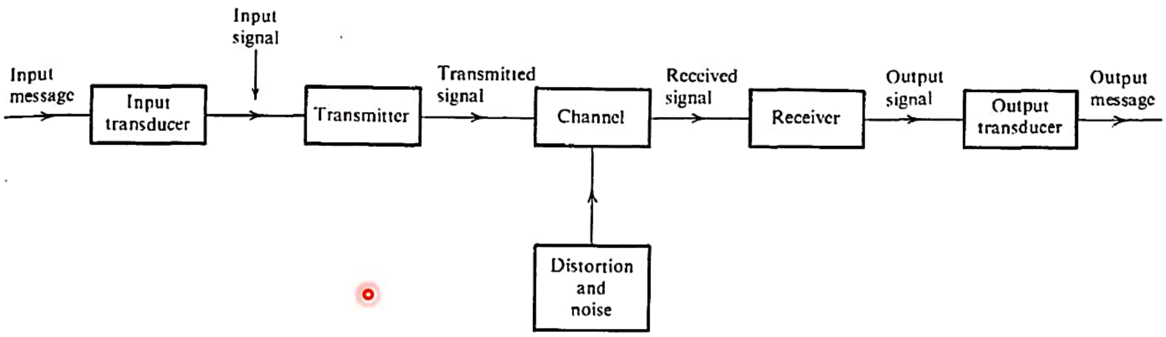
Introduction to Communication Engineering

Communication: Sharing of information

Electronic Communication: Sharing of information electronically that involves transmission of information from one point to another point through a succession of processes such as generation of message signal, encoding of message signal , transmission of encoded signal through a suitable channel, decoding of the received signals and reproduction of the original signal with acceptable quality.



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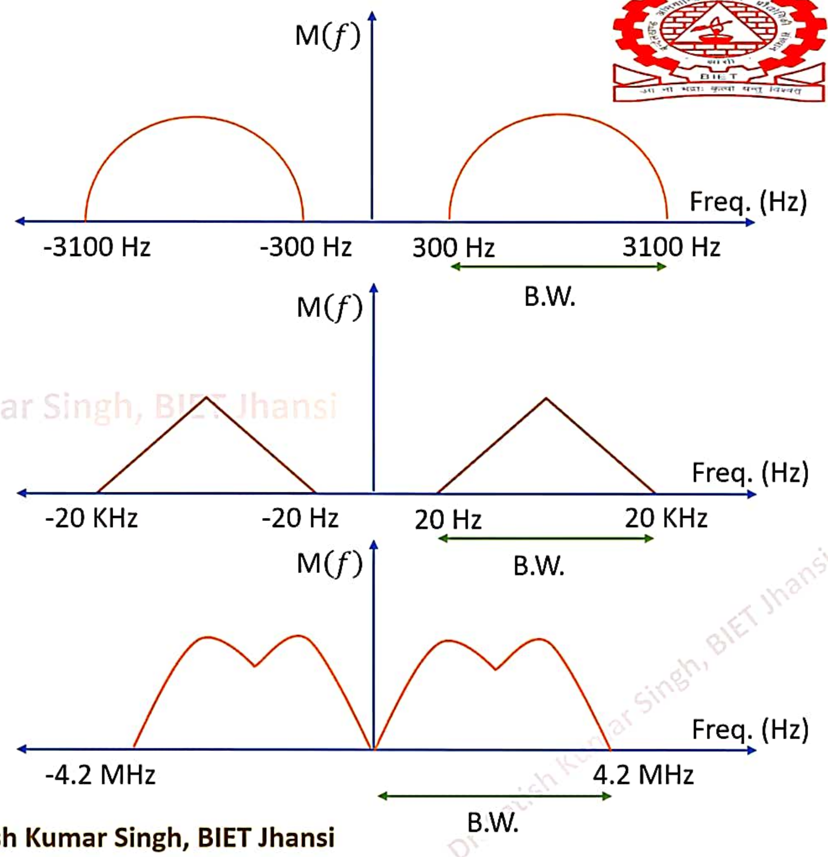
Message Signal: It is the information bearing signal that we want to send from one point to another

Message Signal (m(t))	Approximate Bandwidth
Voice Signal	300 Hz - 3.1 KHz
Audio/Music Signal	20 Hz - 20 KHz
Picture/Video Signal	0 - 4.2 MHz
Computer Data	Depends on various data



