extension diagram of a body under tensile test upto elastic limit.
2. The tensile load P increases gradually from zero to the value of P and the extension of the body increases from zero to the value of x.
3. The load P performs work in stretching the body. This work will be stored in the body as strain energy which is recoverable after the load P is removed.
4. Let, P = Gradually applied load, x = Extension of the body, A = Cross-sectional area, L = Length of the body, V = Volume of the body,

E = Young's modulus, U = Strain energy stored in the body, and σ = Stress induced in the body.

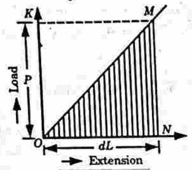


Fig. 1.27.1.

5. Now, work done by the load = Area of load extension curve (Shaded area in Fig. 1.27.1)

$$= \frac{1}{2} \times P \times dL. \qquad ...(1.27.1)$$

6. Load, $\quad dL$ = Stress × Area = $\sigma \times A$
   Extension, $\quad dL$ = Strain × Length

$$= \frac{\text{Stress}}{E} \times L = \frac{\sigma}{E} \times L$$

7. Substituting the values of P and x in eq. (1.27.1), we get

$$\text{Work done by the load} = \frac{1}{2} \times \sigma \times A \times \frac{\sigma}{E} \times L = \frac{1}{2}\frac{\sigma^2}{E} \times A \times L$$

$$= \frac{\sigma^2}{2E} \times V \qquad (\because \text{ Volume, } V = A \times L)$$

8. The work done by the load in stretching the body is equal to the strain energy stored in the body.

   Therefore, energy stored in the body, $U = \dfrac{\sigma^2}{2E} \times V$

9. As the load is applied suddenly, the load, P is constant when the extension of the bar takes place.

   $\therefore$ Work done by the load = Load × Extension = $P \times dL$

10. The maximum strain energy stored (*i.e.*, Energy stored upto elastic limit) in a body is given by,

$$U = \frac{\sigma^2}{2E} \times \text{Volume of the body}$$

$$= \frac{\sigma^2}{2E} \times A \times L \qquad (\because \text{ Volume} = A \times L)$$

11. Equating the strain energy stored in the body to the work done, we get

$$\frac{\sigma^2}{2E} \times A \times L = P \times dL = P \times \frac{\sigma}{E} \times L \qquad \left(\because dL = \frac{\sigma}{E} \times L\right)$$

$$\sigma = 2 \times \frac{P}{A} \times \frac{\sigma \times dL}{2}$$

12. From the above equation it is clear that the maximum stress induced due to suddenly applied load is twice the stress induced when the same load is applied gradually.

Que 1.28. **The load to be carried by a lift may be dropped 10 cm on to the floor. The cage itself weighs 100 kg and is supported by 25 m of wire rope weighing 0.9 kg/m, consisting of 49 wires each 1.6 mm diameter. The maximum stress in the wire is limited to 90 N/mm² and E for the rope is 70000 N/mm². Find the maximum load which can be carried.**

AKTU 2018-19, Marks 07

Answer

Given : Weight of cage = 100 kg, Length of rope = 25 m, Unit weight of rope = 0.9 kg/m, Number of wire = 49, Diameter of wire = 1.6 mm, Maximum stress = 90 N/mm², E = 70000 N/mm²
To Find : Maximum load.

1. Initial stress in the rope $= \dfrac{mg}{A} = \dfrac{(100 + 25 \times 0.9)\, 9.81}{49 \times (\pi/4) \times 1.6^2} = 12.4 \text{ N/mm}^2$
2. Increase in stress, $\sigma = 90 - 12.4 = 77.8$ N/mm²
3. Load due to increased stress,
   $p = 77.8 \times 49 \times (\pi/4) \times 1.6^2 = 7665$ N = 782 kg
4. Increase in length of rope, $x = \sigma \times L/E$

$$x = \frac{77.8 \times 25 \times 100}{70000} = 27.8 \text{ mm} = 2.78 \text{ cm}$$

5. If W is the load dropped, applying the energy equation, we get

$$W(h + x) = (1/2)\, px$$

$$W(10 + 2.78) = \frac{1}{2} \times 782 \times 2.78$$

   Maximum load, $\quad W$ = 85 kg

Que 1.29. **A bar of uniform cross sectional area A and length L hangs vertically, subjected to its own weight. Prove that the strain energy stored within the bar is given by**

$$U = \frac{A \times \rho^2 \times L^3}{6E}$$

Where E is modulus of elasticity and ρ is weight per unit volume.

AKTU 2016-17, Marks 10

Answer

1. Fig. 1.29.1 shows a bar AB fixed at end A and hanging freely under its own weight.

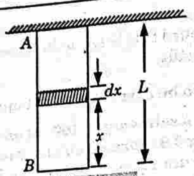


Fig. 1.29.1.

2. Let L = Length of bar, A = Area of cross-section, E = Young's modulus for the bar material, ρ = Weight per unit volume of the bar material.
3. Consider a small strip of thickness dx at a distance x from the lower end. A weight of ρAx is acting in the downward direction. Due to this weight, there will be some increase in the length of element.
4. Elongation of the element = $\dfrac{\rho x}{E} dx$
5. Therefore, strain energy stored in the element;

$$dU = \frac{1}{2} \times \text{Force} \times \text{Elongation}$$

$$= \frac{1}{2} \times \rho \times A \times x \times \frac{\rho x}{E} dx = \frac{1}{2} \frac{\rho^2 A}{E} x^2 \, dx$$

6. Total strain energy stored in the bar, $U = \displaystyle\int_0^L \frac{1}{2} \frac{\rho^2 A}{E} x^2 \, dx = \frac{A \rho^2 L^3}{6E}$

Que 1.30. **A weight W = 5 kN attached to the end of a steel wire rope moves downward with constant velocity 1 m/sec. What stresses are produced in the rope when its upper end is suddenly stripped ? The free length of rope at the moment of impact is 20 m, its net cross-sectional area is 10 sq. cm and $E = 2 \times 10^5$ N/mm².**

Answer

Given : Weight, W = 5 kN = 5000 N, Velocity, v = 1 m/sec,
Cross-sectional area of rope, $A = 10 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2$,
Length of rope at the moment of impact, h = 20 m,
$E = 2.00 \times 10^5 \text{ N/mm}^2 = 2 \times 10^{11} \text{ N/m}^2$
To Find : Stresses in rope when its upper end is suddenly stripped.

1. Initial stress = $\dfrac{W}{A} = \dfrac{5000}{10 \times 10^{-4}} = 5 \times 10^6 \text{ N/m}^2$
2. Let, σ_2 be the additional stress produced in wire rope at the sudden impact.
3. Kinetic energy = Strain energy stored in rope

$$\therefore \quad \frac{1}{2} \times \frac{W}{g} v^2 = \frac{\sigma^2}{2E} V \qquad (\because V = A \times h)$$

$$\frac{1}{2} \times \frac{5000}{9.81} \times 1^2 = \frac{\sigma^2}{2 \times 2 \times 10^{11}} \times (10 \times 10^{-4} \times 20)$$

$$\sigma = 71392156 \text{ N/m}^2 \Rightarrow \sigma = 71392.156 \text{ kN/m}^2$$

Que 1.31. **Obtain a relation for the stress induced in a body, if a load P is applied with an impact.**

Answer

1. Consider a vertical rod fixed at the upper end and having a collar at the lower end as shown in Fig. 1.31.1.

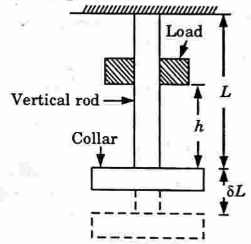


Fig. 1.31.1.

2. Let the load be dropped from a height on the collar. Due to this impact load, there will be some extension in the rod.
3. Let, P = Load dropped (*i.e.*, load applied with impact), L = Length of the rod, A = Cross-sectional area of the rod, V = Volume of rod = $A \times L$, δL = Extension of the rod due to load P, E = Modulus of elasticity of the material of rod, σ = Stress induced in the rod due to impact load.
4. The strain in the bar is given by, Strain = $\dfrac{\text{Stress}}{E}$

$$\frac{\delta L}{L} = \frac{\sigma}{E}$$

$$\therefore \quad \delta L = \frac{\sigma}{E} \times L$$

5. Work done by the load = Load × Distance moved

$$= P(h + \delta L) \qquad \text{...(1.31.1)}$$

6. The strain energy stored by the rod,

$$U = \frac{\sigma^2}{2E} \times V = \frac{\sigma^2}{2E} \times AL \qquad \text{...(1.31.2)}$$

7. Equating the eq. (1.31.1) and eq. (1.31.2), we get

$$P(h + \delta L) = \frac{\sigma^2}{2E} \times AL$$

After putting the value of δL, we get

$$\frac{\sigma^2}{2E} \times AL - P \times \frac{\sigma}{E} \times L - Ph = 0$$

8. Multiplying both sides by $\frac{2E}{AL}$, we get

$$\sigma^2 - P \times \frac{\sigma}{E} \times L \times \frac{2E}{AL} - Ph \times \frac{2E}{AL} = 0 \Rightarrow \sigma^2 - \frac{2P}{A} \times \sigma - \frac{2PEh}{AL} = 0$$

9. The above equation is a quadratic equation in 'σ',

$$\therefore \quad \sigma = \frac{\frac{2P}{A} \pm \sqrt{\left(\frac{2P}{A}\right)^2 + 4 \times \frac{2PEh}{AL}}}{2 \times 1}$$

$$\sigma = \frac{P}{A} + \sqrt{\left(\frac{P}{A}\right)^2 + \frac{2PEh}{A \times L}} = \frac{P}{A}\left(1 + \sqrt{1 + \frac{2AEh}{PL}}\right)$$

PART-8

Normal Stress and Strain, Shear Stress and Strain, Stresses on Inclined Section, Principal Stress and Strain, Maximum Shear Stress.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.32. **Analyze when a body is subjected to a direct tensile stress (σ) in one plane and accompanied by a single shear stress (τ).**

OR

Prove that the maximum shear stress in the body is the half of the difference between maximum principal and minimum principal stress.

AKTU 2014-15, Marks 05

Answer

1. Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to tensile stresses along x-x and y-y axes and accompanied by a positive (*i.e.*, clockwise) shear stress along x-x axis as shown in Fig. 1.32.1(a).

2. Now let us consider an oblique section AB inclined with x-x axis on which we are required to find out the stresses as shown in the Fig. 1.32.1.
3. Let, σ_x = Tensile stress along x-x axis, σ_y = Tensile stress along y-y axis, τ_{xy} = Positive (*i.e.* clockwise) shear stress along x-x axis, and θ = Angle, which the oblique section AB makes with x-x axis in an anticlockwise direction.

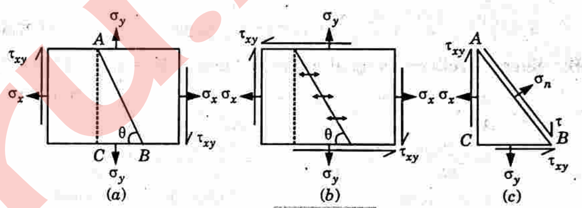


Fig. 1.32.1.

4. First of all, consider the equilibrium of the wedge ABC. We know that as per the principle of simple shear, the face BC of the wedge will be subjected to an anticlockwise shear stress equal to τ_{xy} as shown in Fig. 1.32.1(b).
5. We know that horizontal force acting on the face AC,

$$P_1 = \sigma_x \times AC\ (\leftarrow) \qquad ...(1.32.1)$$

and vertical force acting on the face AC,

$$P_2 = \tau_{xy} \times AC\ (\uparrow) \qquad ...(1.32.1)$$

6. Similarly, vertical force acting on the face BC,

$$P_3 = \sigma_y \times BC\ (\downarrow) \qquad ...(1.32.3)$$

and horizontal force on the face BC,

$$P_4 = \tau_{xy} \times BC\ (\rightarrow) \qquad ...(1.32.4)$$

7. Now resolving the forces perpendicular to the section AB,

$$P_n = P_1 \sin\theta - P_2 \cos\theta + P_3 \cos\theta - P_4 \sin\theta$$
$$= \sigma_x AC \sin\theta - \tau_{xy} AC \cos\theta + \sigma_y BC \cos\theta - \tau_{xy} BC \sin\theta$$

and now resolving the forces tangential to AB,

$$P_t = P_1 \cos\theta + P_2 \sin\theta - P_3 \sin\theta - P_4 \cos\theta$$
$$= \sigma_x AC \cos\theta + \tau_{xy} AC \sin\theta - \sigma_y BC \sin\theta - \tau_{xy} BC \cos\theta$$

A. **Normal Stress (across the section AB) :**

$$\sigma_n = \frac{P_n}{AB} = \frac{\sigma_x AC \sin\theta - \tau_{xy} AC \cos\theta + \sigma_y BC \cos\theta - \tau_{xy} BC \sin\theta}{AB}$$

$$= \frac{\sigma_x AC \sin\theta}{AB} - \frac{\tau_{xy} AC\cos\theta}{AB} + \frac{\sigma_y BC\cos\theta}{AB} - \frac{\tau_{xy} BC\sin\theta}{AB}$$

$$= \frac{\sigma_x AC\sin\theta}{\frac{AC}{\sin\theta}} - \frac{\tau_{xy} AC\cos\theta}{\frac{AC}{\sin\theta}} + \frac{\sigma_y BC\cos\theta}{\frac{BC}{\cos\theta}} - \frac{\tau_{xy} BC\sin\theta}{\frac{BC}{\cos\theta}}$$

$$= \sigma_x \sin^2\theta - \tau_{xy}\sin\theta\cos\theta + \sigma_y\cos^2\theta - \tau_{xy}\sin\theta\cos\theta$$

$$\sigma_n = \left(\frac{\sigma_x+\sigma_y}{2}\right) - \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta \quad ...(1.32.5)$$

B. Shear Stress or Tangential Stress (across the section *AB*) :

$$\tau = \frac{P_t}{AB} = \frac{\sigma_x AC\cos\theta + \tau_{xy} AC\sin\theta - \sigma_y BC\sin\theta - \tau_{xy} BC\cos\theta}{AB}$$

$$= \frac{\sigma_x AC\cos\theta}{AB} + \frac{\tau_{xy} AC\sin\theta}{AB} - \frac{\sigma_y BC\sin\theta}{AB} - \frac{\tau_{xy} BC\cos\theta}{AB}$$

$$= \frac{\sigma_x AC\cos\theta}{\frac{AC}{\sin\theta}} + \frac{\tau_{xy} AC\sin\theta}{\frac{AC}{\sin\theta}} - \frac{\sigma_y BC\sin\theta}{\frac{BC}{\cos\theta}} - \frac{\tau_{xy} BC\cos\theta}{\frac{BC}{\cos\theta}}$$

$$= \sigma_x\sin\theta\cos\theta + \tau_{xy}\sin^2\theta - \sigma_y\sin\theta\cos\theta - \tau_{xy}\cos^2\theta$$

$$\tau = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta \quad ...(1.32.6)$$

8. Now the planes of maximum and minimum normal stresses (*i.e.*, principal planes) may be find out by equating the shear stress to zero.
9. Now, let θ_p be the value of the angle for which the shear stress is zero.

$$\therefore \quad \frac{\sigma_x-\sigma_y}{2}\sin 2\theta_p - \tau_{xy}\cos 2\theta_p = 0$$

$$\frac{\sigma_x-\sigma_y}{2}\sin 2\theta_p = \tau_{xy}\cos 2\theta_p \Rightarrow \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

10. From the above equation, we find that the following two cases satisfy this condition as shown in Fig. 1.32.2(*a*) and (*b*).

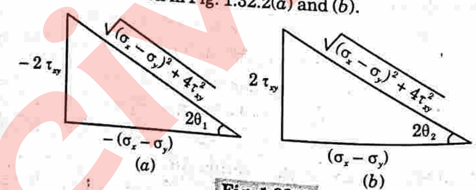


Fig. 1.32.2.

11. Thus we find that there are two principal planes, at right angles to each other, their inclinations with *x-x* axis being θ_1 and θ_2.

12. Now, for case 1,

$$\sin 2\theta_1 = \frac{-2\tau_{xy}}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}} \quad \text{and} \quad \cos 2\theta_1 = \frac{-(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

Similarly for case 2,

$$\sin 2\theta_2 = \frac{2\tau_{xy}}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}} \quad \text{and} \quad \cos 2\theta_2 = \frac{(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

13. Now the values of principal stresses may be find out by substituting the above values of $2\theta_1$ and $2\theta_2$ in eq. (1.32.5).

C. Maximum Principal Stress :

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta_1 - \tau_{xy}\sin 2\theta_1$$

$$= \frac{\sigma_x+\sigma_y}{2} - \left(\frac{\sigma_x-\sigma_y}{2}\times\frac{-(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma_y^2)+4\tau_{xy}^2}}\right) - \left(\tau_{xy}\times\frac{-2\tau_{xy}}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}\right)$$

$$\text{or} \quad \sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

D. Minimum Principal Stress :

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \frac{(\sigma_x-\sigma_y)}{2}\cos 2\theta_2 - \tau_{xy}\sin 2\theta_2$$

$$= \frac{\sigma_x+\sigma_y}{2} - \left(\frac{\sigma_x-\sigma_y}{2}\times\frac{(\sigma_x-\sigma_y)}{\sqrt{(\sigma_x-\sigma^2)+4\tau_{xy}^2}}\right) - \left(\tau_{xy}\times\frac{2\tau_{xy}}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}\right)$$

$$\text{or} \quad \sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

E. Maximum Shear Stress :

1. Maximum shear is given by,

$$(\sigma_t)_{max} = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

Putting the value of sin 2θ and cos 2θ

$$= \pm\frac{\sigma_x-\sigma_y}{2}\times\frac{\sigma_x-\sigma_y}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}} \pm \tau_{xy}\times\frac{2\tau_{xy}}{\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}}$$

$$= \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}$$

2. According to question, $\tau_{max} = \left(\frac{\sigma_1-\sigma_2}{2}\right)$

Putting the value of σ_1 and σ_2

$$= \frac{1}{2}\left[\frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} - \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}\right]$$

$$= \frac{1}{2}\left[\frac{2}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}\right]$$

3. Hence, $\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2} = \left(\frac{\sigma_1-\sigma_2}{2}\right)$

Que 1.33. **For a given loading conditions the state of stress in the wall of a cylinder is expressed as follows :**

i. 85 MN/m² tensile,

ii. 25 MN/ m² tensile at right angles to (i), and

iii. Shear stresses of 60 MN/m² on the planes on which the stresses (i) and (ii) act ; the shear couple acting on planes carrying the 25 MN/m² stress is clockwise in effect. Calculate principal stresses and principal planes.

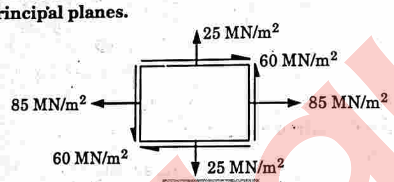


Fig. 1.33.1.

AKTU 2015-16, Marks 10

Answer

Given : $\sigma_x = +85$ MN/m², $\sigma_y = +25$ MN/m², $\tau_{xy} = 60$ MN/m²

To Find : Principal stresses and principal planes.

1. Principal stresses,

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{85+25}{2} \pm \sqrt{\left(\frac{85-25}{2}\right)^2+60^2} = 55 \pm 67.082$$

2. Maximum principal stress,

$$\sigma_1 = 55 + 67.082 = 122.082 \text{ MN/m}^2 \text{ (Tensile)}$$

3. Minimum principal stress,

$$\sigma_2 = 55 - 67.082 = 12.082 \text{ MN/m}^2 \text{ (Compressive)}$$

5. Principal plane, $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x-\sigma_y} = \frac{2\times 60}{85-25} = 2$

$$\theta_p = 31.72°$$

Que 1.34. **A rectangular block of material is subjected to a tensile stress of 110 MPa on one plane and a tensile stress of 47 MPa at right angles to the former. Each of the above stresses is accompanied by a shear stress of 63 MPa and that associated with the former tensile stress tends to rotate the block anticlockwise. Find :**

i. The direction and magnitude of each of the principal stress.

ii. Magnitude of greatest shear stress. **AKTU 2016-17, Marks 10**

Answer

Given : Major tensile stress, $\sigma_x = 110$ N/mm²
Minor tensile stress, $\sigma_y = 47$ N/mm², Shear stress, $\tau_{xy} = 63$ N/mm²

To Find : i. Direction and magnitude of each principal stress.
ii. Magnitude of greatest shear stress.

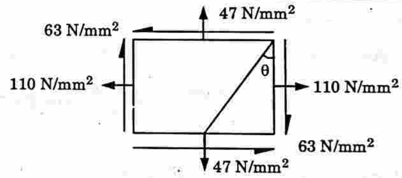


Fig. 1.34.1.

1. Major principal stress, σ_1

$$= \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = \frac{110+47}{2} + \sqrt{\left(\frac{110-47}{2}\right)^2+63^2}$$

$= 148.936$ N/mm² (tensile)

2. Minor principal stress, σ_2

$$= \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = \frac{110+47}{2} - \sqrt{\left(\frac{110-47}{2}\right)^2+63^2}$$

$= 8.064$ N/mm² (tensile)

3. The directions of principal stresses are given by,

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y} = \frac{2\times 63}{110-47} = 2.0$$

$$\theta = 31° 43' \text{ or } 121° 43'$$

4. Greatest shear stress is given by,

$$(\sigma_t)_{max} = \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}$$

$$= \frac{1}{2}\sqrt{(110-47)^2+4\times 63^2} = 70.436 \text{ N/mm}^2$$

Que 1.35. **At a point in a strained material the principal stresses are 100 MPa (tensile) and 60 MPa (compressive). Determine the normal stress, shear stress and resultant stress on a plane inclined at 50° to the axis of major principal stress. Also determine the maximum shear stress at a point.**

AKTU 2016-17, Marks 05

Answer

Given : Major principal stress, $\sigma_x = 100$ N/mm²
Minor principal stress, $\sigma_y = -60$ N/mm², Angle of plane = 50°
To Find : Normal stress, shear stress, resultant stress, and maximum shear stress at a point.

1. Angle of the inclined plane with the axis of minor principal stress,

$$\theta = 90° - 50° = 40°.$$

1. Normal stress, $\sigma_n = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2} \cos 2\theta$

$$= \frac{100 + (-60)}{2} + \frac{100 - (-60)}{2} \cos (2 \times 40°)$$

$$= 33.89 \text{ N/mm}^2$$

2. Shear stress, $\sigma_t = \dfrac{\sigma_x - \sigma_y}{2} \sin 2\theta$

$$= \frac{100 - (-60)}{2} \sin (2 \times 40°) = 78.785 \text{ N/mm}^2$$

3. Resultant stress, $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2} = \sqrt{33.89^2 + 78.785^2} = 85.765$ N/mm²

4. Maximum shear stress at a point,

$$\sigma_{t,\,max} = \frac{\sigma_x - \sigma_y}{2} = \frac{100 - (-60)}{2} = 80 \text{ N/mm}^2$$

Que 1.36. **At a point in a material under stress, the intensity of the resultant stress on a certain plane is 50 MPa (tensile) inclined at 30° to the normal of that plane. The stress on a plane at right angle to this has a normal tensile component of intensity of 30 MPa, find :**
i. The resultant stress on the second plane.
ii. The principal planes and stresses.
iii. The plane of maximum shear and its intensity.

AKTU 2017-18, Marks 07

Answer

Given : Resultant stress on plane *AB* = 50 MPa
Angle of inclination, θ = 30°, Normal stress on plane *BC*, $\sigma_y = 30$ MPa
To Find : i. The resultant stress on the second plane.
ii. Principal plane and stresses.
iii. Plane of maximum shear and its intensity.

1. The resultant stress 50 MPa on plane *AB* is resolved into normal and tangential stress.
The normal stress on plane *AB*, $\sigma_x = 50 \times \cos 30° = 43.30$ N/mm²
The tangential stress on plane *AB*, $\tau_{xy} = 50 \times \sin 30° = 25$ N/mm²
2. Resultant stress on the second plane $BC = \sqrt{\sigma_y^2 + \tau_{xy}^2}$

$$= \sqrt{30^2 + 25^2} = 39.05 \text{ N/mm}^2$$

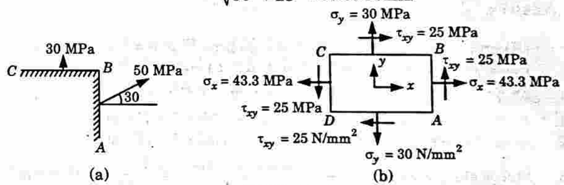


Fig. 1.36.1.

3. Principal stresses,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{43.30 + 30}{2} \pm \sqrt{\left(\frac{43.30 - 30}{2}\right)^2 + (25)^2} = 36.65 \pm 25.87$$

4. Major principal stress,

$$\sigma_1 = 36.65 \pm 25.87 = 62.52 \text{ N/mm}^2 \text{ (tensile)}$$

5. Manor principle stress, $\sigma_2 = 36.65 - 25.87 = 10.78$ N/mm²
6. The direction of principal stress,

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2 \times 25}{43.3 - 30}$$

$$\theta = 37.55° \text{ or } 127.55°$$

6. Maximum shear stress, $(\sigma_t)_{max} = \dfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$

$$= \sqrt{\left(\frac{43.3 - 30}{2}\right)^2 + (25)^2} = 25.87 \text{ N/mm}^2$$

Que 1.37. **A small block is 40 mm long, 30 mm high and 5 mm in thick. It is subjected to uniformly distributed tensile forces having the resultant values in N shown in Fig. 1.37.1. Compute the stress components developed along the diagonal AC.**

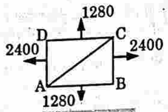


Fig. 1.37.1.

AKTU 2018-19, Marks 07

Answer

Given : Length = 40 mm, Height = 30 mm, Width = 5 mm, Force along x-axis = 2400 N, Force along y-axis = 1280 N
To Find : Stress component along diagonal AC.

1. Area of cross-section normal to X-axis = 30 × 5 = 150 mm^2
2. Area of cross-section normal to Y-axis = 40 × 5 = 200 mm^2
3. Stress along x-axis, $= \dfrac{\text{Force along } X\text{-axis}}{\text{Area normal to } X\text{-axis}} = \dfrac{2400}{150} = 16 \text{ N/mm}^2$
4. Stress along y-axis, $\sigma_y = \dfrac{\text{Force along } Y\text{-axis}}{\text{Area normal to } Y\text{-axis}}$

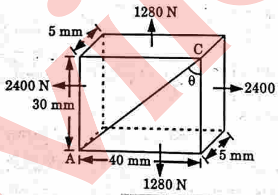


Fig. 1.37.2.

$$\sigma_y = \frac{1280}{200} = 6.4 \text{ N/mm}^2$$

5. Inclination of diagonal AC, $\tan\theta = 40/30 \Rightarrow \theta = 53° \, 7'$
6. Normal stress, on AC

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$\sigma_n = \frac{16 + 6.4}{2} + \frac{16 - 6.4}{2}\cos(2 \times 53° \, 7') = 9.858 \text{ N/mm}^2$$

7. Tangential stress on AC,

$$\sigma_t = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta = \frac{16 - 6.4}{2}\sin(2 \times 53° \, 7') = 4.608 \text{ N/mm}^2$$

Que 1.38. **What are complementary shear stresses ? Explain with diagram.**

Answer

Complementary Shear Stress :

1. Whenever a shear stress τ is applied on parallel surface of body then to keep the body in equilibrium a shear stress τ is induced on remaining surface of body. These stresses form a couple. The couple form due to shear stress τ produces clockwise moment. For equilibrium this couple is balanced by couple developed by τ'. This resisting shear stress τ' is known as complementary shear stress.

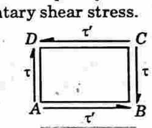


Fig. 1.38.1.

2. Couple produced by τ, $(\tau BC) \times AB$
3. Couple produced by τ', $\tau'(CD) \times BC$
4. For equilibrium,

$$(\tau BC) \times AB = (\tau' CD) \times BC$$

$$\Rightarrow \quad \tau = \tau' \qquad (\because \; AB = CD)$$

Que 1.39. **In a strained material at a point, the strains are ε_{xx} = 600 μ strain, ε_{yy} = 200 μ strain and ε_{xy} = 300 μ strain. What is the maximum principal strain at a point ?**

Answer

Given : $\varepsilon_{xx} = 600\,\mu$, $\varepsilon_{yy} = 200\,\mu$, $\varepsilon_{xy} = 300\,\mu$
To Find : Maximum principal strain at a point.

Maximum principal strain is given by,

$$\varepsilon_1 = \left(\frac{\varepsilon_{xx} + \varepsilon_{yy}}{2}\right) + \sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\right)^2 + \left(\frac{\varepsilon_{xy}}{2}\right)^2}$$

$$= \left(\frac{600\mu + 200\mu}{2}\right) + \sqrt{\left(\frac{600\,\mu - 200\mu}{2}\right)^2 + \left(\frac{300\,\mu}{2}\right)^2} = 650\,\mu$$

Que 1.40. At a point in a strained material, stresses are applied as shown in Fig. 1.40.1 find out the normal and shear stress on the oblique plane, principal stresses and principal strain.

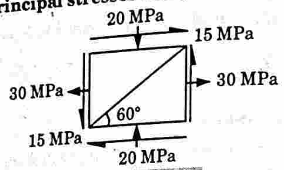


Fig. 1.40.1.

AKTU 2014-15, Marks 10

Answer

Given : $\sigma_x = 30$ MPa, $\sigma_y = -20$ MPa, $\tau_{xy} = 15$ MPa.
Oblique angle, $\theta = 90° - 60° = 30°$ from major axes of plane.
To Find : Normal and shear stress on oblique plane, principal stresses and principal strain.
Data Assume : $\mu = 0.5$ and $E = 200$ MPa

1. Normal stress, $\sigma_n = \dfrac{\sigma_x + \sigma_y}{2} - \dfrac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta.$

$$= \frac{30 + (-20)}{2} - \frac{30 + 20}{2} \cos 60° - 15 \sin 60°$$

$\sigma_n = -20.5$ MPa $= 20.5$ MPa (Compressive)

2. Shear stress, $\tau = \dfrac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$

$$= \frac{30 + 20}{2} \sin 60° - 15 \cos 60° = 14.150 \text{ MPa}$$

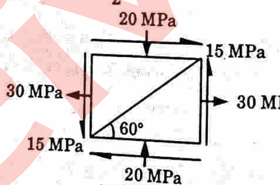


Fig. 1.40.2.

3. Principal stresses, $\sigma_{1,2} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$= \frac{30 - 20}{2} \pm \sqrt{\left(\frac{30 + 20}{2}\right)^2 + (15)^2} = 5 \pm 29.15$$

i. Major principle stress, $\sigma_1 = 5 + 29.15 = 34.15$ MPa (Tensile)
ii. Minor principle stress, $\sigma_2 = 5 - 29.15 = -24.15$ MPa (Compressive)

4. Principal strain :

i. Major principal strain, $\epsilon_1 = \dfrac{\sigma_1}{E} - \mu\dfrac{\sigma_2}{E} = \dfrac{1}{E}(\sigma_1 - \mu\sigma_2)$

$$\epsilon_1 = \frac{1}{200}([4.15 - 0.5 \times (-24.15)] = 0.23$$

ii. Minor principal strain, $\epsilon_2 = \dfrac{\sigma_2}{E} - \mu\dfrac{\sigma_1}{E} = \dfrac{1}{E}[\sigma_2 - \mu\sigma_1]$

$$\epsilon_2 = \frac{1}{200}(-24.15 - 0.5 \times 34.15) = -0.23$$

PART-9

Three Dimensions State of Stress and Strain, Equilibrium Equations, Generalized Hook's Law.3D, Mohr's Stress Circle.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.41. A body is subjected to direct stresses in two mutually perpendicular directions accompanied by a simple shear stress. Draw the Mohr's circle of stresses and explain how will you obtain the principal stresses and principal planes ?

Answer

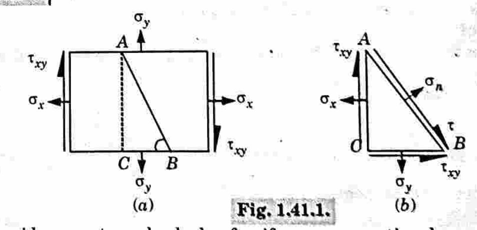


Fig. 1.41.1.

1. Consider a rectangular body of uniform cross-sectional area and unit thickness subjected to direct tensile stresses in two mutually

perpendicular directions along X-X and Y-Y axis accompanied by a positive (*i.e.*, clockwise) shear stress along X-X axis as shown in Fig. 1.41.1(*a*) and (*b*).

2. Now draw the Mohr's circle of stresses as shown in Fig. 1.41.2 and as discussed below :

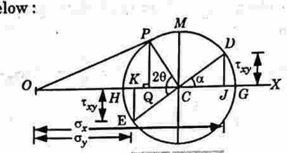


Fig. 1.41.2.

i. First of all, take some suitable point O and through it draw a horizontal line OX.

ii. Cut off OJ and OK equal to the tensile stresses σ_x and σ_y respectively to some suitable scale and towards right (because both the stresses are tensile).

iii. Now erect a perpendicular at J above the line OX (because τ_{xy} is positive along X-X axis) and cut off JD equal to the shear stress τ_{xy} to the scale. The point D represents the stress system on plane AC.

iv. Similarly, erect perpendicular below the line OX (because τ_{xy} is negative along Y-Y axis) and cut off KE equal to the shear stress τ_{xy} to the scale. The point E represents the plane BC. Join DE which bisects it at C.

v. Now with C as centre and radius equal to CD or CE draw a circle. It is known as Mohr's circle of stresses.

vi. Now through C, draw a line CP making an angle 2θ with CE in clockwise direction meeting the circle at P. The point P represents the stress system on section AB. Through P, draw PQ perpendicular to the line OX, join OP.

vii. Now OQ, QP and OP will give the normal stress, shear stress and resultant stress respectively to the scale. Similarly OG and OH will give the maximum and minimum principal shear stresses to the scale. The angle POC is called the angle of obliquity.

A. Proof :

1. From the geometry of the Mohr's Circle of stresses, we find that

$$OC = \frac{\sigma_x + \sigma_y}{2}$$

and radius of the circle, $R = EC = CD = CP = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

2. Now in the right angled triangle DCJ

$$\sin\alpha = \frac{JD}{DC} = \frac{\tau_{xy}}{R} \quad \text{and} \quad \cos\alpha = \frac{JC}{DC} = \frac{\sigma_x - \sigma_y}{2} \times \frac{1}{R} = \frac{\sigma_x - \sigma_y}{2R}$$

3. Similarly in right angled triangle CPQ, $\angle PCQ = (2\theta - \alpha)$

$$CQ = CP\cos(2\theta - \alpha) = R\,[\cos(2\theta - \alpha)]$$
$$= R\cos\alpha\cos 2\theta + R\sin\alpha\sin 2\theta$$
$$= R \times \frac{\sigma_x - \sigma_y}{2R}\cos 2\theta + R \times \frac{\tau_{xy}}{R}\sin 2\theta$$
$$= \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

4. **Normal Stress (across the section AB) :**

$$\sigma_n = OQ = OC - CQ = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

5. **Shear Stress or Tangential Stress (across the section AB) :**

$$\tau = QP = CP\sin[(2\theta - \alpha)] = R\sin(2\theta - \alpha)$$
$$= R\cos\alpha\sin 2\theta - R\sin\alpha\cos 2\theta$$
$$= R \times \frac{\sigma_x - \sigma_y}{2R}\sin 2\theta - R \times \frac{\tau_{xy}}{R}\cos 2\theta$$
$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

6. **Maximum Principal Stress :**

$$\sigma_{max} = OG = OC + CG = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

7. **Minimum Principal Stress :**

$$\sigma_{min} = OH = OC - CH = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Que 1.42. **State the generalized Hooke's law and prove for an isotropic elastic material that the maximum number of independent elastic constants is 21 only. Also show that for isotropic materials it is 2.**

AKTU 2015-16, Marks 10

Answer

1. Since any direct stress produces a strain in its own direction and an opposite kind of strain in every direction at right angle to this we have,

2. Longitudinal strain, $\varepsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} = \frac{\sigma_1}{E} - \mu\frac{(\sigma_2 + \sigma_3)}{E}$ ...(1.42.1)

3. Similarly, $\varepsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} = \frac{\sigma_2}{E} - \mu\frac{(\sigma_3 + \sigma_1)}{E}$ ...(1.42.2)

$\varepsilon_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} = \frac{\sigma_3}{E} - \mu\frac{(\sigma_1 + \sigma_2)}{E}$ ...(1.42.3)

4. Above three equations are known as general equations of Hooke's law or generalized Hooke's law.

5. Hook's law is the constitutive law for a Hookean or linear elastic material.

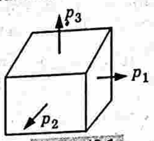


Fig. 1.42.1.

6. It can be given a precise expression in terms of stress and strain by stating, in the most general form, that, if $\tau_{ij} \neq \tau_{ji}$.

$$\sigma_x = f_1(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}, \gamma_{yx}, \gamma_{zy}, \gamma_{xz})$$
$$\sigma_y = f_2(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}, \gamma_{yx}, \gamma_{zy}, \gamma_{xz})$$
$$\sigma_z = f_3(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}, \gamma_{yx}, \gamma_{zy}, \gamma_{xz})$$
$$\tau_{xy} = f_4(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}, \gamma_{yx}, \gamma_{zy}, \gamma_{xz})$$
$$\tau_{yz} = f_5(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}, \gamma_{yx}, \gamma_{zy}, \gamma_{xz})$$
$$\tau_{zx} = f_6(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}, \gamma_{yx}, \gamma_{zy}, \gamma_{xz})$$
$$\tau_{yx} = f_7(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}, \gamma_{yx}, \gamma_{zy}, \gamma_{xz})$$
$$\tau_{zy} = f_8(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}, \gamma_{yx}, \gamma_{zy}, \gamma_{xz})$$
$$\tau_{xz} = f_9(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{zx}, \gamma_{yx}, \gamma_{zy}, \gamma_{xz})$$

7. These functions could be linear or non-linear.
8. For a small deformation an elastic material can be considered to be linearly elastic and in that case the functions become linear. Hence, we can write in tensor form,

$$\sigma^{ij} = C^{ijkl}\varepsilon^{kl}$$

This is the Cauchy's formulation for generalized Hook's law. In most of the general cases σ^{ij} and ε^{kl} will have 9 components each and C^{ijkl} will have 81 components.
9. So we have, in this way only 21 independent elastic constants. This is the maximum number of elastic constants for a completely anisotropic elastic material.
10. If we consider isotropy as well about various axes, the number of independent elastic constants reduces considerably, till for a perfectly isotropic material there are only 2 independent elastic constants.

PART-10

Theories of Failure and Factor of Safety.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.43. Name different theories of failure and represent them graphically.

AKTU 2015-16, Marks 05

OR

Explain different theories of failure along with their graphical representation.

AKTU 2017-18, Marks 07

OR

What are the various theories of failure ? Explain with diagram.

AKTU 2016-17, Marks 10

Answer

Types : Following are the types of failure theories :

A. Maximum Principal Stress Theory (Rankine's Theory) :

1. According to this theory, failure will occur when the maximum principal tensile stress (σ_1) in the system reaches the value of maximum stress at elastic limit (σ_{et}) in simple tension or minimum principal stress reaches the elastic limit stress (σ_{ec}) in simple compression.

$$\sigma_1 \geq \sigma_{et} \text{ (in simple tension)}$$
$$\sigma_3 < \sigma_{ec} \text{ (in simple compression)}$$

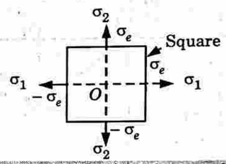


Fig. 1.43.1. Representation of maximum principal stress theory.

B. Maximum Principal Shear Stress Theory (Guest's or Tresca's Theory) :

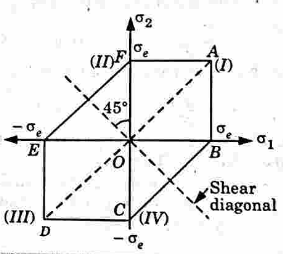


Fig. 1.43.2. Representation of maximum shear stress theory.

1. According to this theory, failure will occur when maximum shear stress τ_{max} in the system reaches the value of maximum shear stress in simple tension at elastic limit.

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_{et}}{2} \text{ (In simple tension)}$$

$$\sigma_1 - \sigma_3 = \sigma_{et}$$

2. According to condition of failure, $(\sigma_1 - \sigma_3) > \sigma_{et}$

C. Maximum Principal Strain Theory (Saint Venant's Theory) :

1. According to this theory, failure of a material will take place when principal tensile strain in the material reaches the strain at elastic limit in the simple tension or when the minimum strain reaches the elastic limit strain in simple compression.

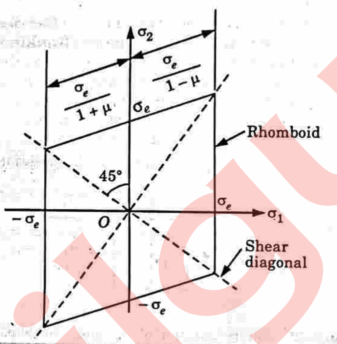


Fig. 1.43.3. Representation of maximum principal strain theory.

2. Principal strain in direction of principal stress σ_1,

$$\epsilon_1 = \frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)] \quad (\mu = \text{Poisson's ratio})$$

3. Principal strain in direction of principal stress σ_3,

$$\epsilon_3 = \frac{1}{E}[\sigma_3 - \mu(\sigma_2 + \sigma_1)]$$

D. Maximum Shear Strain Energy Theory (Distortion Energy Theory or Mises Hencky Theory) :

1. According to this theory, the failure occurs when the shear strain energy per unit volume in the stressed material reaches a value equal to the shear strain energy per unit volume at the elastic limit point in the simple tension test.

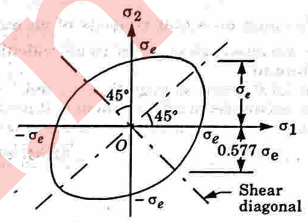


Fig. 1.43.4. Graphical representation of maximum shear strain energy theory.

2. Shear strain energy due to principal stress σ_1, σ_2 and σ_3 per unit volume of the stress material,

$$U_S = \frac{1}{12G}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \geq 2\sigma_1^2$$

E. Maximum Strain Energy Theory (Haigh's Theory) :

1. According to this theory, failure of a material occurs when the total strain energy in the material reaches the total strain energy of the material at elastic limit in simple tension.
2. In three dimensional stress system the strain energy per unit volume is given by,

$$U = \frac{1}{2E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$$

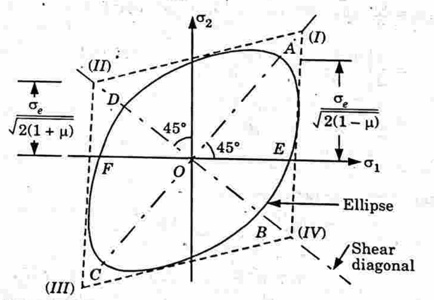


Fig. 1.43.5. Representation of maximum strain energy theory.

Que 1.44. **The load on a bolt consists of an axial pull of 20 kN together with a transverse shear of 10 kN, calculate the diameter of bolt according to :**
1. **Maximum total strain energy theory, and**
2. **Maximum shear strain energy theory (if $\mu = 0.3$).**
(Take elastic limit in tension 280 MPa and factor of safety = 3)

AKTU 2014-15, Marks 10

Answer

Given : Permissible tensile stress, $\sigma_{ut} = \frac{280}{3} = 93.33$ N/mm^2,
Factor of safety = 3, Axial pull = 20 kN = 20×10^3 N,
Transverse shear load = 10 kN = 10×10^3 N
To Find : Bolt diameter according to :
i. Maximum total strain energy theory, and
ii. Maximum shear strain energy theory.