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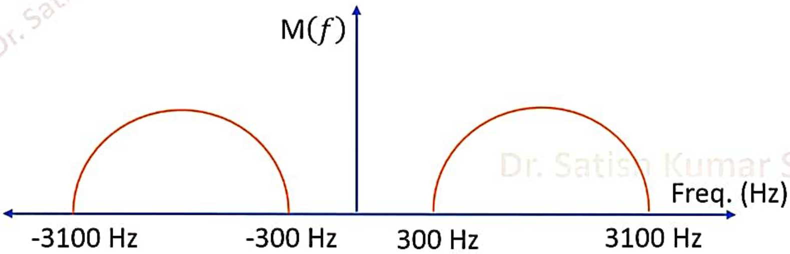
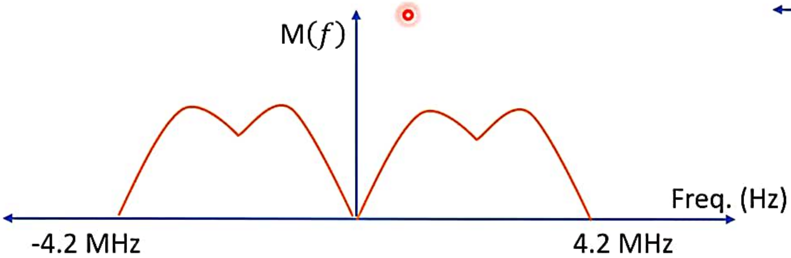
Baseband Vs. Bandpass Signal

Baseband Signal

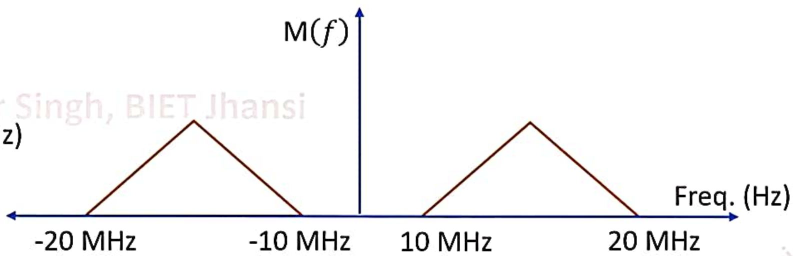
It is the band of frequencies representing the original signal as delivered by a source of information. It contains significant low frequency component in it's spectrum.





Bandpass Signal

It doesn't contains significant low frequency component in it's spectrum



- **Message signals are generally baseband in nature.**

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Communication Channel

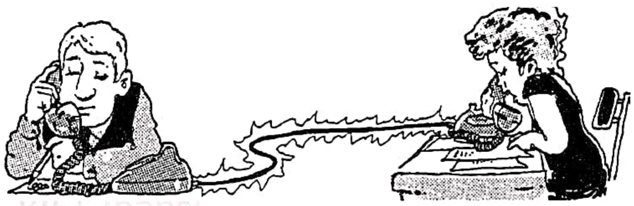
It is the medium through which the signal travels from one point to another point.

Guided or Wired Channel (Man made media)

Twisted Pair Cable
(~ 1 MHz)

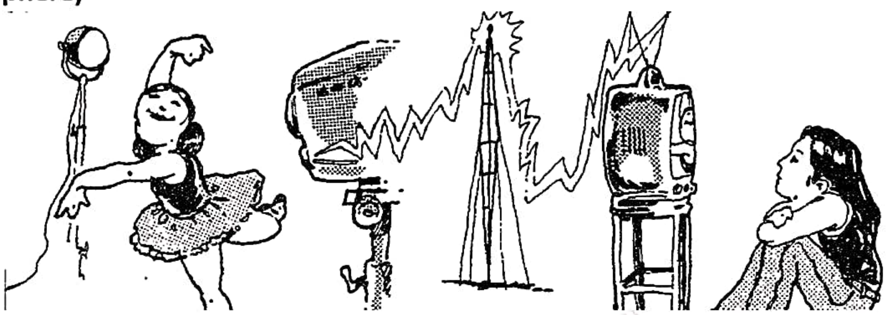
Co-axial Cable
(~ 500 MHz)

Optical Fibre Cable
(~ THz)



Unguided or Wireless Channel (Using earth's atmosphere)

- Wireless broadcast channel
- Radio mobile channel
- Satellite channel



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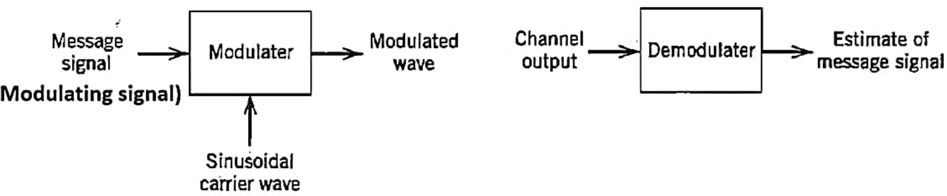
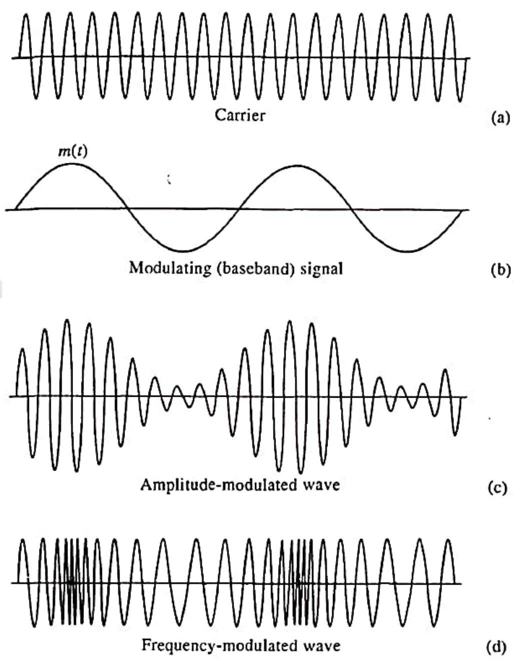
Modulation

- **Definition 1:** It is the process by which one of the properties (Amplitude, frequency and phase) of a carrier signal is varied in a systematic way with the amplitude of the message signal.
- **Definition 2:** It is the process by which the baseband message signal is translated into bandpass signal for it's efficient transmission.
- **Carrier Signal:** It is the high frequency signal, most often it is a sinusoidal signal.
- **Message Signal:** It is the information bearing signal or a baseband signal such as speech, audio and video signal that we want to send.





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Need of Modulation

Antenna height for efficient radiation should be $\lambda/10$ or more (preferably $\lambda/4$) where λ is the wavelength of signal to be transmitted.

$$h = \frac{c}{10f}$$

Let us assume that we want to transmit an audio signal which is having frequency component from 20 Hz to 20 KHz.

For efficient transmission of lowest frequency component the minimum height of the antenna is

$$h = \frac{c}{10f} = \frac{3 \times 10^8}{10 \times 20} = 1500\ km$$

For efficient transmission of highest frequency component the minimum height of the antenna is

$$h = \frac{c}{10f} = \frac{3 \times 10^8}{10 \times 20 \times 10^3} = 1500\ m$$

Antenna height of $h = 1500\ km$ can take care of entire audible range, however, the antenna height is not feasible.

Baseband signal cannot be transmitted directly because of antenna height is not feasible

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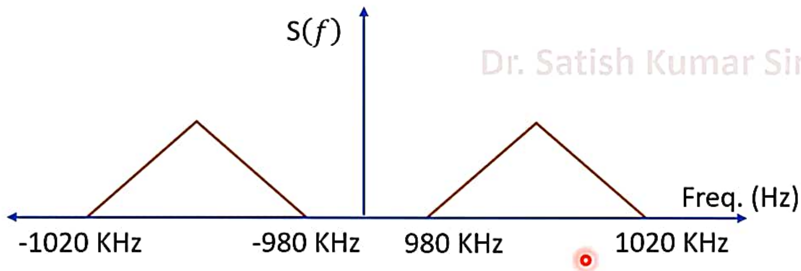
Need of Modulation

1. Modulation for ease of transmission (To reduce the antenna height)

Assume we choose the carrier frequency to be 1 MHz. The linear modulation schemes that would be discussed shortly give rise to a maximum frequency spread (of the modulated signal) of 40 kHz, the spectrum of the modulated signal extending from (1000 -20) = 980 kHz to (1000 + 20) = 1020 kHz. If the antenna is designed for 1000 kHz, it can easily take care of the entire range of frequencies involved



Required antenna height

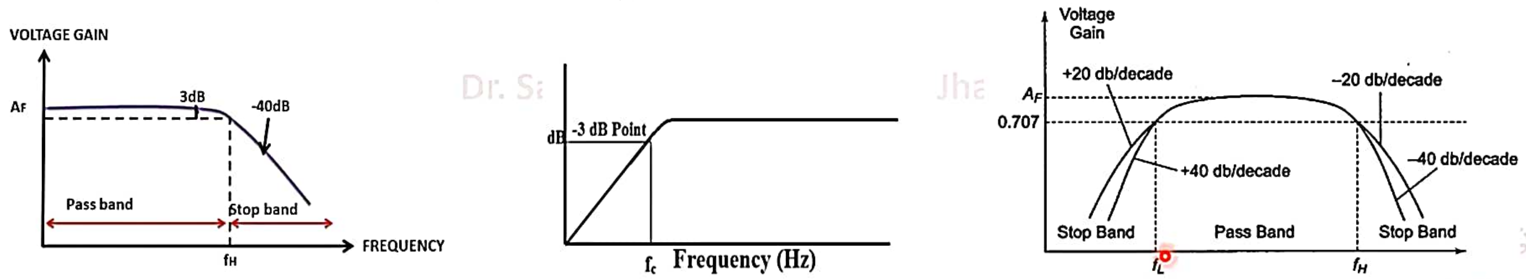
$$h = \frac{c}{10f} = \frac{3 \times 10^8}{10 \times 980 \times 10^3} = 30.6\ m$$

- **Narrow banding** of the signal makes the antenna height feasible.