1. Let d be the diameter of the bolt, in mm.

Hence, applied stress, $\sigma = \frac{20 \times 10^3}{\pi d^2/4} = \frac{80 \times 10^3}{\pi d^2} = \frac{25464.8}{d^2}$ N/mm^2

Applied shear stress, $\tau = \frac{10 \times 10^3}{\pi d^2/4} = \frac{12732.4}{d^2}$ N/mm^2

2. The principal stress,

$$\sigma_{1,2} = \frac{\sigma}{2} - \sqrt{(\sigma/2)^2 + \tau^2} = \frac{1}{2}[\sigma \pm \sqrt{\sigma^2 + 4\tau^2}]$$

$$= \frac{1}{2}\left[\frac{25464.8}{d^2} \pm \sqrt{\left(\frac{25464.8}{d^2}\right)^2 + 4 \times \left(\frac{12732.4}{d^2}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{25464.8}{d^2} \pm \frac{36012.7}{d^2}\right)$$

$$\sigma_1 = \frac{30738.75}{d^2}, \sigma_2 = -\frac{5273.95}{d^2}$$

3. Total strain energy theory :

$$\therefore \quad U = \frac{1}{2E}\left[\left(\frac{30738.75}{d^2}\right)^2 + \left(\frac{-5273.95}{d^2}\right)^2 + 2 \times 0.3 \times \left(\frac{30738.75}{d^2}\right) \times \left(\frac{-5273.95}{d^2}\right)\right] = \frac{1}{2E}\left(\frac{87542 \times 10^4}{d^4}\right)$$

Strain energy in simple tension $= \frac{\sigma_{ut}^2}{2E} = \left(\frac{(93.33)^2}{2 \times E}\right) = \frac{8710.5}{2E}$

$$\therefore \quad \frac{1}{2E}\left(\frac{87542 \times 10^4}{d^4}\right) = \frac{8710.5}{2E}$$

$$d = \left(\frac{87542 \times 10^4}{8710.5}\right)^{1/4} = 17.8 \text{ mm}$$

4. Maximum shear strain energy theory :

$$U_s = \left(\frac{1+\mu}{6E}\right)[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2],$$

Where, $\sigma_3 = 0$

$$\therefore \quad U_s = \left(\frac{\mu+1}{3E}\right) \times \frac{1}{2}\left[\left(\frac{30738.75}{d^2} + \frac{5273.95}{d^2}\right)^2 + \left(\frac{-5273.95}{d^2} - 0\right)^2 + \left(0 - \frac{30738.75}{d^2}\right)^2\right]$$

$$U_s = \left(\frac{\mu+1}{3E}\right) \times \frac{11.35 \times 10^8}{d^4}$$

5. Shear strain under axial stress $= \left(\frac{\mu+1}{3E}\right) \times \sigma_{ut}^2 = \left(\frac{\mu+1}{3E}\right) \times 93.33^2$

$$\therefore \quad \frac{11.35 \times 10^8}{d^4} = 93.33^2 \Rightarrow d = 18.999 \approx 19 \text{ mm}$$

Que 1.45. **A solid circular shaft is subjected to a bending moment of 3 kN-m and a torque of 1 kN-m. The shaft is to be made in carbon steel for which the yield strength in tension and in shear is 480 MPa and 265 MPa respectively. Calculate the diameter of the shaft using distortion energy theory.**

Answer

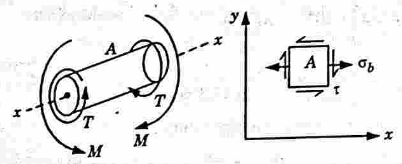
Given : $M = 3$ kN-m, $T = 1$ kN-m, $\sigma_{yt} = 480$ MPa, $\sigma_{st} = 265$ MPa
To Find : Diameter of shaft.

1. Bending stress, $\sigma_b = \frac{32M}{\pi d^3}$
2. The expressions for principal stresses and maximum shearing stress for solid circular shaft subjected to bending moment and torque are written as,

$$\sigma_{p1} = \frac{16}{\pi d^3}[M + \sqrt{M^2 + T^2}] \quad ...(1.45.1)$$

$$\sigma_{p2} = \frac{16}{\pi d^3}[M - \sqrt{M^2 + T^2}] \quad ...(1.45.2)$$

$$\tau_{max} = \frac{16}{\pi d^3}[\sqrt{M^2 + T^2}] \quad ...(1.45.3)$$



(a) Shaft under bending and torsion (b) State of stress at a point A on the surface of the shaft.

Fig. 1.45.1.

4. By substituting the values of M and T in eq. (1.45.1), (1.45.2) and (1.45.3), we get

$$\sigma_{p1} = \frac{16}{\pi d^3}[3 \times 10^6 + 10^6\sqrt{(3)^2 + (1)^2}]$$

$$= \frac{16 \times 6.162 \times 10^6}{\pi d^3} \text{ N/mm}^2 \quad ...(1.45.4)$$

Similarly, $$\sigma_{p2} = -\frac{16 \times 0.162 \times 10^6}{\pi d^3} \text{ N/mm}^2 \quad ...(1.45.5)$$

$\therefore$ $$\tau_{max} = \frac{16 \times 3.162 \times 10^6}{\pi d^3} \text{ N/mm}^2 \quad ...(1.45.6)$$

5. Using distortion energy theory :
$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_{yt}^2$
Here, $\sigma_1 = \sigma_{p1}, \sigma_2 = 0, \sigma_3 = \sigma_{p2}$

$$\therefore \left(\frac{16 \times 6.162 \times 10^6}{\pi d^3}\right)^2 + \left(\frac{16 \times 0.162 \times 10^6}{\pi d^3}\right)^2 + \left(\frac{16 \times 10^6 \times 6.324}{\pi d^3}\right)^2 = 2(480)^2$$

$$\frac{16 \times 10^6}{\pi d^3} = 76.871 \Rightarrow d = 40.5 \text{ mm}$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Draw the stress-strain diagram for mild steel under tensile load.

Ans. Refer Q. 1.3, Unit-1.

Q. 2. Determine the Poisson's ratio and bulk modulus of a material, for which Young's modulus is 120 GPa and modulus of rigidity 48 GPa.

Ans. Refer Q. 1.13, Unit-1.

Q. 3. A steel rod of 3 cm diameter is enclosed centrally in a hollow copper tube of external diameter 5 cm and internal diameter 4 cm. The composite bar is then subjected to an axial pull of 45000 N. If the length of each bar is equal to 15 cm. Determine :

i. The stresses in the rod and the tube.

ii. Load carried by each rod.

***E* for steel = 200 GPa and for Copper = 100 GPa.**

Ans. Refer Q. 1.24, Unit-1.

Q. 4. Analyze when a body is subjected to a direct tensile stress (σ) in one plane and accompanied by a single shear stress (τ).

Ans. Refer Q. 1.32, Unit-1.

Q. 5. At a point in a strained material the principal stresses are 100 MPa (tensile) and 60 MPa (compressive). Determine the normal stress, shear stress and resultant stress on a plane inclined at 50° to the axis of major principal stress. Also determine the maximum shear stress at a point.

Ans. Refer Q. 1.35, Unit-1.

Q. 6. At a point in a strained material, stresses are applied as shown in Fig. 1 find out the normal and shear stress on the oblique plane, principal stresses and principal strain.

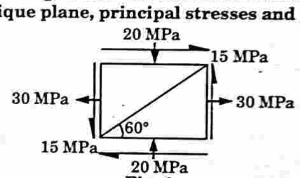


Fig. 1.

Ans. Refer Q. 1.40, Unit-1.

Q. 7. What are the various theories of failure ? Explain with diagram.

Ans. Refer Q. 1.43, Unit-1.

Q. 8. The load on a bolt consists of an axial pull of 20 kN together with a transverse shear of 10 kN, calculate the diameter of bolt according to :

1. Maximum total strain energy theory, and

2. Maximum shear strain energy theory (if μ = 0.3).

(Take elastic limit in tension 280 MPa and factor of safety = 3)

Ans. Refer Q. 1.44, Unit-1.

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2
UNIT

Shear Force and Bending Moment Diagrams

CONTENTS

PART-1

Shear Force and Bending Moment Diagram for Simply Supported Beam.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. **Describe the various types of beams.**

Answer

Types of Beams : The following are the important types of beams :

1. **Cantilever Beam :** A beam which is fixed at one end and free at the other end is known as cantilever beam. Such beam is shown in Fig. 2.1.1.

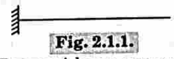


Fig. 2.1.1.

2. **Simply Supported Beam :** A beam supported or resting freely on the supports at its both ends is known as simply supported beam. Such beam is shown in Fig. 2.1.2.

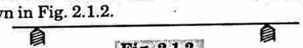


Fig. 2.1.2.

3. **Overhanging Beam :** If the end portion of a beam is extended beyond the support, such beam is known as overhanging beam. Overhanging beam is shown in Fig. 2.1.3.

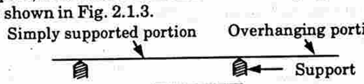


Fig. 2.1.3.

4. **Fixed Beam :** A beam whose both ends are fixed or built-in walls, is known as fixed beam. Such beam is shown in Fig. 2.1.4. A fixed beam is also known as a built-in or encastred beam.

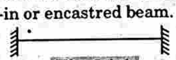


Fig. 2.1.4.

5. **Continuous Beam :** A beam which is provided more than two supports as shown in Fig. 2.1.5 is known as continuous beam.

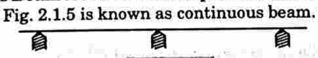


Fig. 2.1.5.

Que 2.2. **What do you mean by shear force ? Also define shear force diagram.**

Answer

A. Shear Force (SF) :

1. Shear is a strain produced by pressure in the structure of a beam (or structure), when its layers are laterally shifted in relation to each other.
2. Shear force is the force that tries to shear off the section of a beam (or structure).
3. It is obtained as algebraic sum of all forces acting normal to axis of beam, either to the left or to the right of section.
4. If shear force (F) tries to push left portion upward with respect to right portion then shear force is taken as positive and vice versa.

Fig. 2.2.1. Internal forces at section; positive shear force.

B. Shear Force Diagram (SFD) :

1. SFD represents the variation of shear force along the length of a beam.

Que 2.3. **Describe bending moment and define bending moment diagram.**

Answer

A. Bending Moment : Bending moment is the moment that tries to bend the beam (or structure) and is obtained as algebraic sum of moment of all forces about the section; acting either to left or to the right of section.

Sign Convention :

Fig. 2.3.1. External effect of bending moment.

B. Bending Moment Diagram (BMD) : BMD represents the variation of bending moment along the length of beam.

Que 2.4. **Write the steps involve in drawing SFD and BMD. Also define point of contraflexure.**

Answer

A. Steps Involve in Drawing SFD and BMD :

1. Obtain the values of all external support reactions on the beam by applying equilibrium equations.

2. Now cut the beam either to the right or to the left of an arbitrary transverse section with a free body diagram.
3. Assume that the shear force and bending moment acting on the cut section are in positive direction.
4. Now use the equilibrium equation to the section and get the expression for the shear force and bending moment acting at the cut section.
5. Following are some important points :
   i. Always choose that part of the beam which involves smaller number of forces either to the right or to the left of the arbitrary section.
   ii. Always avoid using a transverse section which coincides with the location of concentrated load or couple.

B. Sign Convention for Shear Force and Bending Moment :

1. For left side portion downward shear force and anticlockwise moment are positive.
2. And for right side portion upward shear force and clockwise moment are positive.

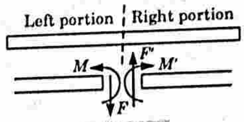


Fig. 2.4.1.

C. Point of Contraflexure : The point of contraflexure is a point which represents the section on the beam where bending moment is zero or bending moment changes its sign.

Que 2.5. **Draw the shear force and bending moment diagrams for simply supported beam of length L carrying a point load W at its middle point.**

Answer

A. Shear Force Diagram :

1. Fig. 2.5.1 shows a beam AB of length L simply supported at the ends A and B and carrying a point load W at its middle point C.
2. The reactions at the support will be equal to $W/2$ as the load is acting at the middle point of the beam. Hence $R_A = R_B = W/2$.
3. Take a section X at a distance x from the end A between A and C.
   Let F_x = Shear force at X,
   and M_x = Bending moment at X.
4. Here we have considered the left portion of the section. The shear force at X will be equal to the resultant force acting on the left portion of the section. But the resultant force on the left portion is $W/2$ acting upwards.
5. But according to the sign convention, the resultant force on the left portion acting upwards is considered positive. Hence shear force at X is positive and its magnitude is $W/2$.

∴ $F_x = +W/2$

Hence the shear force between A and C is constant and equal to $+W/2$.

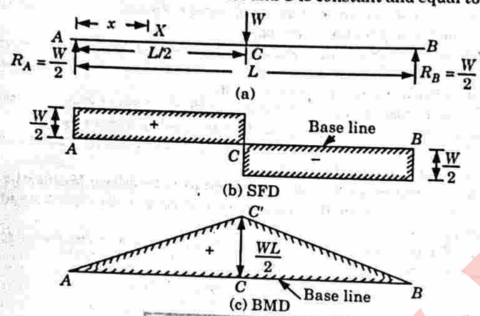


Fig. 2.5.1. BM and SF diagram.

6. Now consider any section between C and B at distance x from end A. The resultant force on the left portion will be

$(W/2 - W) = -W/2.$

7. This force will also remain constant between C and B. Hence shear force between C and B is equal to $-W/2$.
At the section C the shear force changes from $+W/2$ to $-W/2$.

B. Bending Moment Diagram :

1. The bending moment at any section between A and C at a distance of x from the end A is given by,

$M_x = R_A x$ or $M_x = +(W/2)\,x$ ...(2.5.1)

Bending moment will be positive as for the left portion of the section, the moment of all forces at X is clockwise.

2. Moreover, the bending of beam takes place in such a manner that concavity is at the top of the beam.
At A, $x = 0$ hence, $M_A = (W/2)\times 0 = 0$

At C, $x = \frac{L}{2}$ hence, $M_C = \frac{W}{2}\times\frac{L}{2} = \frac{W\times L}{4}$.

From eq. (2.5.1), it is clear that BM varies according to straight line law between A and C. BM is zero at A and it increases to $W\times L/4$ at C.

3. The bending moment at any section between C and B at a distance x from the end A is given by,

$$M_x = R_A x - W\times\left(x-\frac{L}{2}\right)$$

$$= \frac{W}{2}x - Wx + W\times\frac{L}{2} = \frac{WL}{2} - \frac{Wx}{2}$$

At C, $x = \frac{L}{2}$ hence $M_C = \frac{WL}{2} - \frac{W}{2}\times\frac{L}{2} = \frac{W\times L}{4}$

At B, $x = L$ hence $M_B = \frac{WL}{2} - \frac{W}{2}\times L = 0$.

4. Hence, bending moment at C is $WL/4$ and it decreases to zero at B. Now the BM diagram can be completed as shown in Fig. 2.5.1(c).
Note : The bending moment is maximum at the middle point C, where the shear force changes its sign.

Que 2.6. **Draw the SF and BM diagrams for a simply supported beam carrying a uniformly distributed load of w per unit length over the entire span. Also calculate the maximum BM.**

Answer

A. Shear Force Diagram :

1. Fig. 2.6.1 shows a beam AB of length L simply supported at the ends A and B and carrying a uniformly distributed load of w per unit length over the entire length.
2. The reactions at the supports will be equal and their magnitude will be half the total load on the entire length.

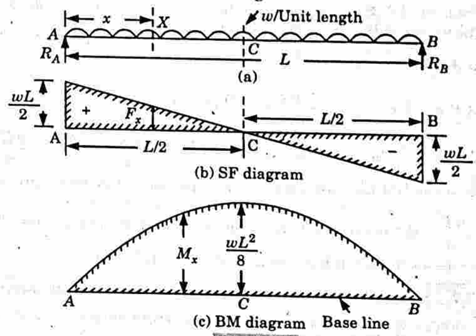


Fig. 2.6.1.

Let, R_A = Reaction at A, and R_B = Reaction at B

∴ $R_A = R_B = \frac{wL}{2}$

3. Consider any section X at a distance x from the left end A. The shear force at the section (*i.e.*, F_x) is given by,

$$F_x = +R_A - wx = +\frac{wL}{2} - wx \quad ...(2.6.1)$$

From eq. (2.6.1), it is clear that the shear force varies according to straight line law.

4. The values of shear force at different points are :

At A, $x = 0$ hence $F_A = +\frac{wL}{2} - \frac{w0}{2} = +\frac{wL}{2}$

At B, $x = L$ hence $F_B = +\frac{wL}{2} - wL = -\frac{wL}{2}$

At C, $x = \frac{L}{2}$ hence $F_C = +\frac{wL}{2} - w\frac{L}{2} = 0$

The shear force diagram is drawn as shown in Fig. 2.6.1(b).

B. Bending Moment Diagram :

1. The bending moment at the section X at a distance x from left end A is given by,

$$M_x = +R_A\, x - wx\, \frac{x}{2}$$

$$= \frac{wL}{2}x - \frac{wx^2}{2} \quad \left(\because R_A = \frac{wL}{2}\right) \quad ...(2.6.2)$$

From eq. (1.6.2), it is clear that BM varies according to parabolic law.

2. The values of BM at different points are :

At A, $x = 0$ hence $M_A = \frac{wL}{2}0 - \frac{w0}{2} = 0$

At B, $x = L$ hence $M_B = \frac{wL}{2}L - \frac{w}{2}L^2 = 0$

At C, $x = \frac{L}{2}$ hence $M_C = \frac{wL}{2}\frac{L}{2} - \frac{w}{2}\left(\frac{L}{2}\right)^2 = \frac{wL^2}{4} - \frac{wL^2}{8} = +\frac{wL^2}{8}$

Thus the BM increases according to parabolic law from zero at A to $+\frac{wL^2}{8}$ at the middle point of the beam and from this value the BM decreases to zero at B according to the parabolic law.

Que 2.7. **Draw the SF and BM diagrams for a simply supported beam carrying a uniformly varying load from zero at one end to w per unit length at the other end.**

Answer

A. Shear Force Diagram :

1. Fig. 2.7.1 shows a beam AB of length L simply supported at the ends A and B and carrying a uniformly varying load from zero at end A to w per unit length at B. First calculate the reactions R_A and R_B.
2. Taking moments about A, we get

$$R_B \times L = \left(\frac{wL}{2}\right)\frac{2}{3}L, \quad \left[\text{Total load}\left(=\frac{wL}{2}\right) \text{ is acting } \frac{2}{3}L \text{ from } A\right]$$

$$\therefore \quad R_B = \frac{wL}{3} \text{ and } R_A = \frac{wL}{2} - \frac{wL}{3} = \frac{wL}{6}$$

Consider any section X at a distance x from end A. The shear force at X is given by,

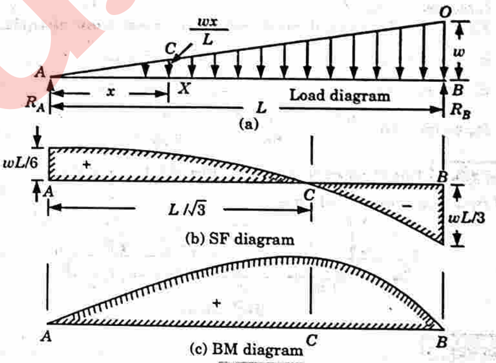


Fig. 2.7.1.

$$F_x = R_A - \text{Load on length } AX = \frac{wL}{6} - \frac{wx}{L} \times \frac{x}{2}$$

$$= \frac{wL}{6} - \frac{wx^2}{2L} \quad ...(2.7.1)$$

Eq. (2.7.1) shows that SF varies according to parabolic law.

At A, $x = 0$ hence, $F_A = \frac{wL}{6} - \frac{w}{2L} \times 0 = \frac{wL}{6}$

At B, $x = L$ hence, $F_B = \frac{wL}{6} - \frac{wL^2}{2L} = \frac{wL}{6} - \frac{wL}{2} = -\frac{wL}{3}$

3. The shear force is $+wL/6$ at A and it decreases to $-wL/3$ at B according to parabolic law. Somewhere between A and B, the SF must be zero.
4. Let, the SF be zero to a distance x from A.

Equating the SF to zero in eq. (2.7.1), we get,

$$0 = \frac{wL}{6} - \frac{wx^2}{2L} \quad x = \frac{L}{\sqrt{3}} = 0.577\,L$$

B. Bending Moment Diagram :

1. The BM is zero at A and B.
2. The BM at the section X at a distance x from the end A is given by,

$M_x = R_A x$ – Load on length AX $(x/3)$

$\left(\because \text{Load on } AX \text{ is acting at } \frac{x}{3} \text{ from } X\right)$

$$= \frac{wL}{6}x - \frac{wx^2}{2L}\cdot\frac{x}{3} = \frac{wL}{6}x - \frac{wx^3}{6L} \qquad ...(2.7.2)$$

3. Equation (2.7.2) shows the BM varies between A and B according to cubic law.
4. Maximum BM occurs at a point where SF becomes zero after changing its sign. That point is at a distance of $L/\sqrt{3}$ from A.
5. Hence, substituting, $x = L/\sqrt{3}$ in eq. (2.7.2), we get

$$\text{Maximum BM} = \frac{wL}{6}\cdot\frac{L}{\sqrt{3}} - \frac{w}{6L}\left(\frac{L}{\sqrt{3}}\right)^3 = \frac{wL^2}{9\sqrt{3}}$$

Que 2.8. For the beam shown in Fig. 2.8.1, draw the shear force and bending moment diagram.

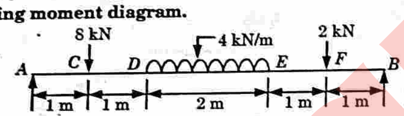


Fig. 2.8.1.

Answer

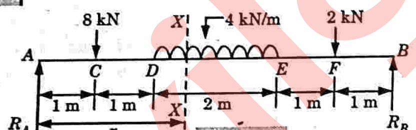


Fig. 2.8.2.

1. Considering, equilibrium of beam,

i. $\Sigma F_y = 0 \Rightarrow R_A + R_B = 8 + 4 \times 2 + 2 = 18$ kN

ii. $\Sigma M_A = 0 \Rightarrow 8 \times 1 + 4 \times 2 \times (2/2 + 2) + 5 \times 2 - R_B \times 6 = 0$

$R_B = 7$ kN and $R_A = 11$ kN

2. **Shear Force :**

i. **Portion AC :** At point A SF = 11 kN and it remains constant upto just left point C because there is no loading between this segment.

SF just on right side of $C = 11 - 8 = 3$ kN

ii. **Portion CD :**

SF just left point of D is constant

SF at point $D = 3$ kN

iii. **Portion DE :** Consider any section at distance x from A

$$\text{SF} = 11 - 8 - (x - 2) \times 4 = 11 - 8 - 4x + 8 = 11 - 4x$$

SF just left side of E at $x = 4$ m

$$\text{SF} = 11 - 4x = 11 - 4 \times 4 = -5 \text{ kN}$$

SF just Right side of $E = -5$ kN

Shear force will be zero at $x = 11/4 = 2.75$ m

iv. **Portion EF :** SF at point $F = -5 - 2 = -7$ kN

v. **Portion FB :** SF at point $B = 0$

3. **Bending Moment :**

i. **Portion AC :** BM at point $A = 0$

BM at point $C = 11 \times 1 = 11$ kN-m

ii. **Portion CD :** BM at point $D = 11 \times 2 - 8 \times 1 = 14$ kN-m

iii. **Portion DE :** Consider any section at distance x from the end point A.

$$M_x = 11x - 8(x - 1) - 4(x - 2) \times \left(\frac{x-2}{2}\right)$$

$$M_x = 3x + 8 - 2(x - 2)^2$$

At point D, $x = 2$ $M_D = 3 \times 2 + 8 = 14$ kN-m

At point D', $x = 2.75$, $M_{D'} = 3 \times 2.75 + 8 - 2(2.75 - 2)^2 = 15.125$ kN-m

At point E, $x = 4$, $M_E = 12 + 8 - 8 = 12$ kN-m

iv. **Portion EF :**

BM at point F, $x = 5$, $= 11 \times 5 - 8 \times 4 - (4 \times 2) \times 2 = 7$ kN-m

v. **Portion FB :**

BM at point B, $M_B = 11 \times 6 - 8 \times 5 - (4 \times 2) \times 3 - 2 \times 1 = 0$

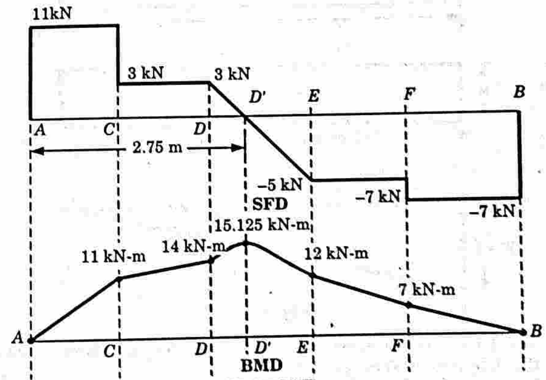


Fig. 2.8.3.

PART-2

Shear Force and Bending Moment Diagram for Cantilever Beam.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.9. **Draw the SF and BM diagrams for a cantilever of length *L* carrying a point load *W* at the free end.**

Answer

A. Shear Force Diagram :

1. Fig. 2.9.1 shows a cantilever *AB* of length *L* fixed at *A* and free at *B* and carrying a point load *W* at the free end *B*.

   Let F_x = Shear force at *X*.

   M_x = Bending moment at *X*.

2. Take a section *X* at a distance *x* from the free end. Consider the right portion of the section.

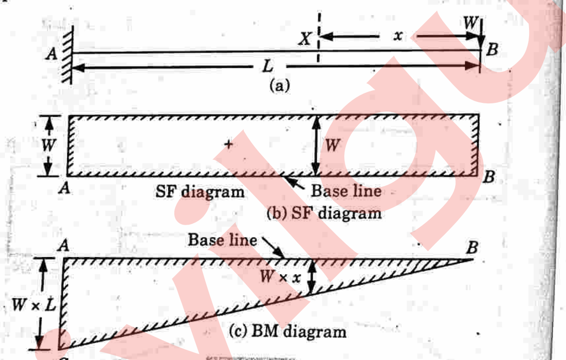


Fig. 2.9.1.

3. The shear force at this section is equal to the resultant force acting on the right portion at the given section.
4. But the resultant force acting on the right portion at the section *X* is *W* and acting in the downward direction.
5. But a force on the right portion acting downwards is considered positive. Hence shear force at *X* is positive.

   $\therefore \quad F_x = + W$

6. The shear force will be constant at all sections of the cantilever between *A* and *B* as there is no other load between *A* and *B*. The shear force diagram is shown in Fig. 2.9.1(b).

B. Bending Moment Diagram :

1. The bending moment at the section *X* is given by,

   $$M_x = - W \times x \qquad ...(2.9.1)$$

2. From eq. (2.9.1), it is clear that BM at any section is proportional to the distance of the section from the free end.

   At $x = 0$ *i.e.*, at *B*, BM = 0

   At $x = L$ *i.e.*, at *A*, BM = *WL*

   Hence BM follows the straight line law.

3. The BM diagram is shown in Fig. 2.9.1(c). At point *A*, take *AC* = *WL* in the downward direction. Joint *B* to *C*.

Que 2.10. **Draw the SF and BM diagrams for a cantilever of length *L* carrying a uniformly distributed load of *w* per m length over its entire length.**

Answer

A. Shear Force Diagram :

1. Fig. 2.10.1 shows a cantilever of length *L* fixed at *A* and carrying a uniformly distributed load of *w* per unit length over the entire length of the cantilever.
2. Take a section *X* at a distance of *x* from the free end *B*.

   Let F_x = Shear force at *X*.

   M_x = Bending moment at *X*.

3. Here we have considered the right portion of the section. The shear force at the section *X* will be equal to the resultant force acting on the right portion of the section. But the resultant force on the right portion

   $= w \times$ Length of right portion $= wx$.

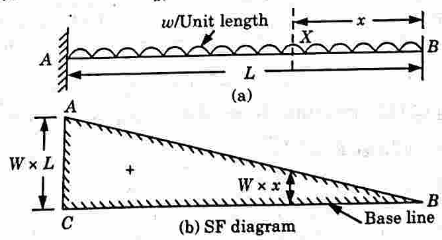
4. This resultant force is acting downwards. But the resultant force on the right portion acting downwards is considered positive. Hence shear force at *X* is positive.

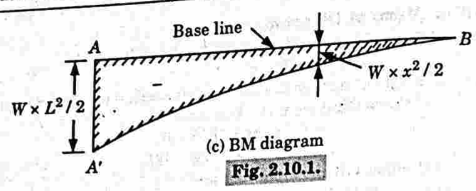
   $\therefore \quad F_x = + wx$

5. The above equation shows that the shear force follows a straight line law.

   At *B*, $x = 0$ and hence $F_x = 0$

   At *A*, $x = L$ and hence $F_x = wL$





(c) BM diagram

Fig. 2.10.1.

B. Bending Moment Diagram :

1. The bending moment at the section X is given by,

$$M_x = -(wx)\,\frac{x}{2} = -w\,\frac{x^2}{2} \qquad ...(2.10.1)$$

2. From eq. (2.10.1), it is clear that BM at any section is proportional to the square of the distance of the section from the free end. This follows a parabolic law.
   At B, $x = 0$, hence, $M_x = 0$
   At A, $x = L$, hence, $M_x = -w\,(L^2/2)$

Que 2.11. **Draw the SF and BM diagrams for a cantilever of length *L* carrying a gradually varying load from zero at the free end to *w* per unit length at the fixed end.**

Answer

A. Shear Force Diagram :

1. Fig. 2.11.1 shows a cantilever of length L fixed at A and carrying a gradually varying load from zero at the free end to w per unit length at the fixed end.
2. Take a section X at a distance x from the free end B.
   Let, F_x = Shear force at the section X.
   M_x = Bending moment at the section X.
   Let us first find the rate of loading at the section X. The rate of loading is zero at B and is w per metre run at A. This means that rate of loading for a length L is w per unit length. Hence rate of loading for a length of x will be wx/L per unit length. This is shown in Fig. 2.11.1(a) by CX, which is also known as load diagram. Hence $CX = wx/L$.
3. The shear force and the section X at a distance x from free end is given by, F_x = Total load on the cantilever for a length x from the free end B

$$= \text{Area of triangle } BCX = \frac{1}{2}x\left(\frac{wx}{L}\right) = \frac{wx^2}{2L}$$

4. The eq. (2.11.1) shows that the SF varies according to the parabolic law.

$$\text{At } B,\ x = 0 \text{ hence } F_B = \frac{w \times 0^2}{2L} = 0$$

$$\text{At } A,\ x = L \text{ hence } F_A = \frac{wL^2}{2L} = \frac{wL}{2}$$

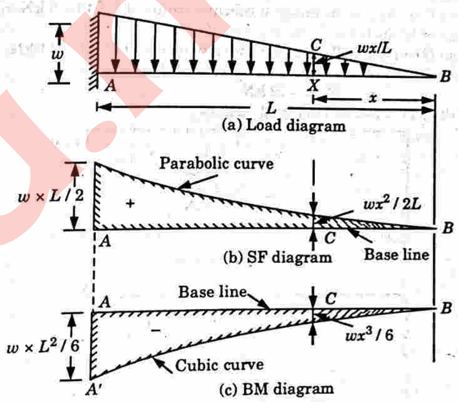


Fig. 2.11.1.

B. Bending Moment Diagram :

1. The bending moment at the section X at a distance x from the free end B is given by,

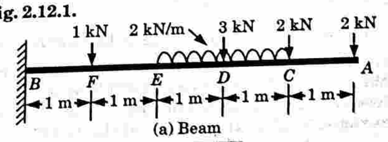
$$M_x = -\left(\frac{wx^2}{2L}\right) \times \frac{x}{3} = -\frac{wx^3}{6L} \qquad ...(2.11.2)$$

2. The eq. (2.11.2) shows that the BM varies according to the cubic law.

$$\text{At } B,\ x = 0 \text{ hence } M_B = -\frac{w \times 0}{6L} = 0$$

$$\text{At } A,\ x = L \text{ hence } M_A = -wL^3/6L = -wL^2/6$$

Que 2.12. **Draw the SF and BM diagrams for a cantilever loaded as shows in Fig. 2.12.1.**



(a) Beam

Fig. 2.12.1.

Answer

A. Calculations of Shear force from A to C :

$S_A = -2$ kN
$S_C = -2 - 2 = -4$ kN

1. From C to D SF will change uniformly from – 4 kN to – 6 kN and total value of SF at D *i.e.* $S_D = -6 - 3 = -9$ kN.
2. From D to E SF will change uniformly from – 9 kN to – 11 kN.
3. Shear force just after point F, $S_F = -11 - 1 = -12$ kN

$$S_B = -12 \text{ kN}$$

4. SF diagram is shown in Fig. 2.12.2(a)

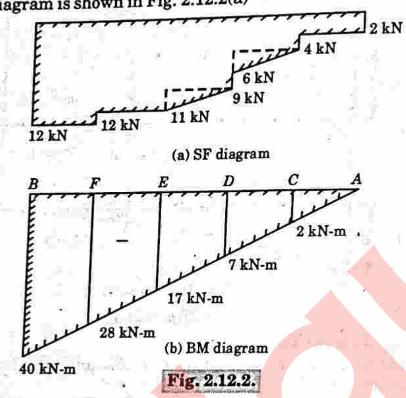


Fig. 2.12.2.

B. Calculations of Bending Moment :

$M_A = 0$, $M_C = -2 \times 1 = -2$ kN-m

$M_D = -2(1+1) - 2 \times 1 - 2 \times 1 \times 1/2 = -7$ kN-m

$M_E = -2(1+1+1) - 2 \times (1+1) - 3 \times 1 - 2 \times 2 \times 2/2 = -17$ kN-m

$M_F = -2(1+1+1+1) - 2 \times (1+1+1) - 3(1+1) - 2 \times 2(2/2+1)$
$= -8 - 6 - 6 - 8 = -28$ kN-m

$M_B = -2(1+1+1+1+1) - 2(1+1+1+1) - 3(1+1+1) - 2 \times 2(2/2+1+1) - 1 \times 1$
$= -10 - 8 - 9 - 12 - 1 = -40$ kN-m

BM diagram is shown in Fig. 2.12.1(b).

Que 2.13. **The cantilever beam in Fig. 2.13.1 carries a triangular load, the intensity of which varies from zero at the right end to 360 N/m at the left end. In addition, a 1000 N upward vertical load acts at the free end of the beam. Draw the shear force and bending moment diagrams. Neglect the weight of the beam.**

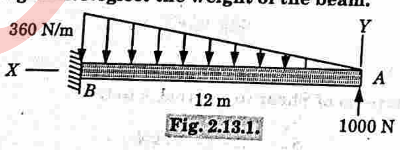


Fig. 2.13.1.

Answer

A. Shear Force Diagram :

1. At section 1-1,

$+\downarrow \Sigma F = 0 \qquad F_x = -1000 + \dfrac{1}{2} \times x \times \dfrac{wx}{l} = -1000 + \dfrac{wx^2}{2l}$

2. At A, $x = 0$ $\qquad F_A = -1000$ N

3. At B, $x = 12$, $\qquad F_B = -1000 + \dfrac{360 \times 12^2}{2 \times 12} = 1160$ N

B. Bending Moment Diagram :

1. $+\circlearrowleft \Sigma M = 0$, $\qquad M_x = 1000 \times x - \dfrac{wx^2}{2l} \times \dfrac{x}{3}$

2. At A, $x = 0$, $\qquad M_A = 0$

3. At B, $x = 12$ $\qquad M_B = 1000 \times 12 - \dfrac{360 \times 12^2}{2 \times 12} \times \dfrac{12}{3} = 3360$ N-m

4. Position of maximum bending moment, $-1000 + \dfrac{wx^2}{2l} = 0$

$$x = 8.164 \text{ m}$$

5. At $x = 8.164$ m, $M_{max} = 1000 \times 8.164 - \dfrac{360 \times 8.164^2}{2 \times 12} \times \dfrac{8.164}{3}$

$$M_{max} = 8164 - 2720.689 = 5443.310 \text{ N-m}$$

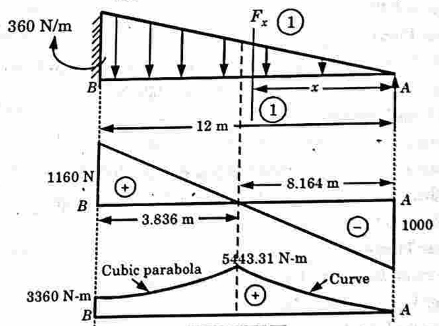


Fig. 2.13.3.

Que 2.14. Determine the support reactions for the cantilever beam shown in Fig. 2.14.1 and sketch shear force and bending moment diagrams showing key points.

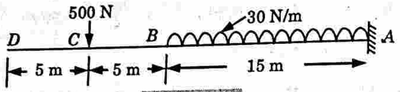


Fig. 2.14.1.

Answer

A. Support Reactions and Moment :

Let, support reaction be $R_A = 500 + 30 \times 15 = 950$ N

B. Portion *DC* :

1. Shear Force :

i. Shear force at $D = 0$

As no load is acting in portion *DC* so the shear force till point $C = 0$

ii. At point *C* a downward force of 500 N is acting hence shear force of *C* will change suddenly = – 500 N

2. Bending Moment :

i. Bending moment at $D = 0$

ii. Bending moment till point $C = 0$.

C. Portion *CB* :

1. Shear Force :

i. In portion *CB*, shear force will be constant till point *B*.

ii. Shear force at *B* = Shear force at *C* = – 500 N

2. Bending Moment :

i. Bending moment at point $C = 0$

ii. Bending moment in portion *CB* is a straight line.

iii. Bending moment at point $B = -500 \times 5 = -2500$ N-m

D. Portion *BA* :

1. Shear Force :

i. In portion *BA*, shear force will be a straight line.

ii. Shear force at $B = -500$ N

iii. Shear force at $A = -500 - 30 \times 15 = -950$ N

2. Bending Moment :

i. Bending moment in portion *BA* will be a parabola due to UDL.

Bending moment at $A = -500 \times 20 - (30 \times 15) \times 15/2 = -13375$ N-m

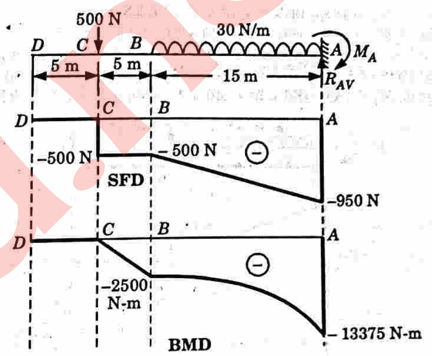


Fig. 2.14.2.

Que 2.15. Draw the SFD and BMD of the loaded beam as shown in Fig. 2.15.1.

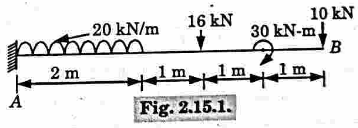


Fig. 2.15.1.

Answer

A. Support Reactions :

1. $\Sigma F_V = 0,$

$R_A - 2 \times 20 - 16 - 10 = 0 \Rightarrow R_A = 66$ kN

2. $+\circlearrowleft \Sigma M_A = 0 \Rightarrow M_A - (2 \times 20 \times 1) - (16 \times 3) - 30 - (10 \times 5) = 0$

$M_A = 168$ kN-m

B. Shear Force :

1. Shear force at $A = R_A = 66$ kN
2. Shear force in portion AC, $F_x = R_A - 20x = 66 - 20x$
3. At point C, Shear force, $F_{AC} = 66 - 20 \times 2 = 26$ kN
4. Shear force in portion *CD*, $F_{CD} = 26$ kN
5. Shear force at *D*, $F_D = 26 - 16 = 10$ kN
6. Shear force in portion *DE* and *EB*, $F_{DE} = 10$ kN
7. Shear force at point *B*, $F_B = 10 - 10 = 0$ kN

C. Bending Moment :

1. Bending moment at *A*, $M_A = 168$ kN-m
2. Bending Moment in portion *AC*,

$M_x = 168 - R_A \times x + 20 \times x \times x / 2 = 168 - 66x + 10x^2$

3. BM at C, $M_C = 168 - 66 \times 2 + 10 \times 2^2 = 76$ kN-m
4. BM at D, $M_D = 168 - (66 \times 3) + (40 \times 2) = 50$ kN-m
5. BM at LHS of E, $M_E = 168 - (66 \times 4) + (40 \times 3) + 16 \times 1 = 40$ kN-m
6. BM at RHS of E, $M_E = 168 - (66 \times 4) + (40 \times 3) + (16 \times 1) - 30 = 10$ kN-m
7. BM at B, $M_B = 168 - (66 \times 5) + (40 \times 4) + (16 \times 2) - 30 = 0$ kN-m

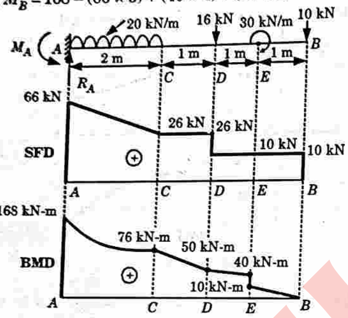


Fig. 2.15.3.

PART-3

Shear Force and Bending Moment Diagram for Overhanging Beams, Points of Contraflexure.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.16. Draw the SFD and BMD and locate the point of contraflexures for the loaded beam in Fig. 2.16.1.

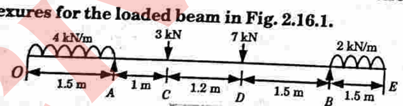


Fig. 2.16.1.

Answer

A. Support Reactions :

1. $\Sigma F_V = 0$

$R_A + R_B = 4 \times 1.5 + 3 + 7 + 2 \times 1.5 = 19$ kN ...(2.16.1)

2. $+\circlearrowleft \Sigma M_O = 0$

$$-4 \times 1.5 \times \left(\frac{1.5}{2}\right) + R_A \times 1.5 - 3 \times 2.5 - 7 \times 3.7 + R_B \times 5.2 - 2 \times 1.5\,(5.95) = 0$$

$$1.5\,R_A + 5.2\,R_B = 55.75 \text{ kN} \quad ...(2.16.2)$$

3. On solving eq. (2.16.1) and eq. (2.16.2), we get

$R_A = 11.635$ kN, $R_B = 7.365$ kN

B. Shear Force Diagram :

1. Shear force at O, $F_O = 0$
2. Shear force till A, $F_{O\to A} = 0 - 4 \times 1.5 = -6$ kN
3. At point A, $F_A = F_{O\to A} + R_A = -6 + 11.635 = 5.635$ kN
4. Shear force till point C, $F_{A\to C} = 5.635$ kN
5. At point C, $F_C = F_{A\to C} - 3 = 5.635 - 3 = 2.635$ kN
6. Shear force till point D, $F_{C\to D} = 2.635$ kN
7. At point D, $F_D = F_{C\to D} - 7 = 2.635 - 7 = -4.365$ kN
8. Shear force till point B, $E_{D\to B} = -4.365$ kN
9. At point B, $F_B = F_{D\to B} + 7.364 = -4.365 + 7.365 = 3$ kN
10. Shear force at point E, $F_E = 3 - 1.5 \times 2 = 0$ kN

The values of shear forces are being drawn in the Fig. 2.16.2 as SFD.

C. Bending Moment Diagram :

1. Bending moment at O, $M_O = 0$
2. Bending moment till A, $M_A = -4 \times 1.5 \times \dfrac{1.5}{2} = -4.5$ kN-m
3. Bending moment till C, $M_C = -4 \times 1.5 \times \left(\dfrac{1.5}{2} + 1\right) + 11.635 \times 1$

$M_C = 1.135$ kN-m

4. Bending moment till D,

$$M_D = -4 \times 1.5 \left(\frac{1.5}{2} + 2.2\right) + 11.635 \times (1 + 1.2) - 3 \times 1.2$$

$M_D = 4.297$ kN-m

5. Bending moment till B,

$$M_B = -4 \times 1.5 \left(\frac{1.5}{2} + 3.7\right) + 11.635 \times 3.7 - 3 \times 2.7 - 7 \times 1.5$$

$M_B = -2.251$ kN-m

6. Bending moment till E,

$$M_E = -4 \times 1.5 \left(\frac{1.5}{2} + 5.2\right) + 11.635 \times 5.2 - 3 \times 4.2 - 7 \times 3 + 7.365 \times 1.5 - 2 \times 1.5 \times (1.5/2)$$

$= -5 \times 10^{-4} \approx 0$ kN-m

The values of bending moments are being drawn in the Fig. 2.16.2 as BMD.

D. Points of Contraflexures :

Points P and Q shown in the Fig. 2.16.2 BMD are the points of contraflexures.

1. **Position of Point P :**
As ΔPTU and ΔPSR are similar triangles.

$$\frac{PT}{PS} = \frac{UT}{SR} = \frac{x}{1-x} = \frac{1.14}{4.5} \qquad \text{(Let, } PT = x,\ PS = 1-x\text{)}$$

$$4.5\,x = 1.14 - 1.14\,x \Rightarrow x = 0.202 \text{ m}$$

So point of contraflexure P will be at $(1.5 + (1 - 0.202))$ m = 2.298 m

2. **Position of point Q :** As ΔQVW and ΔQXY are similar triangles.

$$\frac{WQ}{QX} = \frac{VW}{XY} \Rightarrow \frac{x}{1.5-x} = \frac{4.308}{2.232} \qquad \text{(Let, } WQ = x,\ QX = 1.5-x\text{)}$$

$$2.232\,x = 6.462 - 4.308\,x$$

$$x = 0.988 \text{ m}$$

So point of contraflexure Q will be at $(1.5 + 1 + 1.2 + 0.988$ m$)$ = 4.688 m

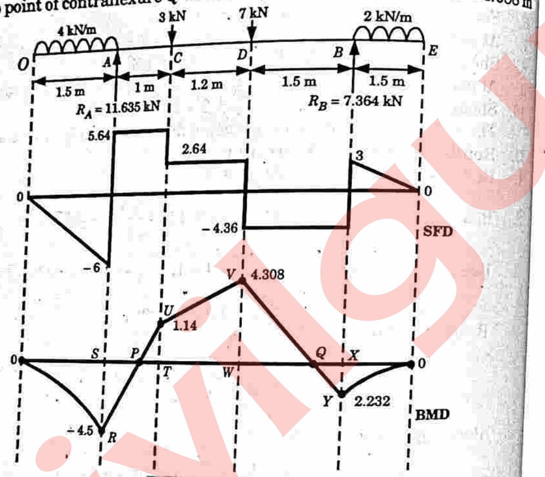


Fig. 2.16.2.

Que 2.17. **Draw the shear and moment diagrams for the beam loaded as shown in Fig. 2.17.1.**

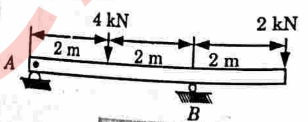


Fig. 2.17.1.

Answer

A. **Support Reactions :**

1. $+\uparrow \Sigma F_V = 0;\ R_A + R_B = 4 + 2 = 6$ kN ...(2.17.1)
2. Taking moment about point A,

$$\curvearrowright + \Sigma M_A = 0\ ;\ -4 \times 2 + 4R_B - 2 \times 6 = 0 \Rightarrow R_B = 5 \text{ kN}$$

3. From equation (2.17.1), we get, $R_A = 6 - R_B = 6 - 5 = 1$ kN

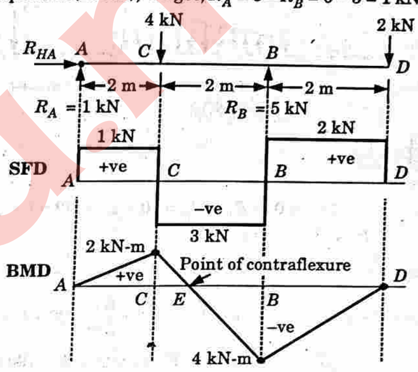


Fig. 2.17.2.

B. **Portion AC :**
$F_{AC} = 1$ kN
$F_A = 1$ kN
$F_C = 1 - 4 = -3$ kN
$M_{AC} = +1 \times x$
At $x = 0$; $M_A = 0$
At $x = 2$; $M_C = 2$ kN-m

C. **Portion CB :**
$F_{CB} = 1 - 4 = -3$ kN
$F_C = -3$ kN
$F_B = -3 + 5 = 2$ kN
$M_{CB} = 1 \times x - 4(x - 2)$
At $x = 2$; $M_C = 2$ kN-m
At $x = 4$; $M_B = 4 - 4 \times 2 = -4$ kN-m

D. **Portion BD :**
$F_{BD} = 1 - 4 + 5 = 2$ kN
$F_B = 2$ kN
$F_D = 2 - 2 = 0$
$M_{BD} = 1 \times x - 4\,(x - 2) + 5\,(x - 4)$
At $x = 4$, $M_B = 1 \times 4 - 4 \times 2 = -4$ kN-m
At $x = 6$, $M_D = 1 \times 6 - 4 \times 4 + 5 \times 2$
$= 6 - 16 + 10 = 0$

E. **Point of Contraflexure :**
In the portion BC, bending moment will be zero as its sign change, $M = 0$

$$M = 1 \times x - 4\,(x - 2) = 0 \Rightarrow x - 4\,x + 8 = 0$$

$$3x = 8 \Rightarrow x = 8/3 = 2.667 \text{ m}$$

At $x = 2.667$ from point E, point of contraflexure.

Que 2.18. **Draw the SF diagram and BM diagram for the beam shown in Fig. 2.18.1.**

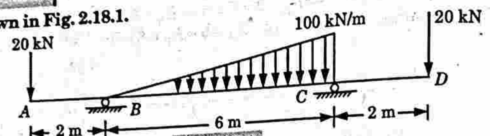


Fig. 2.18.1.

Answer

A. Reactions :

1. $\Sigma F_V = 0 \Rightarrow R_B + R_C = 20 + 20 + 1/2 \times 6 \times 100$

$$R_B + R_C = 340 \text{ kN} \quad ...(2.18.1)$$

2. Taken moment at B, $\Sigma M_B = 0$

$$20 \times 2 + R_C \times 6 - \frac{1}{2} \times 6 \times 100 \times \left(2 \times \frac{6}{3}\right) - 20 \times 8 = 0$$

$$R_C = 220 \text{ kN and } R_B = 340 - 220 = 120 \text{ kN}$$

B. SFD and BMD :

1. Consider Portion AB :

Shear force at point $A = -20$ kN

At point B, Shear force $= -20 + R_B = -20 + 120 = 100$ kN

Bending moment at A, $M_A = 0$

At point B, $M_B = -20 \times 2 = -40$ kN-m

2. Consider Portion BC :

i. Shear force at section X-$X = -20 + 120 - (1/2)\,xy$

From similar Δ property, $\frac{100}{6} = \frac{y}{x} \Rightarrow y = \frac{50}{3}x$

Shear force, $SF_x = -20 + 120 - \frac{1}{2}x\left(\frac{50}{3}x\right) = 100 - \frac{25}{3}x^2$

ii. For zero shear force, $SF_x = 0$; $x^2 = \frac{100 \times 3}{25} \Rightarrow x = 3.464$ m

iii. Shear force upto point C ($x = 6$ m) $= 100 - \frac{25}{3}(6)^2 = -200$ kN

iv. At point C, Shear force $= -200 + R_C = -200 + 220 = 20$ kN-m

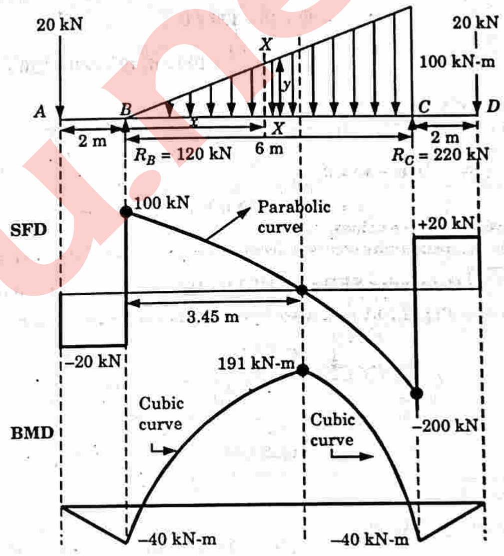


Fig. 2.18.2.

v. Bending moment at section $XX = -20(x+2) + 120(x) - \left(\frac{1}{2}xy\right)\frac{x}{3}$

$$= -20x - 40 + 120x - \frac{1}{6}x^2\left(\frac{50}{3}x\right)$$

$$= -\frac{25}{9}x^3 + 100x - 40$$

vi. Moment at $x = 3.464$ m from $B = -\frac{25}{9}(3.464)^3 + 100 \times 3.464 - 40$

$$= 191 \text{ kN-m}$$

vii. Now bending moment at point $C = -\frac{25}{9}(6)^3 + 100(6) - 40 = -40$ kN-m

3. Consider Portion CD :

i. Shear force at point $C = -200 + 220 = 20$ kN

ii. Shear force at $D = 20 - 20 = 0$ kN

X-X is a section between CD portion from point C at x distance.

iii. Bending moment at point *CD*

$$= -20 \times 10 + 120 \times 8 - \left(\frac{1}{2} \times 100 \times 6\right)(2 + 6/3) + 220 \times 2 = 0$$

4. **Point of Contraflexure :**

i. For point of contra flexure, BM in portion BC is quest to zero

$$\frac{25}{9}x^3 - 100x + 40 = 0$$

$$x = -6319, 5.79, 0.4018$$

Consider positive values.

ii. Point of contraflexure occurs at distance 0.4018 m and 5.79 m from point B.

Que 2.19. **Determine SFD and BMD for the simply supported beam as shown in Fig. 2.19.1 and also find maximum bending moment.**

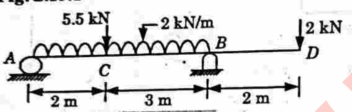


Fig. 2.19.1.

Answer

A. **Support Reactions :**

1. $+\Sigma F_V = 0;\ R_A + R_B = 5.5 + 2 \times 5 + 2 = 17.5$ kN ...(2.19.1)
2. Taking moment about *A*,

$$+\circlearrowleft M_A = 0.$$

$$R_B \times 5 = 5.5 \times 2 + 2 \times 5 \times 5/2 + 2 \times 7 = 11 + 25 + 14 = 50$$

$$R_B = 10 \text{ kN}$$

3. Put this value in equation (2.19.1), we get

$$R_A = 17.5 - 10 = 7.5 \text{ kN}$$

B. **Portion BD :**

$$+\uparrow \Sigma F_Y = 0 \Rightarrow -2 + F_{BD} = 0$$

$$F_{BD} = +2 \text{ kN}$$

Before *B*, $F_B = +2$ kN

$$+\circlearrowleft \Sigma M = 0 \Rightarrow M_{BD} = -2 \times x$$

At *D*, $x = 0$, $M_D = 0$

At *B*, $x = 2$ m, $M_B = -2 \times 2 = -4$ kN-m

C. **Portion BC :**

$+\uparrow \Sigma F_Y = 0,$ $F_{BC} = +2 - 10 + 2x$

At point *B*, $x = 0$, $F_B = +2 - 10 = 8$ kN

Before *C*, $x = 3$ m, $F_C = +2 - 10 + 2 \times 3 = -2$ kN

$$+\circlearrowleft \Sigma M = 0 \Rightarrow M_{BC} = -2 \times x + 10(x-2) - 2 \times (x-2) \times \frac{(x-2)}{2}$$

At point *C*, $x = 5$ m, $M_C = -2 \times 5 + 10(5-2) - 2 \times (5-2) \times \frac{(5-2)}{2} = 11$ kN-m

D. **Portion *AC* :**

1. $+\uparrow \Sigma F_Y = 0 \Rightarrow F_{AC} = 7.5 - 2x$

   At *C*, $x = 0$, $F_{AC} = 7.5$ kN

   At $x = 2$ m, $F_C = 7.5 - 2 \times 2 = 3.5$ kN

   just after *C*, $F_C = 7.5 - 2 \times 2 - 5.5 = -2$ kN
2. At *A*, $x = 2$ from point *A* (from left)

   $F_A = -2 + 10 - 2 \times 3 - 5.5 - 2 \times 2 = -7.5$ kN
3. $+\circlearrowleft \Sigma M = 0,$

   From left, $M_{AC} = +7.5 - 2 \times x \times x/2$

   At *Q*, $x = 1$ m, $M_Q = +7.5 \times 1 - 2 \times 1 \times 1/2 = 6.5$ kN-m
4. Since *BM* at *B* is negative and at *Q* is positive, therefore the *BM* will cross zero line between them.
5. Let, the point of contraflexure lie at a distance *x* from *B*.

$$M_x = -2(2 + x) + 10 \times x - 2 \times x \times x/2 = 0$$

$$-4 - 2x + 10x - x^2 = 0$$

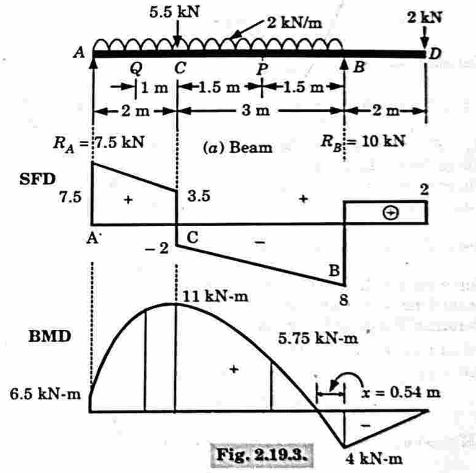


Fig. 2.19.3.

$$x^2 - 8x + 4 = 0 \Rightarrow x = \frac{8 \pm \sqrt{64 - 16}}{2} = \frac{8 \pm 6.93}{2}$$

$$x = 0.54 \text{ m (from B)}$$

6. SFD and BMD are shown in Fig. 2.19.3.

Que 2.20. For the beam shown in Fig. 2.20.1, draw the shear force and bending moment diagram. Determine the position of maximum bending moment. Also determine the point of contraflexure if any.

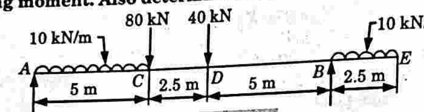


Fig. 2.20.1.

Answer

A. Support Reactions :

1. $R_A + R_B = 10 \times 5 + 80 + 40 + 10 \times 2.5 = 195$ kN

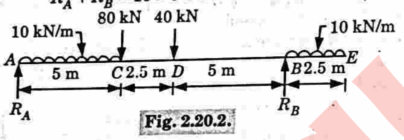


Fig. 2.20.2.

2. Taking moment at A; $+\circlearrowleft \Sigma M_A = 0$

$$-10 \times 5 \times 2.5 - 80 \times 5 - 40 \times 7.5 + R_B \times 12.5 - 10 \times 2.5 \times \left(\frac{2.5}{2} + 12.5\right) = 0$$

$R_B = 93.5$ kN and $R_A = 195 - 93.5 = 101.5$ kN

B. Shear Force Diagram :

1. Shear force at point $C = 101.5 - 10 \times 5 = 51.5$ kN
2. Considering 80 kN, Net shear force at point C
   $= 51.5 - 80 = -28.5$ kN
3. Shear force at point D (including 40 kN)
   $= -28.5 - 40 = -68.5$ kN
4. Shear force at $B = -68.5 + 93.5 = 25$ kN
   Shear force at $E = 25 - 25 = 0$ kN.

E. Bending Moment Diagram :

1. BM at point A, $M_A = 0$ (from left side $\circlearrowright$)
2. BM at point C, $M_C = 101.5 \times 5 - 10 \times 5 \times 2.5$
   $= 382.5$ kN-m (from left side $\circlearrowright$)
3. BM at point D, $M_D = 93.5 \times 5 - 10 \times 2.5 \times \left(\frac{2.5}{2} + 5\right)$
   $= 311.25$ kN-m (from right side $\circlearrowleft$)
4. BM at point B, $M_B = -10 \times \left(\frac{2.5}{2}\right) \times 2.5$
   $= 31.25$ kN-m (from right side $\circlearrowleft$)

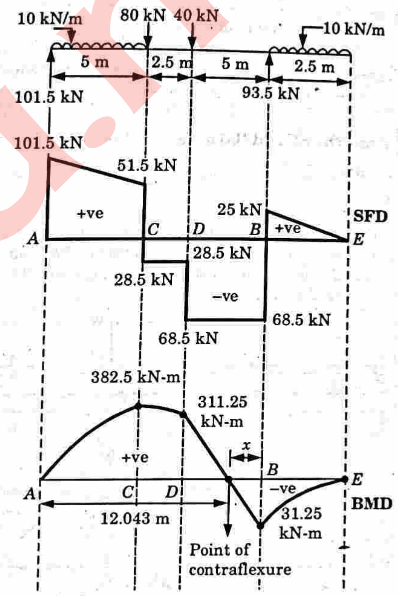


Fig. 2.20.7.

5. Hence, the point of contraflexure lies in portion BD.
   So, $M_x = -10 \times 2.5 \times \left(\frac{2.5}{2} + x\right) + 93.5 \times x$ (from right side)
   at point of contraflexure, $M_x = 0$
   $$-10 \times 2.5 \times \left(\frac{2.5}{2} + x\right) + 93.5 \times x = 0$$
   $x = 0.456$ m (from point B)
6. Hence, maximum bending moment will occur at point C and point of contraflexure will lie at a distance of 12.043 m from left support.

PART-4

Shear Force and Bending Moment Diagram for Fixed Beams.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.21. Draw the SF and BM diagrams for a fixed beam, carrying an eccentric load.

Answer

Fig. 2.21.1(a) shows a fixed beam AB of length L, carrying a point load W at C at a distance of 'a' from A and at a distance of 'b' from B. The fixed end moments M_A and M_B and also reactions at A and B i.e., R_A and R_B are shown in Fig. 2.21.1(a).

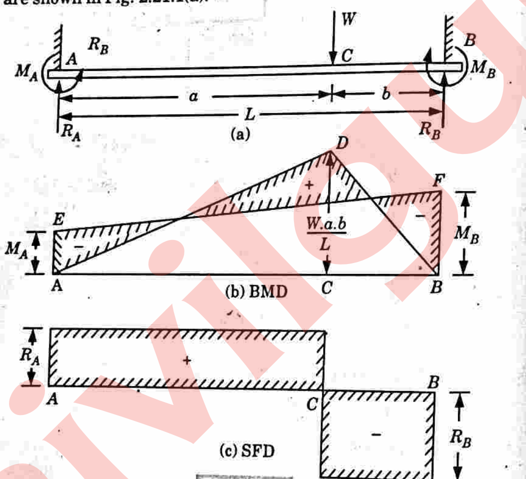


Fig. 2.21.1.

A. BM Diagram :

1. As the load is not acting symmetrically, therefore M_A and M_B will be different. In this case M_B will be more than M_A as the load is nearer to point B. The BM diagram due to end moments will be trapezium as shown in Fig. 2.21.1(b) by $AEFB$. Here the length AE (i.e., M_A) and BF (i.e., M_B) are unknown.
2. The BM diagram for a simply supported beam carrying an eccentric point load will be triangle with maximum BM under the point load equal to $\frac{Wab}{L}$. The BM diagram for this case is shown in Fig. 2.21.1(b) by a triangle ADB in which $CD = \frac{Wab}{L}$.

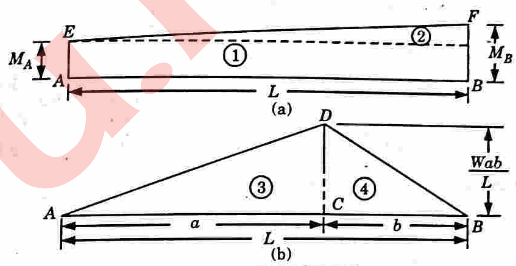


Fig. 2.21.2.

3. Equating the areas of the two bending moment diagrams, we get
Area of trapezium $AEFB$ = Area of triangle ADB

$$\frac{1}{2}(AE + BF)\,AB = \frac{1}{2} \times AB \times CD$$

$$\frac{1}{2}(M_A + M_B)\,L = \frac{1}{2} \times L \times \frac{Wab}{L}$$

$$M_A + M_B = \frac{Wab}{L} \quad \text{...(2.21.1)}$$

Now using eq. (2.21.1),

$$\bar{x} = \bar{x}'$$

4. Distance of CG of BM diagram due to vertical loads from A = Distance of CG of BM diagram due to end moments from A.

Now $$\bar{x}' = \frac{A_1 x_1 + A_2 x_2}{(A_1 + A_2)} \quad \text{[Fig. 2.21.2(a)]}$$

$$= \frac{(M_A L)\frac{L}{2} + \frac{1}{2}L(M_B - M_A) \times \frac{2L}{3}}{M_A L + \frac{1}{2}L(M_B - M_A)} = \frac{(M_A + 2M_B)L}{3(M_A + M_B)}$$

and $$\bar{x} = \frac{A_3 x_3 + A_4 x_4}{(A_3 + A_4)} \quad \text{[Fig. 2.21.2(b)]}$$

$$= \frac{\left(\frac{1}{2} \times a \times CD\right) \times \frac{2a}{3} + \frac{1}{2}\,b\,CD \times \left(a + \frac{b}{3}\right)}{\frac{1}{2}\,a\,CD + \frac{1}{2}\,b\,CD} = \frac{\frac{2a^2}{3} + b\left(a + \frac{b}{3}\right)}{a + b}$$

$$= \frac{2a^2 + 3ab + b^2}{3(a+b)} = \frac{a+L}{3} \quad (\because a + b = L)$$

But $\bar{x}' = \bar{x}$

$$\therefore \quad \frac{(M_A + 2M_B)L}{3(M_A + M_B)} = \frac{a+L}{3}$$

or
$$M_A + 2M_B = \frac{(a+L)(M_A + M_B)}{L}$$

$$= \frac{(a+L)}{L}\frac{Wab}{L} \quad \left[\because M_A + M_B = \frac{Wab}{L}\right]$$

$$= (a+L)\frac{Wab}{L^2} \quad ...(2.21.2)$$

Subtracting eq. (2.21.1) from eq. (2.21.2), we get

$$M_B = (a+L)\frac{Wab}{L^2} - \frac{Wab}{L}$$

$$= \frac{Wab}{L}\left(\frac{a+L-L}{L}\right) = \frac{Wa^2b}{L^2} \quad ...(2.21.3)$$

Substituting the value of M_B in eq. (2.21.1), we get

$$M_A + \frac{Wa^2b}{L^2} = \frac{Wab}{L}$$

$$\therefore \quad M_A = \frac{Wab}{L} - \frac{Wa^2b}{L^2} = \frac{Wab}{L^2}(L-a) = \frac{Wab^2}{L^2} \quad (\because L - a = b)$$

...(2.21.4)

Now M_A and M_B are known and hence bending moment diagram can be drawn. From eq. (2.21.3) and (2.21.4), it is clear that if $a > b$ then $M_B > M_A$.

B. SF Diagram :

1. Equating the clockwise and anticlockwise moments about A,

$$R_B \times L + M_A = M_B + Wa$$

$$\therefore \quad R_B = \frac{(M_B - M_A) + Wa}{L}$$

Similarly,
$$R_A = \frac{(M_A - M_B) + Wa}{L}$$

By substituting the values of M_A and M_B from eq. (2.21.3) and (2.21.4), in the above equations, we shall get R_A and R_B. Now SF can be drawn as shown in Fig. 2.21.2(c).

Que 2.22. **A fixed beam *AB* of length 6 m carries point loads of 160 kN and 120 kN at a distance of 2 m and 4 m from the left end *A*. Find the fixed end moments and the reactions at the supports. Draw BM and SF diagrams.**

Answer

Given : Length = 6 m, Load at C, W_C = 160 kN
Load at D, W_D = 120 kN, Distance, AC = 2 m, Distance, AD = 4 m
To Find : SFD and BMD.

A. Bending Moment Diagram :

1. For the sake of convenience, let us first calculate the fixed end moments due to loads at C and D and then add up the moments.
2. Fixed end moments due to load at C.
   For the load at C, a = 2 m and b = 4 m

$$\therefore \quad M_{A_1} = \frac{W_C ab^2}{L^2} = \frac{160 \times 2 \times 4^2}{6^2} = 142.22 \text{ kN-m}$$

$$M_{B_1} = \frac{W_C a^2 b}{L^2} = \frac{160 \times 2^2 \times 4}{6^2} = 71.11 \text{ kN-m}$$

3. Fixed end moments due to load at D.
   Similarly, for the load at D, a = 4 m and b = 2 m

$$\therefore \quad M_{A_2} = \frac{W_D ab^2}{L^2} = \frac{120 \times 4 \times 2^2}{6^2} = 53.33 \text{ kN-m}$$

and
$$M_{B_2} = \frac{W_D a^2 b}{L^2} = \frac{160 \times 4^2 \times 2}{6^2} = 106.66 \text{ kN-m}$$

4. Total fixing moment at A,

$$M_A = M_{A_1} + M_{A_2} = 142.22 + 53.33 = 195.55 \text{ kN-m.}$$

   and total fixing moment at B,

$$M_B = M_{B_1} + M_{B_2} = 71.11 + 106.66 = 177.77 \text{ kN-m}$$

5. BM diagram due to vertical loads :
   Consider the beam AB as simply supported. Let R_A and R_B are the reactions at A and B due to simply supported beam. Taking moments about A, we get

$$R_B \times 6 = 160 \times 2 + 120 \times 4 = 133.33 \text{ kN}$$

and
$$R_A = \text{Total load} - R_B = (160 + 120) - 133.33 = 146.67 \text{ kN}$$

6. Taken bending moment at A, $M_A = 0$
   BM at $C = R_A \times 2 = 146.67 \times 2 = 293.34$ kN-m
   BM at $D = R_B \times 2 = 133.33 \times 2 = 266.66$ kN-m
   BM at $B = 0$.
7. Now the BM diagram due to vertical loads can be drawn as shown in Fig. 2.22.1(b). In the same figure the BM diagram due to fixed end moments is also shown.

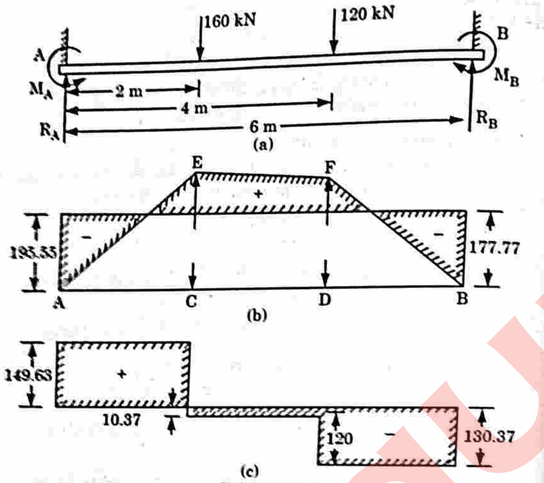


Fig. 2.22.1.

B. **SF Diagram :**

1. Let R_A = Resultant reaction at A due to fixed end moments and vertical loads.
   R_B = Resultant reaction at B.
2. Equating the clockwise moments and anticlockwise moments about A, we get, $R_B \times 6 + M_A = 160 \times 2 + 120 \times 4 + M_B$
   $$R_B \times 6 + 195.55 = 320 + 480 + 177.77 \Rightarrow R_B = 130.37 \text{ kN}$$
   and $$R_A = \text{Total load} - R_B = (160 + 120) - 130.37 = 149.63 \text{ kN}$$
3. **SF Calculations :**
   $$\text{SF at } A = R_A = 149.63 \text{ kN}$$
   $$\text{SF at } C = 149.63 - 160 = -10.37 \text{ kN}$$
   $$\text{SF at } D = -10.37 - 120 = -130.37 \text{ kN}$$
   $$\text{SF at } B = -130.37 \text{ kN}$$
   Now SF diagram can be drawn as shown in Fig. 2.22.1(c)

Que 2.23. **Draw the SFD and BMD for shown in Fig. 2.23.1(a). Also give the position of contraflexure point form end A.**

Answer

Given : Fig. 2.23.1(a) shows a fixed beam of length L, intensity of UDL = w, length of UDL = L.
To Find : SFD and BMD.

Let M_A = Fixed end moment at A, M_B = Fixed end moment at B
R_A = Reaction at A, R_B = Reaction at B.

A. **Bending Moment Diagram :**

1. Since the loading on the beam is symmetrical, hence $M_A = M_B$. The BM diagram due to end moments will be a rectangle as shown in Fig. 2.23.1(b) by $AEFB$. The magnitude M_A or M_B is unknown.
2. The BM diagram for a simply supported beam carrying a uniformly distributed load will be parabola whose central ordinate will be $wL^2/8$.
3. The BM diagram for this case is shown in Fig. 2.23.1(b) by parabola ADB in which $CD = wL^2/8$.
4. Equating the areas of the two bending moment diagrams, we get
   Area of rectangle $AEFB$ = Area of parabola ADB
   $$AB \times AE = 2/3 \times [AB \times CD]$$
   $$L \times M_A = \frac{2}{3} \times L \times \frac{wL^2}{8} \Rightarrow M_A = \frac{wL^2}{12}$$
   $$\therefore \quad M_B = M_A = \frac{wL^2}{12} \qquad \text{...(2.23.1)}$$
5. Now the B.M. diagram can be drawn as shown in Fig. 2.23.1(b).

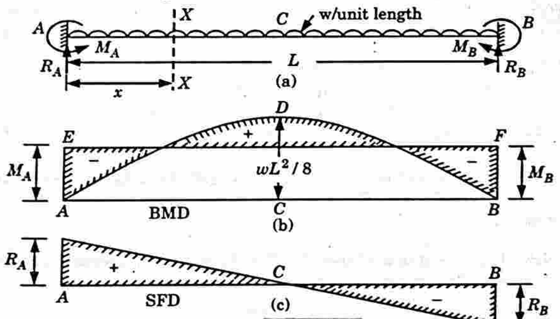


Fig. 2.23.1.

B. **Shear Force Diagram :**

1. Equating the clockwise moments and anticlockwise moments about A,
   we get, $R_B \times L + M_A = wL\frac{L}{2} + M_B$

But $M_A = M_B$

$\therefore$ $R_B \times L = wL\dfrac{L}{2} \Rightarrow R_B = \dfrac{wL}{2}$

Due to symmetry, $R_A = R_B = \dfrac{wL}{2}$

Now the SF diagram can be drawn as shown in Fig. 2.23.1(c).

C. Points of Contraflexures :

1. For the points of contraflexures, BM at any section in beam should be zero. BM at a section X-X, $M_x = 0$

$$\frac{wLx}{2} - \frac{wx^2}{2} - \frac{wL^2}{12} = 0$$

$$x^2 - Lx + \frac{L^2}{6} = 0$$

2. Solving the above quadratic equation, we get

$$x = \frac{+L \pm \sqrt{L^2 - \frac{4 \times L^2}{6}}}{2} = \frac{L \pm \sqrt{\frac{L^2}{3}}}{2} = \frac{L}{2} \pm \frac{L}{2\sqrt{3}}$$

As, $L/2$ represents the centre of the beam, hence, the two points of contraflexures occur at a distance of $L/2\sqrt{3}$ from the centre of the beam.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Write the steps involve in drawing SFD and BMD. Also define point of contraflexure.

Ans. Refer Q. 2.4, Unit-2.

Q. 2. For the beam shown in Fig. 1, draw the shear force and bending moment diagram.

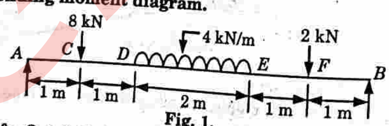


Fig. 1.

Ans. Refer Q. 2.8, Unit-2.

Q. 3. Draw the SF and BM diagrams for a cantilever of length L carrying a gradually varying load from zero at the free end to w per unit length at the fixed end.

Ans. Refer Q. 2.11, Unit-2.

Q. 4. Draw the S.F. and B.M. diagrams for a cantilever loaded as shows in Fig. 2.

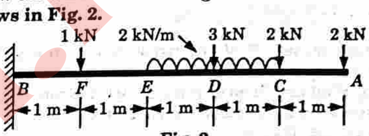


Fig. 2.

Ans. Refer Q. 2.12, Unit-2.

Q. 5. Draw the SFD and BMD of the loaded beam as shown in Fig. 3 below.

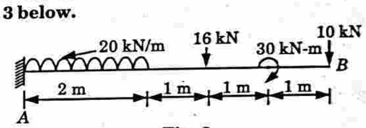


Fig. 3.

Ans. Refer Q. 2.15, Unit-2.

Q. 6. Draw the SFD and BMD and locate the point of contraflexures for the loaded beam in Fig. 4.

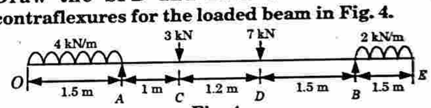


Fig. 4.

Ans. Refer Q. 2.16, Unit-2.

Q. 7. Determine SFD and BMD for the simply supported beam as shown in Fig. 5 and also find maximum bending moment.

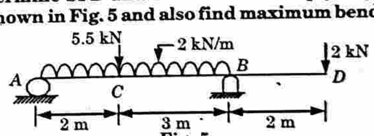


Fig. 5.

Ans. Refer Q. 2.19, Unit-2.

Q. 8. For the beam shown in Fig. 6, draw the shear force and bending moment diagram. Determine the position of maximum bending moment. Also determine the point of contraflexure if any.

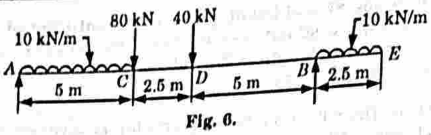


Fig. 6.

Ans. Refer Q. 2.20, Unit-2.

Q. 9. ***A* fixed beam *AB* of length 6 m carries point loads of 160 kN and 120 kN at a distance of 2 m and 4 m from the left end *A*. Find the fixed end moments and the reactions at the supports. Draw BM and SF diagrams.**

Ans. Refer Q. 2.22, Unit-2.

☺☺☺

3 UNIT

Flexure Stresses, Torsion and Shear Stresses

CONTENTS

PART-1

Theory of Simple Bending, Assumption, Derivation of Equation $M/I = \sigma/y = E/R$.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. **Prove that the bending stress in any fibre is proportional to the distance of that fibre from neutral layer in a beam.**

OR

Write the assumptions for pure bending and also derive the equation for bending.

AKTU 2014-15, Marks 10

OR

Derive the bending equation for a beam subjected to bending moment M in pure bending condition. Also state the assumptions.

AKTU 2015-16, Marks 10

Answer

A. Pure Bending : The state of loading where the beam segment is obviously free from shear force (or shear stress) and subjected only to constant bending moment is called as pure bending or simple bending.

B. Assumptions : Following are the assumptions used in deriving the bending equation :

1. Material of beam is homogeneous and isotropic.
2. Loading is within elastic limit.
3. Beam is initially straight and is of uniform section throughout.
4. Radius of curvature of beam during bending is large compared to its transverse dimensions.
5. Transverse section of beam remains plane and perpendicular to neutral surface after bending.
6. There is no lateral strain, only longitudinal stress is there.

C. Derivation :

1. Consider a beam segment $ABCD$ subjected to pure bending.
2. Let, θ = Angle subtended at centre by arc.
   R = Radius of curvature.

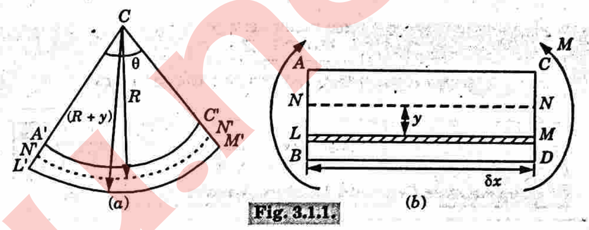


Fig. 3.1.1.

3. Before bending, $NN = LM = \delta x$
   After bending, the length of neutral layer $N'N'$ will increase
   Hence, $N'N' = NN = \delta x = R\theta$ (from Fig. 3.1.1)
   $L'M' = (R + y)\theta$
4. Change in length of strip $LM = L'M' - LM = (R + y)\theta - R\theta$
5. Strain in LM, $\varepsilon = \dfrac{(R+y)\theta - R\theta}{R\theta} = \dfrac{y\theta}{R\theta} = \dfrac{y}{R}$ ...(3.1.1)
6. From Hooke's law, $\varepsilon = \dfrac{\sigma}{E}$ ...(3.1.2)
7. From equation (3.1.1) and equation (3.1.2), we get,
   $$\frac{\sigma}{E} = \frac{y}{R} \Rightarrow \frac{\sigma}{y} = \frac{E}{R} \quad ...(3.1.3)$$
8. Now, $\sigma = \dfrac{E}{R} y \Rightarrow \sigma \propto y$ $\left[\because \dfrac{E}{R} \text{ is constant}\right]$
   Hence stress in any fibre is proportional to distance of that fibre 'y' from neutral axis.
9. Bending force, in consider strip is given by $dF = \sigma\, dA = \dfrac{E}{R} y\, dA$
10. Moment at element (Moment of resistance),
   $$dM = d \times Fy = \frac{E}{R} y\, dAy = \frac{E}{R} y^2 dA \quad (\because dF = \sigma dA)$$
   $$\int dM = \int \frac{E}{R} y^2\, dA \Rightarrow M = \frac{E}{R}\int y^2 dA = \frac{E}{R} I \quad \left(\because I = \int y^2 dA\right)$$
   $$\frac{M}{I} = \frac{E}{R} \quad ...(3.1.4)$$
11. From eq. (3.1.3) and eq. (3.1.4), we get
   $$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

PART-2

Neutral Axis, Determination Bending Stresses.

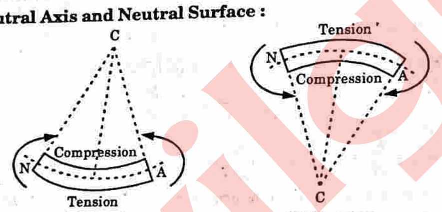
Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.2. What do you understand by the term neutral axis and neutral surface ? A steel beam of hollow square section of 60 mm outer side and 50 mm inner side is simply supported on a span of 4 meters. Find the maximum concentrated load the beam can carry at the middle of the span if the bending stress is not to exceed 120 N/mm^2.

Answer

A. **Neutral Axis and Neutral Surface :**



(a) Sagging (b) Hogging

Fig. 3.2.1.

1. During sagging of beam, the fibres at the upper portion of the beam section get compressed and accordingly upper side of beam is in compression.
2. The fibres at the lower portion of the beam get stretched and are in tension.
3. At a particular section of the beam there exists a surface where the fibres are neither in compression nor in tension.
4. Such a surface is called neutral surface or neutral plane. A trace of this plane on any transverse plane of the beam is called neutral axis (NA).

B. **Numerical :**

Given : Hollow square section of 60 mm outer side and 50 mm inner side, span of beam, $L = 4$ m, Bending stress, $\sigma = 120$ N/mm^2.
To Find : Maximum concentrated load (beam can carry at the middle of the span).

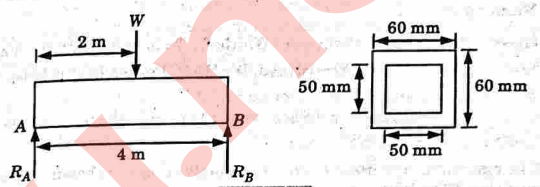


Fig. 3.2.2.

1. Maximum bending moment due to the concentrated load,

$$M = \frac{WL}{4} = \frac{W \times 4}{4} = W \text{ N-m}$$

2. Moment of inertia of the cross-section of the beam,

$$I = \frac{(60)^4}{12} - \frac{(50)^4}{12} = 559166.67 \text{ mm}^4$$

3. Now, from the bending equation, $\dfrac{M}{I} = \dfrac{\sigma}{y}$, where, $y = \dfrac{60}{2} = 30$ mm

$$\frac{W \times 10^3}{559166.67} = \frac{120}{30} \Rightarrow W = 2236.67 \text{ N}$$

Que 3.3. Why I-section beam is preferred over a rectangular section beam ? A simply supported beam, 2 cm wide by 4 cm high and 1.5 m long is subjected to a concentrated load of 2 kN (perpendicular to beam) at a point 0.5 m from one of the supports.
Determine :
i. The maximum fiber stress, and
ii. The stress in a fiber located 1 cm from the top of the beam at mid-span.

Answer

1. I-section beam is preferred over a rectangular-section beam because of the following reasons :
   i. According to beam theory, I-shaped section is very efficient form for carrying both bending and shear loads in the plane of the web.
   ii. I-section beam also saves material and weight as it has the same weight carrying strength as a rectangular beam having same height and widest width of the I-section.
   iii. I-section also maximizes the moment of inertia.
   iv. For purpose of economy and weight reduction, the material should be concentrated as much as possible at the greatest distance from neutral axis. Hence, an I-section is preferred over a rectangular-section beam.

2. **Numerical :**

Given : Simply supported beam (Width, $b = 2$ cm and depth, $d = 4$ cm), Span, $L = 1.5$ m, Concentrated load, $W = 2$ kN (Perpendicular to beam).
To Find : i. The maximum fiber stress.
ii. The stress in a fiber located 1 cm from the top of the beam at mid-span.

i. The maximum bending moment for a simply supported beam subjected to an eccentric load,

$$M = \frac{Wab}{L} = \frac{2 \times 10^3 \times 1 \times 0.5}{1.5} \text{ N-m} = 666.67 \text{ N-m}$$

ii. Section modulus for the beam section,

$$Z = \frac{bd^2}{6} = \frac{2 \times (4)^2}{6} = 5.33 \text{ cm}^3$$

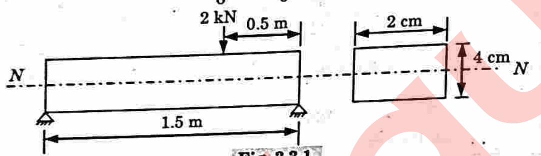


Fig. 3.3.1.

iii. The maximum fiber stress, at top surface of the beam,

$$\sigma_{max} = \frac{M}{Z} = \frac{666.67 \times 10^3}{5.33 \times 10^3} = 125.08 \text{ MPa}$$

iv. Let, the stress in fiber located 1 cm from the top of the beam at mid span = σ
Hence, distance from neutral axis, $y = 2 - 1 = 1$ cm

v. So, $\frac{\sigma}{y}$ = Constant (from bending moment equation)

$$\frac{\sigma_{max}}{y_{max}} = \frac{125.08}{2 \times 10^{-2}} = \frac{\sigma}{1 \times 10^{-2}} \Rightarrow \sigma = 62.54 \text{ MPa}$$

Que 3.4. **Calculate the maximum stress induced in a cast iron pipe of external diameter 40 mm, of internal diameter 20 mm and of length 4 metre when the pipe is supported at its ends and carries a point load of 80 N at its centre.**

Answer

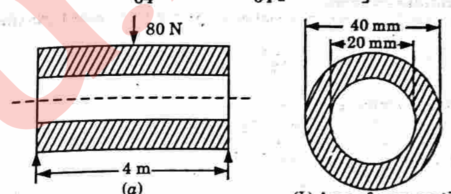
Given : External diameter, $D = 40$ mm, Internal diameter, $d = 20$ mm, Length, $L = 4$ m = 4000 mm, Point load, $W = 80$ N (at mid-span)
To Find : Maximum stress induced in cast iron pipe.

1. In case of simply supported beam carrying a point load at the centre, the maximum bending moment is at the centre of the beam. Hence, the

$$\text{Maximum BM} = \frac{W \times L}{4} = \frac{80 \times 4000}{4} = 8 \times 10^4 \text{ N-mm}$$

2. Moment of inertia of hollow pipe,

$$I = \frac{\pi}{64}[D^4 - d^4] = \frac{\pi}{64}\left[40^4 - 20^4\right] = 117809.72 \text{ mm}^4$$



(b) Area of cross-section

Fig. 3.4.1.

3. The distance the top layer is maximum from NA

$$y_{max} = \frac{D}{2} = \frac{40}{2} = 20 \text{ mm}$$

4. Maximum bending stress is given by,

$$\sigma_{max} = \frac{M}{I} \times y_{max} = \frac{8 \times 10^4 \times 20}{117809.72} = 13.58 \text{ N/mm}^2$$

PART-3

Section Modulus of Rectangular and Circular Sections (Solid and Hollow), I, T, Angle and Channel Sections.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.5. **What do you mean by section modulus ? Find an expression for section modulus for a rectangular, circular and hollow circular section.**

Answer

1. **Section Modulus :** It is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the distance of the outermost layer from the neutral axis. Section modulus is given by,

$$Z = \frac{I}{y_{max}} \qquad ...(3.5.1)$$

where, I = MOI about neutral axis.

y_{max} = Distance of the outermost layer from the neutral axis.

2. Section Modulus for Various Sections :

i. Rectangular Section :
For rectangular section, moment of inertia about an axis through its CG or NA, $I = bd^3/12$ and $y_{max} = \frac{d}{2}$

$\therefore$ Section modulus, $Z = \frac{I}{y_{max}} = \frac{bd^3}{12 \times \left(\frac{d}{2}\right)} = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6}$

ii. Circular Section : For a circular section,

$$I = \frac{\pi}{64} d^4 \text{ and } y_{max} = \frac{d}{2}$$

$$\therefore \quad Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64} d^4}{\left(\frac{d}{2}\right)} = \frac{\pi}{32} d^3$$

iii. Hollow Circular Section : For a hollow circular section,

$$I = \frac{\pi}{64}[D^4 - d^4] \text{ and } y_{max} = D/2$$

$$\therefore \quad Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64}[D^4 - d^4]}{\left(\frac{D}{2}\right)} = \frac{\pi}{32D}[D^4 - d^4]$$

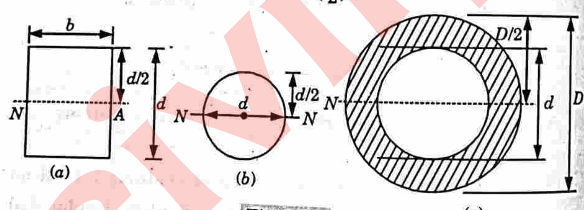


Fig. 3.5.1.

Que 3.6. Prove that the ratio of depth to width of the strongest beam that can be cut from a circular log of diameter d is 1.414. Hence calculate the depth and width of the strongest beam that can be cut of a cylindrical log of wood whose diameter is 300 mm.

Answer

1. Let, $ABCD$ be the strongest rectangular section which can be cut out of the cylindrical log of diameter d.
2. Let, b = Width of strongest section.
   d = Depth of strongest section.

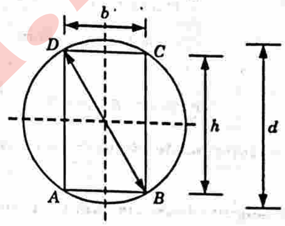


Fig. 3.6.1.

3. Now, section modulus of the rectangular section,
$$Z = \frac{I}{y_{max}} = \frac{(bh^3/12)}{(h/2)} = \frac{bh^2}{6} \qquad ...(3.6.1)$$
In the above equation, b and h are variable.
4. From ΔBCD, $b^2 + h^2 = d^2 \Rightarrow h^2 = d^2 - b^2$
5. Substituting the value of h^2 in eq. (3.6.1), we get
$$Z = \frac{b}{6}[d^2 - b^2] = \frac{1}{6}[bd^2 - b^3] \qquad ...(3.6.2)$$
In the above equation, d is constant and hence only variable is b.
6. Now for the beam to be strongest, the section modulus should be maximum. For maximum value of Z, $dZ/db = 0$
$$\frac{d}{db}\left[\frac{bd^2 - b^3}{6}\right] = 0 \Rightarrow d^2 = 3b^2 \qquad ...(3.6.3)$$
7. Substituting the value of d^2 in eq. (3.6.3), we get
$$b^2 + h^2 = 3b^2 \Rightarrow h^2 = 2b^2$$
$$\frac{h}{b} = \sqrt{2} = 1.414$$

Numerical :

Given : Diameter of circular log, d = 300 mm
To Find : The width and depth of the beam.

We know that width of beam,

$$d^2 = 3b^2 \Rightarrow 3b^2 = 300^2$$

$$b = 173.2 \text{ mm}$$

Depth of beam is given by,

$$h = \sqrt{2} \times b = 1.414 \times 173.2 = 249.95 \text{ mm}$$

PART-4

Design of Simple Beam Sections.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.7. **A rectangular beam 200 mm deep and 300 mm wide is simply supported over a span of 8 m. What uniformly distributed load per metre the beam may carry, if the bending stress is not to exceed 120 N/mm².**

Answer

Given : Depth of beam, d = 200 mm, Width of beam, b = 300 mm, Length of beam, L = 8 m, Maximum bending stress, σ_{max} = 120 N/mm²
To Find : Uniformly distributed load per metre.