2. Modulation for efficient transmission

Channels through which the signal is transmitted is having it's own passband within which it can transmit signal efficiently.

- Ground wave propagation (from lower atmosphere) can take upto 2 MHz.
- Ionospheric propagation is possible in the range of 2 MHz to 30 MHz.
- Line of sight propagation is possible beyond 30 MHz.
- Satellite communication takes place in the range 3 to 6 GHz.



3. Modulation for Multiplexing

4. Modulation for frequency assignment

5. Modulation to improve the signal to noise ratio

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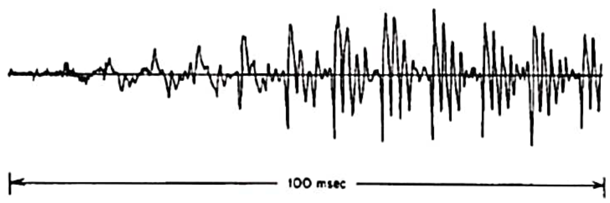
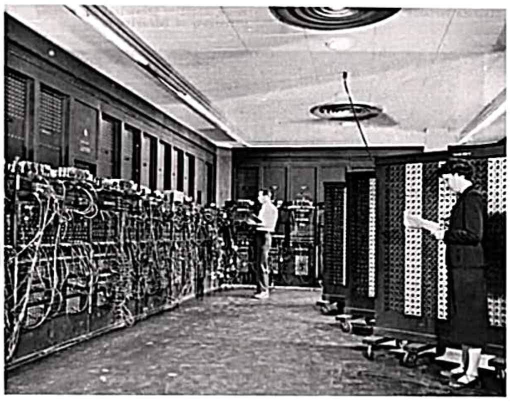


Speech signal is represented mathematically by acoustic pressure as a function of time.



A picture is represented as a brightness function of two spatial variable

Classification of Signals

- **In this course the signal that we consider will be a function of only a single independent variable (time). Therefore, a signal is a single valued function of time that is at every instant of time the function will has a unique value.**

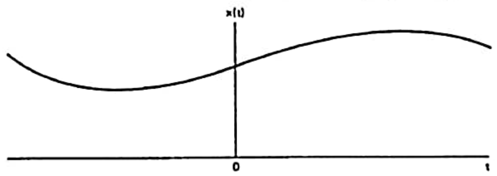
- There are signals which are function of some other variable for example the variation of air pressure, temperature and wind speed with altitude.

- One dimensional signal: Speech signal, music signal or computer data
- Two dimensional signal: Still images/pictures
- Three dimensional signal: Video signal
- Four dimensional signal : Volume data over time

Classification of Signals

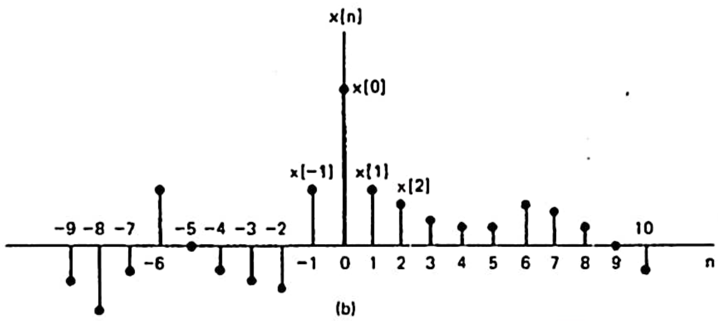
- Continuous time and discrete time signal
- Analog and digital signal
- Periodic and aperiodic signal
- Energy and power signals
- Deterministic and Probabilistic signals

Continuous time signal



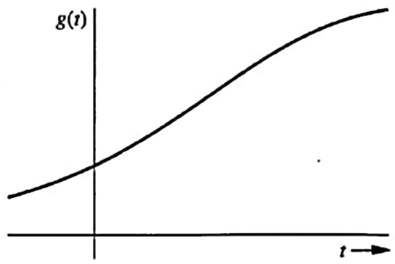
(a)

Discrete time signal



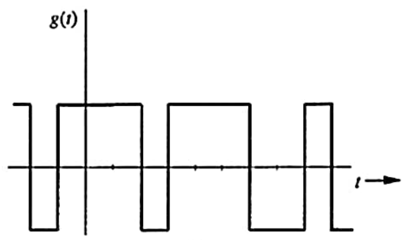
(b)