1. Let, w = Uniformly distributed load per metre length over the beam.
2. For a rectangular section,

Section modulus, $$Z = \frac{bd^2}{6} = \frac{300 \times 200^2}{6} = 2 \times 10^6 \text{ mm}^3$$

3. Maximum BM for a simply supported beam carrying uniformly distributed load as shown in Fig. 3.7.1 is at the centre of the beam. Hence, the maximum BM,

$$M = \frac{w \times L^2}{8} = \frac{w \times 8^2}{8} = 8\,w \text{ N-m} = 8000\,w \text{ N-mm}$$

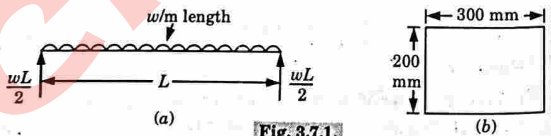


Fig. 3.7.1.

4. We know that, $M = \sigma_{max} \times Z$

$$8000\,w = 120 \times 2 \times 10^6$$

Intensity of UDL, w = 30 kN/m.

Que 3.8. **A steel beam of hollow square section of 60 mm outer side and 50 mm inner side is simply supported on a span of 4 metres. Find the maximum concentrated load on the beam at the middle of the span if the bending stress is not to exceed 120 N/mm².**

Answer

Given : Steel beam of hollow square section (60 mm outer side and 50 mm inner side), L = 4 m, σ_b = 120 N/mm²
To Find : Maximum concentrated load on the mid-span.

1. Let, W = Maximum concentrated load.
2. Maximum bending moment of the centre,

$$M = \frac{W \times L}{4} = \frac{W \times 4}{4} = W \text{ N-m.}$$

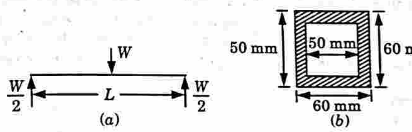


Fig. 3.8.1.

$$M = W \times 1000 = 1000\,W \text{ N-mm}$$

3. Moment of inertia for hollow square section,

$$I = \left[\frac{60 \times 60^3}{12} - \frac{50 \times 50^3}{12}\right] \text{mm}^4 = 55.91 \times 10^4 \text{ mm}^4$$

4. $$y_{max} = \frac{60}{2} = 30 \text{ mm}$$

5. We know that, $$\frac{M}{I} = \frac{\sigma_b}{y_{max}}$$

$$\therefore \quad \frac{1000\,W}{55.91 \times 10^4} = \frac{120}{30} \Rightarrow W = 2236.40 \text{ N.}$$

Que 3.9. **A wooden beam of rectangular cross section is subjected to a bending moment of 5 kN-m. If the depth of the section is to be twice the breadth and stress in wood is 60 N/cm². Find the dimensions of the cross section of the beam.**

Answer

Given : Bending moment, $M = 5$ kN-m $= 5 \times 10^6$ N-mm,
Stress, $\sigma_{max} = 60$ N/cm² $= 0.6$ N/mm², $d = 2b$
To Find : Dimension of the cross-section of the beam.

1. Section modulus for rectangular section,

$$Z = \frac{bd^2}{6} = \frac{b(2b)^2}{6} = \frac{2b^3}{3}$$

2. Bending stress, $\sigma_{max} = \dfrac{M}{Z} \Rightarrow 0.6 = \dfrac{5 \times 10^6}{\dfrac{2b^3}{3}}$

Width of beam, $b = 232$ mm
Depth of beam, $d = 2b = 2 \times 232 = 464$ mm

Que 3.10. **A simply supported beam of 8 m length is subjected to a uniformly distributed load of 6 kN/m over the half of the span length of the beam from the left support. Select a suitable beam of rectangular cross-section having depth to width ratio of 1 : 2 considering the allowable stress for beam material not to exceed 40 MPa.**

Answer

Given : Span of beam, $L = 8$ m, UDL = 6 kN/m, Length of *UDL* = 4 m
$d/b = 1/2$, Allowable stress, $\sigma_{max} = 40$ MPa.
To Find : Design of beam of rectangular cross section.

1. **Support reactions :**
i. $\Sigma F_g = 0;\ R_A + R_B = 6 \times 4 = 24$ kN ...(3.10.1)

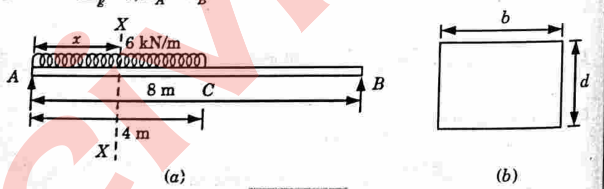


Fig. 3.10.1.

ii. $$\Sigma M_B = 0 \Rightarrow R_A \times 8 - (6 \times 4) \times \left(\frac{4}{2} + 4\right) = 0$$

$$R_A = 18 \text{ kN and } R_B = 24 - 18 = 6 \text{ kN}$$

2. Moment at distance x from left support,

$$M_x = R_A x - 6x \times \frac{x}{2}$$

$$M_x = 18x - 3x^2$$

3. For maximum value of M_x,

$$\frac{dM_x}{dx} = 0$$

$$18 - 6x = 0 = 3 \text{ m}$$

4. At $x = 3$ m, maximum BM, $M = 18(3) - 3(3)^2 = 27$ kN-m

5. Section modulus, $Z = \dfrac{bd^2}{6} = \dfrac{2d \times d^2}{6} = \dfrac{d^3}{3}$

6. We know that, $M = \sigma Z$

$$27 \times 10^3 = 40 \times 10^6 \times d^3/3$$

Depth, $d = 0.127$ m
Width, $b = 2 \times 0.127 = 0.254$ m

PART-5

Derivation of Torsion Equation and its Assumption.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.11. **What are the assumptions made in the derivations of shear stress produced in a circular shaft subjected to torsion ?**

Answer

Assumptions : Following are the assumptions made in derivation as shear stress produced in a circular shaft subjected to torsion :

1. The material of the shaft is uniform throughout.
2. The twist along the shaft is uniform.
3. The shaft is of uniform circular section throughout.
4. Cross-sections of the shaft which are plane before twist remain plain after twist.
5. All radii which are straight before twist remain straight after twist.

Que 3.12. **Derive the expression of shear stress produced in a circular shaft subjected to torsion.**

Answer

Shear Stress Produced in a Circular Shaft :

1. When a circular shaft is subjected to torsion, shear stresses are set up in the material of the shaft. To determine the magnitude of shear stress at any point on the shaft, consider a shaft fixed at one end AA and free at the end BB as shown in Fig. 3.12.1.
2. Let CD is any line on the outer surface of the shaft. Now let the shaft is subjected to a torque T at the end BB as shown in Fig. 3.12.2. As a result of this torque T, the shaft at the end BB will rotate clockwise and every cross-section of the shaft will be subjected to shear stresses.
3. The point D will shift to D' and hence line CD will be deflected to CD' as shown in Fig. 3.12.2(a). The line OD will be shifted to OD' as shown in Fig. 3.12.2(*b*).
4. Let,
   R = Radius of shaft, L = Length of shaft.
   T = Torque applied at the end BB.
   τ = Shear stress induced at the surface of the shaft due to torque T.
   G = Modulus of rigidity of the material of the shaft.
   $\phi = \angle DCD'$ also equal to shear strain.
   $\theta = \angle DOD'$ also called angle of twist.
5. Now distortion at the outer surface due to torque $T = DD'$
   $\therefore$ Shear strain at outer surface = Distortion per unit length
   $$= \frac{\text{Distortion at the outer surface}}{\text{Length of shaft}} = \frac{DD'}{L} = \tan\phi$$
   $\therefore$ Shear strain at outer surface,
   $$\phi = \frac{DD'}{L} \text{ (if } \phi \text{ is very small then } \tan\phi = \phi) \quad ...(3.12.1)$$
6. Now from Fig. 3.12.1(*b*).
   $$\text{Arc } DD' = OD \times \theta = R\theta \quad (\because OD = R = \text{Radius of shaft})$$
7. Substituting the value of DD' in eq. (3.12.1), we get
   $$\text{Shear strain at outer surface, } \phi = \frac{R \times \theta}{L} \quad ...(3.12.2)$$

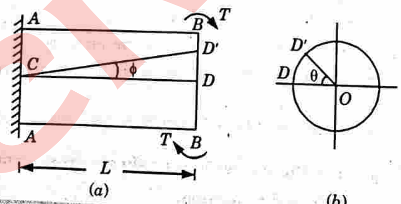


Fig. 3.12.1. Shaft fixed at end AA subjected to torque T at BB.

8. Now the modulus of rigidity (G) of the material of the shaft is given as
   $$G = \frac{\text{Shear stress at the outer surface}}{\text{Shear strain at outer surface}}$$
   $$G = \frac{\tau}{\phi} = \frac{\tau}{(R\theta)/L} = \frac{\tau \times L}{R\theta}$$
   $$\frac{G\theta}{L} = \frac{\tau}{R} \Rightarrow \tau = \frac{R \times G \times \theta}{L} \quad ...(3.12.3)$$
9. Now for a given shaft subjected to a given torque (T), the values of G, θ and L are constant. Hence, shear stress produced is proportional to the radius R. *i.e.*, $\tau \propto R \Rightarrow \tau / R$ = constant ...(3.12.4)
10. If q is shear stress induced at a radius 'r' from the centre of the shaft then,
    $$\tau / R = q / r \quad ...(3.12.5)$$
    From eq. (3.12.4) and eq. (2.12.5), we get,
    $$\frac{\tau}{R} = \frac{G\theta}{L} = \frac{q}{r} \quad (3.12.6)$$
11. From eq. (3.12.4), it is clear that shear stress at any point in the shaft is proportional to the distance of the point from the axis of the shaft.
12. Hence the shear stress is maximum at the outer surface and zero at the axis of the shaft.

PART-6

Application of the Equation of the Hollow and Solid Circular Shaft.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.13. **Prove that the torque transmitted by a solid shaft when subjected to torsion is given by,**

$$T = \frac{\pi}{16} \tau D^3$$

where, D = Diameter of solid shaft
τ = Maximum shear stress.

Answer

1. The maximum torque transmitted by a circular solid shaft, is obtained from the maximum shear stress induced at the outer surface of the solid shaft.
2. Consider a shaft subjected to a torque T as shown in Fig. 3.13.1.

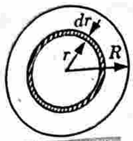


Fig. 3.13.1.

3. Let, τ = Maximum shear stress induced at the outer surface.
   R = Radius of the shaft.
   q = Shear stress at a radius 'r' from the centre.
4. Consider an elementary circular ring of thickness 'dr' at a distance r from the centre as shown in Fig. 3.13.1.
   Then the area of the ring, $dA = 2\pi r dr$
5. We know that, $\dfrac{\tau}{R} = \dfrac{q}{r} \Rightarrow q = \dfrac{\tau}{R} r = \tau \dfrac{r}{R}$
6. Turning force on the elementary circular ring
   = Shear stress acting on the ring × Area of ring
   $= q \times dA = (\tau \times r / R) \times 2\pi r dr = (\tau / R) \times 2\pi r^2 dr$
7. Now turning moment due to the turning force on the elementary ring.
   dT = Turning force on the ring × Distance of the ring from the axis
   $= \tau / R \times 2\pi r^2 dr \times r$
   $= \tau / R \times 2\pi r^3 dr$
8. The total turning moment (or total torque) is obtained by integrating the above equation between the limits 0 and R.

$$T = \int_0^R dT = \int_0^R \frac{\tau}{R} \times 2\pi r^3\, dr = \frac{\tau}{R} \times 2\pi \left[\frac{r^4}{4}\right]_0^R = \frac{\pi}{16} \tau D^3$$

Que 3.14. **Find an expression for the torque transmitted by a hollow circular shaft of external diameter = D_0 and internal diameter = D_i.**

Answer

1. Consider a hollow shaft which is subjected to a torque T as shown in Fig. 3.14.1.
2. Take an elementary circular ring of thickness 'dr' at a distance r from the centre as shown in Fig. 3.14.1.
   Let, r = Radius of elementary circular ring.
   dr = Thickness of the ring.
   τ = Maximum shear stress induced at outer surface of the shaft.
   q = Shear stress induced on the elementary ring.
   dA = Area of the elementary circular ring = $2\pi r \times dr$

3. Shear stress at the elementary ring is obtained by,
$$\frac{\tau}{R_0} = \frac{q}{r} \Rightarrow q = \frac{\tau}{R_0} \times r \qquad (\because \text{Here outer radius } R = R_0)$$
4. Turning force on the ring = Stress × Area = $q \times dA$
$$= \frac{\tau}{R_0} r \times 2\pi r dr = 2\pi \frac{\tau}{R_0} r^2\, dr \qquad \left(\because q = \frac{\tau}{R_0} r\right)$$
5. Turning Moment on the ring,
   dT = Turning force × Distance of the ring from centre.
$$= 2\pi \frac{\tau}{R_0} r^2 dr \times r = 2\pi \frac{\tau}{R_0} r^3\, dr$$

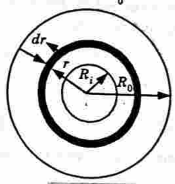


Fig. 3.14.1.

Let, R_0 = Outer radius of the shaft.
R_i = Inner radius of the shaft.

6. The total turning moment (or total torque T) is obtained by integrating the above equation between the limits R_i and R_0,
$$T = \int_{R_i}^{R_0} dT = \int_{R_i}^{R_0} 2\pi \frac{\tau}{R_0} r^3\, dr = 2\pi \frac{\tau}{R_0} \int_{R_i}^{R_0} r^3\, dr$$
$(\because \tau \text{ and } R_0 \text{ are constant})$
$$= 2\pi \frac{\tau}{R_0} \left[\frac{r^4}{4}\right]_{R_i}^{R_0} = \frac{\pi}{2} \tau \left[\frac{R_0^4 - R_i^4}{R_0}\right]$$
7. Let, D_0 and D_i = Outer and inner diameter of the shaft.
   Then, $R_0 = \dfrac{D_0}{2}$ and $R_i = \dfrac{D_i}{2}$
   Substituting the values of R_0 and R_i in equation (1),
$$T = \frac{\pi}{16} \tau \left[\frac{D_0^4 - D_i^4}{D_0}\right]$$

Que 3.15. **A solid shaft of 200 mm diameter has the same cross sectional area as the hollow shaft of the same material with inside diameter of 150 mm. Find the ratio of :**

i. **Powers transmitted by both the shafts at the same angular velocity.**

ii. Angles of twists in equal length of these shafts, when stressed to same intensity.

AKTU 2016-17, Marks 10

Answer

Given : Diameter of solid shaft (D_1) = 200 mm
Inside diameter of hollow shaft (d) = 150 mm.
To Find : The ratio of :
i. Powers transmitted by both of the same angular velocity.
ii. Angles of twists in equal length, when stressed to same intensity.

1. **Ratio of Powers Transmitted by Both the Shafts :**

i. Cross sectional area of the solid shaft,

$$A_1 = \frac{\pi}{4} \times D_1^2 = \frac{\pi}{4} \times 200^2 = 10000\pi \text{ mm}^2$$

Cross sectional area of hollow shaft,

$$A_2 = \frac{\pi}{4} \times (D^2 - d^2) = \frac{\pi}{4} \times [D^2 - (150)^2] = \frac{\pi}{4}(D^2 - 22500)$$

ii. Since the cross sectional area of both the shafts are same, therefore equating A_1 and A_2, $10000\,\pi = \frac{\pi}{4}(D^2 - 22500) \Rightarrow D = 250$ mm

iii. Torque transmitted by the solid shaft,

$$T_1 = \frac{\pi}{16}\tau D_1^3 = \frac{\pi}{16} \times \tau \times 200^3 = 500 \times 10^3\, \pi\,\tau \text{ N-mm}$$

iv. Similarly, torque transmitted by the hollow shaft,

$$T_2 = \frac{\pi}{16}\tau\left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau \times \left[\frac{(250)^4 - (150)^4}{250}\right] \text{N-mm}$$
$$= 850 \times 10^3\, \pi\,\tau \text{ N-mm}$$

v. Therefore, $\frac{\text{Power transmitted by hollow shaft}}{\text{Power transmitted by solid shaft}} = \frac{T_2}{T_1}$

$$= \frac{850 \times 10^3\,\pi\,\tau}{500 \times 10^3\,\pi\,\tau} = 1.7$$

2. **Angles of Twist in Equal Length of these Shafts, when Stressed to Same Intensity :**

i. Angle of twist for a shaft, $\frac{\tau}{R} = \frac{G\theta}{L}$

ii. Therefore, angle of twist for the solid shaft,

$$\theta = \frac{\tau L}{RG} = \frac{\tau L}{100G} \qquad \left(\text{where, } R = \frac{D_1}{2} = \frac{200}{2} = 100 \text{ mm}\right)$$

iii. Similarly angle of twist for the hollow shaft,

$$\theta_2 = \frac{\tau L}{RG} = \frac{\tau L}{125G} \qquad \left(\text{where, } R = \frac{D_1}{2} = \frac{250}{2} = 125 \text{ mm}\right)$$

iv. Therefore, $\frac{\text{Angle of twist of hollow shaft}}{\text{Angle of twist of solid shaft}} = \frac{\theta_2}{\theta_1} = \frac{\frac{\tau l}{125G}}{\frac{\tau l}{100G}} = \frac{100}{125} = 0.8$

Que 3.16. In a hollow circular shaft of outer and inner diameters of 20 cm and 10 cm respectively, the shear stress is not to exceed 40 MPa. Find the maximum torque which can be safely transmitted.

AKTU 2016-17, Marks 05

Answer

Given : Outer diameter, D_0 = 20 cm = 200 mm, Inner diameter, D_i = 10 cm = 100 mm, Maximum shear stress, τ = 40 N/mm²
To Find : Maximum torque.

1. Maximum torque transmitted by the hollow shaft,

$$T = \frac{\pi}{16}\tau\left[\frac{D_0^4 - D_i^4}{D_0}\right] = \frac{\pi}{16} \times 40 \times \left[\frac{200^4 - 100^4}{200}\right]$$
$$= 58904862.25 \text{ N-mm} = 58904.86 \text{ N-m}$$

Que 3.17. Determine the dimensions of hollow shaft with a diameter ratio of 3:4, which is to transmit 60 kN at 200 rpm. The maximum shear stress is limited to 70 MN/m² and angle of twist is 3.8° in a length of 4 m. G = 80 GPa.

AKTU 2017-18, Marks 07

Answer

Given : Diameter ratio, $D_i : D_o$ = 3:4, Power, P = 60 kN, N = 200 rpm
τ_{max} = 70 MN/m², θ = 3.8°, l = 4 m, G = 80 GPa
To Find : Dimension of hollow shaft.

1. Power is given by, $P = \frac{2\pi NT}{60} \Rightarrow 60 \times 10^3 = \frac{2\pi \times 200 \times T}{60}$

$$T = 2864.78 \text{ N-m} = 2.86 \times 10^6 \text{ N-mm}$$

2. Diameter of the shaft when shear stress is not to exceed 70 MN/m²,

$$T = \frac{\pi}{16}\tau\left(\frac{D_0^4 - D_i^4}{D_0}\right) \Rightarrow 2864.78 \times 10^3 = \frac{\pi}{16} \times 70 \times \frac{\left[D_0^4 - \left(\frac{3}{4}D_0\right)^4\right]}{D_0}$$

Outer diameter, $D_0 = 67.3$ mm

Inner diameter, $D_i = \frac{3}{4}D_0 = \frac{3}{4} \times 67.3 = 50.47$ mm

4. Diameters of shaft when the twist is not to exceed 3.8°.

We know that, $\frac{T}{J} = \frac{G\theta}{L}$

$$\frac{2864.78 \times 10^3}{\frac{\pi}{32}[D_0^4 - D_i^4]} = \frac{80 \times 10^3 \times 3.8 \times \pi}{4000 \times 180} \Rightarrow D_0^4 - D_i^4 = 21998872.46$$

$$D_0^4 - \left(\frac{3}{4}D_0\right)^4 = 21998872.46$$

$$D_0 = 75.32 \text{ mm and } D_i = \frac{3}{4} \times 75.32 = 56.49 \text{ mm}$$

5. The diameter of the shaft, which would satisfy both the conditions are the greater of the two values.

∴ External Diameter, $D_0 = 75.32$ mm

Internal Diameter, $D_i = 56.49$ mm

Que 3.18. **A solid circular shaft of length of 3 m and diameter of 50 mm rotates at 1200 rpm by a 400 HP electric motor at its middle. It derives two machines of 150 HP and 250 HP at left and right ends of the shaft, respectively. Determine the maximum shear stress and relative displacement of the two ends of the shaft.**

Take $G = 85$ GPa. **AKTU 2018-19, Marks 07**

Answer

Given : Length of shaft, $L = 3$ m Diameter of shaft = 50 mm, speed, $N = 1200$ rpm, $G = 85$ GPa

To Find : Maximum shear stress and relative displacement

1. Power transmitted across BC = Power applied at B = 400 HP

For Part BC : $P = 400 \times 746 = 298.4$ kW

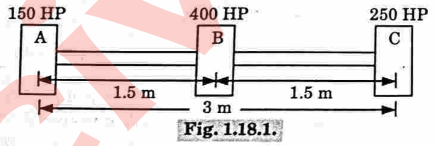


Fig. 1.18.1.

2. Torque transmitted across BC is given by,

$$P = \frac{2\pi NT}{60 \times 1000}$$

$$T = \frac{298.4 \times 60 \times 1000}{2 \times 1200} = 2374.6 \text{ N-m}$$

3. The maximum shear stress in BC is given by,

$$T = \frac{\tau \times \pi}{16} D^3$$

$$2374.6 = \frac{\tau \times \pi}{16} \times 0.05^3 \Rightarrow \tau = 96.75 \text{ MN/m}^2$$

4. The maximum angle of twist θ_1 in BC of B relative to C is given by,

$$\frac{T}{I_p} = \frac{G\theta}{L}$$

$$\theta_1 = \frac{2374.6 \times 1.5}{85 \times 10^9 \times \pi / 32 \times 0.05^4} = 0.0683 \text{ radian}$$

For Part AB : 5 Power transmitted across AB,

$$P = 150 \times 746 = 111.9 \text{ kW}$$

5. Torque transmitted across AB is given by,

$$T = \frac{111.9 \times 60 \times 1000}{2\pi \times 1200} = 890.47 \text{ N-m}$$

6. The maximum shear stress in AB is given by,

$$\tau = \frac{16 \times 890.47}{\pi \times 0.05^3} \times 10^{-6} = 36.28 \text{ MN/m}^2$$

7. The maximum angle of twist θ_2 in AB of A relative to B is given by,

$$\theta_2 = \frac{890.47 \times 1.5}{85 \times 10^9 \times \pi / 32 \times (0.05)^4} = 0.0256 \text{ radian}$$

The shear stress in the shaft is developed in BC being equal to 96.75 MN/m²

ii. **Angle of Twist :**

Angle of twist of A relative to $C = \theta_1 + \theta_2 = 0.0683 + 0.0256 = 0.0939$ radian.

Que 3.19. **A torque of 4 kN-m is applied on a shaft of diameter 60 mm. Calculate the shearing stress at a point just below the surface and at another point which is at distance of 20 mm from the axis. Consider the cylindrical region of radius 15 mm and calculate the torque carried by this cylinder.**

Answer

Given : Torque, $T = 4$ kN-m, Diameter, $d = 60$ mm

To Find : i. The shearing stress at a point just below the surface.
ii. The shearing stress at distance of 20 mm from the axis.
iii. Torque carried by the cylinder.

1. Shear stress at a point just below the surface,

$$\tau_{max} = \frac{Tr}{J} = \frac{4 \times 10^6 \times 30}{\frac{\pi}{32} \times (60)^4} = 94.3140 \text{ N/mm}^2$$

2. Shear stress at a distance 20 mm from the axis of shaft,

From similar ΔOAB and ΔOCD,

$$\frac{94.3140}{30} = \frac{\tau}{20} \Rightarrow \tau = 62.876 \text{ N/mm}^2$$

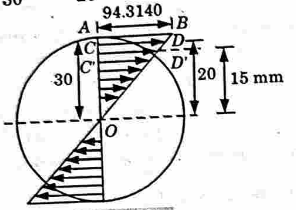


Fig. 3.19.1.

3. Shear stress at a distance 15 mm,

$$\frac{94.3140}{30} = \frac{\tau}{15} \Rightarrow \tau = 47.157 \text{ N/mm}^2$$

4. Now torque acting on the radius of 15 mm,

$$\frac{T}{J} = \frac{\tau}{r} \Rightarrow T = \tau\frac{J}{r} = \tau\frac{(\pi/32)\,d^4}{d/2} = \tau\frac{\pi}{16}d^3$$

$$T = 47.157 \times \frac{\pi}{16} \times (30)^3 = 0.25 \text{ kN-m}$$

Que 3.20. **A solid steel shaft 60 mm diameter is fixed rigidly and coaxially inside a bronze sleeve 90 mm external diameter. Calculate the angle of twist in a length of 2 m of the composite shaft due to action of a torque of 1 kN-m. Take G (steel) = 80 GPa, G (bronze) = 42 GPa.**

AKTU 2014-15, Marks 10

Answer

Given : Diameter of steel shaft, $D_1 = 60$ mm
Diameter of bronze shaft, $D_2 = 90$ mm, Length of shaft, $l = 2$ m
Torque, $T = 1$ kN-m, Modulus of rigidity for steel, $G_1 = 80$ GPa
Modulus of rigidity for bronze, $G_2 = 42$ GPa
To Find : The angle of twist.

1. Polar moment of inertia for steel,

$$J_1 = \frac{\pi}{32} \times d^4 = \frac{\pi}{32} \times (0.06)^4 = 1.27 \times 10^{-6} \text{ m}^4$$

2. Polar moment of inertia for bronze,

$$J_2 = \frac{\pi}{32} \times (D_2^4 - D_1^4) = \frac{\pi}{32} \times (0.09^4 - 0.06^4) = 5.17 \times 10^{-6} \text{ m}^4$$

3. Therefore, shafts are coaxially mounted. So the angle of twist is common.
4. Angle of twist is given by,

$$\theta = \frac{Tl}{JG} = \frac{1 \times 10^3 \times 2}{1.27 \times 10^{-6} \times 80 \times 10^9} = 0.019685 \text{ rad}$$

PART-7
Torsional Rigidity.

Questions-Answers
Long Answer Type and Medium Answer Type Questions

Que 3.21. **What do you understand by torsional rigidity or stiffness of the shaft ?**

Answer

Torsional Rigidity :

1. Torsional rigidity or stiffness of the shaft is defined as the product of modulus of rigidity (G) and polar moment of inertia of the shaft (J). Hence, mathematically, the torsional rigidity is given as,

   Torsional rigidity = $G \times J$

2. Torsional rigidity is also defined as the torque required to produce a twist of one radian per unit length of the shaft.
3. Let, a twisting moment T produces a twist of θ radians in a shaft of length L.

We know that, $\frac{T}{J} = \frac{G\theta}{L} \Rightarrow G \times J = \frac{T \times L}{\theta}$

Torsional rigidity $= \frac{T \times L}{\theta}$ ($\because$ $G \times J$ = Torsional rigidity)

If $L = 1$ metre and $\theta = 1$ radian then

Torsional rigidity = Torque = T

Que 3.22. **A hollow cylindrical steel shaft is 1.5 m long and has inner and outer diameters respectively equal to 40 mm and 60 mm Fig. 3.22.1. (*a*) What is the largest torque that can be applied to the shaft if the shearing stress is not to exceed 120 MPa ? (*b*) What is the corresponding minimum value of the shearing stress in the shaft ?**

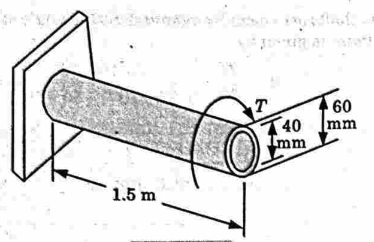


Fig. 3.22.1.

Answer

Given : Length of the shaft, L = 1.5 m, Inner diameter, d_i = 40 mm = 0.04 m, Outer diameter, d_o = 60 mm = 0.06 m, Maximum shear stress, τ_{max} = 120 MPa = 120 × 10^6 N/m^2

To Find : a. The largest torque applied to the shaft.
b. Minimum value of shearing stress in the shaft.

1. For a hollow shaft, the largest torque,

$$T_{max} = \frac{\pi}{16} \times \tau_{max} \times \frac{d_o^4 - d_i^4}{d_o}$$

$$T_{max} = \frac{\pi}{16} \times 120 \times 10^6 \left(\frac{0.06^4 - 0.04^4}{0.06}\right) = 4084.07 \text{ N-m}$$

2. Minimum value of the shearing stress, τ_{min} using torsion equation

$$\frac{T_{max}}{J} = \frac{\tau_{min}}{r_{min}} \Rightarrow \tau_{min} = \frac{T_{max}}{J}\left(\frac{d_i}{2}\right)$$

$$= \frac{4084.07}{\frac{\pi}{32} \times [(0.06)^4 - (0.04)^4]} \times \left(\frac{0.04}{2}\right)$$

$$\tau_{min} = 79.999 \times 10^6 \text{ N/m}^2 \approx 80 \text{ MPa}$$

PART-8

Combined Torsion and Bending of Circular Shafts, Principal Stress and maximum Shear under Combined Loading of Bending and Torsion.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.23. **Determine equivalent bending moment and equivalent torque for the shafts subjected to combined bending and torsion.**

AKTU 2015-16, Marks 10

Answer

1. When a shaft is transmitting torque, it is subjected to shear stresses. At the same time, shaft is also subjected to bending moment due to inertia loads. Due to bending moment, bending stresses are also setup.
2. Now, from torsion equation, $\frac{T}{J} = \frac{\tau}{R} \Rightarrow T = \tau \times \frac{\pi}{16} D^3$
3. From bending equation, $\frac{M}{I} = \frac{\sigma_b}{y}$

$$M = \frac{\sigma_b}{y} \times I = \frac{\sigma_b}{D/2} \times \frac{\pi D^4}{64} = \sigma_b \times \frac{\pi D^3}{32}$$

4. If a certain material loaded in such a way that at a point, bending stress σ_b and shear stress τ are present then principal stress σ_{max} and maximum shear stress τ_{max} are given as,

$$\sigma_{max} = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

5. Multiplying both sides by $\frac{\pi D^3}{32}$, we get

$$\sigma_{max} \times \frac{\pi D^3}{32} = \frac{\sigma_b}{2} \times \frac{\pi D^3}{32} + \sqrt{\left(\frac{\sigma_b}{2} \times \frac{\pi D^3}{32}\right)^2 + \left(\tau \times \frac{\pi D^3}{32}\right)^2}$$

Equivalent bending moment, $M_e = \frac{M}{2} + \sqrt{\left(\frac{M}{2}\right)^2 + \left(\frac{T}{2}\right)^2} = \frac{M + \sqrt{M^2 + T^2}}{2}$

6. Now, $$\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$
7. Multiplying both sides by $\frac{\pi D^3}{16} \Rightarrow \tau_{max} \times \frac{\pi D^3}{16} = \sqrt{\left(\frac{\sigma_b}{2} \times \frac{\pi D^3}{16}\right)^2 + \left(\tau \times \frac{\pi D^3}{16}\right)^2}$

Equivalent torque, $T_e = \sqrt{M^2 + T^2}$

Que 3.24. **How would you calculate the principle stresses in the shaft subjected to combined bending and torsion.**

Answer

1. The principle stresses and maximum shear stress when a shaft is subjected to bending and torsion, are obtained as :

Consider any point on the cross-section of a shaft.

Let, T = Torque at the section.
D = Diameter of the shaft.
M = BM at the section.

2. The torque T will produce shear stress at the point whereas the BM will produce bending stress.
Let, q = Shear stress at the point produced by torque T
σ = Bending stress at the point produced by BM (M)

3. The shear stress at any point due to torque (T) is given by,
$$\frac{q}{r} = \frac{T}{J} \Rightarrow q = \frac{T}{J} \times r \qquad \left(\because \frac{q}{r} = \frac{\tau}{R} = \frac{T}{J}\right)$$

4. The bending stress at any point due to bending moment (M) is given by,
$$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma = \frac{M \times y}{I}$$

5. We know that the angle θ made by the plane of maximum shear with the normal cross-section is given by, $\tan 2\theta = \frac{2\tau}{\sigma}$

6. The bending stress and shear stress is maximum at a point on the surface of the shaft,

i. Maximum bending stress, $\sigma_b = \frac{M}{I} \times \frac{D}{2} = \frac{M}{\frac{\pi}{64}D^4} \times \frac{D}{2} = \frac{32M}{\pi D^3}$

ii. Maximum shear stress, $\tau_c = \frac{T}{J} \times R = \frac{T}{\frac{\pi}{32}D^4} \times \frac{D}{2} = \frac{16T}{\pi D^3}$

7. $\therefore$ $$\tan 2\theta = \frac{2\tau}{\sigma} = \frac{2\tau_c}{\sigma_b} = \frac{2 \times \frac{16T}{\pi D^3}}{\frac{32M}{\pi D^3}} = \frac{T}{M}$$

8. Major Principal stress,
$$= \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_s^2}$$
$$= \frac{32M}{2 \times \pi D^3} + \sqrt{\left(\frac{32M}{2 \times \pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$
$$= \frac{16}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right)$$

9. Similarly, minor principal stress $= \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)$

10. Maximum shear stress $= \frac{\text{Major principal stress} - \text{Minor principal stress}}{2}$

$$= \frac{16}{\pi D^3}\left(\sqrt{M^2 + T^2}\right)$$

11. For a hollow shaft,

i. Major principal stress $= \frac{16D_0}{\pi[D_0^4 - D_i^4]}\left(M + \sqrt{M^2 + T^2}\right)$

ii. Minor principal stress $= \frac{16D_0}{\pi[D_0^4 - D_i^4]}\left(M - \sqrt{M^2 + T^2}\right)$

iii. Maximum shear stress $= \frac{16D_0}{\pi[D_0^4 - D_i^4]}\left(\sqrt{M^2 + T^2}\right)$

Que 3.25. **A solid shaft of diameter 80 mm is subjected to a twisting moment of 8 MN-mm and a bending moment of 5 MN-mm at a point. Determine :**
i. **Principal stresses.**
ii. **Position of the plane on which they act.**

Answer

Given : Diameter of shaft, D = 80 mm, Twisting moment, T = 8 MN-mm
Bending moment, M = 5 MN-mm
To Find : i. Principal stresses, ii. Position of the plane.

1. The major principal stress is given by, $\sigma_{max} = \frac{16}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right)$
$$= \frac{16}{\pi \times 80^3}\left(5 \times 10^6 + \sqrt{(5 \times 10^6)^2 + (8 \times 10^6)^2}\right) = 143.57 \text{ N/mm}^2$$

2. Minor principal stress $= \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)$
$$= \frac{16}{\pi \times 80^3}\left(5 \times 10^6 - \sqrt{(5 \times 10^6)^2 + (8 \times 10^6)^2}\right) = -44.12 \text{ MN/mm}^2$$

3. Position of plane, $\tan 2\theta = \frac{T}{M} = \frac{8 \times 10^6}{5 \times 10^6} = 1.6 \Rightarrow \theta = 29° \text{ or } 119°$

Que 3.26. **Determine the internal and external diameter of a hollow shaft whose internal diameter is 0.6 times external diameter and transmits 120 kW at 210 rpm and the allowable stress is limited to 75 MPa. If bending moment of 2800 N-m is applied to the shaft, find the speed at which the shaft must rotate to transmit the same power for the same value of maximum shear stress.**

AKTU 2017-18, Marks 07

Answer

Given : $D_i = 0.6D_o$, $P = 120$ kW, $N = 210$ rpm, $\tau = 75$ MPa, $M = 2800$ N-m

To Find : i. Internal and external diameter of hollow shaft.
ii. Speed of shaft at applied bending moment.

1. Power, $P = \dfrac{2\pi NT}{60 \times 10^3} \Rightarrow 120 = \dfrac{2\pi \times 210 \times T}{60 \times 10^3}$

$$T = \frac{120 \times 10^3 \times 60}{2 \times \pi \times 210} = 5456.7 \text{ N-m}$$
$$= 5456.7 \times 10^3 \text{ N-mm}$$

Assume, $T_{max} = 1.4\,T$
$$= 1.4 \times 5456.7 \times 10^3 = 7639.4 \times 10^3 \text{ N-mm}$$

2. $$T_{max} = \frac{\pi}{16}\tau\left[\frac{D_0^4 - D_i^4}{D_0}\right]$$

$$7639.4 \times 10^3 = \frac{\pi}{16} \times 75 \times \left[\frac{D_0^4 - (0.6\,D_0)^4}{D_0}\right]$$

$$D_o = 84.17 \text{ mm}$$
$$\therefore \quad D_i = 0.6\,D_o = 0.6 \times 84.17 = 50.5 \text{ mm}$$

3. Maximum shear stress,
$$\tau = \frac{16\,D0}{\pi\,[D_0^4 - D_i^4]}\left(\sqrt{M^2 + T^2}\right)$$

$$75 = \frac{16 \times 84.17}{\pi\,[(84.17)^4 - (50.5)^4]} \times \sqrt{(2800 \times 10^3)^2 + T^2}$$

$$T = 711217 \text{ N-mm} = 7.1122 \text{ kN-m}$$

4. Power, $P = \dfrac{2\,\pi\,NT}{60 \times 10^3}$

$$N = \frac{120 \times 10^3 \times 60}{2\,\pi \times 7112.2} = 161.12 \text{ rpm}$$

PART-9

Shear Stresses, Derivation of Formula.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.27. **Derive the expression for shearing stress at any section on a beam, also show the distribution of shearing stress over a rectangular section.**

AKTU 2016-17, Marks 10

Answer

A. Shearing Stress on Beam :

1. In Fig. 3.27.1, a simply supported beam carrying a uniformly distributed load. Due to UDL, the shear force and bending moment will vary along the length of the beam.
2. Consider, two sections AB and CD of this beam at a distance dx apart.
3. Let, at section AB,
   F = Shear force.
   M = Bending moment.
   At section CD,
   $F + dF$ = Shear force.
   $M + dM$ = Bending moment.
   I = moment of inertia of the section.

   Now, it is required to find shear stress intensity on the section AB at a distance y from neutral axis.
4. Bending stress at distance y from the neutral axis is given by,

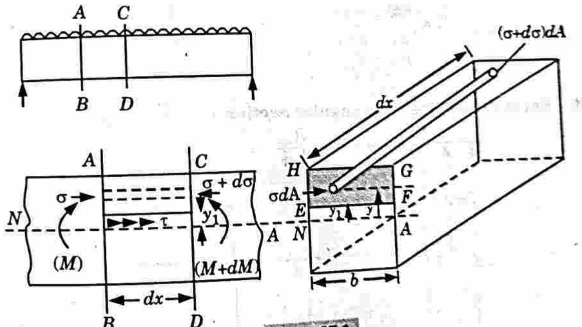


Fig. 3.27.1.

$$\frac{M}{I} = \frac{\sigma}{y}$$

5. Intensity of stress on section AB, $\sigma = \dfrac{M}{I}y$
6. Intensity of stress on section CD, $\sigma + d\sigma = \dfrac{(M + dM)}{I} \times y$
7. Force on the section $AB = \sigma \times dA = \dfrac{M}{I} \times y \times dA$
8. Force on the section $CD = (\sigma + d\sigma) \times dA = \dfrac{(M + dM)}{I} \times y \times dA$

9. Net unbalanced force on the section = Force on the section CD – Force on the section $AB = \frac{(M+dM)}{I} \times y \times dA - \frac{M}{I} \times y \times dA = \frac{dM}{I} \times y \times dA$

10. Total unbalanced force $= \int \frac{dM}{I} \times y \times dA = \frac{dM}{I} \int y \times dA$

Since, $\int y \times dA = \bar{y} \times A$

Where, A = area of the section above EF,

$\bar{y}$ = distance of the CG of the area A from the neutral axis.

$$= \frac{dM}{I} \times A \times \bar{y} \quad ...(3.27.3)$$

This is total unbalanced force or shear resistance at the level EF.

11. At the level EF, shear force is given as

= Shear stress × Shear area

$= \tau \times b \times dx$ ...(3.27.4)

12. Eq. (3.27.3) and eq. (3.27.4) are equal, so

$$\tau \times b \times dx = \frac{dM}{I} \times A \times \bar{y}$$

$$\tau = \frac{dM}{dx} \times \frac{A \times \bar{y}}{b \times I} \quad \left\{\frac{dM}{dx} = F \text{ (shear force)}\right\}$$

$$\tau = \frac{F \times A \times \bar{y}}{b \times I}$$

B. Shear Stress Over Rectangular Section :

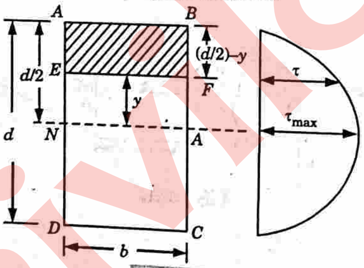


Fig. 3.27.2.

PART-10

Shear Stress Distribution across various Beam Sections like Rectangular, Circular, Triangular, I, T Angle Section.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.28. **Show that for a rectangular section the maximum shear stress is 1.5 times the average stress.**

Answer

1. Let, a rectangular section of width b and depth d is shown in Fig. 3.28.1 and this section is subjected to shear force F. Consider a section EF at distance y from neutral axis.

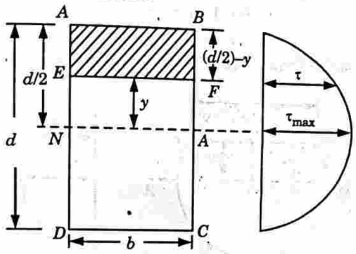


Fig. 3.28.1.

2. Shear stress at layer EF is given by, $\tau = \frac{F \times A \times \bar{y}}{I \times b}$ ...(3.28.1)

Area of section above y, $A = \left(\frac{d}{2} - y\right) \times b$

3. Distance of CG of area A from neutral axis, $\bar{y}$

$$= y + \frac{1}{2}\left(\frac{d}{2} - y\right) = \frac{1}{2}\left(y + \frac{d}{2}\right)$$

4. Putting the value of A and $\bar{y}$ in eq. (3.28.1),

$$\tau = \frac{F \times \left(\frac{d}{2} - y\right) \times b \times \frac{1}{2}\left(y + \frac{d}{2}\right)}{I \times b} = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

At $y = 0$, $$\tau_{max} = \frac{F}{2I}\left(\frac{d^2}{4} - 0\right) = \frac{Fd^2}{8 \times \frac{1}{12} \times bd^3} \quad \left[\because I = \frac{1}{12} \times bd^3\right]$$

$$\tau_{max} = 1.5\frac{F}{bd} \quad ...(3.28.2)$$

5. Average shear stress is given as,

$$\tau_{avg} = \frac{\text{Shear force}}{\text{Area}} = \frac{F}{bd} \quad ...(3.28.3)$$

6. From eq. (3.28.2) and eq. (3.28.3), we get

$$\tau_{max} = 1.5\ \tau_{avg} = (3/2)\ \tau_{avg}$$

Que 3.29. **Prove that the maximum shear stress in a circular section of a beam is 4/3 times the average shear stress.**

Answer

1. Consider, a solid circular section of radius R and a layer EF at a distance y from neutral axis (NA).
2. The shear stress at layer EF is given as,

$$\tau = \frac{F \times A \times \bar{y}}{I \times b} \quad ...(3.29.1)$$

3. Consider a strip of thickness dy at a distance y from NA.

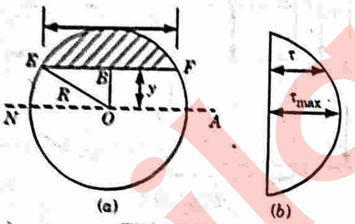


Fig. 3.29.1.

4. Area of this strip, $dA = b \times dy$

$$= EF \times dy = 2 \times EB \times dy = 2\sqrt{R^2 - y^2} \times dy$$

5. Moment of this area dA about NA

$$= y \times dA = y \times 2\sqrt{R^2 - y^2}\ dy = 2y\sqrt{R^2 - y^2}\ dy$$

6. Moment of the whole shaded area about NA

$$A\bar{y} = \int_y^R 2y\sqrt{R^2 - y^2}\,dy$$

7. Put $R^2 - y^2 = x^2, -2y\,dy = 2x\,dx$

$$= \int_{\sqrt{R^2-y^2}}^{0} x(-2x\,dx) = \left[-\frac{2x^3}{3}\right]_{\sqrt{R^2-y^2}}^{0} = \frac{2}{3}\ (R^2 - x^2)^{3/2}$$

8. Now, put the value of $A\bar{y}$ and b in eq. (3.29.1),

$$\tau = \frac{F \times \frac{2}{3}(R^2 - y^2)^{3/2}}{I \times 2\sqrt{R^2 - y^2}} = \frac{F}{3I}(R^2 - y^2)$$

$$\text{At } y = 0,\ \tau_{max} = \frac{F}{3I}(R^2 - 0) = \frac{F \times R^2}{3 \times \frac{\pi}{4}R^4} = \frac{4}{3} \times \frac{F}{\pi R^2} \left[\because I = \frac{\pi}{64}D^4 = \frac{\pi R^4}{4}\right] \quad ...(3.29.2)$$

10. Average shear stress is given as, $\tau_{avg} = \dfrac{\text{Shear force}}{\text{Area}} = \dfrac{F}{\pi R^2}$ ...(3.29.3)

11. From eq. (3.29.2) and eq. (3.29.3), we get, $\tau_{max} = \dfrac{4}{3}\tau_{avg}$

Que 3.30. **A shearing force of 180 kN acts over a *T*-section shown in Fig. 3.30.1. Draw the shear stress distribution curve. (Take $I = 1.134 \times 10^8$ mm⁴).**

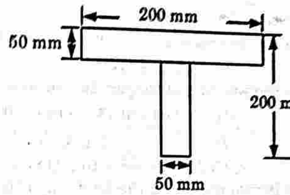


Fig. 3.30.1.

AKTU 2014-15, Marks 10

Answer

Given : Shearing force, $F = 180$ kN
Moment of inertia, $I = 1.134 \times 10^8\ \text{mm}^4 = 1.134 \times 10^{-4}\ \text{m}^4$
To Find : The shear stress distribution curve.

1. Distance of the neutral axis from the top edge,

$$y = \frac{(0.20 \times 0.05) \times 0.025 + (0.15 \times 0.05)\left(\frac{0.15}{2} + 0.05\right)}{(0.2 \times 0.05) + (0.15 \times 0.05)}$$

$$= 0.067857\ \text{m} = 67.85\ \text{mm}$$

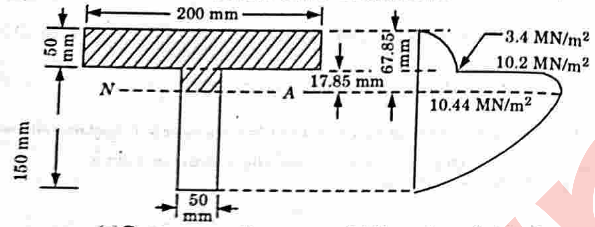
2. We know that the shear stress at the top edge of the flange and bottom of the web is zero.

3. Shear stress at the neutral axis, $\tau_{NA} = \dfrac{FA\bar{y}}{Ib}$

$$A\bar{y}_1 = (0.15 \times 0.05) \times (0.0678 - 0.025) + \left(0.01785 \times 0.05 \times \frac{0.01785}{2}\right)$$

$$= 3.29 \times 10^{-4}\ \text{m}^3$$

$$\tau_{NA} = \frac{180 \times 3.29 \times 10^{-4}}{1.134 \times 10^{-4} \times 0.05}$$
$$= 10444.4 \text{ kN/m}^2 = 10.44 \text{ MN/m}^2$$



(a) Beam cross-section. (b) Shear stress distribution.

Fig. 3.30.2.

4. Shear stress in the web just at the junction of web and flange

$$= \frac{FA\bar{y}}{Ib} = \frac{180 \times (0.15 \times 0.05) \times (0.025 + 0.01785)}{1.134 \times 10^{-4} \times 0.05}$$
$$= 10202.38 \text{ kN/m}^2 = 10.2 \text{ MN/m}^2$$

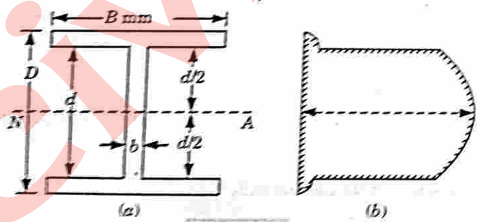
5. Shear stress in the flange just at the junction of the flange and web

$$= \frac{180 \times (0.2 \times 0.05) \times (0.025 + 0.01785)}{1.134 \times 10^{-4} \times 0.2}$$
$$= 3400.79 \text{ kN/m}^2 = 3.4 \text{ MN/m}^2$$

Que 3.31. **How will prove that the shear stress changes abruptly at the junction of the flange and the web of an *I*-section ?**

Answer

Fig. 3.31.1 shows the I-section of a beam.



(a) (b)

Fig. 1.31.1.

Let, B = Overall width of the section.

D = Overall depth of the section.

b = Thickness of the web.

d = Depth of web.

The shear stress at a distance y from the N.A.,

$$\tau = F \times \frac{A\bar{y}}{I \times b}$$

1. **Shear Stress Distribution in the Flange :**

i. Consider, a section at a distance y from NA in the flange as shown in Fig. 3.31.1(a) shaded area of flange, $A = B\left(\frac{D}{2} - y\right)$

ii. Distance of the CG of the shaded area from neutral axis

$$\bar{y} = y + \frac{1}{2}\left(\frac{D}{2} - y\right) = \frac{1}{2}\left(\frac{D}{2} + y\right)$$

iii. Hence shear stress in the flange becomes,

$$\tau = \frac{F \times A\bar{y}}{I \times B} = \frac{F \times B(D/2 - y) \times 1/2(D/2 + y)}{I \times B}$$
$$= \frac{F}{2I}\left(\frac{D^2}{2} - y^2\right)$$

iv. Hence, the variation of shear stress (τ) with respect to y in the flange is parabolic. It is also clear that with the increase of y, shear stress decreases.

a. For the upper edge of the flange, $y = \frac{D}{2}$

Hence, shear stress, $\tau = \frac{F}{2I}\left[\frac{D^2}{4} - \left(\frac{D}{2}\right)^2\right] = 0$

b. For the lower edge of the flange, $y = \frac{d}{2}$

Hence, shear stress, $\tau = \frac{F}{2I}\left[\frac{D^2}{4} - \left(\frac{d}{2}\right)^2\right] = \frac{F}{2I}\left(\frac{D^2}{4} - \frac{d^2}{2}\right) = \frac{F}{8I}(D^2 - d^2)$

2. **Shear Stress Distribution in the Web :**

i. Consider a section at a distance y in the web from the NA as shown in Fig. 3.32.1(a)

$\therefore$ $A\bar{y}$ = Moment of the flange area about NA + Moment of the shaded area of web area about NA

$$A\bar{y} = B\left(\frac{D}{2} - \frac{d}{2}\right) \times \frac{1}{2}\left(\frac{D}{2} + \frac{d}{2}\right) + b\left(\frac{d}{2} - y\right) \times \frac{1}{2}\left(\frac{d}{2} + y\right)$$
$$= \frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$$

ii. The shear stress in the web,

$$\tau = \frac{F \times A\bar{y}}{I \times b} = \frac{F}{I \times b} \times \left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

Hence, it is clear that variation of τ with respect to y is parabolic. Also with the increase of y, τ decreases.

iii. At the neutral axis, $y = 0$

$$\tau_{max} = \frac{F}{I \times b}\left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2} \times \frac{d^2}{4}\right] = \frac{F}{I \times b}\left[\frac{B(D^2 - d^2)}{8} + \frac{bd^2}{8}\right]$$

iv. At the junction of top of the web and bottom of flange, $y = \frac{d}{2}$

Shear stress, $\tau = \frac{F}{I \times b}\left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - \left(\frac{d}{2}\right)^2\right)\right] = \frac{F \times B \times (D^2 - d^2)}{8I \times b}$

Hence, the shear stress distribution for the web and flange is shown in Fig. 3.31(b). The shear stress at the junction of the flange and the web changes abruptly.

Que 3.32. The shear force action on a beam at a section is *F*. The section of the beam is triangular base *b* and of an altitude *h*. The beam is placed with its base horizontal. Find the maximum shear stress and the shear stress at the NA.

Answer

Given : Shear force = F, Base of triangular section = b, Altitude of triangular section = h
To Find : The maximum shear stress and the shear stress at the NA.

1. The NA of the triangle ABC will lie at the CG of the triangle. But the CG of the triangle will be at a distance of $2h/3$ from the top.
   $\therefore$ Neutral axis will be at a distance of $2h/3$ from the top.
2. Consider, a level EF at a distance y from the NA. The shear stress at this level is given by, $\tau = \frac{F \times A\bar{y}}{I \times b}$ ...(3.32.1)

   $A\bar{y}$ = Moment of the shaded area about the neutral axis.
   = Area of triangle CEF × Distance of CG of triangle CEF from NA

$$= \left(\frac{1}{2} \times EF \times x\right) \times \left(\frac{2h}{3} - \frac{2x}{3}\right) = \frac{1}{3} \times \frac{bx^2}{h} \times (h - x)$$

3. From the similar triangle property, we get
   $EF = x \times b/h$

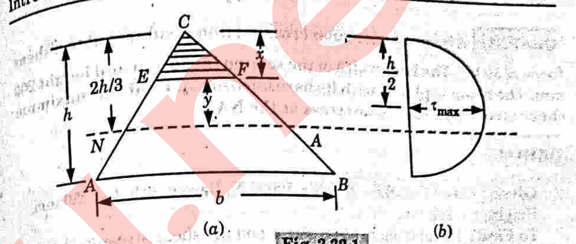


Fig. 3.32.1.

4. Substituting the value of $A\bar{y}$ and EF in eq. (3.32.1), we get

$$\tau = \frac{F \times \frac{1}{3}\frac{bx^2}{h}(h - x)}{I \times \frac{x \times b}{h}} = \frac{F}{3I}(xh - x^2) \quad ...(3.32.2)$$

5. From eq. (3.32.2) it is clear that variation of τ with respect to x is parabolic.
   i. At the top, $x = 0 \Rightarrow \tau = 0$
   ii. At the bottom, $x = h \Rightarrow \tau = 0$
   iii. At the NA., $\left(x = \frac{2h}{3}\right)$, $\tau = \frac{F}{3I}\left[\frac{2h}{3} \times h - \left(\frac{2h}{3}\right)^2\right] = \frac{2}{27} \times \frac{Fh^2}{I}$
   iv. For triangular section, $I = \frac{bh^3}{36}$

$$\therefore \quad \tau = \frac{2}{27} \times \frac{Fh^2}{\left(\frac{bh^3}{36}\right)} = \frac{8}{3}\frac{F}{bh}$$

6. For the maximum shear stress, $\frac{d\tau}{dx} = 0$

$$\frac{d}{dx}\left[\frac{F}{3I}(xh - x^2)\right] = 0$$

$$\frac{F}{3I}(h - 2x) = 0 \Rightarrow h - 2x = 0 \Rightarrow x = \frac{h}{2}$$

7. Now substituting this value of x in eq. (3.33.2), we get

$$\tau_{max} = \frac{F}{3I}\left[\frac{h}{2} \times h - \left(\frac{h}{2}\right)^2\right] = \frac{Fh^2}{12I} = \frac{Fh^2}{12 \times \frac{bh^3}{36}} = \frac{3F}{bh}$$

Hence, the shear stress is not maximum at the NA but it is maximum at a depth of $h/2$ from the top. In all other cases the shear stress was maximum at the NA.

Que 3.33. A beam of triangular cross-section is subjected to a shear force of 50 kN. The base width of the section is 250 mm and height 200 mm. The beam is placed with its base horizontal. Find the maximum shear stress and the shear stress at the NA.

Answer

Given : Shear Force, $F = 50$ kN $= 50000$ N, Base width, $b = 250$ mm, Height, $h = 200$ mm,
To Find : The maximum shear stress and the shear stress at the NA.

Maximum shear stress in triangular section,

$$\tau_{max} = \frac{3F}{bh} = \frac{3 \times 50000}{250 \times 200} = 3 \text{ N/mm}^2$$

Shear stress at NA in triangular section,

$$\therefore \quad \tau = \frac{8F}{3bh} = \frac{8 \times 50000}{3 \times 250 \times 200} = 2.67 \text{ N/mm}^2$$

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. What do you mean by Pure bending ? What assumptions are made in simple bending ?

Ans. Refer Q. 3.1, Unit-3.

Q. 2. What do you mean by section modulus ? Find an expression for section modulus for a rectangular, circular and hollow circular section.

Ans. Refer Q. 3.5, Unit-3.

Q. 3. A solid shaft of 200 mm diameter has the same cross sectional area as the hollow shaft of the same material with inside diameter of 150 mm. Find the ratio of
i. Powers transmitted by both the shafts at the same angular velocity.
ii. Angles of twists in equal length of these shafts, when stressed to same intensity.

Ans. Refer Q. 3.15, Unit-3.

Q. 4. Determine the dimensions of hollow shaft with a diameter ratio of 3:4, which is to transmit 60 kN at 200 rpm. The maximum shear stress is limited to 70 MN/m² and angle of twist is 3.8° in a length of 4 m. $G = 80$ GPa.

Ans. Refer Q. 3.17, Unit-3.

Q. 5. A solid steel shaft 60 mm diameter is fixed rigidly and coaxially inside a bronze sleeve 90 mm external diameter. Calculate the angle of twist in a length of 2 m of the composite shaft due to action of a torque of 1 kN-m. Take G (steel) = 80 GPa, G (bronze) = 42 GPa.

Ans. Refer Q. 3.20, Unit-3.

Q. 6. Determine equivalent bending moment and equivalent torque for the shafts subjected to combined bending and torsion.

Ans. Refer Q. 3.23, Unit-3.

Q. 7. Determine the internal and external diameter of a hollow shaft whose internal diameter is 0.6 times external diameter and transmits 120 kW at 210 rpm and the allowable stress is limited to 75 MPa. If bending moment of 2800 N-m is applied to the shaft, find the speed at which the shaft must rotate to transmit the same power for the same value of maximum shear stress.

Ans. Refer Q. 3.26, Unit-3.

Q. 8. Derive the expression for shearing stress at any section on a beam, also show the distribution of shearing stress over a rectangular section.

Ans. Refer Q. 3.27, Unit-3.

Q. 9. A shearing force of 180 kN acts over a *T*-section shown in Fig. 1. Draw the shear stress distribution curve. (Take $I = 1.134 \times 10^8$ mm⁴).

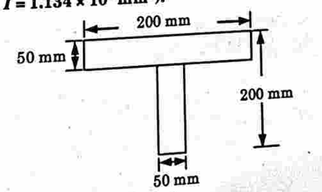


Fig. 1.

Ans. Refer Q. 3.30, Unit-3.

☺☺☺

4 UNIT

Deflection of Beams, Short Columns and Struts

CONTENTS

PART-1

Slope and Deflection, Relationship between Moment, Slope and Deflection.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. **Derive an expression for the slope and deflection of a beam subjected to uniform bending moment.**

Answer

1. A beam AB of length L is subject to uniform bending moment M.
2. As shown in Fig. 4.1.1 beam is subjected to a constant bending moment so it will bend into a circular arc.

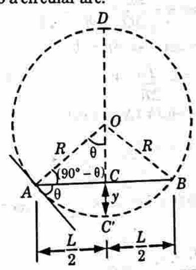


Fig. 4.1.1. Curvature of the beam.

Here,
R = Radius of curvature of the deflected beam.
y = Deflection of beam at the center.
I = Moment of inertia of the beam.
E = Young's modulus for beam material.
θ = Slope of the beam at the end.

A. Value of Deflection :

1. Now, from the geometry of the circle

$$AC \times CB = CC' \times CD$$

$$\frac{L}{2} \times \frac{L}{2} = y\,(2R - y)$$

$$\frac{L^2}{4} = 2yR - y^2$$

2. If deflection y is very small, then y^2 is too small so we neglect it.

$$\Rightarrow \quad \frac{L^2}{4} = 2yR$$

$$y = \frac{L^2}{8R} \quad ...(4.1.1)$$

3. Now from bending equation,

$$\frac{M}{I} = \frac{E}{R} \Rightarrow R = \frac{EI}{M}$$

4. Put the value of R in eq. (4.1.1), we get

$$\text{Deflection of beam, } y = \frac{L^2}{8 \times \frac{EI}{M}} = \frac{ML^2}{8EI}$$

5. This equation gives the value of central deflection of beam.

B. Value of Slope :

1. From triangle AOC,

$$\sin\theta = \frac{AC}{AO} = \frac{L/2}{R} = \frac{L}{2R}$$

2. Since angle θ is very small so $\sin\theta \approx \theta$

$$\theta = \frac{L}{2R} \quad ...(4.1.2)$$

3. Put the value of R in eq. (4.1.2), we get

$$\theta = \frac{L}{2 \times \frac{EI}{M}} = \frac{LM}{2EI}$$

4. This is the value of slope of beam. Due to symmetry, slope at point A and B should be equal.

So, $$\theta_A = \theta_B = \frac{ML}{2EI}$$

Que 4.2. **Derive the relationship between slope, deflection and radius of curvature.**

OR

Derive the differential equation of deflection curve.

AKTU 2018-19, Marks 07

Answer

1. Consider a small portion PQ of a beam, bent into an arc as shown in Fig. 4.2.1.
2. Let, ds = Length of the beam PQ.

R = Radius of the arc, into which the beam has been bent.
C = Centre of the arc.
Ψ = Angle, which the tangent at P makes with O-X axis.
$\Psi + d\Psi$ = Angle which the tangent at Q makes with O-X axis.

3. From the geometry of the figure, we find that

$$\angle PCQ = d\Psi$$

and $$ds = Rd\Psi$$

$$\therefore \quad R = \frac{ds}{d\Psi} = \frac{dx}{d\Psi} \quad \text{(Considering } ds = dx\text{)}$$

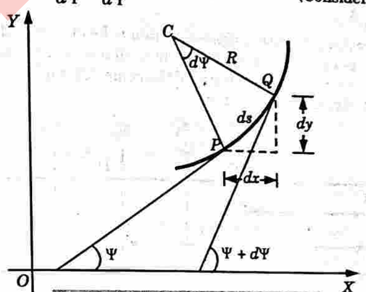


Fig. 4.2.1. Beam bent into an arc.

or $$\frac{1}{R} = \frac{d\Psi}{dx}$$

4. We know that if x and y be the co-ordinates of point P, then

$$\tan\Psi = \frac{dy}{dx}$$

5. Since Ψ is a very small angle, therefore taking $\tan\Psi = \Psi$,

$$\Psi = \frac{dy}{dx} \Rightarrow \frac{d\Psi}{dx} = \frac{d^2y}{dx^2}$$

6. We also know that

$$\frac{M}{I} = \frac{E}{R} \quad \text{or} \quad M = EI \times \frac{1}{R} \quad \left(\because \frac{1}{R} = \frac{d\Psi}{dx}\right)$$

$$\therefore \quad M = EI \times \frac{d^2y}{dx^2}$$

PART-2

Macaulay's Method, Calculating of Slope and Deflection for Determinant Beams.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.3. **Explain the Macaulay's method.**

Answer

1. In Macaulay's method a single equation is formed for all loadings on a beam, the equation is constructed in such a way that the constants of integration apply to all portions of the beam. This method is also called method of singularity functions.

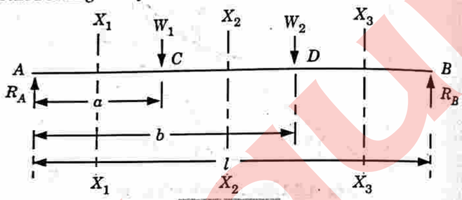


Fig. 4.3.1.

2. Fig. 4.3.1 shows a beam of span l simply supported at A and B and carrying the point loads W_1 and W_2 at distances a and b from the end A.
3. Let R_A and R_B be the reactions at A and B respectively.
4. Consider a section X_1X_1 between A and C at a distance x from A. The bending moment is given by,
$$M_x = R_A \times x \quad ...(4.3.1)$$
This expression (for the bending moment) holds good for all values of x between $x = 0$ and $x = a$.
5. Consider a section X_2X_2 between C and D at a distance x from end A. The bending moment is given by,
$$M_x = R_A \times x - W_1 (x - a) \quad ...(4.3.2)$$
This expression holds good for all values of x between $x = a$ and $x = b$.
6. Consider a section X_3X_3 between D and B at a distance x from A. The bending moment is given by,
$$M_x = R_A x - W_1 (x - a) - W_2 (x - b) \quad ...(4.3.3)$$
This expression holds good for all values of x between $x = b$ and $x = l$.
7. At any section, in general, the bending moment is given by,

$$M_x = EI\frac{d^2y}{dx^2} = R_A \times x \,|\, - W_1(x - a) \,|\, - W_2 (x - b) \quad ...(4.3.4)$$

8. Integrating eq. (4.3.4), we get the general expression for slope as follows :
$$EI\frac{dy}{dx} = R_A \frac{x^2}{2} + C_1 \left| -\frac{W_1(x-a)^2}{2} \right| -\frac{W_2(x-b)^2}{2} \quad ...(4.3.5)$$
9. Integrating eq. (4.3.5), we get the deflection equation,
$$EIy = R_A \times \frac{x^3}{6} + C_1x + C_2 \left| -\frac{W_1(x-a)^3}{6} \right| -\frac{W_2(x-b)^3}{6} \quad ...(4.3.6)$$

Que 4.4. **Find the free end deflection in cantilever beam with uniformly distributed load by Macaulay's method.**

AKTU 2014-15, Marks 05

OR

Derive the deflection for cantilever beam loaded with uniformly distributed load.

AKTU 2015-16, Marks 10

Answer

Given : Cantilever beam with uniformly distributed load.
To Find : The free end deflection of beam.

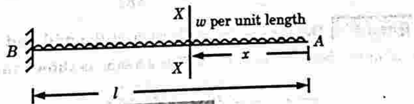


Fig. 4.4.1.

1. Let us consider a beam AB length l carrying uniformly distributed load w per unit length. Take section XX at a distance x from the free end A.
2. Moment at XX section,
$$M_x = -wx\frac{x}{2} = -\frac{wx^2}{2} \quad ...(4.4.1)$$
3. We know that, $M_x = EI\dfrac{d^2y}{dx^2}$ ...(4.4.2)
4. Eq. (4.4.1) and eq. (4.4.2), both are equal.
$$EI\frac{d^2y}{dx^2} = -\frac{wx^2}{2}$$

5. Integrate the equation,

$$EI\frac{dy}{dx} = -\frac{wx^3}{6} + C_1 \quad ...(4.4.3)$$

6. Again integrate $EIy = -\frac{wx^4}{24} + C_1x + C_2$ ...(4.4.4)

7. Boundary conditions,

When $x = l, y = 0, \frac{dy}{dx} = 0$

8. So from eq. (4.4.3), applying boundary condition

$$EI \times 0 = -\frac{wl^3}{6} + C_1$$

$$C_1 = \frac{wl^3}{6}$$

9. Applying boundary condition in eq. (4.4.4) after putting the value of C_1, we get

$$EI \times 0 = -\frac{wl^4}{24} + \frac{wl^3}{6} \times l + C_2$$

$$C_2 = -\frac{wl^4}{8}$$

10. Put the values of C_1 and C_2 in eq. (4.4.4),

$$EIy = -\frac{wx^4}{24} + \frac{wl^3}{6}x - \frac{wl^4}{8}$$

This is deflection equation.

11. Deflection at free end ($x = 0$), $y = -\frac{wl^4}{8EI}$

Que 4.5. **Determine the deflection at the mid and slope at the end of the beam in terms of *EI* for a beam as shown in Fig. 4.5.1.**

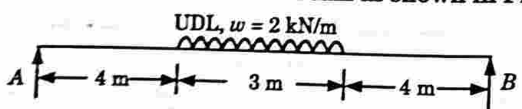


Fig. 4.5.1.

AKTU 2014-15, Marks 10

Answer

Given : Intensity of UDL, w = 2 kN/m, Length of beam, l = 11 m
To Find : The deflection at the mid and slope at the end of the beam.

1. **Support Reaction :**

$$\Sigma F_v = 0; R_A + R_B = 2 \times 3 = 6 \text{ kN}$$

$$\Sigma M_B = 0; R_A \times 11 = 2 \times 3 \times 5.5$$

$$R_A = \frac{33}{11} = 3 \text{ kN}$$

$$R_B = 3 \text{ kN}$$

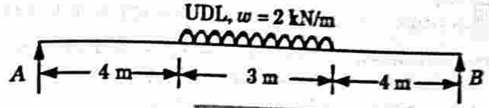


Fig. 4.5.2.

2. Now taking a section X-X at a distance x from end A, the moment at this section,

$$M_x = 3x - \frac{2 \times (x-4)^2}{2} + \frac{2 \times (x-7)^2}{2}$$

3. By Macaulay's method, $EI\frac{d^2y}{dx^2} = 3x \left| -\frac{2(x-4)^2}{2} \right| + \frac{2(x-7)^2}{2}$ ...(4.5.1)

4. On integrating, $EI\frac{dy}{dx} = \frac{3x^2}{2} + C_1 \left| -\frac{2(x-4)^3}{6} \right| + \frac{2(x-7)^3}{6}$ ...(4.5.2)

5. Again integrating it, $EIy = \frac{3x^3}{6} + C_1x + C_2 \left| -\frac{2(x-4)^4}{24} \right| + \frac{2(x-7)^4}{24}$ ...(4.5.3)

6. Boundary conditions, $x = 0, y = 0$ in eq. (4.5.3), we get

$$C_2 = 0$$

7. Now, putting boundary condition, $x = 11, y = 0$ in eq. (4.5.3), we get

$$EI \times 0 = \frac{3 \times 11^3}{6} + C_1 \times 11 + 0 - \frac{2(11-4)^4}{24} + \frac{2 \times (11-7)^4}{24}$$

$$0 = 665.5 + 11\,C_1 - 200.08 + 21.33 \Rightarrow C_1 = -44.25$$

8. Put the value of C_1 and C_2 in eq. (4.5.3), deflection equation becomes

$$EIy = \frac{3x^3}{6} - 44.25x + 0 \left| -\frac{2 \times (x-4)^4}{24} \right| + \frac{2(x-7)^4}{24} \quad ...(4.5.4)$$

9. Deflection at mid-span ($x = 5.5$), put in eq. (4.5.4)

$$EIy = \frac{3 \times 5.5^3}{6} - 44.25 \times 5.5 - \frac{2 \times (5.5-4)^4}{24}$$

$$EIy = -160.61 \Rightarrow y = \frac{-160.61}{EI}$$

10. From eq. (4.5.2), put the value of $x = 11$ at the end point of the beam.

$$EI\frac{dy}{dx} = \frac{3x^2}{2} - 44.25 \left| -\frac{2(x-4)^3}{6} \right| + \frac{2(x-7)^3}{6}$$

$$EI\frac{dy}{dx} = \frac{3 \times 11^2}{2} - 44.25 - \frac{2(11-4)^3}{6} + \frac{2 \times (11-7)^3}{6} = 44.25$$

$\therefore$ Slope, $\theta = \left(\frac{dy}{dx}\right)_{x=11} = \frac{44.25}{EI}$ radian

Que 4.6. **A beam of length 6 m is simply supported at its ends and carries two point loads of 48 kN and 40 kN at a distance of 1 m and 3 m respectively from the left support. Find :**

i. **Deflection under each load.**
ii. **Maximum deflection.**
iii. **The point at which maximum deflection occurs.**
Given E = 200 GPa and $I = 85 \times 10^6$ mm^4.

AKTU 2016-17, Marks 10

Answer

Given : Length of beam, l = 6 m, Point loads, 48 kN and 40 kN
$I = 85 \times 10^6$ mm^4, $E = 2 \times 10^5$ N/mm^2
To Find : i. Deflection under each load, ii. Maximum deflection.
iii. The point at which maximum deflection occurs.

1. First calculate the reactions R_A and R_B.
Taking moments about A, we get

$$R_B \times 6 = 48 \times 1 + 40 \times 3 = 168$$

$$R_B = \frac{168}{6} = 28 \text{ kN}$$

$$R_A = \text{Total load} - R_B = (48 + 40) - 28 = 60 \text{ kN}$$

2. Consider the section X in the last part of the beam (*i.e.*, in length DB) at a distance x from the left support A. The BM at this section is given by,

$$M_x = EI\frac{d^2y}{dx^2} = R_A x \,|- 48(x-1)\,|- 40(x-3)$$
$$= 60x \,|- 48(x-1)\,|- 40(x-3)$$

3. Integrating the above equation, we get

$$EI\frac{dy}{dx} = \frac{60x^2}{2} + C_1 \left|- 48\frac{(x-1)^2}{2}\right| - 40\frac{(x-3)^2}{2}$$
$$= 30x^2 + C_1 \,|- 24(x-1)^2\,|- 20(x-3)^2 \quad \text{...(4.6.1)}$$

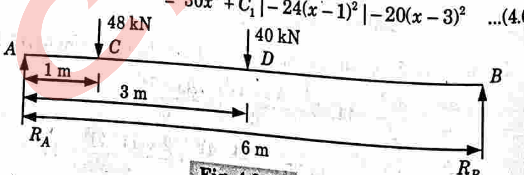


Fig. 4.6.1.

4. Again integrating the above equation, we get

$$EIy = \frac{30x^3}{3} + C_1 x + C_2 \left|\frac{-24(x-1)^3}{3}\right| \frac{-20(x-3)^3}{3}$$
$$= 10x^3 + C_1 x + C_2 \,|- 8(x-1)^3\,| \frac{-20}{3}(x-3)^3 \quad \text{...(4.6.2)}$$

5. To find the values of C_1 and C_2, use boundary conditions. The boundary conditions are :
At $x = 0$, $y = 0$, and at $x = 6$ m, $y = 0$.

i. Substituting the first boundary condition *i.e.*, at $x = 0$, $y = 0$ in eq. (4.6.2) and considering the equation upto first lines (as $x = 0$ lies in the first part of the beam), we get

$$0 = 0 + 0 + C_2 \quad \therefore \quad C_2 = 0$$

ii. Substituting the first boundary condition *i.e.*, at $x = 6$, $y = 0$ in eq. (4.6.2) and considering the complete equation (as $x = 6$ lies in the last part of the beam), we get

$$0 = 10 \times 6^3 + C_1 \times 6 + 0 - 8(6-1)^3 - \frac{20}{3}(6-3)^3 \quad (\because C_2 = 0)$$

$$\therefore \quad C_1 = \frac{-980}{6} = -163.33$$

6. Now substituting the values of C_1 and C_2 in eq. (4.6.2), we get

$$EIy = 10x^3 - 163.33x \,|- 8(x-1)^3\,| - \frac{20}{3}(x-3)^3 \quad \text{...(4.6.3)}$$

A. Deflection under each load :

1. **Deflection under first load *i.e.*, at point C :** This is obtained by substituting $x = 1$ in eq. (4.6.3) upto the first line (as the point C lies in the first part of the beam). Hence, we get

$$EIy_c = 10 \times 1^3 - 163.33 \times 1$$
$$= 10 - 163.33 = -153.33 \text{ kN-m}^3 = -153.33 \times 10^3 \times 10^9 \text{ Nmm}^3$$
$$= -153.33 \times 10^{12} \text{ N-mm}^3$$

$$y_c = \frac{-153.33 \times 10^{12}}{EI} = \frac{-153.33 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6} \text{ mm} = -9.019 \text{ mm.}$$

(Negative sign shows that deflection is downwards).

2. **Deflection under second load *i.e.*, at point D :** This is obtained by substituting $x = 3$ m in eq. (4.6.3) upto the second line (as the point D lies in the second part of the beam). Hence, we get

$$EIy_D = 10 \times 3^3 - 163.33 \times 3 - 8(3-1)^3$$
$$= -283.99 \text{ kN-m}^3 = -283.99 \times 10^{12} \text{ N-mm}^3$$

$$\therefore \quad y_D = \frac{-283.99 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6} = -16.7 \text{ mm.}$$

B. Maximum Deflection :

1. The deflection is likely to be maximum at a section between C and D.
For maximum deflection, $\frac{dy}{dx}$ should be zero. Hence equate the eq. (4.6.1) equal to zero upto the second line.

$$\therefore \quad 30x^2 + C_1 - 24(x-1)^2 = 0$$

PART-3

Moment Area Method, Calculating of Slope and Deflection of Determinate Beam.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.9. Explain moment area method for deflection and slope of determinate beam.

Answer

1. Let us consider a beam AB carrying some type of load. Due to the bending moment diagram of beam is shown in Fig. 4.9.1(b).

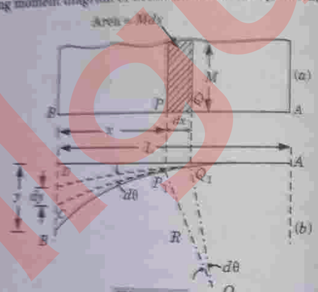


Fig. 4.9.1.

2. Let P and Q be two points on the beam. After deflection these points are P_1 and Q_1.
3. Let normal at these points will meet at point O and distance of point P and Q from free point is x and $x + dx$.
4. So length of the section PQ is dx and angle between the tangent at point P and Q will $d\theta$ which is same as angle between the normal at point P_1 and Q_1. R is radius of curvature.
5. Now $\angle P_1OQ_1$ is $d\theta$.

So
$$d\theta = \frac{P_1Q_1}{R} = \frac{PQ}{R} \qquad (P_1Q_1 = PQ)$$

$$d\theta = \frac{dx}{R} \qquad (PQ = dx)$$

$$\frac{1}{R} = \frac{d\theta}{dx} \qquad ...(4.9.1)$$

6. From bending equation,
$$\frac{M}{I} = \frac{E}{R} \Rightarrow \frac{M}{EI} = \frac{1}{R} \qquad ...(4.9.2)$$
7. Eq. (4.9.1) and eq. (4.9.2) are equal, we get
$$\frac{M}{EI} = \frac{d\theta}{dx} \Rightarrow d\theta = \frac{M}{EI}dx$$
8. Now total change of slope may be calculated by integrating the above equation between limit 0 to L, so
$$\theta = \frac{1}{EI}\int_0^L M\,dx$$
but $\int_0^L M\,dx$ = area of BMD over the entire span = A
$$\therefore \qquad \theta = \frac{A}{EI} \qquad ...(4.9.3)$$
9. Deflection due to bending in portion CD,
$$dy = x\,d\theta = x\frac{M}{EI}dx \qquad \left[d\theta = \frac{M}{EI}dx\right]$$
$$dy = \frac{Mx\,dx}{EI}$$
10. Total deflection is the integration of above equation between limit 0 to L,
$$y = \int_0^L \frac{Mx\,dx}{EI} = \frac{1}{EI}\int_0^L Mx\,dx$$
$\int_0^L Mx\,dx$ = moment of area of BMD over the entire span = $A\bar{x}$

($\bar{x}$ = centroid of BMD)

So,
$$y = \frac{A\bar{x}}{EI} \qquad ...(4.9.4)$$
11. From eq. (4.9.3) and eq. (4.9.4), we can find the slope and deflection for a beam subjected to the loading.

Que 4.10. A simply supported beam of span L is carrying a uniformly distributed load of w per unit length over the entire span. Find the maximum slope and deflection of the beam.

AKTU 2016-17, Marks 10

Answer

Given : Length of beam = L, Intensity of load = w per unit length.
To Find : Maximum slope and deflection.

or $30x^2 - 163.33 - 24(x^2 + 1 - 2x) = 0$ $(\because \; C_1 = -163.33)$

or $6x^2 + 48x - 187.33 = 0$

2. The above equation is a quadratic equation. Hence its solution is

$$x = \frac{-48 \pm \sqrt{48^2 + 4 \times 6 \times 187.33}}{2 \times 6} = 2.87 \text{ m.}$$

(Neglecting –ve root)

Now substituting $x = 2.87$ m in eq. (4.6.3) upto the second line, we get maximum deflection as

$$EIy_{max} = 10 \times 2.87^3 - 163.33 \times 2.87 - 8(2.87 - 1)^3$$
$$= -284.67 \text{ kNm}^3 = -284.67 \times 10^{12} \text{ Nmm}^3$$

$$y_{max} = \frac{-284.67 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6} = -16.745 \text{ mm.}$$

C. Point of Maximum Deflection : $x = 2.87$ m

Que 4.7. **A beam of uniform section, 10 m long, is simply supported at its ends. It carries point loads of 150 kN and 65 kN at a distance 2.6 m and 5.5 m respectively from the left end. Calculate :**

i. The maximum deflection, and

ii. Deflection under each load.

AKTU 2017-18, Marks 07

Answer

Procedure : Same as Q. 4.6, Page 4–9A, Unit-4.

(**Ans.** (i) Maximum deflection = $\frac{-3565.645}{EI}$ unit at distance 4.72 m from lift end A, (ii) Deflection under each load, $y_1 = -2776.03/EI$ unit and $y_2 = \frac{-3462.19}{EI}$ unit)

Que 4.8. **Determine the deflection of the beam at midpoint for the beam loading system shown in the figure given below Take : $E = 200$ GN/m² and $I = 83 \times 10^6$ mm⁴.**

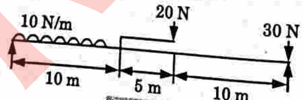


Fig. 4.8.1.

AKTU 2018-19, Marks 07

Answer

Given : $E = 200 \text{ GN/m}^2, I = 83 \times 10^{-6} \text{ m}^4$

To Find : Deflection of midpoint.

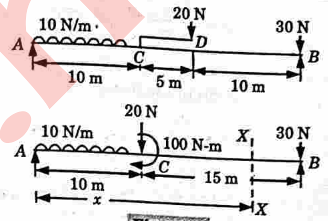


Fig. 4.8.2.

1. Calculate support reaction,

$$R_A + R_B = 10 \times 10 + 20 + 30 = 150 \text{ N}$$

2. Total moment about A, $\Sigma M_A = 0$

$$R_B \times 25 = 20 \times 10 + 10 \times 10 \times 5 + 100 + 30 \times 25$$
$$R_B = 62 \text{ N and } R_A = 150 - 62 = 88 \text{ N}$$

3. Use Macaulay's method,

$$EI\frac{d^2y}{dx^2} = 88x - \frac{10x^2}{2}\Bigg| + \frac{10}{2}(x-10)^2 - 20(x-10) + 100 \quad ...(4.8.1)$$

4. Integrate the eq.(4.8.1), wet get

$$EI\frac{dy}{dx} = 88\frac{x^2}{2} - \frac{5}{3}x^3 + C_1\Bigg| + \frac{5(x-10)^3}{3} - \frac{20(x-10)^2}{2} + 100x \quad ...(4.8.2)$$

5. Again integrate the eq. (4.8.2),

$$EIy = 44\frac{x^3}{3} - \frac{5x^4}{12} + C_1x + C_2\Bigg| + \frac{5(x-10)^4}{12} - \frac{10(x-10)^3}{3} + 50x^2 \quad ...(4.8.3)$$

6. By boundary condition, $x = 0, y = 0, C_2 = 0$

And $x = 25, y = 0$, put in eq. (4.8.3), we get

$$0 = 44\frac{(25)^3}{3} - \frac{5}{12}(25)^4 + C_1 \times 25 + 0\Bigg| + \frac{5(15)^4}{12} - \frac{10(15)^3}{3} + 50(25)^2$$

$$C_1 = -4300$$

7. Putting the value of C_1 and C_2 in eq. (4.8.3)

$$EIy = \frac{44x^3}{3} - \frac{5x^4}{12} - 4300x\Bigg| + \frac{5(x-10)^4}{12} - \frac{10(x-10)^3}{3} + 50x^2 \quad ...(4.8.4)$$

8. Deflection at mid point (Put $x = 25/2 = 12.5$ in eq. (4.8.4)),

$$y = -\frac{27500}{200 \times 10^9 \times 83 \times 10^{-6}} = -1.657 \times 10^{-3} \text{ m} = -1.657 \text{ mm}$$

Here, (–ve) sign shows the downward deflection.

PART-3

Moment Area Method, Calculating of Slope and Deflection of Determinate Beam.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.9. **Explain moment area method for deflection and slope of determinate beam.**

Answer

1. Let us consider a beam AB carrying some type of load. Due to load bending moment diagram of beam is shown in Fig. 4.9.1(b).

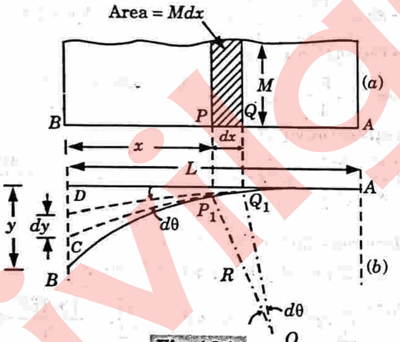


Fig. 4.9.1.

2. Let P and Q be two points on the beam. After deflection these points are P_1 and Q_1.
3. Let normal at these points will meet at point O and distance of point P and Q from free point is x and $x + dx$.
4. So length of the section PQ is dx and angle between the tangent at point P and Q will $d\theta$ which is same as angle between the normal at point P_1 and Q_1. R is radius of curvature.
5. Now $\angle P_1OQ_1$ is $d\theta$,

So $$d\theta = \frac{P_1Q_1}{R} = \frac{PQ}{R} \qquad (P_1Q_1 = PQ)$$

$$d\theta = \frac{dx}{R}$$

$$\frac{1}{R} = \frac{d\theta}{dx} \qquad (PQ = dx) \qquad ...(4.9.1)$$

6. From bending equation,

$$\frac{M}{I} = \frac{E}{R} \Rightarrow \frac{M}{EI} = \frac{1}{R} \qquad ...(4.9.2)$$

7. Eq. (4.9.1) and eq. (4.9.2) are equal, we get

$$\frac{M}{EI} = \frac{d\theta}{dx} \Rightarrow d\theta = \frac{M}{EI}dx$$

8. Now total change of slope may be calculated by integrating the above equation between limit 0 to L, so

$$\theta = \frac{1}{EI}\int_0^L M\,dx$$

but $\int_0^L M\,dx$ = area of BMD over the entire span = A

$$\therefore \qquad \theta = \frac{A}{EI} \qquad ...(4.9.3)$$

9. Deflection due to bending in portion CD,

$$dy = x d\theta = x\frac{M}{EI}dx \qquad \left[d\theta = \frac{M}{EI}dx\right]$$

$$dy = \frac{Mxdx}{EI}$$

10. Total deflection is the integration of above equation between limit 0 to L,

$$y = \int_0^L \frac{Mxdx}{EI} = \frac{1}{EI}\int_0^L Mxdx$$

$\int_0^L Mxdx$ = moment of area of BMD over the entire span = $A\bar{x}$

($\bar{x}$ = centroid of BMD)

So, $$y = \frac{A\bar{x}}{EI} \qquad ...(4.9.4)$$

11. From eq. (4.9.3) and eq. (4.9.4), we can find the slope and deflection for a beam subjected to the loading.

Que 4.10. **A simply supported beam of span L is carrying a uniformly distributed load of w per unit length over the entire span. Find the maximum slope and deflection of the beam.**

AKTU 2016-17, Marks 10

Answer

Given : Length of beam = L, Intensity of load = w per unit length.
To Find : Maximum slope and deflection.

1. Due to symmetry, each vertical reaction, $R_A = R_B = \frac{w \times L}{2}$.

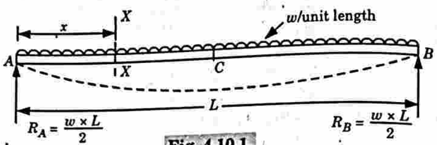


Fig. 4.10.1.

2. Consider a section $X-X$ at a distance x from A. The bending moment at this section is given by,
3. Draw the moment diagram of Fig. 4.10.1.

$$M_{AB} = \frac{wL}{2}x - \frac{wx^2}{2}$$

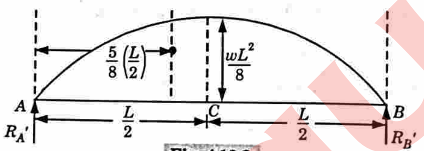


Fig. 4.10.2.

4. Slope at point A and B, $\theta_A = \theta_B =$ (Area of BMD) $\frac{1}{EI}$

$$= \frac{1}{EI}\left(\frac{2}{3}\frac{wL^2}{8} \times \frac{L}{2}\right) = \frac{wL^3}{24\,EI}$$

5. Deflection at midspan, $y_c = \frac{A\bar{x}}{EI} = \frac{wL^3}{24\,EI}\left(\frac{5}{8}\frac{L}{2}\right) = \frac{5\,wL^4}{384\,EI}$

Que 4.11. **Determine the rotation and deflection at the free end of the cantilever beam subjected to uniformly distributed load over in an entire span as shown in Fig. 4.11.1.**

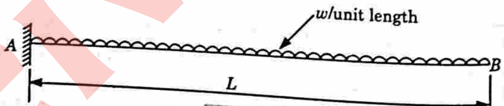


Fig. 4.11.1.

Answer

Given : Intensity of UDL = w, Unit length, length of beam = L
To Find : Slope and deflection at free end.

1. The bending moment diagram is shown in Fig. 4.11.1. At any distance x from free end bending moment is $Awx^2/2$.

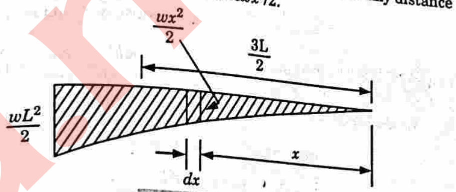


Fig. 4.11.2. BMD.

2. Slope at B, $\theta_B =$ (Area of BMD) $\frac{1}{EI} = \frac{1}{EI}\left(\frac{1}{3}\frac{wL^2}{2} \times L\right) = \frac{wL^3}{6EI}$

3. Deflection at point B, $y_B = \frac{A\bar{x}}{EI} = \frac{wL^3}{6\,EI}\left(\frac{3}{4}L\right) = \frac{wL^4}{8\,EI}$

Que 4.12. **Find the rotation and deflection at the free end in the cantilever beam shown in Fig. 4.12.1**

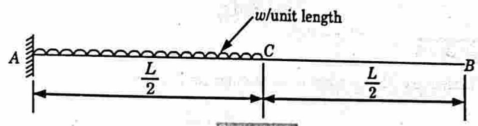


Fig. 4.12.1.

Answer

Given : Length of beam = L, Length of UDL = $L/2$, Intensity of UDL = w.
To Find : Slope and deflected of beam.