Classification of Signals



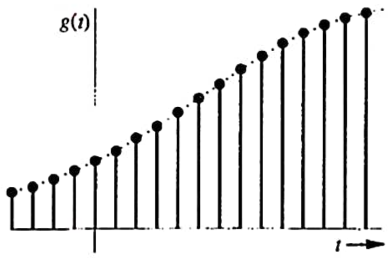
(a)

Continuous time, Analog Signal



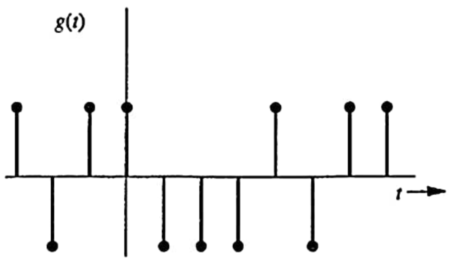
(b)

Continuous time, Digital Signal



(c)

Discrete time, Analog Signal



(d)

Discrete time, Digital Signal

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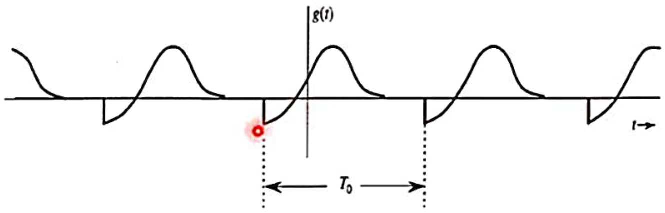
Classification of Signals

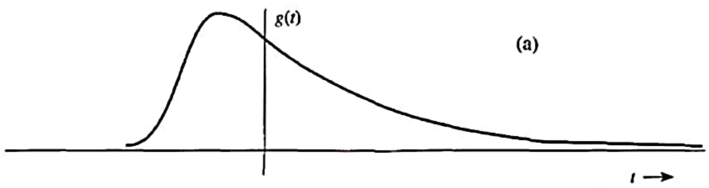
Periodic Signal

For a signal *g(t)* to be periodic, it has to satisfy the following conditions



1. $g(t) = g(t + T_0)$ for all 't'
2. *g(t)* should be defined for $-\infty \leq t \leq \infty$

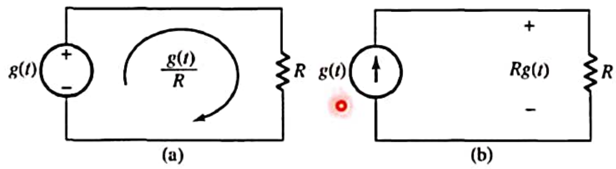
Aperiodic Signal



Calculation of Energy and Power of Signal in Circuit

Power dissipated across the resistor

P = Voltage across the resistor × current through the resistor



Case (a)

$$P = g(t) \times \frac{g(t)}{R}$$

$$P = \frac{g^2(t)}{R}$$

Case (b)

$$P = g(t) \times Rg(t)$$

$$P = g^2(t)\mathrm{R}$$

Normalized power for R = 1 ohm in both the cases

$$P = g^2(t)$$

Normalized energy dissipated in time '*t*' is

$$\mathrm{E} = \int_{-\infty}^{t} g^2(t)dt$$

Total normalized energy dissipated for real signal is

$$\mathrm{E} = \int_{-\infty}^{\infty} g^2(t)dt$$

If $g(t)$ is a complex signal then

$$\mathrm{E} = \int_{-\infty}^{\infty} |g(t)|^2 dt$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt$$

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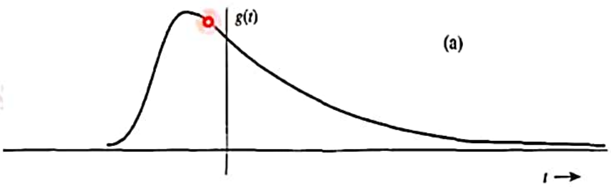
Energy Signal