1. The bending moment diagram is a parabola as shown in Fig. 4.12.2 with maximum ordinate as $\frac{wL}{4} \times \frac{L}{4} = \frac{wL^2}{8}$.

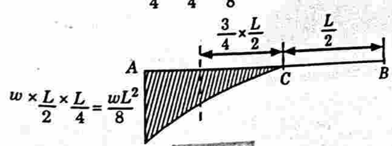


Fig. 4.12.2.

2. From the moment area theorem,

$$\theta_B = \text{Area of } \left(\frac{M}{EI}\right) \text{ diagram between } A \text{ and } B$$

$$= -\left(\frac{1}{3}\frac{wL^2}{8} \times \frac{L}{2}\right) \times \frac{1}{EI} = -\frac{wL^3}{48EI} = \frac{wL^3}{48EI} \text{ (clockwise)}$$

3. Vertical deflection at B

$$= \text{Moment of } \left(\frac{M}{EI}\right) \text{ diagram about } B$$

$$= \left[-\frac{wL^3}{48EI}\right] \times \left[\frac{3}{4}\left(\frac{L}{2}\right) + \frac{L}{2}\right] = -\frac{7wL^4}{384EI} = \frac{7wL^4}{384EI}, \text{ downward}$$

Que 4.13. **Determine the rotation at supports and deflection at mid-span and under the loads in the simply supported beam as shown in Fig. 4.13.1.**

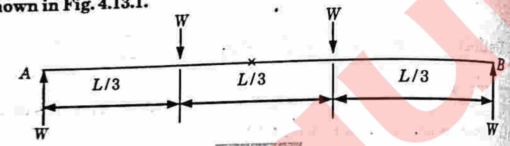


Fig. 4.13.1.

Answer

1. In this case, the bending moment diagram is shown in Fig. 4.13.2.

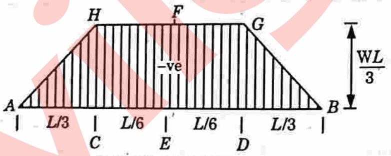


Fig. 4.13.2. BMD.

2. Slope at support A, θ_A = Area of $\left(\frac{M}{EI}\right)$ diagram between A and E.

$$= \frac{1}{EI}\left[\frac{1}{2} \times \frac{L}{3} \times \frac{WL}{3} + \frac{L}{6} \times \frac{WL}{3}\right] = \frac{WL^2}{9EI}$$

3. Deflection under the load at C and D,

$$\Delta_C = \Delta_D = \frac{1}{2} \times \left(\frac{WL}{3}\right) \times \left(\frac{L}{3}\right)\left(\frac{2}{3} \times \frac{L}{3}\right) = \frac{WL^3}{81\,EI}$$

4. Deflection at mid point, $\Delta_E = \frac{1}{EI}$ (moment of BDG + moment of GDEF)

$$= \frac{1}{EI}\left[\frac{1}{2} \times \left(\frac{wL}{3}\right) \times \left(\frac{L}{3}\right) \times \frac{2}{3}\left(\frac{L}{3}\right) + \frac{wL}{3} \times \frac{L}{6} \times \left(\frac{L}{12} + \frac{L}{3}\right)\right]$$

$$= \frac{1}{EI}\left[\frac{wL^3}{81} + \frac{5\,wL^3}{216}\right] = \frac{23\,wL^3}{648\,EI}$$

PART-4

Short Column and Struts : Buckling and Stability, Slenderness Ratio.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.14. **Write short note on**

i. **Column.**
ii. **Strut.**
iii. **Slenderness ratio.**
iv. **Buckling Factor.**

Answer

1. **Column :** A column is a long vertical slender bar or vertical member, subjected to an axial compressive load and fixed rigidly at both circle.
2. **Strut :** It is a slender bar or member in any position other than vertical, subjected to a compressive load and fixed rigidly or hinged or pin joined at one or both the ends.
3. **Slenderness Ratio :** It is the ratio of unsupported length of the column to the minimum radius of gyration of the cross-sectional ends of the column. It has no unit whatsoever.
4. **Buckling Factor :** It is the ratio between the equivalent length of the column to the minimum radius of the gyration.

PART-5

Combined Bending and Direct Stress.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.15. **Derive the expression for resultant stress when a column of rectangular section is subjected to an eccentric load.**

Answer

1. A column of rectangular section subjected to an eccentric load as shown in Fig. 4.15.1. Let the load is eccentric with respect to the axis Y-Y.
2. Calculating direct stress as well as bending stress caused due to eccentric load.

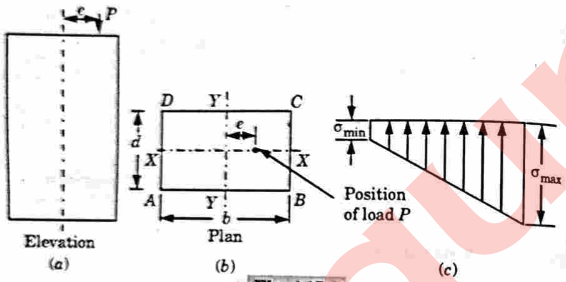


Fig. 4.15.1.

3. Let, P = Eccentric load on column, e = Eccentricity of the load
   σ_0 = Direct stress, σ_b = Bending stress, b = Width of column
   d = Depth of column
4. Area of column section, $A = b \times d$
5. Now moment due to eccentric load P is given by,
   $$M = \text{Load} \times \text{eccentricity} = P \times e$$
6. The direct stress (σ_0) is given by,
   $$\sigma_0 = \frac{\text{Load}}{\text{Area}} = \frac{P}{A} \qquad ...(4.15.1)$$
   This stress is uniform along the cross-section of the column.
7. The bending stress σ_b due to moment at any point of the column section at a distance y from the neutral axis Y-Y is given by,
   $$\frac{M}{I} = \frac{\sigma_b}{\pm y} \Rightarrow \sigma_b = \pm\frac{M}{I} \times y \qquad ...(4.15.2)$$
   Where, I = Moment of inertia of the column section about the neutral axis Y-Y = $\dfrac{db^3}{12}$.
8. Substituting the value of I in eq. (4.15.2), we get
   $$\sigma_b = \pm\frac{M}{\frac{db^3}{12}} \times y = \pm\frac{12M}{db^3} \times y \qquad ...(4.15.3)$$

The bending stress depends upon the value of y from the axis Y-Y.

8. The bending stress at the extreme is obtained by substituting $y = \dfrac{b}{2}$ in the eq. (4.15.3)
   $$\therefore \quad \sigma_b = \pm\frac{12M}{db^3} \times \frac{b}{2} = \pm\frac{6M}{db^2}$$
   $$= \pm\frac{6P \times e}{db^2} = \pm\frac{6P \times e}{dbb} = \pm\frac{6P \times e}{A \times b} \quad (\because M = P \times e)$$
9. The resultant stress at any point will be the algebraic sum of direct stress and bending stress.
10. If y is taken positive on the same side of Y-Y as the load, then bending stress will be of the same type as the direct stress. Here direct stress is compressive and hence bending stress will also be compressive towards the right of the axis Y-Y. Similarly bending stress will be tensile towards the left of the axis Y-Y.
11. Taking compressive stress as positive and tensile stress as negative we can find the maximum and minimum stress at the extremities of the section.
12. The stress will be maximum along layer BC and minimum along layer AD.
13. Maximum stress, σ_{max} = Direct stress + Bending stress = $\sigma_0 + \sigma_b$
   $$= \frac{P}{A} + \frac{6Pe}{Ab} \qquad \text{(Here bending stress is +ve)}$$
   $$= \frac{P}{A}\left(1 + \frac{6 \times e}{b}\right) \qquad ...(4.15.4)$$
   Minimum stress σ_{min} = Direct stress − Bending stress
   $$= \sigma_0 - \sigma_b$$
   $$= \frac{P}{A} - \frac{6Pe}{Ab} = \frac{P}{A}\left(1 - \frac{6 \times e}{b}\right) \qquad ...(4.15.5)$$
14. These stresses are shown in Fig. 4.15.1(c). The resultant stress along the width of the column will vary by a straight line law.
15. If in eq. (4.15.4), σ_{min} is negative then the stress along the layer AD will be tensile.
16. If σ_{min} is zero then there will be no tensile stress along the width of the column. If σ_{min} is positive then there will be only compressive stress along the width of the column.

Que 4.16. **Derive the expression for resultant stress when a column of rectangular section is subjected to a load which is eccentric to both axes.**

Answer

1. A column of rectangular section $ABCD$, subjected to a load which is eccentric to both axes, is shown in Fig. 4.16.1.

2. Let, P = Eccentric load on column.
e_x = Eccentricity of load about X-X axis.
e_y = Eccentricity of load about Y-Y axis, b = Width of column.
d = Depth of column, σ_0 = Direct stress.
σ_{bx} = Bending stress due to eccentricity e_x.
σ_{by} = Bending stress due to eccentricity e_y.
M_x = Moment of load about X-X axis = $P \times e_x$
M_y = Moment of load about Y-Y axis = $P \times e_y$

$$I_{xx} = \text{Moment of inertia about } X\text{-}X \text{ axis} = \frac{bd^3}{12}$$

$$I_{yy} = \text{Moment of inertia about } Y\text{-}Y \text{ axis} = \frac{db^3}{12}$$

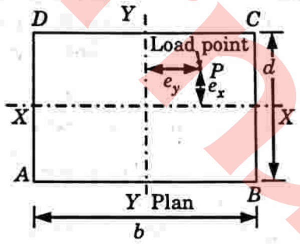


Fig. 4.16.1.

3. Now the eccentric load is equivalent to a central load P, together with a bending moment $P \times e_y$ about Y-Y and a bending moment $P \times e_x$ about X-X.

i. The direct stress (σ_0) is given by,

$$\sigma_0 = \frac{P}{A} \quad ...(4.16.1)$$

ii. The bending stress due to eccentricity e_y is given by,

$$\sigma_{by} = \frac{M_y \times x}{I_{yy}} = \frac{P \times e_y \times x}{I_{yy}} \quad (\because\ M_y = P \times e_y) \quad ...(4.16.2)$$

In the eq. (4.16.2), x varies from $-\frac{b}{2}$ to $+\frac{b}{2}$

iii. The bending stress due to eccentricity e_x is given by,

$$\sigma_{bx} = \frac{M_x \times y}{I_{xx}} = \frac{P \times e_x \times y}{I_{xx}} \quad ...(4.16.3)$$

In the eq. (4.16.3), y varies from $-\frac{d}{2}$ to $+\frac{d}{2}$

4. The resultant stress at any point on the section

$$= \sigma_0 \pm \sigma_{by} \pm \sigma_{bx}$$

$$= \frac{P}{A} \pm \frac{M_y \times x}{I_{yy}} \pm \frac{M_x \times y}{I_{xx}} \quad ...(4.16.4)$$

i. At the point C, the co-ordinates x and y are positive hence the resultant stress will be maximum.

ii. At the point A, the co-ordinates x and y are negative and hence the resultant stress will be minimum.

iii. At the point B, x is +ve and y is –ve and hence resultant stress

$$= \frac{P}{A} + \frac{M_y\, x}{I_{yy}} - \frac{M_x\, y}{I_{xx}}$$

iv. At the point D, x is –ve and y is +ve and hence resultant stress

$$= \frac{P}{A} - \frac{M_y\, x}{I_{yy}} + \frac{M_x\, y}{I_{xx}}$$

Que 4.17. **A short column of rectangular cross section 200 mm by 150 mm carries a load of 400 kN at a point 50 mm from longer side and 87.5 mm from the shorter side. What are the maximum compressive and tensile stresses ?**

Answer

Given : Length b = 200 mm, Width d = 150 mm, Load P = 400 kN
To Find : Maximum compressive and tensile stresses.

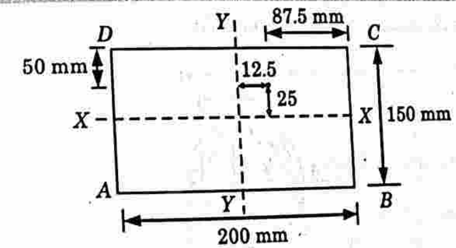


Fig. 4.17.1.

1. Moment of Inertia about axis XX,

$$I_{xx} = \frac{200 \times (150)^3}{12} = 56.25 \times 10^6\ \text{mm}^4$$

2. Moment of Inertia about axis YY,

$$I_{yy} = \frac{150 \times (200)^3}{12} = 100 \times 10^6\ \text{mm}^4$$

3. Direct stress, $\sigma_0 = P/A = \frac{400 \times 10^3}{200 \times 150} = 13.333$ MPa (Compressive)

4. Bending stress at A and C, $\sigma_b = \pm \frac{M_y \times x}{I_y} \pm \frac{M_x \times y}{I_x}$

$$\sigma_b = \pm \frac{400 \times 10^3 \times 12.5 \times 100}{100 \times 10^6} \pm \frac{400 \times 10^3 \times 25 \times 75}{56.25 \times 10^6}$$

i. At C, $\sigma_t = 5 + 13.333 = 18.333$ MPa (Compressive)
ii. At A, $\sigma_t = -5 - 13.333 = -18.333$ MPa (Tensile)
It is compressive at C and tensile at A.

5. Maximum compressive stress is at C

$$= \frac{P}{A} + \frac{M_y \times x}{I_{yy}} + \frac{M_x \times y}{I_{xx}} = 13.333 + 18.333 = 31.666 \text{ MPa}$$

6. Maximum tensile stress is at A

$$= \frac{P}{A} - \frac{M_y \times x}{I_{yy}} - \frac{M_x \times y}{I_{xx}} = 13.333 - 18.333 = -5 \text{ MPa}$$

Que 4.18. **A short column is of hollow circular section, the center of the inside hole being 6 mm eccentric to that of the outside. The outside diameter is 96 mm and the inside 48 mm. The line of action of the load intersects the cross-section at a point in line with the two centers. What are the limiting positions of the load for there to be no tensile stress set up ?**

AKTU 2018-19, Marks 07

Answer

Given : $D = 96$ mm, $d = 48$ mm
To Find : Limiting position of the load.

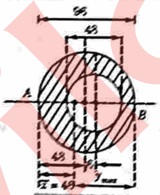


Fig. 4.18.1.

1. Area of hollow section,

$$A = \frac{\pi}{4}(96)^2 - \frac{\pi}{4}(48)^2 = 5428.67 \text{ mm}^2$$

2. CG of hollow section from end A

$$\bar{x} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2} = \frac{\left(\frac{\pi}{4} \times 96^2\right) \times 48 - \left(\frac{\pi}{4} \times 48^2\right) \times 54}{\left(\frac{\pi}{4} \times 96^2\right) - \left(\frac{\pi}{4} \times 48^2\right)} = 46 \text{ mm}$$

3. Moment of inertia of section, $I = I_1 - I_2 = (I_{G_1} + A_1h_1^2) - (I_{G_2} + A_2h_2^2)$

$$= \left(\frac{\pi}{64} \times 96^4 + \frac{\pi}{4} \times 96^2 \times 2^2\right) - \left(\frac{\pi}{64} \times 48^4 + \frac{\pi}{4} \times 48^2 \times 8^2\right)$$

$$= \frac{\pi}{64}(96^4 - 48^4) + \frac{\pi}{4}(96^2 \times 2^2 - 48^2 \times 8^2)$$

$$= 3908643.916 + (-86858.7537) = 3821785.16 \text{ mm}^4$$

4. Section modulus of hollow section, $Z = I/y_{max} = \frac{3821785.16}{50}$

$(\because \; y_{max} = 2 + 48 = 50 \text{ mm})$

$$= 76435.70 \text{ mm}^3$$

5. For not tensile stress, $e \le \frac{Z}{A} \Rightarrow e \le \frac{76435.70}{5428.67} \Rightarrow e \le 14.08$ mm

6. For no tensile stress, place the load within the distance of 14.08 mm either side of the CG of the hollow section.

PART-6

Middle Third and Middle Quarter Rules.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.19. **Derive middle third rule for rectangular sections.**

Answer

1. Consider a rectangular section of width 'b' and depth 'd' as shown in Fig. 4.19.1.
2. Let this section is subjected to a load which is eccentric to the axis Y-Y.
3. Let P = Eccentric load acting on the column,
e = Eccentricity of the load, and A = Area of the section.

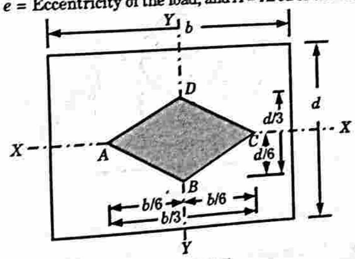


Fig. 4.19.1.

Then from equation, we have the minimum stress as

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6 \times e}{b}\right)$$

4. If σ_{min} is –ve, then stress will be tensile, but if σ_{min} is zero (or positive) then there will be no tensile stress along the width of column. Hence for no tensile stress along the width of column,

$$\sigma_{min} \geq 0$$

$$\frac{P}{A}\left(1 - \frac{6 \times e}{b}\right) \geq 0 \quad \Rightarrow \quad \left(1 - \frac{6 \times e}{b}\right) \geq 0$$

$$1 \geq \frac{6 \times e}{b} \quad \Rightarrow \quad e \leq \frac{b}{6}$$

5. The above result shows that the eccentricity '*e*' must be less than or equal to $\frac{b}{6}$.

6. Hence the greatest eccentricity of the load is $\frac{b}{6}$ from the axis *Y-Y*, the stresses are wholly compressive and the range within which the load can be applied so as not to produce any tensile stress is within the middle third of the base.

Que 4.20. **Explain middle quarter rule for circular sections.**

Answer

1. Consider a circular section of diameter '*d*' as shown in Fig. 4.20.1.
2. Let this section is subjected to a load which is eccentric to the axis *Y-Y*.

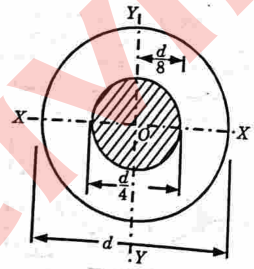


Fig. 4.20.1.

3. Let P = Eccentric load, e = Eccentricity of the load

A = Area of the section = $\frac{\pi}{4} d^2$

4. Now direct stress, $\sigma_0 = \frac{P}{A} = \frac{P}{\frac{\pi}{4} d^2} = \frac{4P}{\pi d^2}$

And moment, $M = P \times e$

5. Bending stress is (σ_b) given by,

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M \times y}{I}$$

6. Maximum bending stress will be when

$$y = \pm \frac{d}{2}$$

∴ Maximum bending stress is given by,

$$\sigma_b = \frac{M}{I} \times \left(\pm \frac{d}{2}\right) = \pm \frac{P \times e \times \frac{d}{2}}{\frac{\pi}{64} d^4} = \pm \frac{32Pe}{\pi d^3}$$

7. Now minimum stress is given by,

$$\sigma_{min} = \sigma_o - \sigma_b = \frac{4P}{\pi d^2} - \frac{32P \times e}{\pi d^3}$$

8. For no tensile stress, $\sigma_{min} \geq 0$

$$\frac{4P}{\pi d^2} - \frac{32P \times e}{\pi d^3} \geq 0 \quad \Rightarrow \quad \frac{4P}{\pi d^2}\left(1 - \frac{8d}{d}\right) \geq 0$$

$$1 - \frac{8 \times e}{d} \geq 0 \quad \Rightarrow \quad 1 \geq \frac{8 \times e}{d} \Rightarrow e \leq \frac{d}{8}$$

9. The above result shows that the eccentricity '*e*' must be lesser than or equal to *d*/8.
10. It means that the load can be eccentric, on any side of the centre of the circle, by an amount equal to *d*/8.
11. Thus, if the line of action of the load is within a circle of diameter equal to one-fourth of the main circle as shown in Fig. 4.20.1 then the stress will be compressive throughout the circular section.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. Derive the relationship between slope, deflection and radius of curvature.

Ans. Refer Q. 4.2, Unit-4.

Q. 2. Find the free end deflection in cantilever beam with uniformly distributed load by Macaulay's method.

Ans. Refer Q. 4.4, Unit-4.

Q. 3. Determine the deflection at the mid and slope at the end of the beam in terms of *EI* for a beam as shown in Fig. 1.

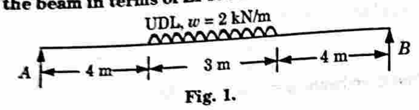


Fig. 1.

Ans. Refer Q. 4.5, Unit-4.

Q. 4. A beam of length 6 m is simply supported at its ends and carries two point loads of 48 kN and 40 kN at a distance of 1 m and 3 m respectively from the left support. Find :

i. **Deflection under each load.**
ii. **Maximum deflection.**
iii. **The point at which maximum deflection occurs.**

Given $E = 200$ GPa and $I = 85 \times 10^6$ mm^4.

Ans. Refer Q. 4.6, Unit-4.

Q. 5. Determine the deflection of the beam at midpoint for the beam loading system shown in the figure given below Take : $E = 200$ GN/m^2 and $I = 83 \times 10^6$ mm^4.

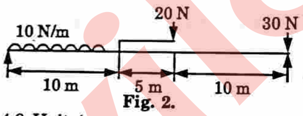


Fig. 2.

Ans. Refer Q. 4.8, Unit-4.

Q. 6. A simply supported beam of span *L* is carrying a uniformly distributed load of *w* per unit length over the entire span. Find the maximum slope and deflection of the beam.

Ans. Refer Q. 4.10, Unit-4.

☺☺☺

5 UNIT

Springs, Cylinders and Spheres

CONTENTS

PART-1

Deflection of Springs by Energy Method Close Helical Spring Acting Axial Load.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. **Derive the expression for shear stress, deflection, energy stored for closed coiled helical spring with axial load.**

Answer

Closed-Coiled Helical Springs :

A. Shear Stress (τ) :

1. In Fig. 5.1.1 a closed coiled helical spring is shown which is loaded with an axial load W.
2. Let, R = Radius of the coil, d = Diameter of the wire of the coil,
   δ = Deflection of coil under the load W, G = Modulus of rigidity,
   n = Number of coils or turns, θ = Angle of twist,
   l = Length of wire = $2\pi Rn$, τ = Shear stress, and
   J = Polar moment of inertia = $\pi d^4/32$
3. It may be noted that each section of the coil is under torsion but there are small bending and shearing stresses which being small are usually neglected.

i. Shear Stress (τ) :

From torsion equation, $\dfrac{T}{J} = \dfrac{G\theta}{l} = \dfrac{\tau}{r} \Rightarrow \dfrac{T}{J} = \dfrac{\tau}{r}$

$$\tau = \frac{Tr}{J} = \frac{T \times d/2}{\frac{\pi}{32}d^4} = \frac{16T}{\pi d^3} = \frac{16 \times WR}{\pi d^3} \quad (\because T = WR) \quad ...(5.1.1)$$

B. Deflection (δ) :

1. We know that, $\dfrac{T}{J} = \dfrac{G\theta}{l}$

$$\theta = \frac{Tl}{Gj} = \frac{WR \times 2\pi Rn \times 32}{G \times \pi d^4} = \frac{64WR^2n}{Gd^4} \quad ...(5.1.2)$$

2. But, $$\delta = R \times \theta = \frac{64WR^3n}{Gd^4} = \frac{8WD^3n}{Gd^4} \quad (\because D = 2R)$$

C. Stiffness of the Spring, k :

$$k = \frac{W}{\delta} = \frac{W}{\frac{64WR^3n}{Gd^4}} = \frac{Gd^4}{64R^3n}$$

D. Energy Stored, U :

1. Energy stored, $U = \dfrac{1}{2} \times T \times \theta = \dfrac{1}{2} \times WR \times \dfrac{64WR^2n}{Gd^4}$

$$= \frac{1}{4G} \times \frac{16WR}{\pi d^3} \times \frac{16WR}{\pi d^3} \times \left[2\pi R n d^2 \times \frac{\pi}{4}\right]$$

$$\tau = \frac{1}{4G} \times \tau^2 \times \text{volume of wire}$$

2. Again, energy stored,

$$U = \frac{1}{2} \times T \times \theta = \frac{1}{2} \times WR \times \frac{\delta}{R} = \frac{1}{2} \times W\delta \quad (\because \delta = R\theta)$$

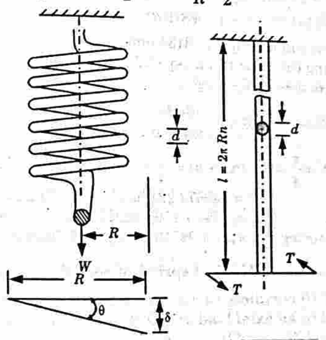


Fig. 5.1.1. Closed-coiled helical spring.

Que 5.2. **A close coiled helical spring is to carry a load of 5000 N with a deflection of 50 mm and a maximum shearing stress of 400 N/mm², if the number of active turns of active coils is 8. Estimate the following :**
a. Wire diameter. b. Mean coil diameter. c. Weight of the spring. Assume G = 83000 N/mm². Specific weight, ρ = 7700 kg/m³

AKTU 2015-16, Marks 10

Answer

Given : Load, $W = 5000$ N, Deflection, $\delta = 50$ mm, Maximum shear stress, $\tau = 400$ N/mm², $n = 8$, $G = 83000$ N/mm², Specific weight, $\rho = 7700$ kg/m³.
To Find : Wire diameter, mean coil diameter and weight of the spring.

1. We know that shear stress, $\tau = \dfrac{16WR}{\pi d^3} \Rightarrow R = \dfrac{\tau \times \pi d^3}{16 \times W}$

$$D = 2R = \frac{\tau \times \pi \times d^3}{8 \times W} = \frac{400 \times \pi \times d^3}{8 \times 5000}$$

Diameter of coil, $D = 0.0314159\, d^3$ ...(5.2.1)

2. Deflection, $\delta = \dfrac{8WD^3n}{Gd^4} \Rightarrow 50 = \dfrac{8 \times 5000 \times D^3 \times 8}{83000 \times d^4}$

$$D^3 = 12.96875\, d^4 \quad ...(5.2.2)$$

3. From eq. (5.2.1) and eq. (5.2.2), we get

$$(0.0314159\, d^3)^3 = 12.96875\, d^4$$

Diameter of coil wire, $d = 13.3134$ mm

4. Now putting the value of d in eq. (5.2.1), we get
Diameter of mean coil spring, $D = 0.0314159(13.3134)^3 = 74.134$ mm

5. We know that density, $\rho = \dfrac{\text{mass}}{\text{volume}}$

Volume, $V = \dfrac{\pi}{4}(d^2) \times (\pi D \times n) = \dfrac{\pi}{4} \times (13.3134)^2 \times \pi \times 74.134 \times 8$

$$V = 259373.221 \text{ mm}^3 = 259373.221 \times 10^{-9} \text{ m}^3$$

Mass $m = 7700 \times 259373.221 \times 10^{-9} = 1.99718$ kg

Weight of spring $= m \times g = 1.99718 \times 9.81 = 19.5923$ N

Que 5.3. **A close coil helical spring of round steel wire 10 mm in diameter and 10 complete turns with a mean diameter of 120 mm and subjected to an axial load of 200 N. Determine (a) deflection of the spring, (b) stiffness of the spring, (c) maximum shear stress, and (d) strain energy stored in spring.** **AKTU 2015-16, Marks 05**

Answer

Given : Diameter of wire, $d = 10$ mm, Number of turns, $n = 10$
Mean diameter of coil, $D = 120$ mm, Radius of coil, $R = D/2 = 60$ mm
Axial load, $W = 200$ N.
To Find : Deflection and stiffness of the spring, maximum shear stress and strain energy stored in spring.
Data Assume : Modulus of rigidity, $G = 8 \times 10^4$ N/mm²

1. Deflection, $\delta = \dfrac{64\, WR^3 \times n}{Gd^4} = \dfrac{64 \times 200 \times 60^3 \times 10}{8 \times 10^4 \times 10^4} = 34.5$ mm

2. Stiffness of the spring, $k = \dfrac{W}{\delta} = \dfrac{200}{34.5} = 5.8$ N/mm

3. Maximum shear stress, $\tau = \dfrac{16\, WR}{\pi d^3} = \dfrac{16 \times 200 \times 60}{\pi \times 10^3} = 61.1$ N/mm²

4. Strain energy stored in the spring,

$$U = \frac{32\, W^2R^3n}{Gd^4} = \frac{32 \times 200^2 \times 60^3 \times 10}{8 \times 10^4 \times 10^4} = 3456 \text{ J}$$

Que 5.4. **A leaf spring is made of plates 50 mm wide and 8 mm thick. The spring has a span of 700 mm. Determine the number of plates required to carry a central load of 45 kN. The maximum allowable stress in the plates is 200 MPa. What is the maximum deflection under this load ?** **AKTU 2015-16, Marks 10**

Answer

Given : Width, $b = 50$ mm, Thickness, $t = 8$ mm, Length, $l = 700$ mm
Central load, $W = 45$ kN, Maximum allowable stress, $\sigma = 200$ MPa
To Find : Number of plates and maximum deflection.
Data Assume : Modulus of Elasticity $E = 200$ MPa

1. We know that bending stress (σ) in leaf spring is given by,

$$\sigma = \frac{3Wl}{2nbt^2}$$

$$200 \times 10^6 = \frac{3 \times 45 \times 10^3 \times 700 \times 10^{-3}}{2 \times n \times 50 \times 10^{-3} \times (8 \times 10^{-3})^2}$$

$$n = 7.38 = 74$$

2. It is too large because given load is 45 kN (too large). So we consider the load is 4.5 kN, then

$$200 \times 10^6 = \frac{3 \times 4.5 \times 10^3 \times 700 \times 10^{-3}}{2 \times n \times 50 \times 10^{-3} \times (8 \times 10^{-3})^2}$$

$$n = 7.38 = 8$$

3. Deflection of the spring, $\delta = \dfrac{3Wl^3}{8nEbt^3}$

$$= \frac{3 \times 4.5 \times 1000 \times (700 \times 10^{-3})^3}{8 \times 8 \times 200 \times 10^9 \times 50 \times 10^{-3} \times (8 \times 10^{-3})^3}$$

$$\delta = 0.0141 \text{ mm} = 0.14 \text{ cm}$$

PART-2

Close Helical Spring Acting Axial Twist.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.5. **Find the expression for energy stored in closed coiled helical spring subjected to an axial torque.**

Answer

Energy Stored in Closed Coiled Helical Spring Subjected to an Axial Torque :

1. The axial torque T tends to wind up the spring by producing approximately a pure bending moment at all cross-sections (Fig. 5.5.1).

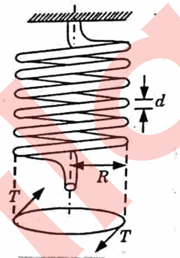


Fig. 5.5.1.

2. When bending moment is applied to curved bars with small curvature (T is the applied bending moment),

$$T = EI\left(\frac{1}{R'} - \frac{1}{R}\right)$$

Where R and R' are the radii of curvature before and after applying the bending moment.

3. Let n and n' be the number of turns of the spring before and after applying the bending moment.
Then length of spring, $l = 2\pi nR = 2\pi n'R' \Rightarrow n' = n(R/R')$

4. If ϕ is the angle of rotation of one end of the spring relative to the other and about the axis,

$$\phi = 2\pi \times \text{Increase in number of turns of the spring}$$

$$= 2\pi(n' - n) = 2\pi[n(R/R') - n] = 2\pi nR\left(\frac{1}{R'} - \frac{1}{R}\right) = \frac{Tl}{EI}$$

$$= \frac{Tl}{E(\pi d^4/64)} = \frac{64T(\pi Dn)}{E\pi d^4} = \frac{64TDn}{Ed^4}$$

5. Then strain energy due to torsional effect, $U = \frac{1}{2}T\phi$

Que 5.6. **A closed coil helical spring made of 8 mm diameter has 12 coils of 150 mm mean diameter. Calculate the elongation, torsional stress and strain energy per unit volume when the spring is subjected to an axial load of 120 kN. Take modulus of rigidity as 80 GPa. If a torque of 9 kN-m is applied in place of axial load, find axial twist, bending stress and strain energy per unit volume. Take modulus of elasticity as 200 GPa.** **AKTU 2014-15, Marks 10**

Answer

Given : Wire diameter, $d = 8$ mm, Number of coils, $n = 12$, Mean coil diameter, $D = 150$ mm, Torque, $T = 9$ kN-m, Axial load, $W = 120$ kN, Modulus of rigidity, $G = 80$ Gpa, Modulus of elasticity, $E = 200$ GPa.

To Find :

i. When applied axial load :	ii. When applied torque :
a. Elongation.	a. Axial twist.
b. Torsional stress.	b. Bending stress.
c. Strain energy per unit volume.	c. Strain energy per unit volume.

A. When applied Axial Load :

1. Elongation of spring, $\delta = \dfrac{64\,WR^3n}{Gd^4} = \dfrac{64 \times 120 \times 10^3 \times \left(\frac{150}{2}\right)^3 \times 12}{80 \times 10^3 \times 8^4}$

$$= 118652 \text{ mm} = 118.652 \text{ m}$$

2. Torsional stress, $\tau = \dfrac{16WR}{\pi d^3} = \dfrac{16 \times 120 \times 10^3 \times \left(\frac{150}{2}\right)}{\pi(8)^3} = 89.52 \text{ kN/mm}^2$

3. Strain energy per unit volume,

$$U = \frac{\tau^2}{4G} = \frac{(89.52 \times 10^3)^2}{4 \times 80 \times 10^3} = 24 \times 10^3 \text{ N/mm}^2$$

B. When applied Torque :

1. Axial twist, $\phi = \dfrac{64TDn}{Ed^4} = \dfrac{64 \times 9 \times 10^6 \times 150 \times 12}{200 \times 10^3 \times 8^4} = 1265.625 \text{ rad}$

2. Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{T \times y}{I} = \frac{32T}{\pi d^3} \quad (\because M = T \text{ and } Z = \pi d^3/32)$$

$$= \frac{32 \times 9 \times 10^6}{\pi \times 8^3} = 179.05 \text{ kN/mm}^2$$

2. Energy stored per unit volume,

$$U = \frac{\sigma_b^2}{2E} = \frac{(179.05 \times 10^3)^2}{2 \times 200 \times 10^3} = 80.146 \times 10^3 \text{ N/mm}^2$$

Que 5.7. **A close coiled helical spring is fixed at one end and subjected to axial twist at the other. When the spring is in use the axial torque varies from 0.75 N-m to 3 N-m, the working angular deflection between these torques being 35°. The spring is to be made from rod of circular section, the maximum permissible stress being 150 MN/m². The mean diameter of the coils is 8 times the rod diameter. Calculate the mean coil diameter, the number of turns and the wire diameter.**

AKTU 2018-19, Marks 07

Answer

Given : Axial torque range = 0.75 N-m to 3 N-m, Angular deflection = 35°, Maximum stress, σ = 150 MN/m², Mean coil diameter, $D = 8d$
To Find : Mean coil diameter, wire diameter and number of turns.
Data Assume : Modulus of elasticity, E = 200 GPa.

1. Maximum bending stress, $\sigma_{max} = \frac{32T}{\pi d^3}$

$$150 \times 10^6 = \frac{32 \times 3}{\pi d^3}$$

Wire diameter, $d = 5.884 \times 10^{-3}$ m $\approx$ 6 mm

2. Mean coil diameter, $D = 8d = 8 \times 6 = 48$ mm

3. Angular deflection, $\theta = \frac{64TDn}{Ed^4}$

$$35° \times \frac{\pi}{180°} = \frac{64 \times 0.75 \times 0.048 \times n}{200 \times 10^9 \times 0.006^4}$$

$$n = 68.72 = 69$$

PART-3

Open Helical Spring for Axial Load Only.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.8. **Derive an equation for the deflection of an open coiled helical spring subjected to axial load.**

Answer

Deflection of an Open Coiled Helical Spring Subjected to Axial Load :

1. In an open helical spring, the spring wire is coiled in such a way, that there is large gap between the two consecutive turns. As a result of this, the spring can take compressive load also.
2. An open helical spring may be subjected to (a) axial loading or (b) axial twist.
3. Now consider an open coiled helical spring subjected to an axial load as shown in Fig. 5.8.1.
4. Let, d = Diameter of the spring wire.
   R = Mean radius of the spring coil.
   P = Pitch of the spring coils.
   n = Number of turns of coils.

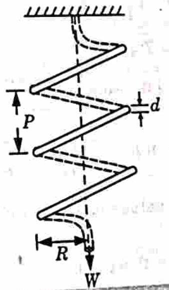


Fig. 5.8.1. Open coiled helical spring.

G = Modulus of rigidity for the spring materials.
W = Axial load on the spring.
τ = Maximum shear stress induced in the spring wire due to loading.
σ_b = Bending stress induced in the spring wire due to bending.
δ = Deflection of the spring as a result of axial load.

α = Angle of helix.

5. A little consideration will show that the load W will cause a moment WR. This moment may be resolved into the following two components,

$$T = WR\cos\alpha \quad \text{...(It causes twisting of coils)}$$
$$M = WR\sin\alpha \quad \text{...(It causes bending of coils)}$$

6. Let ϕ = Angle of twist, as a result of twisting moment.
β = Angle of bend, as a result of bending moment.

7. The length of the spring wire, $l = 2\pi nR\sec\alpha$ ...(5.8.1)

Twisting moment, $WR\cos\alpha = \dfrac{\pi}{16}\times\tau\times d^3$ ...(5.8.2)

8. Bending stress, $\sigma_b = \dfrac{My}{I} = \dfrac{WR\sin\alpha\times\dfrac{d}{2}}{\dfrac{\pi}{64}\times d^4} = \dfrac{32\,WR\sin\alpha}{\pi d^3}$ ...(5.8.3)

Angle of twist, $\phi = \dfrac{Tl}{JG} = \dfrac{WR\cos\alpha\, l}{JG}$

9. Angle of bend due to bending moment,

$$\beta = \frac{Ml}{EI} = \frac{WR\sin\alpha\, l}{EI}$$

10. We know that the work done by the load in deflecting the spring is equal to the stress energy of the spring.

$$\therefore \quad \frac{1}{2}W\delta = \frac{1}{2}T\phi + \frac{1}{2}M\beta$$
$$W\delta = T\phi + M\beta$$
$$W\delta = \left[WR\cos\alpha\times\frac{WR\cos\alpha\, l}{JG}\right] + \left[WR\sin\alpha\times\frac{WR\sin\alpha\, l}{EI}\right]$$
$$\delta = WR^2 l\left[\frac{\cos^2\alpha}{JG} + \frac{\sin^2\alpha}{EI}\right] \quad \text{...(5.8.4)}$$

11. Now substituting the values of $l = 2\pi nR\sec\alpha$, $J = \dfrac{\pi}{32}(d)^4$ and $I = \dfrac{\pi}{64}(d)^4$ in the eq. (5.8.4), we get

$$\delta = WR^2\times 2\pi nR\sec\alpha\left[\frac{\cos^2\alpha}{\dfrac{\pi}{32}d^4G} + \frac{\sin^2\alpha}{E\times\dfrac{\pi}{64}d^4}\right]$$
$$\delta = \frac{64WR^3\, n\sec\alpha}{d^4}\left[\frac{\cos^2\alpha}{G} + \frac{2\sin^2\alpha}{E}\right]$$

Que 5.9. **Derive expression of deflection, angular rotation and stresses in case of open coil helical spring subjected to axial load. Calculate what % the axial extension is underestimated if the inclination of the coil is neglected for a spring in which α = 25°. Assume n and R remain same.**

AKTU 2017-18, Marks 07

Answer

A. **Derivation :** Refer Q. 5.8, Page 5–9A, Unit-5.

B. **Numerical :**

Given : α = 25° (In first condition), α = 0 (In second condition), n and R remains same.
To Find : % change in axial extension.
Data Assume : E/G = 2.5.

1. The axial deflection of open coiled helical spring is given by,

$$\delta = \frac{64WR^3 n\sec\alpha}{d^4}\left[\frac{\cos^2\alpha}{G} + \frac{2\sin^2\alpha}{E}\right]$$
$$(\because E/G = 2.5 \text{ and } \cos^2\alpha = 1-\sin^2\alpha)$$
$$\delta = \frac{64WR^3 n\sec\alpha}{Gd^4}[1 - 0.2\sin^2\alpha]$$

2. When α = 25°, $\delta = \dfrac{64WR^3 n\sec 25°}{Gd^4}[1 - 0.2\sin^2 25°]$

$$\delta = 1.064\times\frac{64WR^3 n}{Gd^4}$$

3. When inclination of the coil is neglected i.e., $\alpha = 0$, $\delta' = \dfrac{64WR^3 n}{Gd^4}$

4. Percentage change in axial extension, $= \dfrac{\delta - \delta'}{\delta} = \dfrac{1.064 - 1}{1.064} = 6\,\%$

Que 5.10. **Find the mean radius of an open coiled spring of helix angle 30°, to give a vertical displacement of 23 mm and an angular rotation of the load end of 0.02 radian under an axial load of 35 N. The material available is steel rod 6 mm diameter. $E = 2\times10^5$ N/mm², $G = 8.0\times10^4$ N/mm².**

Answer

Given : Helix angle, α = 30°, Angular rotation, ϕ = 0.02 rad
Vertical displacement, δ = 23 mm, Load, W = 35 N
Wire diameter, d = 6 mm, $E = 2\times10^5$ N/mm², $G = 8.0\times10^4$ N/mm²
To Find : Mean radius of open coiled spring.

1. Polar moment of inertia, $J = \frac{\pi}{32} d^4 = \frac{\pi}{32} \times 6^4 = 127.23 \text{ mm}^4$

2. Moment of inertia, $I = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times 6^4 = 63.62 \text{ mm}^4$

3. We know that, $\delta = 2\, WR^3 n\pi \sec\alpha \left[\frac{\cos^2\alpha}{GJ} + \frac{\sin^2\alpha}{EI}\right]$ ...(5.10.1)

Let, $n = 1$

Putting the value of n, J and I in eq. (5.10.1), we get

$$23 = 2 \times 35 \times R^3 \times 1 \times \pi \times \sec 30° \times \left[\frac{(\cos 30°)^2}{8 \times 10^4 \times 127.23} + \frac{(\sin 30°)^2}{2 \times 10^5 \times 63.62}\right]$$

$$R = 99 \text{ mm}$$

PART-4

Open Coiled Spring for Twist Axial Axial Load and Twisting Acting Simultaneously for Both Open and Closed Spring.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.11. **Derive equation for deflection of an open coiled helical spring subjected to axial thrust.**

OR

Derive the relation to find deflection induced in the open coiled helical spring subjected to axial Torque.

AKTU 2018-19, Marks 07

Answer

Deflection of an Open Coiled Helical Spring Subjected to Axial Thrust :

1. Let the torque T applied about axis of spring OY' [Fig. 5.11.1(*a*)] be resolved about OX' and OY';
   Component about OX', $T' = T \sin\alpha$ ....Causes torsion of spring
   Component about OY', $M' = T\cos\alpha$ ....Causes bending of coil.
2. If ϕ' be the angular twist due to T', β' due to M' and ϕ due to T then, by principle of conservation of energy, we have,

$$\frac{1}{2}T\phi = \frac{1}{2}T'\phi' + \frac{1}{2}M'\beta' = \frac{1}{2}T' \times \frac{T'l}{GJ} + \frac{1}{2}M' \times \frac{M'l}{EI}$$

$$\frac{1}{2}T\phi = \frac{1}{2}\frac{T^2 \sin^2\alpha \times l}{GJ_p} + \frac{1}{2}\frac{T^2\cos^2\alpha \times l}{EI}$$

$$\phi = \frac{T\sin^2\alpha \times l}{GJ} + \frac{T\cos^2\alpha \times l}{EI}$$

$$\phi = 2\,TR\,n\,\pi \sec\alpha \left[\frac{\sin^2\alpha}{GJ} + \frac{\cos^2\alpha}{EI}\right] \quad (\because\ l = 2\pi R \sec\alpha \times n)$$

3. For axial deflection/extension resolve rotations as before :

$$\delta = TRl \sin\alpha\cos\alpha\left(\frac{1}{GJ} - \frac{1}{EI}\right)$$

$$= TR \times 2\pi\, Rn \sec\alpha \times \sin\alpha\cos\alpha\left(\frac{1}{GJ} - \frac{1}{EI}\right)$$

$$= 2TR^2 n\,\pi \sin\alpha\left(\frac{1}{GJ} - \frac{1}{EI}\right)$$

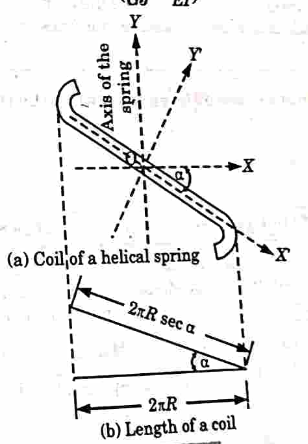


Fig. 5.11.1.

PART-5

Introduction of Thin Cylinder, Thick Cylinder and Spheres, Difference between Thin and Thick Walled Pressure Vessels, Thin Walled Sphere and Cylinder.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.12. **What are the difference between thin cylinder and thick cylinders ?**

Answer

S. No.	Thin Cylinder	Thick Cylinder
1.	The wall thickness is less than 1/20th of their diameter (or 1/10th of the their radius).	The wall thickness is more than 1/20th of their diameter (or 1/10th of the their radius).
2.	The distribution of stress is approximately uniform throughout the thickness of the wall.	The distribution of stress is not uniform throughout the thickness of the wall.

Que 5.13. **What are the difference between the thin walled sphere and cylinders ?**

Answer

S. No.	Thin Cylinder	Thin Sphere
1.	In thin cylinder hoop longitudinal and radial stress are generated simultaneously.	In thin sphere hoop and radial stress are generated simultaneously.
2.	Less pressure exerted.	More pressure exerted.
3.	Cylindrical shell is easy to casting.	Spherical shell is more difficult in casting.
4.	In thin cylinder shear stress generated.	In thin sphere shear stress will be zero.
5.	It is used for storing fluids under pressure e.g. steam boilers, tanks and water tanks.	It is used for containing gas under pressure e.g. air compressors.

PART-6

Hoop and Axial Stresses and Strain Volumetric Strain for Sphere.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.14. **Derive the expression for circumferential stress and longitudinal stress for a thin shell subjected to an internal pressure.**

Answer

A. Circumferential Stress or Hoop Stress :

1. Consider a thin cylindrical shell subjected to an internal pressure as shown in Fig. 5.14.1(*a*) and (*b*).
2. We know that as a result of the internal pressure, the cylinder has a tendency to split up into two troughs as shown in the Fig. 5.14.1.

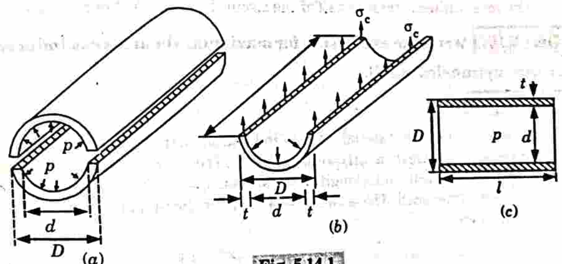


Fig. 5.14.1.

3. Let, l = Length of the shell. d = Diameter of the shell.
   t = Thickness of the shell. p = Intensity of internal pressure.
4. Bursting force (pressure) = Resisting strength

$$p \times (dl) = (2tl) \times \sigma_c$$

Circumferential stress, $\sigma_c = pd / 2t$ ($\because$ There are two sections)

5. If η is the efficiency of the riveted joints of the shell, then stress,

$$\sigma_c = \frac{pd}{2t\eta}$$

B. Longitudinal Stress :

1. Consider the same cylinder under consideration has its two ends covered with two end plates connected to them as shown in Fig. 5.14.1(a) and (b).
3. Let, σ_l = Longitudinal stress produced in the shell.

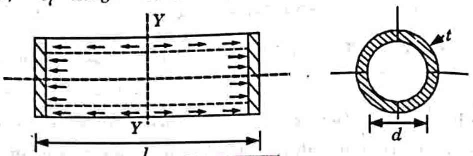


Fig. 5.14.2.

4. Pressure on the ends = Resisting force

$$p \times \frac{\pi}{4} d^2 = (\pi d t) \times \sigma_l$$

Longitudinal stress, $\sigma_l = \frac{pd}{4t} = \frac{1}{2}\left(\frac{pd}{2t}\right) = \frac{1}{2}\sigma_c$ $\quad (\because \sigma_c = pd/2t)$

6. This is also a tensile stress across the section Y-Y. It may be noted that the longitudinal stress is half of the circumferential or hoop stress.

Que 5.15. **Write the expression for maximum shear stress induced in thin cylindrical shell.**

Answer

1. At any point in the material of the cylindrical shell, there are two principal stresses, namely a circumferential stress, $\sigma_c = pd/2t$ acting circumferentially and a longitudinal stress, $\sigma_l = pd/4t$ acting parallel to the axis of the shell. These two stresses are tensile and perpendicular to each other.
2. Maximum shear stress, $\tau_{max} = \frac{\sigma_c - \sigma_l}{2} = \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2} = \frac{pd}{8t}$

Que 5.16. **For a tube having $E = 2 \times 10^5$ N/mm² and $\mu = 0.3$ the hoop stress at the inner surface is twice the internal pressure. Find the thickness of the wall if internal radius is 60 mm.**

AKTU 2017-18, Marks 07

Answer

Given : $E = 2 \times 10^5$ N/mm², $\mu = 0.3$, $r_i = 60$ mm, $\sigma_c = 2p$
To Find : Thickness of wall.

Hoop stress, $\sigma_c = \frac{pd}{2t} \Rightarrow 2p = \frac{p \times 120}{2t}$

Thickness of wall, $t = \frac{120}{4} = 30$ mm

Que 5.17. **The cylinder for a hydraulic press has an inside diameter of 300 mm. Determine the wall thickness required if the cylinder is to withstand an internal pressure of 60 MPa without exceeding a shearing stress of 90 MPa.** **AKTU 2018-19, Marks 07**

Answer

Given : $d = 300$ mm, $p = 60$ MPa, $\tau_{max} = 90$ MPa
To Find : Wall thickness.

1. Hoop stress, $\sigma_c = \frac{pd}{2t} = \frac{60 \times 300}{2t} = \frac{9000}{t}$
2. Longitudinal stress, $\sigma_l = \frac{pd}{4t} = \frac{60 \times 300}{4t} = \frac{4500}{t}$
3. Maximum shear stress, $\tau_{max} = \frac{\sigma_c - \sigma_l}{2}$

$$90 = \frac{(9000/t) - (4500/t)}{2} \Rightarrow 90 = \frac{4500}{2t}$$

Thickness of wall, $t = 25$ mm

Que 5.18. **What is the effect of internal pressure on the dimensions of thin cylindrical shell and also strain induced in it ?**

Answer

Effect of Internal Pressure on the Dimension of Thin Cylindrical Shell :

1. When a fluid having internal pressure (p) is stored in a thin cylindrical shell, due to internal pressure of the fluid the stresses set up at point of the material of the shell are :
   i. Hoop or circumferential stress (σ_c), acting on longitudinal section.
   ii. Longitudinal stress (σ_l), acting on the circumferential section.
2. These are principal stresses, as they are acting on principal planes.
3. The stress in the third principal plane is zero as the thickness (t) of the cylinder is very small. Actually the stress in the third principal plane is radial stress which is very small for thin cylinders and can be neglected.

4. Let p = Internal pressure of fluid, L = Length of cylindrical shell.
d = Diameter of the cylindrical shell.
t = Thickness of the cylindrical shell.
E = Modulus of elasticity for the material of the shell.
σ_c = Hoop stress in the material.
σ_l = Longitudinal stress in the material, μ = Poisson's ratio.
δd = Change in diameter due to stresses set up in the material.
δL = Change in length, δV = Change in volume.
ε_c and ε_c = Circumferential and longitudinal strain.

5. Circumferential strain, $\varepsilon_c = \dfrac{\sigma_c}{E} - \dfrac{\mu\sigma_l}{E}$

$$= \frac{pd}{2tE} - \frac{\mu pd}{4tE} \qquad \left(\because \sigma_c = \frac{pd}{2t} \text{ and } \sigma_l = \frac{pd}{4t}\right)$$

$$= \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right) \qquad ...(5.18.1)$$

Longitudinal strain, $\varepsilon_l = \dfrac{\sigma_l}{E} - \dfrac{\mu\sigma_c}{E} = \dfrac{pd}{4tE} - \dfrac{\mu pd}{2tE}$

$$= \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right) \qquad ...(5.18.2)$$

6. But circumferential strain is also given as,

$$\varepsilon_c = \frac{\text{Change in circumference due to pressure}}{\text{Original circumference}}$$

$$= \frac{\text{Final circumference} - \text{Original circumference}}{\text{Original circumference}}$$

$$= \frac{\pi(d + \delta d) - \pi d}{\pi d}$$

$$= \frac{\delta d}{d}\left(\text{or } \frac{\text{Change in diameter}}{\text{Original diameter}}\right) \qquad ...(5.18.3)$$

7. Equating the two values of ε_c given by eq. (5.18.1) and eq. (5.18.3), we get

$$\frac{\delta d}{d} = \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right) \qquad ...(5.18.4)$$

Change in diameter, $\delta d = \dfrac{pd^2}{2tE}\left(1 - \dfrac{\mu}{2}\right)$ ...(5.18.5)

8. Similarly longitudinal strain is also given as,

$$\varepsilon_l = \frac{\text{Change in length due to pressure}}{\text{Original length}} = \frac{\delta L}{L} \qquad ...(5.18.6)$$

9. From eq. (5.18.2) and eq. (5.18.6)

$$\frac{\delta L}{L} = \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

$\therefore$ Change in length, $\delta L = \dfrac{p \times d \times L}{2tE}\left(\dfrac{1}{2} - \mu\right)$

10. **Volumetric Strains :** It is defined as change in volume divided by original volume.

$\therefore$ Volumetric strain, $\varepsilon_V = \dfrac{\delta V}{V}$

11. Change in volume δV = Final volume – Original volume

Original volume, V = Area of cylindrical shell × Length = $\dfrac{\pi}{4} d^2 \times L$

Final volume = (Final area of cross-section) × Final length

$$= \frac{\pi}{4}[d + \delta d]^2 \times [L + \delta L]$$

$$= \frac{\pi}{4}[d^2 L + (\delta d)^2 L + 2d\,L\delta d + \delta L d^2 + \delta L(\delta d)^2 + 2d\,\delta d\delta L]$$

Neglecting the smaller quantities such as $(\delta d^2)L$, $\delta L(\delta d)^2$ and $2d\,\delta d\delta L$, we get

$$\text{Final volume} = \frac{\pi}{4}[d^2L + 2d\,L\delta d + \delta L d^2]$$

$$\therefore \text{ Change in volume } (\delta V) = \frac{\pi}{4}[d^2L + 2dL\delta d + \delta L d^2] - \frac{\pi}{4}d^2 \times L$$

$$= \frac{\pi}{4}[2d\,L\delta d + \delta L d^2]$$

12. Volumetric strain, $\varepsilon_V = \dfrac{\delta V}{V} = \dfrac{\frac{\pi}{4}[2d\,L\delta d + \delta L d^2]}{\frac{\pi}{4}d^2 \times L}$

$$\varepsilon_V = \frac{2\delta d}{d} + \frac{\delta L}{L} \qquad ...(5.18.7)$$

$$\varepsilon_V = 2\,\varepsilon_c + \varepsilon_l \qquad \left(\because \frac{\delta d}{d} = \varepsilon_c, \frac{\delta L}{L} = \varepsilon_l\right) \qquad ...(5.18.8)$$

13. Substituting the values of ε_c and ε_l in eq. 5.18.8, we get

$$= 2 \times \frac{pd}{2Et}\left(1 - \frac{\mu}{2}\right) + \frac{pd}{2Et}\left(\frac{1}{2} - \mu\right) = \frac{pd}{2Et}\left(\frac{5}{2} - 2\mu\right)$$

Que 5.19. A mild steel hollow cylinder has diameter to thickness ratio of 25. Find the internal pressure to which the cylinder should be subjected so that its volume is increased by 5×10^{-4} of its original volume. Take $E = 2 \times 10^5$ and $\mu = 0.3$.

AKTU 2014-15, Marks 05

Answer

Given : $d/t = 25$, $\frac{\delta V}{V} = 5 \times 10^{-4}$, $E = 2 \times 10^5$, $\mu = 0.3$

To Find : Internal pressure.

Volumetric strain is given by,

$$\varepsilon_v = \frac{\delta V}{V} = \frac{pd}{2\,EI}\left(\frac{5}{2} - 2\mu\right)$$

$$5 \times 10^{-4} = \frac{p \times 25}{2 \times 2 \times 10^5}\left(\frac{5}{2} - 2 \times 0.3\right)$$

$$p = 4.21 \text{ N/mm}^2$$

Que 5.20. A cylindrical shell 90 cm long 20 cm internal diameter having thickness of metal as 8 mm is filled with fluid at atmospheric pressure. If an additional 20 cm³ of fluid is pumped into the cylinder, find

A. The pressure exerted by the fluid on the cylinder, and

B. The hoop stress induced. Take $E = 200$ GPa and $\mu = 0.3$

AKTU 2016-17, Marks 10

Answer

Given : Length of cylinder, $l = 90$ cm, Diameter of cylinder, $d = 20$ cm
Thickness of cylinder, $t = 8$ mm $= 0.8$ cm, Volume of additional fluid $\delta_V = 20$ cm³, $E = 200$ GPa $= 2 \times 10^5$ N/mm², $\mu = 0.3$

To Find : Pressure exerted by the fluid on cylinder and hoop stress induced.

A. Pressure Exerted by the Fluid on the Cylinder :

1. Volume of cylinder, $V = \frac{\pi}{4} d^2 \times l = \frac{\pi}{4} \times 20^2 \times 90 = 28274.33 \text{ cm}^3$

2. Volumetric strain is given by, $\varepsilon_V = \frac{\delta V}{V} = \frac{pd}{2Et}\left(\frac{5}{2} - 2u\right)$

$$\frac{20}{28274.33} = \frac{P \times 20}{2 \times 2 \times 10^5 \times 0.8}\left(\frac{5}{2} - 2 \times 0.3\right)$$

$$p = 5.957 \text{ N/mm}^2$$

B. Hoop Stress :

Hoop stress, $\sigma_c = \frac{pd}{2t} = \frac{5.957 \times 20}{2 \times 0.8} = 74.46 \text{ N/mm}^2$.

PART-7

Hoop and Axial Stresses and Strain Volumetric Strain for Sphere.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.21. Derive the equations for circumferential stress and volumetric strain in a thin spherical shell under internal pressure.

AKTU 2015-16, Marks 10

Answer

A. Circumferential Stress :

1. Fig. 5.21.1 shows a thin spherical shell of internal diameter d and thickness t and subjected to an internal fluid pressure p.
2. The fluid inside the shell has a tendency to split the shell into two hemispheres along X-X axis. The cross sectional area of thin spherical shell resisted the bursting force.
3. Now, Bursting force = Resisting force

$$p \times \frac{\pi}{4} \times d^2 = (\pi dt) \times \sigma_c$$

Circumferential stress, $\sigma_c = pd/4t$ ...(5.21.1)

4. The above eq. (5.22.1) for stress is true only when the shell is seamless, but in case of built up sphere shell,

$$\sigma_c = \frac{pd}{4t\eta}$$

where, η = Joint efficiency

5. The stress σ_c is tensile in nature.

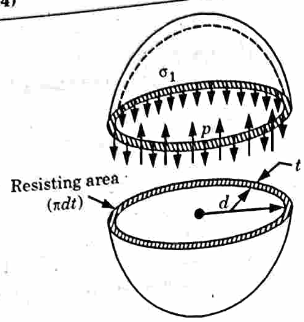


Fig. 5.21.1.

6. For thin spherical shell,
   Longitudinal stress = Circumferential stress

$$\therefore \quad \sigma_l = \sigma_c = \frac{pd}{4t}$$

The stress σ_l will be at right angles to σ_c.

B. Volumetric Strain $\left(\frac{\delta V}{V}\right)$:

1. Volume of sphere, $V = \frac{4}{3}\pi r^3 = \frac{\pi}{6}d^3$
2. Taking the differential of the above equation,

$$\delta V = \frac{\pi}{6} \times 3d^2 \times \delta d$$

3. Volumetric strain, $\varepsilon_V = \frac{\delta V}{V} = \frac{\frac{\pi}{6} \times 3d^2 \times \delta d}{\frac{\pi}{6} \times d^3} = 3\,\frac{\delta d}{d}$ ...(5.21.1)

4. Circumferential strain is given by,

$$\varepsilon_c = \frac{\delta d}{d} = \frac{pd\,(1-\mu)}{4tE}$$

5. Substituting the value of ε_c in eq. (5.21.1), we get

$$\varepsilon_V = \frac{\delta V}{V} = \frac{3pd\,(1-\mu)}{4tE}$$

PART-8

Radial, Axial and Circumferential Stress in Thick Cylinders Subjected to Internal or External Pressure.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.22. **Write down the assumption in Lame's theory and also derive its equation for thick shell.**

AKTU 2017-18, Marks 07

OR

Derive an expression for maximum principal stress in a thick cylindrical shell subjected to internal pressure only.

AKTU 2017-18, Marks 07

Answer

A. Assumptions of Lame's Theory :

Following are the assumptions made in Lame's theory :

1. The material is homogeneous and isotropic.
2. Plane sections perpendicular to the longitudinal axis of the cylinder remain plane after the application of internal pressure.
3. The material is stressed within elastic limit.
4. All the fibres of material are free to expand or contract independently without being constrained by adjacent fibres.

B. Derivation :

1. Consider a thick cylinder subject to internal and external radial stress (pressure) is shown in Fig. 5.22.1. Consider an element ring of internal radius r and thickness dr.
2. Let,
   r_1 = Internal radius of the cylinder.
   r_2 = External radius of the cylinder.
   l = Length of the cylinder.
   p_1 = Pressure on the inner surface of the cylinder.
   p_2 = Pressure on the outer surface of the cylinder.
   σ_r = Internal radial stress on the elemental ring.
   $(\sigma_r + d\sigma_r)$ = External radial stress on the elemental ring.
   σ_c = Circumferential stress on elemental ring.

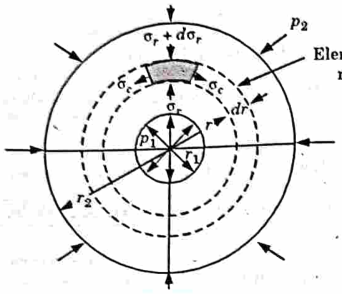


Fig. 5.22.1.

3. The conditions for equilibrium on one half of the elemental ring (similar to those in the case of thin cylinder) are as follows :

$$\text{Bursting force} = (\sigma_r \times 2rl) - [(\sigma_r + d\sigma_r) \times 2(r + dr)\, l]$$
$$= 2l[-\sigma_r dr - r d\sigma_r - dr\, d\sigma_r] = -2l\,(\sigma_r dr + r d\sigma_r)$$

(Neglecting the product of small quantities)

$$\text{Resisting force} = 2\sigma_c\, l\, dr$$

6. Equating the resisting force to bursting force (for equilibrium), we get

$$2\sigma_c\, l\, dr = -2l(\sigma_r dr + r d\sigma_r)$$

$$\sigma_c = -\sigma_r - r\frac{d\sigma_r}{dr} \quad \text{...(5.22.1)}$$

7. Now let us obtain another relation between the radial stress (pressure) and circumferential (or hoop) stress by using the condition that the longitudinal strain (ε_l) at any point in the section is same.
The Longitudinal stress,

$$\sigma_l = \frac{p_1 \times \pi r_1^2}{\pi(r_2^2 - r_1^2)} = \frac{p_1 r_1^2}{(r_2^2 - r_1^2)}$$

8. Hence at any point in the section of the element ring considered above, the following three principal stresses exist,

i. The radial stress (pressure), σ_r.
ii. The circumferential stress, σ_c.
iii. The longitudinal tensile stress, σ_l.

9. Since the longitudinal strain (ε_l) is constant, we have,

$$\varepsilon_l = \frac{\sigma_l}{E} - \frac{\mu\sigma_c}{E} + \frac{\mu\sigma_r}{E} = \text{constant}$$

But, since σ_l, μ and E are constant

$$\therefore \quad \sigma_c - \sigma_r = \text{constant}$$

10. Let, $\sigma_c - \sigma_r = 2a$ ...(5.22.2)
Putting $\sigma_c = (\sigma_r + 2a)$ in eq. (5.22.1), we get

$$(\sigma_r + 2a) = -\sigma_r - r\frac{d\sigma_r}{dr}; \quad \frac{d\sigma_r}{dr} = \frac{-2(\sigma_r + a)}{r}$$

$$\frac{d\sigma_r}{\sigma_r + a} = -\frac{2\,dr}{r} \quad \text{...(5.22.3)}$$

Integrating both sides, we get

$$\log_e(\sigma_r + a) = -2\log_e r + \log_e b \quad (\because \log_e b = \text{Constant})$$

$$\therefore \quad \log_e(\sigma_r + a) = \log_e \frac{b}{r^2}$$

$$\sigma_r = \frac{b}{r^2} - a \quad \text{...(5.22.4)}$$

11. Similarly, $$\sigma_c = \frac{b}{r^2} + a \quad \text{...(5.22.5)}$$

12. The eq. (5.22.4) and eq. (5.22.5) are called Lame's equations.
13. The constant a and b can be evaluated from the known internal and external radial pressure and radius.
14. It may be noted that in the above equations σ_r is compressive and σ_c is tensile.

Que 5.23. **In an experiment on a thick cylinder of 100 mm external diameter and 50 mm internal diameter the hoop and longitudinal strains as measured by strain gauges applied to the outer surface of the cylinder were 240×10^{-6} and 60×10^{-6} respectively, for an internal pressure of 90 MN/m^2, the external pressure being zero. Determine the actual hoop and longitudinal stresses present in the cylinder if E = 208 GN/m^2 and μ = 0.29. Compare the hoop stress value so obtained with the theoretical value given by the Lame's equations.**

AKTU 2018-19, Marks 07

Answer

Given : d_2 = 100 mm, d_1 = 50 mm, $\varepsilon_c = 240 \times 10^{-6}$, $\varepsilon_l = 60 \times 10^{-6}$, σ_r = 90 MN/m^2, E = 208 GN/m^2, μ = 0.29, $p_2 = 0$
To Find : Compare the hoop stress.

1. Radius ratio, $\frac{r_2}{r_1} = \frac{50}{25} = 2$
2. We know that, Lame's equation, $\sigma_r = \frac{b}{r^2} - a$

at $r = r_2$, $\sigma_r = 0 \Rightarrow 0 = b/r_2^2 - a \Rightarrow b = ar_2^2$

at $r = r_1$, $\sigma_r = 90 \Rightarrow 90 = \dfrac{b}{r_1^2} - a$

$$90 = \frac{ar_2^2}{r_1^2} - a = a\left[\frac{r_2}{r_1}\right]^2 - a = a[2^2 - 1] = 3a$$

$$a = 90 / 3 = 30 \text{ MN/m}^2$$

3. Again at, $r = r_2$

$$\sigma_c = \frac{b}{r_2^2} + a = a + a = 2a = 2 \times 30 = 60 \text{ MN/m}^2$$

$$\sigma_l = \frac{p_1 r_1^2}{r_2^2 - r_1^2} = \frac{p_1}{\left(\frac{r_2}{r_1}\right)^2 - 1} = \frac{90}{2^2 - 1} = 30 \text{ MN/m}^2$$

At $r = r_2$; $\sigma_r = 0$ $(\because p_2 = 0)$

4. Circumferential strain, $\varepsilon_c = \dfrac{1}{E}[\sigma_c - \mu(\sigma_l + \sigma_r)]$

$$240 \times 10^{-6} = \frac{1}{208 \times 10^3}[\sigma_c - 0.29\sigma_l]$$

$$\sigma_c - 0.29\,\sigma_l = 49.92 \quad \text{...(5.23.1)}$$

5. Similarly longitudinal strain, $\varepsilon_l = \dfrac{1}{E}[\sigma_l - \mu(\sigma_c + \sigma_r)]$

$$60 \times 10^{-6} = \frac{1}{208 \times 10^3}[\sigma_l - 0.29(\sigma_c)]$$

$$\sigma_l - 0.29\,\sigma_c = 12.48 \quad \text{...(5.23.2)}$$

6. From eq. (5.23.1) and eq. (5.23.2), we get

$\sigma_c = 58.45$ MN/m² (Compressive)

$\sigma_l = 29.43$ MN/m² (Compressive)

PART-9
Compound Cylinders.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.24. **Discuss about the stresses induced in the compound thick cylindrical shells.**

Answer

A. Stresses in Compound Thick Cylindrical Shells :

1. When the compound shell is subjected to an internal pressure, both the inner and outer shells will be subjected to hoop tensile stress.
2. The net effect of the initial stresses and those due to internal pressure is to make the resultant stresses more or less uniform.
3. Now consider a compound thick cylindrical shell made up of two tubes as shown in Fig. 5.24.1.
4. Let, r_1 = Outer radius of the outer shell.

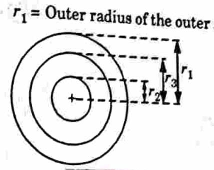


Fig. 5.24.1.

r_2 = Inner radius of the inner shell.
r_3 = Outer radius of the inner shell, (also inner radius of the outer shell).
p_1 = Radial pressure at the junction of the two shells (*i.e.*, at radius r_3).

5. Now the Lame's equations may be applied in this case for the initial conditions *i.e.*, when the outer tube exerts pressure on the inside shell, or in other words before the fluid under pressure is admitted into the inner shell.

i. **For Inner Tube :** $p_x = \dfrac{b_1}{x^2} - a_1$

When $x = r_2$, then $p_x = 0$, $0 = \dfrac{b_1}{r_2^2} - a_1$ ...(5.24.1)

When $x = r_3$, then $p_x = p_1$, $p_1 = \dfrac{b_1}{r_3^2} - a_1$ ...(5.24.2)

ii. **For Outer Tube :** $p_x = \dfrac{b_2}{x^2} - a_2$ and $\sigma_x = \dfrac{b_2}{x^2} + a_2$

When $x = r_3$, then $p_x = p_1$, $p_1 = \dfrac{b_2}{r_3^2} - a_2$ ...(5.24.3)

When $x = r_1$, then $p_x = 0$, $0 = \dfrac{b_2}{r_1^2} - a_2$ ...(5.24.4)

6. The values of a_1, b_1, a_2 and b_2 may be find out from the above four equations, if the radial pressure p_1 at the junction of the two shells is known.

7. The hoop stress (σ_x) may also be obtained with the help of relative expressions.

Que 5.25. **A compound steel tube is composed of a tube 200 mm internal diameter and 30 mm thickness, shrunk on a tube of 200 mm external diameter and 25 mm thickness. The radial pressure of the junction is 12 N/mm². The composed tube is subjected to an internal fluid pressure of 80 N/mm². Find the variation of the hoop stress over the wall of the compound tube.**

AKTU 2014-15, Marks 10

Answer

Given : Inner diameter of outer cylinder, $d_3 = 200$ mm, Thickness of outer cylinder = 30 mm, Thickness of inner cylinder = 25 mm, Pressure due to shrinkage at the junction of two cylinder, $p_1 = 12$ N/mm², Internal fluid pressure $p_x = 80$ N/mm²
Outer diameter of outer cylinder $d_1 = 200 + 30 + 25 = 255$ mm
Inner diameter of inner cylinder $d_2 = 200 - 30 - 25 = 145$ mm
To Find : Variation of the hoop stress.

1. Let, σ_x = Hoop stress at a radius x in the compound cylinder.
2. First of all, let us apply all the Lame's equations for the inner and outer cylinders before the fluid under pressure is admitted.

i.
$$0 = \frac{b_1}{r_2^2} - a_1 = \frac{b_1}{(72.5)^2} - a_1$$

$$0 = \frac{b_1}{5256.25} - a_1 \quad ...(5.25.1)$$

ii. Similarly,
$$p_1 = \frac{b_1}{r_3^2} - a_1 = \frac{b_1}{(100)^2} - a_1$$

$$12 = \frac{b_1}{10000} - a_1 \quad ...(5.25.2)$$

iii.
$$p_1 = \frac{b_2}{r_3^2} - a_2 \Rightarrow 12 = \frac{b_2}{10000} - a_2 \quad ...(5.25.3)$$

iv. and
$$0 = \frac{b_2}{r_1^2} - a_2 = \frac{b_2}{(127.5)^2} - a_2$$

$$0 = \frac{b_2}{16256.25} - a_2 \quad ...(5.25.4)$$

3. Solving eq. (5.25.1) and eq. (5.25.2) simultaneously, we get
$b_1 = 132964.4$ and $a_1 = 25.3$.
4. Similarly solving eq. (5.25.3) and eq. (5.25.4) simultaneously, we find that $b_2 = 311808.19$ and $a_2 = 19.18$

4. We know from Lame's equation that permissible stress (σ_x)

$$\sigma_{72.5} = \frac{-132964.4}{(72.5)^2} + (-25.3) \quad \text{(For inner tube)}$$
$$= -50.6 \text{ N/mm}^2$$

$$\sigma_{100} = \frac{-132964.4}{(100)^2} + (-25.3) \quad \text{(For inner tube)}$$
$$= -38.59 \text{ N/mm}^2$$

Similarly,
$$\sigma_{100} = \frac{311808.19}{100^2} + 19.18 \quad \text{(For outer tube)}$$
$$= 50.36 \text{ N/mm}^2$$

and
$$\sigma_{127.5} = \frac{311808.19}{127.5^2} + 19.18 \quad \text{(For outer tube)}$$
$$= 38.36 \text{ N/mm}^2$$

5. Now let us apply Lame's equation for the inner cylinder only after the fluid under pressure 80 N/mm² is admitted, *i.e.*,

$$p_x = \frac{b}{x^2} - a$$

$$80 = \frac{b}{72.5^2} - a = \frac{b}{5256.25} - a \quad ...(5.25.5)$$

and
$$0 = \frac{b}{127.5^2} - a = \frac{b}{16256.25} - a \quad ...(5.25.6)$$

6. Subtracting eq. (5.25.6) from eq. (5.25.5), we get

$$\frac{b}{5256.25} - \frac{b}{16256.25} = 80 \Rightarrow b = 621432.1023$$

and
$$a = \frac{b}{16256.25} = \frac{621432.1023}{16256.25} = 38.23$$

7. From Lame's equation, the permissible stress

$$\sigma_x = \frac{b}{x^2} + a = \frac{621432.1023}{x^2} + 38.23$$

$$\therefore \quad \sigma_{72.5} = \frac{621432.1023}{72.5^2} + 38.23 = 156.46 \text{ N/mm}^2$$

$$\sigma_{100} = \frac{621432.1023}{100^2} + 38.23 = 100.37 \text{ N/mm}^2$$

$$\sigma_{127.5} = \frac{621432.1023}{127.5^2} + 38.23 = 76.46 \text{ N/mm}^2$$

8. Now tabulate the hoop (*i.e.*, circumferential) stress at different point as given below.
[(+ve for tension, –ve for compression)

	Inner Cylinder		Outer Cylinder	
	$x = 72.5$ mm	$x = 100$ mm	$x = 100$ mm	$x = 127.5$ mm
Initial	– 50.6	– 38.59	+ 50.36	+ 38.36
Due to fluid pressure	+ 156.46	+ 100.37	+ 100.37	76.45
Final	+ 105.86	+ 61.78	+ 150.73	114.81

Que 5.26. **A compound cylinder is composed of a tube of 250 mm internal diameter and 25 mm thick, shrunk on a tube of 200 mm internal diameter and 250 mm external diameter. The interface radial pressure at the junction is 8 N/mm² due to shrinking. Then the compound cylinder is subjected to an internal pressure of 60 N/mm². Find the variation in hoop stresses over the thickness of compound cylinder.**

AKTU 2015-16, Marks 10

Answer

Given : A compound cylinder,
Outer cylinder : $d_i = 250$ mm $\quad d_o = 300$ mm
Inner cylinder : $d'_i = 200$ mm $\quad d'_0 = 250$ mm
$P_i = 60$ MPa $\quad P = 8$ MPa
To Find : Variation of hoop stress.

A. Shrinkage Stresses :

1. Outer cylinder, pressure P is internal,

$$\sigma_{c250} = \frac{d_0^2 + d_i^2}{d_0^2 - d_i^2} \times P = \frac{300^2 + 250^2}{300^2 - 250^2} \times 8 = 44.36 \text{ MPa}$$

$$\sigma_{c300} = \frac{2 \times d_i^2}{d_0^2 - d_i^2} \times P = \frac{2 \times 250^2}{300^2 - 250^2} \times 8 = 36.3636 \text{ MPa}$$

2. Inner cylinder, pressure P is external,

$$\sigma_{c250} = \frac{d_0^2 + d_i'^2}{d_0^2 - d_i'^2} \times P = -\frac{250^2 + 200^2}{250^2 - 200^2} \times 8 = -36.44 \text{ MPa}$$

$$\sigma_{c200} = \frac{2\, d_i'^2}{d_0^2 - d_i'^2} \times P = -\frac{2 \times 200}{250^2 - 200^2} \times 8 = -28.44 \text{ MPa}$$

3. Stress due to pressure :

$$\sigma_{c200} = \frac{d_o^2 + d_i'^2}{d_o^2 - d_i'^2} \times P_i = \frac{300^2 + 200^2}{300^2 - 200^2} \times 60 = 156 \text{ MPa}$$

$$\sigma_{c300} = \frac{2d_i'^2}{d_0^2 - d_i'^2} \times P_i = \frac{2 \times 200^2}{300^2 - 200^2} \times 60 = 96 \text{ MPa}$$

$$\sigma_{c250} = \frac{d_0^2 - d_i^2}{d_0^2 - d_i'^2} \times \frac{d_i'^2}{d_i^2} \times P = \frac{300^2 + 250^2}{300^2 - 200^2} \times \frac{200^2}{250^2} \times 60 = 117.12 \text{ MPa}$$

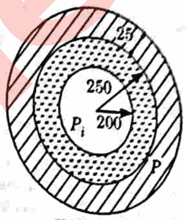


Fig. 5.26.1.

B. Final Stresses :

1. The final stresses are summed in table

	Inner cylinder		Outer Cylinder	
	200 mm	250 mm	250 mm	300 mm
Shrinkage stresses (MPa)	– 28.44	– 36.44	44.36	36.3636
Stresses due to pressure	156	117.12	117.12	96
Final stresses (MPa)	127.56	80.68	161.48	132.3636

2. Variation in hoop stresses over thickness of compound cylinder,

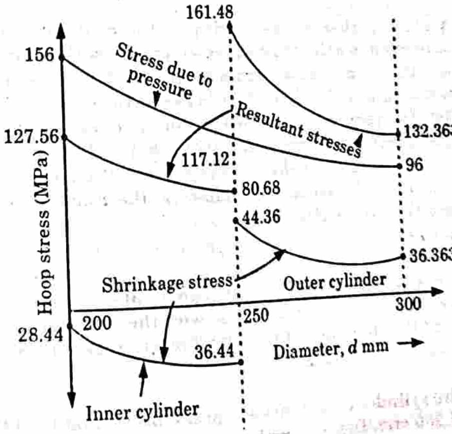


Fig. 5.27.2.

VERY IMPORTANT QUESTIONS

Following questions are very important. These questions may be asked in your SESSIONALS as well as UNIVERSITY EXAMINATION.

Q. 1. A close coil helical spring of round steel wire 10 mm in diameter and 10 complete turns with a mean diameter of 120 mm and subjected to an axial load of 200 N. Determine (a) deflection of the spring (b) stiffness of the spring (c) maximum shear stress, and (d) strain energy stored in spring.

Ans. Refer Q. 5.3, Unit-5.

Q. 2. A closed coil helical spring made of 8 mm diameter has 12 coils of 150 mm mean diameter. Calculate the elongation, torsional stress and strain energy per unit volume when the spring is subjected to an axial load of 120 kN. Take modulus of rigidity as 80 GPa. If a torque of 9 kN-m is applied in place of axial load, find axial twist, bending stress and strain energy per unit volume. Take modulus of elasticity as 200 GPa.

Ans. Refer Q. 5.6, Unit-5.

Q. 3. A close coiled helical spring is fixed at one end and subjected to axial twist at the other. When the spring is in use the axial torque varies from 0.75 N-m to 3 N-m, the working angular deflection between these torques being 35°. The spring is to be made from rod of circular section, the maximum permissible stress being 150 MN/m². The mean diameter of the coils is 8 times the rod diameter. Calculate the mean coil diameter, the number of turns and the wire diameter.

Ans. Refer Q. 5.7, Unit-5.

Q. 4. For a tube having $E = 2 \times 10^5$ N/mm² and $\mu = 0.3$ the hoop stress at the inner surface is twice the internal pressure. Find the thickness of the wall if internal radius is 60 mm.

Ans. Refer Q. 5.16, Unit-5.

Q. 5. The cylinder for a hydraulic press has an inside diameter of 300 mm. Determine the wall thickness required if the cylinder is to withstand an internal pressure of 60 MPa without exceeding a shearing stress of 90 MPa.

Ans. Refer Q. 5.17, Unit-5.

Q. 6. A cylindrical shell 90 cm long 20 cm internal diameter having thickness of metal as 8 mm is filled with fluid at atmospheric pressure. If an additional 20 cm³ of fluid is pumped into the cylinder, find

A. The pressure exerted by the fluid on the cylinder, and

B. The hoop stress induced. Take $E = 200$ GPa and $\mu = 0.3$

Ans. Refer Q. 5.20, Unit-5.

Q. 7. Derive the equations for circumferential stress and volumetric strain in a thin spherical shell under internal pressure.

Ans. Refer Q. 5.21, Unit-5.

Q. 8. Write down the assumption in Lame's theory and also derive its equation for thick shell.

Ans. Refer Q. 5.22, Unit-5.

Q. 9. In an experiment on a thick cylinder of 100 mm external diameter and 50 mm internal diameter the hoop and longitudinal strains as measured by strain gauges applied to the outer surface of the cylinder were 240×10^{-6} and 60×10^{-6} respectively, for an internal pressure of 90 MN/m², the external pressure being zero.
Determine the actual hoop and longitudinal stresses present in the cylinder if $E = 208$ GN/m² and $\upsilon = 0.29$. Compare the hoop stress value so obtained with the theoretical value given by the Lame's equations.

Ans. Refer Q. 5.23, Unit-5.

Q. 9. A compound steel tube is composed of a tube 200 mm internal diameter and 30 mm thickness, shrunk on a tube of 200 mm external diameter and 25 mm thickness. The radial pressure of the junction is 12 N/mm². The composed tube is subjected to an internal fluid pressure of 80 N/mm². Find the variation of the hoop stress over the wall of the compound tube.

Ans. Refer Q. 5.25, Unit-5.

☺☺☺

1 UNIT

Simple and Compound Stress and Strains (2 Marks Questions)

1.1. What do you mean by stress ?

Ans. The force of resistance per unit area, offered by a body against deformation is known as stress.

1.2. What is strain ?

Ans. The ratio of change of dimension of the body to the original dimension of body is known as strain.

1.3. What do you mean by shear strain ?

Ans. Shear strain is defined as the change in the right angle of the element measured in radians and is dimensionless.

1.4. Explain the following terms :
i. Young's modulus.
ii. Modulus of rigidity.

Ans.
i. **Young's Modulus :** It is the ratio between tensile stress and tensile strain or compressive stress and compressive strain.
ii. **Modulus of Rigidity :** It is defined as the ratio of shear stress to shear strain.

1.5. Define bulk modulus of elasticity.

Ans. Bulk modulus of elasticity is defined as the ratio of normal stress to volumetric strain.

1.6. Define strain energy.

Ans. The work done by the load in straining the material of a body is stored within it in the form of energy known as strain energy.

1.7. Differentiate between resilience and proof resilience.

AKTU 2016-17, Marks 02

Ans.

Resilience	Proof Resilience
The strain energy stored by the body within elastic limit, when loaded externally is called resilience.	The maximum strain energy stored in a body upto elastic limit is known as proof resilience.

1.8. Define the modulus of resilience.

Ans. Modulus of resilience is defined as the proof resilience of a material per unit volume.

$$\text{Modulus of resilience} = \frac{\text{Proof resilience}}{\text{Volume of the body}}$$

1.9. What is the difference between impact loading and gradual loading ?

Ans. In impact loading, the maximum stress induced is twice the stress induced during gradual loading.

1.10. What are principal stresses and strains ?

AKTU 2016-17, Marks 02

Ans. **Principal Stresses :** In complex systems of loading, there exist three mutually perpendicular planes, on each of which the resultant stress is wholly normal. These are known as principal planes and the normal stresses across these planes are known as principal stresses.

Principal Strain : Maximum and minimum normal strain possible for a specific point on a structure element. Shear strain is zero at the orientation where principal strain occurs.

1.11. Explain Mohr's circle.

Ans. Mohr's circle is a graphical method for finding out the normal and shear stresses on any interface of an element when it is subjected to two perpendicular stresses.

1.12. What do you mean by major principal plane and major principal stress ?

Ans. The plane carrying the maximum normal stress is called the major principal plane and the stress acting on it is called the major principal stress.

1.13. What do you mean by minor principal plane and minor principal stress ?

Ans. The plane carrying the minimum normal stress is called the minor principal plane and the stress acting on it, is called the minor principal stress.

1.14. What is Hook's law ? Explain. **AKTU 2016-17, Marks 02**

Ans. **Hook's Law :** It states, "when a material is loaded, within its elastic limit, the stress is proportional to the strain." Mathematically,

$$\frac{\text{Stress}}{\text{Strain}} = E \text{ (constant)}$$

It may be noted that Hook's law equally hold good for tension as well as compression.

1.15. What are thermal stress and thermal strain ? **AKTU 2016-17, Marks 02**

Ans.

i. Thermal strain, $\varepsilon_T = \dfrac{\text{Change in length due to temperature}}{\text{Original length}}$

$$= \frac{dL}{L} = \frac{\alpha TL}{L} = \alpha T$$

ii. Thermal stress = Thermal strain × $E = \alpha TE$

Thermal stress is also known as temperature stress and thermal strain is also known as temperature strain.

1.16. Differentiate between strain energy and shear strain energy. **AKTU 2018-19, Marks 02**

Ans.

S. No.	Strain Energy	Shear Strain Energy
1.	Energy due to change in size.	Energy due to change in shape.
2.	This is caused by normal stress acting on the material.	This is caused by shear stresses acting on the material.

1.17. An unknown weight falls by 30 mm onto a collar rigidly attached to the lower end of a vertical bar 4 m long and 1000 mm² in section. If the maximum instantaneous extension is found to be 3.66 mm. Find corresponding weight. $E = 2 \times 10^5$ N/mm². **AKTU 2015-16, Marks 02**

Ans.

Given : $h = 30$ mm, $l = 4$ m, $A = 1000$ mm², $\Delta l = 3.66$ mm
$E = 2 \times 10^5$ N/mm²
To Find : Unknown weight.

1. Stress due to impact load is given by,

$$\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2hAE}{Wl}}\right]$$

We know that, $\sigma = E \times \left(\dfrac{\Delta l}{l}\right)$

then, $E \times \left(\dfrac{\Delta l}{l}\right) = \dfrac{W}{A}\left[1 + \sqrt{1 + \dfrac{2hAE}{Wl}}\right]$

$$2 \times 10^5 \times \frac{3.66}{4000} = \frac{W}{1000}\left[1 + \sqrt{1 + \frac{2 \times 30 \times 1000 \times 2 \times 10^5}{W \times 4000}}\right]$$

$$\frac{183000}{W} - 1 = \sqrt{1 + \frac{3000000}{W}}$$

$$W = 9.949 \text{ kN} \approx 10 \text{ kN}$$

1.18. How does the complimentary shear stress affects the anisotropic materials ?

Ans. The presence of complimentary shear stress may cause an early failure of anisotropic materials.

1.19. When does the principle of superposition is not applicable to materials ?

Ans. The principle of superposition is not applicable to materials with non-linear stress-strain characteristics which do not follow Hooke's law.

1.20. Why stresses are called tensor ? **AKTU 2015-16, Marks 02**

Ans. Stresses are called tensor because they can produce strain in all three directions.

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2 UNIT

Shear Force and Bending Moment Diagrams (2 Marks Questions)

2.1. Define shear force.

Ans. Shear force is the force that tries to shear off the section of a beam (or structure). It is obtained as algebraic sum of all forces acting normal to axis of beam, either to the left or to the right of section.

2.2. Define bending moment.

Ans. Bending moment is the moment that tries to bend the beam (or structure) and is obtained as algebraic sum of moment of all forces about the section acting either left or to the right of section.

2.3. Define SFD and BMD.

Ans. **SFD :** SFD stands for Shear Force Diagram. It represents the variation of shear force along the length of the beam.

BMD : BMD stands for Bending Moment Diagram. It represents the variation of bending moment along the length of beam.

2.4. What is the relationship between load, shear force and bending moment ?

Ans. **Relationship between Load (w), Shear Force (F) and Bending Moment (M) :**

$$\frac{dF}{dx} = -w$$

Here, $\frac{dF}{dx}$ is the slope of shear force diagram and its numerical value at any point is equal to the load per unit length at that point.

$$\frac{dM}{dx} = F$$

Here, $\frac{dM}{dx}$ is the slope of bending moment diagram and its value is equal to shear force at that point.

2.5. What do you understand by the term point of contraflexure ?

Ans. The point of contraflexure is a point which represents the section on the beam where bending moment is zero or bending moment changes its sign.

2.6. Define point load, UDL and UVL.

Ans.

i. **Point Load :** A point load is one which acts over a very small area or portion.
Here, W is a point load.

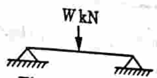

Fig. 1. Point load.

ii. **UDL :** UDL acts over a finite length of a beam. UDL implies the intensity of loading is constant over a finite length and its unit is kN / m.

iii. **UVL :** UVL implies the intensity of loading increases or decreases linearly (at constant rate) along the length of beam.

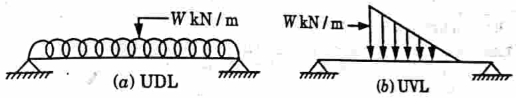

Fig. 2.

2.7. What is the shape of bending moment diagram for a cantilever beam subjected to bending moment at end of the beam ?

Ans. The shape of bending moment diagram for a cantilever beam subjected to bending moment at end of the beam is rectangle.

2.8. If only couples are applied at the end of the beam (*i.e.*, no forces), which type of stress will get produced in beam ?

Ans. A beam subjected to only couples is known as a pure bending. In case of pure bending only normal stresses are produced.

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3 UNIT Flexure Stresses, Torsion and Shear Stresses (2 Marks Questions)

3.1. What do you mean by bending stresses ?

Ans. The stresses produced due to bending moment are known as bending stresses.