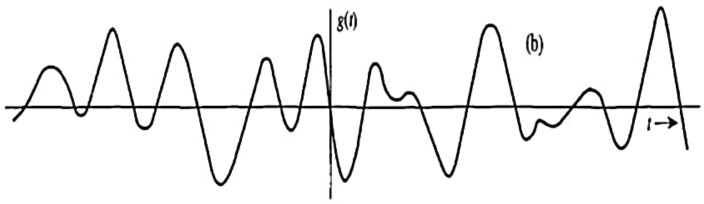


Signal with finite energy

$$\int_{-\infty}^{\infty} |g(t)|^2 \, dt < \infty$$

Power = 0

Power Signal



Signal with finite power

$$0 < \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 \, dt < \infty$$

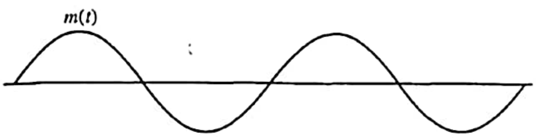
Energy = ∞

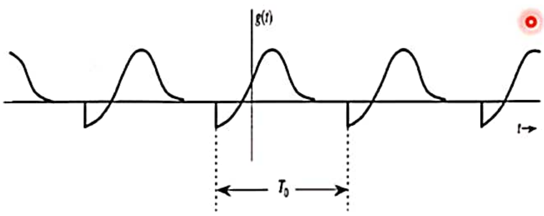
- All periodic signals are power signal but all power signal need not be periodic.

Deterministic and Probabilistic Signals

Deterministic Signals

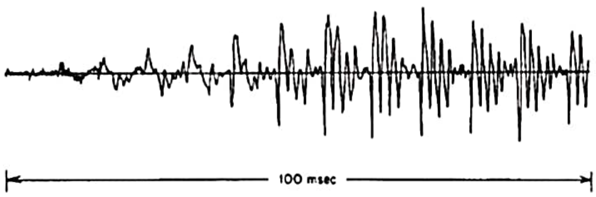
- In this case the signal physical description is known completely, in either mathematical or graphical form.





Probabilistic/Random Signals

- In this case only probabilistic description of the signal such as mean value, mean squared value etc are known.



Some Important Fourier Transform and Inverse Trans...

Fourier transform of a time-domain signal is defined as (Analysis equation)

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

Inverse Fourier transform (Synthesis equation) is defined as

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

1. $x(t) = \delta(t)$

$$\Rightarrow X(\omega) = \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt$$

$$\Rightarrow X(\omega) = \int_{-\infty}^{\infty} \delta(t)e^{-j0}dt$$

$$\Rightarrow X(\omega) = e^{-j0}\int_{-\infty}^{\infty} \delta(t)\,dt$$

$$\Rightarrow X(\omega) = 1$$

$$\Rightarrow \delta(t) \leftrightarrow 1$$

2. $X(\omega) = \delta(\omega - \omega_0)$

$$\Rightarrow x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)e^{j\omega t}d\omega$$

$$\Rightarrow x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)e^{j\omega_0 t}d\omega$$

$$\Rightarrow x(t) = \frac{1}{2\pi}e^{j\omega_0 t}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\,d\omega$$

$$\Rightarrow x(t) = \frac{1}{2\pi}e^{j\omega_0 t} \leftrightarrow \delta(\omega-\omega_0)$$

$$\Rightarrow e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega-\omega_0)$$

$$\Rightarrow e^{-j\omega_0 t} \leftrightarrow 2\pi\delta(\omega+\omega_0)$$

$$\Rightarrow e^{j\omega_0 t} + e^{-j\omega_0 t} \leftrightarrow 2\pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$$

$$\Rightarrow \cos\omega_0 t \leftrightarrow \pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$$

$$\Rightarrow \sin\omega_0 t \leftrightarrow \frac{\pi}{j}(\delta(\omega-\omega_0)+\delta(\omega+\omega_0))$$

We have $\delta(2\pi f) = \frac{1}{2\pi}\delta(f)$

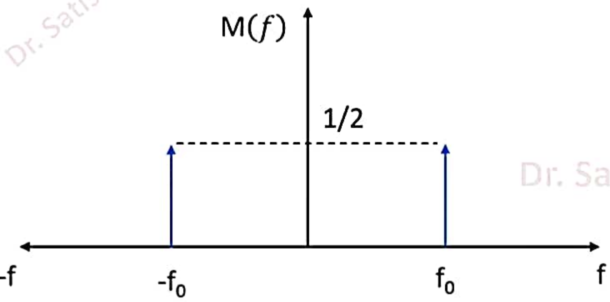
$$\Rightarrow \cos 2\pi f_0 t \leftrightarrow \frac{1}{2}(\delta(f-f_0)+\delta(f+f_0))$$

$$\Rightarrow \sin 2\pi f_0 t \leftrightarrow \frac{1}{2j}(\delta(f-f_0)-\delta(f+f_0))$$