3.2. Write down the bending equation for the beam.

Ans. The bending equation is given by,

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

3.3. Define pure bending.

Ans. When the beam is subjected to a constant bending moment, the beam bends or tries to bend into circular arc, such type of bending is known as pure bending.

3.4. What is neutral axis ?

Ans. Neutral axis is defined as the line of intersection of the neutral layer with the transverse section.

3.5. What do you mean by composite beams ?

Ans. A beam made up of two or more different materials assumed to be rigidly connected together behaving like a single unit, is known as composite beam.

3.6. Define neutral surface.

Ans. The surface of the beam which is neither in tension nor in compression (*i.e.*, bending stresses are zero) is known as neutral surface.

3.7. Write down any two assumptions of simple theory of bending ?

Ans. Following are the two assumptions of simple theory of bending :

i. The material of beam is homogeneous.

ii. A plane before bending remains plane after bending.

3.8. Define torsion.

Ans. Torsion refers to the twisting of a structural member when it is loaded by couples that produce rotation about its longitudinal axis.

3.9. What are the different torques acting on the shaft during power transmission ?

Ans. Shaft is subjected to following torques during power transmission :

i. Driving torque at the input end.

ii. Resisting torque at the output end.

3.10. Write down the torsion equation for circular shafts.

Ans. The torsion equation is given by,

$$\frac{\tau}{R} = \frac{G\theta}{l}$$

3.11. Define modulus of rupture.

Ans. It is defined as the stress at which the material breaks.

3.12. Write down any two assumptions made for a circular shaft subjected to torsion.

Ans. Following assumptions are made for a circular shaft subjected to torsion :

i. The twist along the length of shaft is uniform throughout.

ii. The material of shaft is uniform throughout.

3.13. What do you mean by polar modulus ?

Ans. Polar modulus is defined as the ratio of the polar moment of inertia to the radius of the shaft.

3.14. What do you mean by strength of a shaft ?

Ans. The strength of a shaft means the maximum torque or maximum power that the shaft can transmit.

3.15. What do you understand by torsional rigidity ?

AKTU 2017-18, Marks 02

Ans. Torsional rigidity of the shaft is defined as the product of modulus of rigidity and polar moment of inertia of the shaft.

3.16. Compare the strength of hollow and solid shaft.

AKTU 2015-16, Marks 02

Ans. Let us assume that both the shafts have same length, material, weight and hence the same maximum shear stress.

Then, $$\frac{\text{Strength of hollow shaft}}{\text{Strength of solid shaft}} = \frac{T_H}{T_S} = \frac{n^2+1}{n\sqrt{n^2-1}}$$

where, $n = \frac{\text{External diameter}}{\text{Internal diameter}}$

Since, n is always greater than 1. Hence, strength of hollow shafts is greater than solid shaft.

3.17. How does the bending stress vary with the distance of the layer from the neutral axis ?

Ans. The bending stress in any layer is directly proportional to the distance of the layer from the neutral axis.

3.18. Which shaft is more stronger for same length, material and weight ?

Ans. Hollow shaft is stronger than solid shaft for same length, material and weight.

3.19. Different between the terms section modulus and flexural rigidity.

AKTU 2018-19, Marks 02

Ans.

S. No.	Section Modulus	Flexural Rigidity
1.	Section modulus is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis.	The product EI is an index of the bending (flexural) strength of an element is called the flexural rigidity of the element.
2.	It is given by, $Z = I / y_{max}$.	It is the product of elastic modulus and moment of inertia of the section.

3.20. Explain :

i. Section modulus.

ii. Modular ratio.

AKTU 2016-17, Marks 02

Ans. **Section Modulus :** It is defined as the ratio of total moment resisted by the section to the stress in the extreme fibre which is equal to yield stress.

Modular Ratio : It is the ratio of Young's modulli of elasticity of two different materials in construction by composite materials.

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4 UNIT

Deflection of Beams, Short Columns and Struts (2 Marks Questions)

4.1. Define elastic curves.

Ans. Under the action of load the neutral axis becomes a curved line and is called the elastic curve.

4.2. What are the different methods to determine the deflection of cantilever beam ?

AKTU 2017-18, Marks 02

Ans. Following are the different methods to determine the deflection of cantilever beams :

i. Moment area method.
ii. Macaulay's method.
iii. Double integration method.

4.3. What are flitched beam and fixed beam ?

AKTU 2016-17, Marks 02

Ans. **Flitched Beam :** Beam made up to two different materials such as wooden beams reinforced by steel plates are known as flitched.

Fixed Beam : Fixed beam is a beam in which both ends are constrained or built-in to remain in horizontal position.

4.4. Define continuous beam.

Ans. A continuous beam is one which is supported on more than two supports.

4.5. What are the advantages of fixed beams over simply supported beams ?

Ans. Following are the advantages of a fixed beam over a simply supported beam :

i. For the same loading, the maximum deflection of a fixed beam is less than that of a simply supported beam.
ii. The slope at both ends of a fixed beam is zero.
iii. The beam is more stable and stronger.

4.6. Which method is used to determine the deflection of beam under several loads ?

Ans. Macaulay's method is used to determine the deflection under several loads.

4.7. Which methods are used to determine deflection under single load ?

Ans. Double integration method and moment area method are used to determine deflection under single load.

4.8. Write relation for maximum deflection and slope for simply supported beam subjected to uniformly distributed load over the whole span. **AKTU 2015-16, Marks 02**

Ans. For maximum deflection, $y_{max} = \dfrac{5Wl^4}{384\,EI}$

For maximum slope, $\theta_{max} = \dfrac{Wl^3}{24\,EI}$

4.9. What is the difference between column and strut ? **AKTU 2016-17, Marks 02**

Ans.

Column	Strut
A column is a long vertical slender bar or vertical member, subjected to an axial compressive load and fixed rigidly at both ends.	A strut is a slender bar or member in any position other than vertical, hinged or pin joined at one or both the ends.

4.10. What is slenderness ratio and equivalent length of a column ? **AKTU 2016-17, Marks 02**

OR

Explain slenderness ratio and its importance in case of column. **AKTU 2018-19, Marks 02**

Ans. **Slenderness Ratio :** It is the ratio of unsupported length of the column to the minimum radius of gyration of the cross-sectional ends of the column. It is dimensionless and denoted by '*k*'.

Importance :

i. The slenderness ratio of a column gives an indication of buckling failure in the column.

ii. More the slenderness ratio more is the tendency of column to fail by buckling effect in that direction.

iii. The slenderness ratio is taken in different directions but the direction in which the moment of inertia is minimum gives maximum slenderness ratio as the radius of gyration will be minimum in such case.

Equivalent Length : The distance between adjacent points of inflexion is called equivalent length or effective length or simple column length. A point of inflexion is found at every column end that is free to rotate and at every point where there is a change of the axis.

4.11. What is crippling load ?

Ans. The maximum limiting load at which the column tends to have lateral displacement or tends to buckle is called crippling or buckling load.

4.12. What do you understand by the term "buckling" of columns ? **AKTU 2017-18, Marks 02**

Ans. When the long column is subjected to a compressive load, it is subjected to a compressive stress and if the load is gradually increased, the column will reach a stage when it will start bending. This bending is called buckling.

4.13. What are the limitations of Euler's formula ? **AKTU 2015-16, Marks 02**

Ans. Following are the limitations for the use of Euler's formula :

i. It is applicable to an ideal strut only.

ii. It takes no account of direct stress.

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5 UNIT

Springs, Cylinders and Spheres (2 Marks Questions)

5.1. What is spring ? What are different types of spring ?

AKTU 2016-17, Marks 02

Ans. **Spring :** These are elastic members which distort under load and regain their original shape when load is removed.

Types of Spring : Following are the different types of springs :

i. Helical springs.
ii. Leaf springs.
iii. Torsion springs.
iv. Circular springs.
v. Belleville springs.
vi. Flat springs.

5.2. Write down the functions of springs.

Ans. Following are the functions of spring :

i. To absorb shock or impact loading.
ii. To measure forces as in spring balances.
iii. To store energy as in clock springs.

5.3. What do you mean by leaf spring ?

Ans. A leaf spring is made up of a number of parallel plates of varying length, but having the same width and thickness, strapped together.

5.4. What is bending spring ?

Ans. When a spring is subjected to bending only and the resilience is also due to it, such a spring is known as bending spring.

5.5. What do you mean by resilience of the spring ?

Ans. Resilience of the spring is defined as the capacity of the spring for storing energy without exceeding a certain stress limit.

5.6. What are the various stresses induced in closed coil helical spring ?

AKTU 2015-16, Marks 02

Ans. Following are the various stresses induced in closed coil helical spring :

i. Direct shear stress.
ii. Torsional stress.

5.7. Define stiffness.

Ans. The stiffness of the spring is defined as the load required to produce unit deflection.

5.8. Define spring index.

Ans. Spring index is defined as the ratio of mean coil diameter to wire diameter. It is generally given by, $C = \frac{D}{d}$

5.9. What are closed helical springs ?

Ans. Closed helical springs are those in which the angle of helix is so small that the coils may be assumed to be in a horizontal plane if the axis of the spring is vertical.

5.10. Write the relation for axial deflection for open coil helical spring subjected to axial twist.

AKTU 2015-16, Marks 02

Ans. Axial deflection, $\delta = 2\pi TR^2 n \sin\alpha \left[\frac{1}{GJ} - \frac{1}{EI}\right]$

5.11. What is the difference in analysis of closed and open coiled springs ?

AKTU 2018-19, Marks 02

Ans.

S. No.	Open Coiled Spring	Closed Coiled Spring
1.	It is known as compression spring.	It is known as torsion spring.
2.	Torsional stress is extremely low.	Torsional stress is extremely high.

5.12. Give one application of bending spring and one application of torsion spring ?

Ans.

i. Bending springs are used in railway wagons.
ii. Torsion springs are used in clocks.

5.13. Which type of stress is acted on the leaf spring ?

Ans. Bending stress is acted on the leaf spring.

5.14. Which types of spring are used to carry axial tension ?

Ans. Generally, the closed coiled springs are used to carry axial tension.

5.15. What is thin cylinder ?

Ans. If the thickness of the wall of the cylindrical vessel is less than 1/20 of its internal diameter, the cylindrical vessel is known as a thin cylinder.

5.16. What do you mean by circumferential stress ?

Ans. The stress act in a tangential direction to the circumference of the shell is known as circumferential stress.

5.17. Define longitudinal stress.

Ans. The stress acting along the length of the cylinder *i.e.*, in the longitudinal direction, is known as longitudinal stress.

5.18. Establish the relation between circumferential stress and longitudinal stress.

Ans. Longitudinal stress = $\dfrac{\text{Circumferential stress}}{2}$

5.19. Differentiate between thick and thin cylinder.

AKTU 2015-16, Marks 02

OR

What is the difference between thin and thick cylinder ?

AKTU 2017-18, Marks 02

Ans.

S. No.	Thick Cylinder	Thin Cylinder
1.	The thickness of cylindrical vessel is greater than $\frac{1}{20}$ of its internal diameter.	The thickness of the cylindrical vessel is less than $\frac{1}{15}$ to $\frac{1}{20}$ of its internal diameter.
2.	The stresses are not uniform rather it varies along the thickness.	Stresses are assumed to be uniform throughout the wall thickness

5.20. What is wire winding of thin cylinder ?

AKTU 2015-16, Marks 02

Ans. Wire winding is the process in which a thin cylinder is strengthened against the internal pressure by winding it with wire under tension and putting the cylinder wall in compression.

5.21. Write down any two assumptions of Lame's theory.

Ans. Following assumptions are made in Lame's theory :
i. Material is homogeneous and isotropic.
ii. Material is stressed within elastic limit.

5.22. How can we obtain the variation in the radial as well as circumferential stresses across the thick cylinder ?

Ans. The variation in the radial as well as circumferential stresses across the thick cylinder is obtained with the help of Lame's theory.

5.23. For what purpose, cylinders are compounded ?

Ans. The cylinders are compounded with a purpose to increase the pressure bearing capacity of a single cylinder.

5.24. Why is it necessary to strengthen the cylinder longitudinally ?

Ans. The chances of bursting the cylinder longitudinally are more than those for circumferential failure of the cylinder. Therefore, it is necessary to strengthen the cylinder longitudinally.

5.25. Why is wire winding used in thin cylinders ?

Ans. Wire winding is used in thin cylinders for following purposes :
i. To increase the pressure carrying capacity of the cylinder.
ii. To reduce the chances of longitudinal burst.

5.26. At which part of the thick cylinder the hoop stress is maximum and minimum ?

Ans. The hoop stress is maximum at the inner circumference and minimum at the outer circumference of a thick cylinder.

5.27. What do you understand by autofrettage ? Explain it in brief.

AKTU 2018-19, Marks 02

Ans. **Autofrettage :** It is metal fabrication technique in which a pressure vessel is subjected to enormous pressure, causing internal portions of the part to yield plastically, resulting in internal compressive residual stresses once the pressure is released.

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B. Tech.
(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2014-15
MECHANICS OF SOLIDS

Time : 3 Hours **Max. Marks : 100**

Note : 1. Attempt all the question.
2. Notations used have usual meaning.

1. Attempt any four parts of the following : **(5 × 4 = 20)**

a. **Prove that the maximum shear stress in the body is the half of the difference between maximum principal and minimum principal stress.**

Ans. Refer Q. 1.32, Page 1–31A, Unit-1.

b. **Drive the expression for extension in the vertically suspended bar due to self weight.**

Ans. Refer Q. 1.15, Page 1–14A, Unit-1.

c. **Find the free end deflection in cantilever beam with uniformly distributed load by Macaulay's method.**

Ans. Refer Q. 4.4, Page 4–6A, Unit-4.

d. **A mild steel hollow cylinder has diameter to thickness ratio of 25. Find the internal pressure to which the cylinder should be subjected so that its volume is increased by 5×10^{-4} of its original volume. Take $E = 2 \times 10^5$ and $\mu = 0.3$.**

Ans. Refer Q. 5.19, Page 5–20A, Unit-5.

e. **Under what conditions unsymmetrical bending occurs in a beam. Also state the position of neutral axis.**

Ans. This question is out of syllabus from sessions 2019-20.

f. **Derive the expression of the value of constant (h^2) in curved beam for rectangular cross section area beam.**

Ans. This question is out of syllabus from sessions 2019-20.

2. Attempt any two parts of the following : **(10 × 2 = 20)**

a. **At a point in a strained material, stresses are applied as shown in Fig. 1, find out the normal and shear stress on the oblique plane, principal stresses and principal strain.**

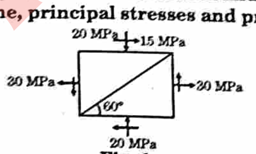

Fig. 1.

Ans. Refer Q. 1.40, Page 1–41A, Unit-1.

b. **The load on a bolt consists of an axial pull of 20 kN together with a transverse shear of 10 kN, calculate the diameter of bolt according to :**
1. **Maximum total strain energy theory, and**
2. **Maximum shear strain energy theory (if $\mu = 0.3$).**

(Take elastic limit in tension 280 MPa and factor of safety = 3)

Ans. Refer Q. 1.44, Page 1–49A, Unit-1.

c. **Write the assumptions for pure bending and also derive the equation for bending.**

Ans. Refer Q. 3.1, Page 3–2A, Unit-3.

3. Attempt any two parts of the following : **(10 × 2 = 20)**

a. **Determine the deflection at the mid and slope at the end of the beam in terms of EI for a beam as shown in Fig. 2.**

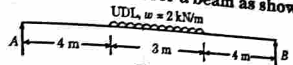

Fig. 2.

Ans. Refer Q. 4.5, Page 4–7A, Unit-4.

b. **A solid steel shaft 60 mm diameter is fixed rigidly and coaxially inside a bronze sleeve 90 mm external diameter. Calculate the angle of twist in a length of 2 m of the composite shaft due to action of a torque of 1 kN-m. Take G (steel) = 80 GPa, G (bronze) = 42 GPa.**

Ans. Refer Q. 3.20, Page 3–22A, Unit-3.

c. **A shearing force of 180 kN acts over a T-section shown in Fig. 3. Draw the shear stress distribution curve. (Take $I = 1.134 \times 10^8$ mm^4).**

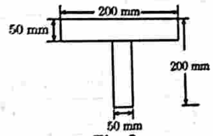

Fig. 3.

Ans. Refer Q. 3.30, Page 3–33A, Unit-3.

4. Attempt any two part of the following : **(10 × 2 = 20)**

a. **From the first principles derive the expression for the critical buckling for a column having both end fixed.**

Ans. This question is out of syllabus from sessions 2019-20.

b. **A closed coil helical spring made of 8 mm diameter has 12 coils of 150 mm mean diameter. Calculate the elongation, torsional stress and strain energy per unit volume when the spring is subjected to an axial load of 120 kN. Take modulus of rigidity as 80 GPa. If a torque of 9 kN-m is applied** in place of axial load, find axial twist, bending stress and strain energy per unit volume. Take modulus of elasticity as 200 GPa.

Ans. Refer Q. 5.6, Page 5–7A, Unit-5.

c. **A compound steel tube is composed of a tube 200 mm internal diameter and 30 mm thickness, shrunk on a tube of 200 mm external diameter and 25 mm thickness. The radial pressure of the junction is 12 N/mm². The composed tube is subjected to an internal fluid pressure of 80 N/mm². Find the variation of the hoop stress over the wall of the compound tube.**

Ans. Refer Q. 5.25, Page 5–28A, Unit-5.

5. Attempt any two parts of the following : **(10 × 2 = 20)**

a. **A crane hook trapezoidal horizontal cross-section is 50 mm wide inside and 30 mm wide outside. Thickness of the section is 60 mm. The crane hook carries a vertical load of 20 kN whose line of action is 50 mm from the inside edge of the section. The center of curvature is 60 mm from the inside edge. Determine the maximum tensile and compressive stresses in the section.**

Ans. This question is out of syllabus from sessions 2019-20.

b. **If principal moments of inertia of section are I_{uu} and I_{vv} and X and Y axes inclined to an angle θ to U-V axis, then prove that**

$$I_{xx} + I_{yy} = I_{uu} + I_{vv}$$

Ans. This question is out of syllabus from sessions 2019-20.

c. **A simply supported I-section beam of span 1.5 m carries a concentrated load of 8 kN at an angle of 20° from vertical as shown in Fig. 4. Load passes through the centroid of the section. Determine the maximum bending stress in the beam.**

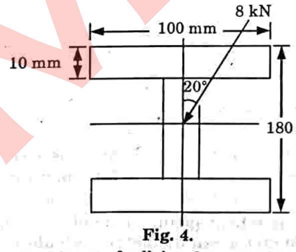

Fig. 4.

Ans. This question is out of syllabus from sessions 2019-20.

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B. Tech.
(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2015-16
MECHANICS OF SOLIDS

Time : 3 Hours **Max. Marks : 100**

Section – A

1. Attempt **all** parts. All parts carry equal marks. Write answer of each part in short. **(2 × 10 = 20)**

a. **Why stresses are called tensor ?**

Ans. Refer Q. 1.20, 2 Marks Questions, Page SQ–4A, Unit-1.

b. **What are the limitations of Euler's formula ?**

Ans. Refer Q. 4.13, 2 Marks Questions, Page SQ–12A, Unit-4.

c. **What is wire winding of thin cylinder ?**

Ans. Refer Q. 5.20, 2 Marks Questions, Page SQ–15A, Unit-5.

d. **What are the various stresses induced in closed coil helical spring ?**

Ans. Refer Q. 5.6, 2 Marks Questions, Page SQ–13A, Unit-5.

e. **Define shear centre.**

Ans. This question is out of syllabus from sessions 2019-20.

f. **Difference between thin and thick cylinder.**

Ans. Refer Q. 5.19, 2 Marks Questions, Page SQ–15A, Unit-5.

g. **Write the relation for axial deflection for open coil helical spring subjected to axial twist.**

Ans. Refer Q. 5.10, 2 Marks Questions, Page SQ–14A, Unit-5.

h. **Compare the strength of hollow and solid shaft.**

Ans. Refer Q. 3.16, 2 Marks Questions, Page SQ–8A, Unit-3.

i. **An unknown weight falls by 30 mm on to a collar rigidly attached to the lower end of a vertical bar 4 m long and 1000 mm² in section. If the maximum instantaneous extension is found to be 3.66 mm. Find corresponding weight. $E = 2 \times 10^5$ N/mm².**

Ans. Refer Q. 1.17, 2 Marks Questions, Page SQ–3A, Unit-1.

j. **Write relation for maximum deflection and slope for simply supported beam subjected to uniformly distributed load over the whole span.**

Ans. Refer Q. 4.8, 2 Marks Questions, Page SQ–11A, Unit-4.

Section – B

Attempt any **five** questions from this section. **(10 × 5 = 50)**

2. For a given loading conditions the state of stress in the wall of a cylinder is expressed as follows :
i. 85 MN/m² tensile,
ii. 25 MN/ m² tensile at right angles to (*i*), and
iii. Shear stresses of 60 MN/m² on the planes on which the stresses (*i*) and (*ii*) act ; the shear couple acting on planes carrying the 25 MN/m² stress is clockwise in effect.
Calculate principal stresses and principal planes.

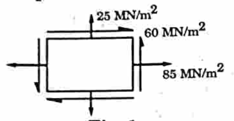

Fig. 1.

Ans. Refer Q. 1.33, Page 1–35A, Unit-1.

3. State the generalized Hooke's law and prove for an anisotropic elastic material that the maximum number of independent elastic constants is 21 only. Also show that for isotropic materials it is 2.

Ans. Refer Q. 1.42, Page 1–44A, Unit-1.

4. Derive the deflection for cantilever beam loaded with uniformly distributed load.

Ans. Refer Q. 4.4, Page 4–6A, Unit-4.

5. Determine equivalent bending moment and equivalent torque for the shafts subjected to combined bending and torsion.

Ans. Refer Q. 3.23, Page 3–25A, Unit-3.

6. A close coiled helical spring is to carry a load of 5000 N with a deflection of 50 mm and a maximum shearing stress of 400 N/mm², if the number of active turns of active coils is 8. Estimate the following :
a. Wire diameter, b. Mean coil diameter, and
c. Weight of the spring.
Assume G = 83000 N/mm², Specific weight ρ = 7700 kg/m³

Ans. Refer Q. 5.2, Page 5–3A, Unit-5.

7. A leaf spring is made of plates 50 mm wide and 8 mm thick. The spring has a span of 700 mm. Determine the number of plates required to carry a central load of 45 kN. The maximum allowable stress in the plates is 200 MPa. What is the maximum deflection under this load ?

Ans. Refer Q. 5.4, Page 5–5A, Unit-5.

8. Derive the equations for circumferential stress and volumetric strain in a thin spherical shell under internal pressure.

Ans. Refer Q. 5.21, Page 5–21A, Unit-5.

9. Determine the shear centre for the channel section as shown in Fig. 2.

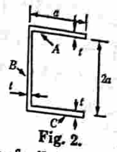

Fig. 2.

Ans. This question is out of syllabus from sessions 2019-20.

Section – C

Attempt any two questions from this section. (15 × 2 = 30)

10. a. A compound cylinder is composed of a tube of 250 mm internal diameter and 25 mm thick, shrunk on a tube of 200 mm internal diameter and 250 mm external diameter. The interface radial pressure at the junction is 8 N/mm² due to shrinking. Then the compound cylinder is subjected to an internal pressure of 60 N/mm². Find the variation in hoop stresses over the thickness of compound cylinder.

Ans. Refer Q. 5.26, Page 5–30A, Unit-5.

b. Define product of inertia and principal moment of inertia.

Ans. This question is out of syllabus from sessions 2019-20.

11. a. Derive the bending equation for a beam subjected to bending moment M in pure bending condition. Also state the assumptions.

Ans. Refer Q. 3.1, Page 3–2A, Unit-3.

b. A close coil helical spring of round steel wire 10 mm in diameter and 10 complete turns with a mean diameter of 120 mm and subjected to an axial load of 200 N. Determine (*i*) deflection of the spring (*ii*) stiffness of the spring (*iii*) maximum shear stress and (*iv*) strain energy stored in spring.

Ans. Refer Q. 5.3, Page 5–4A, Unit-5.

12. Define the following terms :
a. Draw the stress-strain diagram for mild steel under tensile load.

Ans. Refer Q. 1.3, Page 1–4A, Unit-1.

b. Write down assumption in Euler's theory for column.

Ans. This question is out of syllabus from sessions 2019-20.

c. Name different theories of failure and represent them graphically.

Ans. Refer Q. 1.43, Page 1–46A, Unit-1.

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B. Tech.
(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2016-17
MECHANICS OF SOLIDS

Time : 3 Hours **Max. Marks : 100**

Note : Attempt questions as per instructions.

Section – A

1. Attempt all parts of the following : **(2 × 10 = 20)**

a. **What is Hooke's law ? Explain.**
Ans. Refer Q. 1.14, 2 Marks Questions, Page SQ–3A, Unit-1.

b. **What are thermal stress and thermal strain ?**
Ans. Refer Q. 1.15, 2 Marks Questions, Page SQ–3A, Unit-1.

c. **What are principal stresses and strains ?**
Ans. Refer Q. 1.10, 2 Marks Questions, Page SQ–2A, Unit-1.

d. **What is slenderness ratio and equivalent length of a column ?**
Ans. Refer Q. 4.10, 2 Marks Questions, Page SQ–11A, Unit-4.

e. **What are flitched beam and fixed beam ?**
Ans. Refer Q. 4.3, 2 Marks Questions, Page SQ–10A, Unit-4.

f. **Differentiate between resilience and proof resilience.**
Ans. Refer Q. 1.7, 2 Marks Questions, Page SQ–1A, Unit-1.

g. **What is spring ? What are different types of spring ?**
Ans. Refer Q. 5.1, 2 Marks Questions, Page SQ–13A, Unit-5.

h. **What is the difference between column and strut ?**
Ans. Refer Q. 4.9, 2 Marks Questions, Page SQ–11A, Unit-4.

i. **Explain :**
i. **Section Modulus.**
ii. **Modular ratio.**
Ans. Refer Q. 3.20, 2 Marks Questions, Page SQ–9A, Unit-3.

Section-B

Note : Attempt any five questions from this section. **(10 × 5 = 50)**

2. **Derive the expression for shearing stress at any section on a beam, also show the distribution of shearing stress over a rectangular section.**
Ans. Refer Q. 3.27, Page 3–29A, Unit-3.

3. **A simply supported beam of span L is carrying a uniformly distributed load of w per unit length over the entire span. Find the maximum slope and deflection of the beam.**
Ans. Refer Q. 4.10, Page 4–14A, Unit-4.

4. **A solid shaft of 200 mm diameter has the same cross sectional area as the hollow shaft of the same material with inside diameter of 150 mm. Find the ratio of**
i. **Powers transmitted by both the shafts at the same angular velocity.**
ii. **Angles of twists in equal length of these shafts, when stressed to same intensity.**
Ans. Refer Q. 3.15, Page 3–17A, Unit-3.

5. **A cylindrical shell 90 cm long 20 cm internal diameter having thickness of metal as 8 mm is filled with fluid at atmospheric pressure. If an additional 20 cm^3 of fluid is pumped into the cylinder, find**
i. **The pressure exerted by the fluid on the cylinder, and**
ii. **The hoop stress induced. Take E = 200 GPa and μ = 0.3**
Ans. Refer Q. 5.20, Page 5–20A, Unit-5.

6. **A short length of tube, 4 cm internal diameter and 5 cm external diameter, failed in compression at a load of 240 kN. When a 2 m length of the same tube was tested with the fixed ends, the load at failure was 158 kN. Assuming that the ultimate crushing stress in Rankine's formula is given by the first test, find the value of the constant a in the same formula. What will be crippling load of this tube if it is used as a strut 3 m long with one end fixed and other is hinged.**
Ans. This question is out of syllabus from sessions 2019-20.

7. **What are the various theories of failure ? Explain with diagram.**
Ans. Refer Q. 1.43, Page 1–46A, Unit-1.

8. **A bar of uniform cross section area A and length L hangs vertically, subjected to its own weight. Prove that the strain energy stored within the bar is given by**

$$U = \frac{A \times \rho^2 \times L^3}{6E}$$

Where E is modulus of elasticity and ρ is weight per unit volume.

Ans. Refer Q. 1.29, Page 1–28A, Unit-1.

9. **A beam of length 6 m is simply supported at its ends and carries two point loads of 48 kN and 40 kN at a distance of 1 m and 3 m respectively from the left support. Find :**
   i. **Deflection under each load,**
   ii. **Maximum deflection,**
   iii. **The point at which maximum deflection occurs.**
   Given E = 200 GPa and $I = 85 \times 10^6$ mm^4

Ans. Refer Q. 4.6, Page 4–9A, Unit-4.

10. **A rectangular block of material is subjected to a tensile stress of 110 MPa on one plane and a tensile stress of 47 MPa at right angles to the former. Each of the above stresses is accompanied by a shear stress of 63 MPa and that associated with the former tensile stress tends to rotate the block anticlockwise. Find :**
    i. **The direction and magnitude of each of the principal stress.**
    ii. **Magnitude of greatest shear stress**

Ans. Refer Q. 1.34, Page 1–36A, Unit-1.

Section – C

Note : Attempt any **two** questions from this section. **(15 × 2 = 30)**

11. Attempt all parts of the following : **(5 × 3 = 15)**
    a. **In a hollow circular shaft of outer and inner diameters of 20 cm and 10 cm respectively, the shear stress is not to exceed 40 MPa. Find the maximum torque which can be safely transmitted.**

Ans. Refer Q. 3.16, Page 3–19A, Unit-3.

b. **Determine the Poisson's ratio and bulk modulus of a material, for which Young's modulus is 120 GPa and modulus of rigidity 48 GPa.**

Ans. Refer Q. 1.13, Page 1–13A, Unit-1.

c. **At a point in a strained material the principal stresses are 100 MPa (tensile) and 60 MPa (compressive). Determine the normal stress, shear stress and resultant stress on a plane inclined at 50° to the axis of major principal stress. Also determine the maximum shear stress at a point.**

Ans. Refer Q. 1.35, Page 1–37A, Unit-1.

12. Attempt **all** parts of the following : **(5 × 3 = 15)**
    a. **Derive the expression for elongation of a uniform bar due to its self-weight.**

Ans. Refer Q. 1.15, Page 1–14A, Unit-1.

b. **Derive the expression for elongation of a conical bar due to its self-weight.**

Ans. Refer Q. 1.16, Page 1–15A, Unit-1.

c. **A steel rod of 3 cm diameter is enclosed centrally in a hollow copper tube of external diameter 5 cm and internal diameter 4 cm. The composite bar is then subjected to an axial pull of 45000 N. If the length of each bar is equal to 15 cm. Determine :**
   i. **The stresses in the rod and the tube.**
   ii. **Load carried by each rod.**
   E for steel = 200 GPa and for Copper = 100 GPa.

Ans. Refer Q. 1.24, Page 1–23A, Unit-1.

13. **A closely coiled helical spring is to carry a load of 500 N. Its mean coil diameter is to be 10 times that of the wire diameter. Calculate the mean diameter if the maximum shear stress in the material of the spring is to be 80 MPa. If the stiffness of the spring is 20 N/mm deflection and modulus of rigidity = 84000 MPa. Find the number of coils in the closely helical springs.**

Ans.

Given : W = 500 N, τ_{max} = 80 N/mm^2, k = 20 N/mm
$G = 8.4 \times 10^4$ N/mm^2, $D = 10d$
To Find : Mean diameter and number of coils in helical spring.

1. Shear stress, $\tau = \dfrac{16WR}{\pi d^3} \Rightarrow 80 = \dfrac{16 \times 500 \times (D/2)}{\pi d^3}$

   d = 12.6 mm = 1.26 cm
2. $D = 10 \times d = 10 \times 1.26$ = 12.6 cm
3. Stiffness = Load/δ $\Rightarrow$ $20 = 500/\delta$ $\Rightarrow$ δ = 25 mm
4. Deflection, $\delta = \dfrac{64\,WR^3 n}{Gd^4}$

$$25 = \frac{64 \times 500 \times (63)^3 \times n}{8.4 \times 10^4 \times 12.6^4} \quad \left(\because R = \frac{D}{2} = \frac{126}{2} = 63 \text{ mm}\right)$$

$$n = \frac{25 \times 8.4 \times 10^4 \times 12.6^4}{64 \times 500 \times (63)^3} = 6.6 \text{ say } 7.0$$

Number of coils in the spring, $n = 7$

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B. Tech.
(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2017-18
MECHANICS OF SOLIDS

Time : 3 Hours **Max. Marks : 100**

Note : Attempt **all** sections. If require any missing data; then choose suitably.

Section-A

1. Attempt **all** questions in brief : **(2 × 7 = 14)**

a. What are principal planes and principal stresses ?

Ans. **Major Principal Plane and Stress :** Refer Q. 1.12, 2 Marks Questions, Page SQ–2A, Unit-1.
Minor Principal Plane and Stress : Refer Q. 1.13, 2 Marks Questions, Page SQ–2A, Unit-1.

b. Define resilience, proof resilience and modulus of resilience.

Ans. **Resilience and Proof Resilience :** Refer Q. 1.7, 2 Marks Questions, Page SQ–1A, Unit-1.
Modulus of Resilience : Refer Q. 1.8, 2 Marks Questions, Page SQ–2A, Unit-1.

c. What are the different methods of finding slope and deflection of cantilever beam ?

Ans. Refer Q. 4.2, 2 Marks Questions, Page SQ–10A, Unit-4.

d. What do you understand by the term torsional rigidity ?

Ans. Refer Q. 3.15, 2 Marks Questions, Page SQ–8A, Unit-3.

e. What is shear center ?

Ans. This question is out of syllabus from sessions 2019-20.

f. What do you understand by the term "buckling" of columns ?

Ans. Refer Q. 4.12, 2 Marks Questions, Page SQ–12A, Unit-4.

g. What is the difference between thin and thick cylinder ?

Ans. Refer Q. 5.19, 2 Marks Questions, Page SQ–15A, Unit-5.

Section-B

2. Attempt any **three** of the following : **(7 × 3 = 21)**

a. A beam of uniform section, 10 m long, is simply supported at its ends. It carries point loads of 150 kN and 65 kN at a distance 2.6 m and 5.5 m respectively from the left end. Calculate :

i. The maximum deflection, and

ii. Deflection under each load.

Ans. Refer Q. 4.7, Page 4–11A, Unit-4.

b. Determine the internal and external diameter of a hollow shaft whose internal diameter is 0.6 times external diameter and transmits 120 kW at 210 rpm and the allowable stress is limited to 75 MPa. If bending moment of 2800 N-m is applied to the shaft, find the speed at which the shaft must rotate to transmit the same power for the same value of maximum shear stress.

Ans. Refer Q. 3.26, Page 3–27A, Unit-3.

c. A mild steel hollow column, having 100 mm external diameter, 40 mm thick and 4 m long. Determine crippling load using Rankine's formula, when both end fixed. Take σ_c = 320 N/mm² and Rankine constant $\alpha = 1/7500$.

Ans. This question is out of syllabus from sessions 2019-20.

d. Derive an expression for maximum principal stress in a thick cylindrical shell subjected to internal pressure only.

Ans. Refer Q. 5.22, Page 5–23A, Unit-5.

e. For a tube having $E = 2 \times 10^5$ N/mm² and $\mu = 0.3$ the hoop stress at the inner surface is twice the internal pressure. Find the thickness of the wall if internal radius is 60 mm.

Ans. Refer Q. 5.16, Page 5–16A, Unit-5.

Section-C

3. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. Derive Euler's equation for long column having its both end built in.

Ans. This question is out of syllabus from sessions 2019-20.

b. Derive expression of deflection angular rotation and stresses in case of open coil helical spring subjected to axial load. Calculate what % the axial extension is underestimated if the inclination of the coil is neglected for a spring in which $\alpha = 25°$. Assume n and R remain same.

Ans. **Derivation :** Refer Q. 5.8, Page 5–9A, Unit-5.
Numerical : Refer Q. 5.9, Page 5–11A, Unit-5.

4. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. What is shear centre ? Channel section has flanges $b \times t_1$ and web $h \times t_2$. Determine position of its shear centre.

Ans. This question is out of syllabus from sessions 2019-20.

b. A 60 mm × 40 mm × 6 mm unequal angle is placed with longer leg vertical and used as a beam. It is subjected to a bending moment of 12 kN-cm acting in the vertical plane through the centroid of the section. Determine the maximum bending stress induced in the section.

Ans. This question is out of syllabus from sessions 2019-20.

5. Attempt any one part of the following : (7 × 1 = 7)

a. At a point in a material under stress, the intensity of the resultant stress on a certain plane is 50 MPa (tensile) inclined at 30° to the normal of that plane. The stress on a plane at right angle to this has a normal tensile component of intensity of 30 MPa, find :

i. The resultant stress on the second plane.

ii. The principal planes and stresses.

iii. The plane of maximum shear and its intensity.

Ans. Refer Q. 1.36, Page 1–37A, Unit-1.

b. Explain different theories of failure along with their graphical representation.

Ans. Refer Q. 1.43, Page 1–46A, Unit-1.

6. Explain any one part of the following : (7 × 1 = 7)

a. A crane hook is of trapezoidal c/s having inner side 80 mm, outer side 300 mm and depth 120 mm. The radius of curvature of inner side is 80 mm. If a load of 100 kN is applied to the hook passing through the centre of curvature, determine the maximum tensile and compressive stresses at the critical cross section.

Ans. This question is out of syllabus from sessions 2019-20.

b. Determine the dimensions of hollow shaft with a diameter ratio of 3:4, which is to transmit 60 kN at 200 rpm. The maximum shear stress is limited to 70 MN/mm^2 and angle of twist is 3.8° in a length of 4 m. G = 80 GPa.

Ans. Refer Q. 3.17, Page 3–19A, Unit-3.

7. Attempt any one part of the following : (7 × 1 = 1)

a. Write down the assumption in Lame's theory and also derive its equation for thick shell.

Ans. Refer Q. 5.22, Page 5–23A, Unit-5.

b. Find the Euler's buckling load of a long column of length L subjected to compressive load P when one end of column is fixed and other is hinged. Take EI constant.

Ans. This question is out of syllabus from sessions 2019-20.

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B. Tech.
(SEM. III) ODD SEMESTER THEORY EXAMINATION, 2018-19
MECHANICS OF SOLIDS

Time : 3 Hours **Max. Marks : 100**

Note : Attempt all sections. If require any missing data; then choose suitably.

Section-A

1. Attempt all questions in brief : (2 × 7 = 14)

a. Different between the terms section modulus and flexural rigidity.

Ans. Refer Q. 3.19, 2 Marks Questions, Page SQ–9A, Unit-3.

b. What do you understand by autofrettage ? Explain it in brief.

Ans. Refer Q. 5.27, 2 Marks Questions, Page SQ–16A, Unit-5.

c. Analytically differentiate between bending of straight and curved beam.

Ans. This question is out of syllabus from sessions 2019-20.

d. Differentiate between strain energy and shear strain energy.

Ans. Refer Q. 1.16, 2 Marks Questions, Page SQ–3A, Unit-1.

e. What is the difference in analysis of closed and open coiled springs ?

Ans. Refer Q. 5.11, 2 Marks Questions, Page SQ–14A, Unit-5.

f. Define shear center and its importance.

Ans. This question is out of syllabus from sessions 2019-20.

g. Explain slenderness ratio and its importance in case of column.

Ans. Refer Q. 4.10, 2 Marks Questions, Page SQ–11A, Unit-4.

Section-B

2. Attempt any three of the following : (7 × 3 = 21)

a. A beam *ABCDE* is continuous over four supports and carries the loads shown in Fig. 1 given below. Determine the values of the fixing moments at each support.

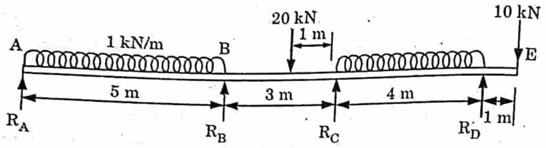

Fig. 1.

Ans. This question is out of syllabus from sessions 2019-20.

b. Derive the differential equation of deflection curve.

Ans. Refer Q. 4.2, Page 4–3A, Unit-4.

c. A close coiled helical spring is fixed at one end and subjected to axial twist at the other. When the spring is in use the axial torque varies from 0.75 N-m to 3 N-m, the working angular deflection between these torques being 35°. The spring is to be made from rod of circular section, the maximum permissible stress being 150 MN/m². The mean diameter of the coils is 8 times the rod diameter. Calculate the mean coil diameter, the number of turns and the wire diameter.

Ans. Refer Q. 5.7, Page 5–8A, Unit-5.

d. The load to be carried by a lift may be dropped 10 cm on to the floor. The cage itself weighs 100 kg and is supported by 25 m of wire rope weighing 0.9 kg/m, consisting of 49 wires each 1.6 mm diameter. The maximum stress in the wire is limited to 90 N/mm² and E for the rope is 70000 N/mm². Find the maximum load which can be carried.

Ans. Refer Q. 1.28, Page 1–28A, Unit-1.

e. A timber beam 6 cm wide and 8 cm deep is to be reinforced by bolting on two steel flitches, each 6 cm by 5 mm in section in the following cases :

i. flitches attached symmetrically at top and bottom.

ii. flitches attached symmetrically at the sides.

Take allowable timber stress as 8 N/mm². What is the maximum stress in the steel in each case ? Take E_{steel} = 210 kN/mm², E_{timber} = 14 kN/mm²

Ans. This question is out of syllabus from sessions 2019-20.

Section-C

3. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. A steel tube of 24 mm external diameter and 18 mm internal diameter encloses a copper rod 15 mm diameter to which it is rigidly attached at each end. If, at a temperature of 10 °C there is no longitudinal stress, calculate the stresses in the tube and rod when the temperature is raised to 200 °C. E_{steel} = 210 kN/mm², E_{copper} = 210 kN/mm²
Coefficients of linear expansion :
$\alpha_{steel} = 11 \times 10^{-6}$ /°C, $\alpha_{copper} = 11 \times 10^{-6}$ /°C

Ans. Refer Q. 1.25, Page 1–24A, Unit-1.

b. A small block is 40 mm long, 30 mm high and 5 mm in thick. It is subjected to uniformly distributed tensile forces having the resultant values in N shown in figure. Compute the stress components developed along the diagonal AC.

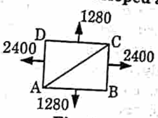

Fig. 2.

Ans. Refer. Q. 1.37, Page 1–39A, Unit-1.

4. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. A solid circular shaft of length of 3 m and diameter of 50 mm rotates at 1200 rpm by a 400 HP electric motor at its middle. It derives two machines of 150 HP and 250 HP at left and right ends of the shaft, respectively. Determine the maximum shear stress and relative displacement of the two ends of the shaft.
Take G = 85 GPa.

Ans. Refer Q. 3.18, Page 3–20A, Unit-3.

b. Determine the deflection of the beam at midpoint for the beam loading system shown in the figure given below :
Take : E = 200 GN/m² and $I = 83 \times 10^6$ m⁴.

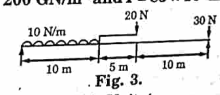

Fig. 3.

Ans. Refer Q. 4.8, Page 4–11A, Unit-4.

5. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. A short column is of hollow circular section, the center of the inside hole being 6 mm eccentric to that of the outside. The outside diameter is 96 mm and the inside 48 mm. The line of action of the load intersects the cross-section at a point in line with the two centers. What are the limiting position of the load for there to be no tensile stress set up ?

Ans. Refer Q. 4.18, Page 4–23A, Unit-4.

b. Derive the relation to find deflection induced in the open coiled helical spring subjected to axial torque.

Ans. Refer Q. 5.11, Page 5–12A, Unit-5.

6. Explain any **one** part of the following : **(7 × 1 = 7)**

a. The cylinder for a hydraulic press has an inside diameter of 300 mm. Determine the wall thickness required if the cylinder is to withstand an internal pressure of 60 MPa without exceeding a shearing stress of 90 MPa.

Ans. Refer Q. 5.17, Page 5–17A, Unit-5.

b. In an experiment on a thick cylinder of 100 mm external diameter and 50 mm internal diameter the hoop and longitudinal strains as measured by strain gauges applied to the outer surface of the cylinder were 240×10^{-6} and 60×10^{-6} respectively, for an internal pressure of 90 MN/m^2, the external pressure being zero.
Determine the actual hoop and longitudinal stresses present in the cylinder if E = 208 GN/m^2 and υ = 0.29. Compare the hoop stress value so obtained with the theoretical value given by the Lame's equations.

Ans. Refer Q. 5.23, Page 5–25A, Unit-5.

7. Attempt any **one** part of the following : **(7 × 1 = 1)**

a. A 75 mm × 75 mm × 12 mm angle is used as a cantilever with the face *AB* horizontal as shown in figure. A vertical load of 3 kN is applied at the tip of the cantilever which is 1 m long. Determine the stresses at *A*, *B* and *C*.

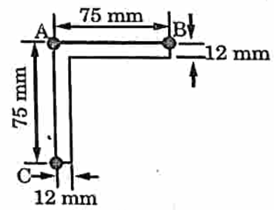

Fig. 4.

Ans. This question is out of syllabus from sessions 2019-20.

b. The bending moment acting on the curved beam with a rectangular cross section is *M* = 8 kN-m. Calculate the bending stress at point B.

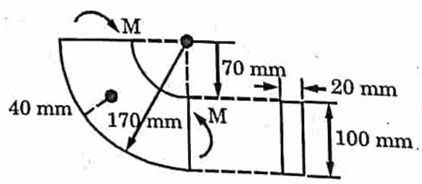

Fig. 5.

Ans. This question is out of syllabus from sessions 2019-20.

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