Single Tone Vs. Multitone Signal

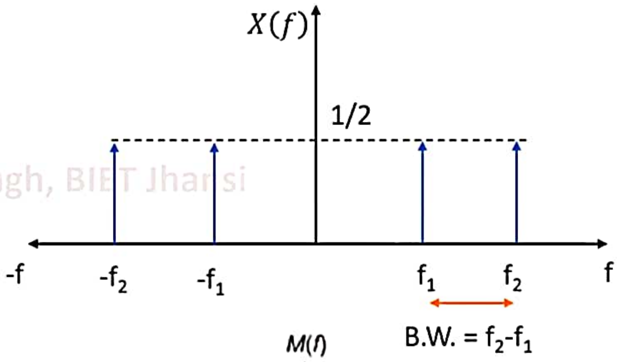
Single Tone Signal

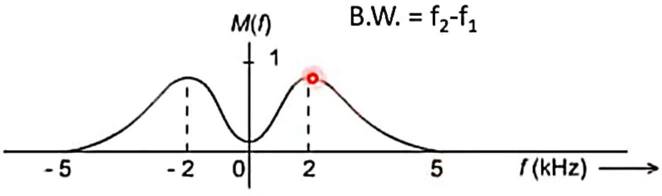
$$\cos 2\pi f_0 t \leftrightarrow \frac{1}{2}(\delta(f-f_0)+\delta(f+f_0))$$



Multi-tone Signal

$$\cos 2\pi f_1 t + \cos 2\pi f_2 t \leftrightarrow \frac{1}{2}(\delta(f-f_1)+\delta(f+f_1)) + \frac{1}{2}(\delta(f-f_2)+\delta(f+f_2))$$





(a)

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Amplitude Modulation

It is the process by which Amplitude of a carrier signal is varied in a systematic way with amplitude of the message signal.

Standard time-domain expression for AM Signal (Conventional AM or DSB-LC)

$$s_{AM}(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t \qquad \text{.........(1)}$$

k_a = Amplitude sensitivity of AM modulator in $Volt^{-1}$

Single-Tone Modulation

$$m(t) = A_m \cos 2\pi f_m t$$

$$\Rightarrow \quad s_{AM}(t) = A_c[1 + k_a A_m \cos 2\pi f_m t] \cos 2\pi f_c t$$

Let $\mu = k_a A_m$ = Modulation index

$$\Rightarrow \quad s_{AM}(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t \qquad \text{.........(2)}$$

$$\Rightarrow \quad s_{AM}(t) = A_c \cos 2\pi f_c t + \mu A_c \cos 2\pi f_c t \cos 2\pi f_m t$$

By using the formula $\quad \mathbf{\cos A \cos B = \frac{1}{2}(\cos(A+B) + \cos(A-B))}$

$$\Rightarrow \quad s_{AM}(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos 2\pi (f_c + f_m)t + \frac{\mu A_c}{2} \cos 2\pi (f_c - f_m)t \qquad \text{.........(3)}$$

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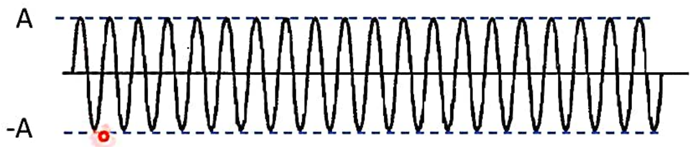
Amplitude Modulation

Time-Domain Waveform of AM Signal

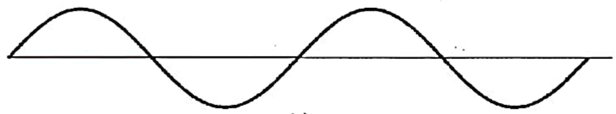
$$x(t) = A\cos 2\pi f t$$



(a)

$$s_{AM}(t) = \underbrace{A_c[1 + \mu\cos 2\pi f_m t]}\cos 2\pi f_c t$$

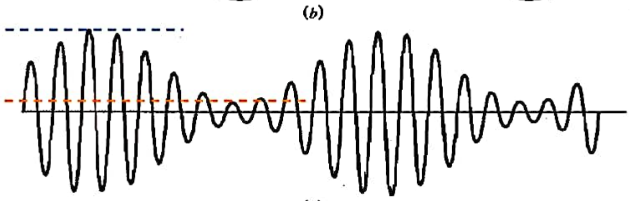
Envelope of the cosine function $\cos 2\pi f_c t$, provided $f_c >> f_m$.



(b)

$V_{max} = A_c[1 + \mu]$

$V_{min} = A_c[1 - \mu]$



(c)

$$\mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

Normally, $\mu \leq 1$

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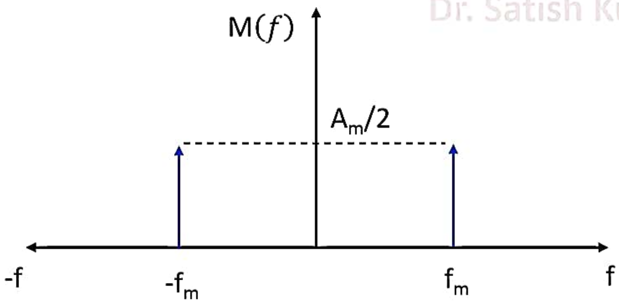
Amplitude Modulation

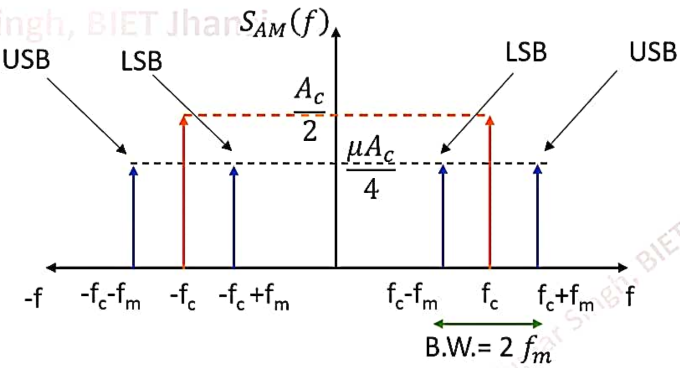
$$\Rightarrow \; s_{AM}(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2}\cos 2\pi(f_c+f_m)t + \frac{\mu A_c}{2}\cos 2\pi(f_c-f_m)t \qquad \text{.........(3)}$$

Taking the Fourier Transform of Eq. (3)

$$\Rightarrow \; S_{AM}(f) = \frac{A_c}{2}\left(\delta(f-f_c)+\delta(f+f_c)\right) + \frac{\mu A_c}{4}\left(\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m)\right) + \frac{\mu A_c}{4}\left(\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m)\right)$$



Spectrum of message signal



Spectrum of AM modulated signal

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Amplitude Modulation

Power Requirement of AM Signal

Time-domain expression of single-tone AM modulated signal is

$$S_{AM}(t) = \underbrace{A_c \cos 2\pi f_c t}_{\text{Carrier}} + \underbrace{\frac{\mu A_c}{2} \cos 2\pi (f_c + f_m) t}_{\text{Upper Side-Band}} + \underbrace{\frac{\mu A_c}{2} \cos 2\pi (f_c - f_m) t}_{\text{Lower Side-Band}}$$

$$P_T = P_C + P_{SB}$$

$$P_T = P_C + P_{USB} + P_{LSB}$$

$$P_T = \frac{A_c^2}{2} + \frac{\mu^2 A_c^2}{8} + \frac{\mu^2 A_c^2}{8}$$

$$P_T = \frac{A_c^2}{2}\left(1 + \frac{\mu^2}{2}\right)$$

$$P_T = P_C\left(1 + \frac{\mu^2}{2}\right)$$

For $\mu = 1$

$$P_T = \frac{3}{2} P_C$$

$$P_C = \frac{2}{3} P_T$$

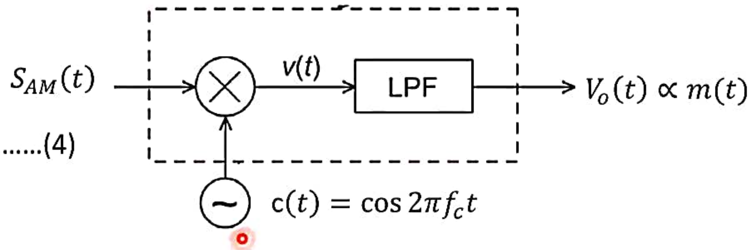
That reflects 66.67 % of total power is wasted in transmitting the carrier component and maximum side band power is limited to $P_{SB} = 33.33\%$ of total power.

Note: AM modulation is neither bandwidth efficient nor power efficient.

Demodulation of AM Signal

Synchronous or Coherent Demodulation of AM Signal

The carrier used for demodulation should be of same frequency and phase that was used for modulation.



$$V(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t \times \cos 2\pi f_c t \qquad \text{.........(4)}$$

$$V(t) = A_c[1 + \mu \cos 2\pi f_m t] \frac{(1 + \cos 4\pi f_c t)}{2}$$

By using the formula $\cos A \cos B = \frac{1}{2}(\cos(A+B) + \cos(A-B))$

$$V(t) = \frac{A_c}{2} + \frac{A_c}{2}\cos 4\pi f_c t + \frac{\mu A_c}{2}\cos 2\pi f_m t + \frac{\mu A_c}{4}\cos 2\pi(2f_c + f_m)t + \frac{\mu A_c}{4}\cos 2\pi(2f_c - f_m)t \qquad \text{.........(5)}$$

After passing through the LPF of certain cut-off frequency, higher frequency components can be filtered out

$$V_o(t) = \frac{\mu A_c}{2}\cos 2\pi f_m t = \frac{k_a A_c}{2} A_m \cos 2\pi f_m t$$

$$V_o(t) \propto m(t)$$

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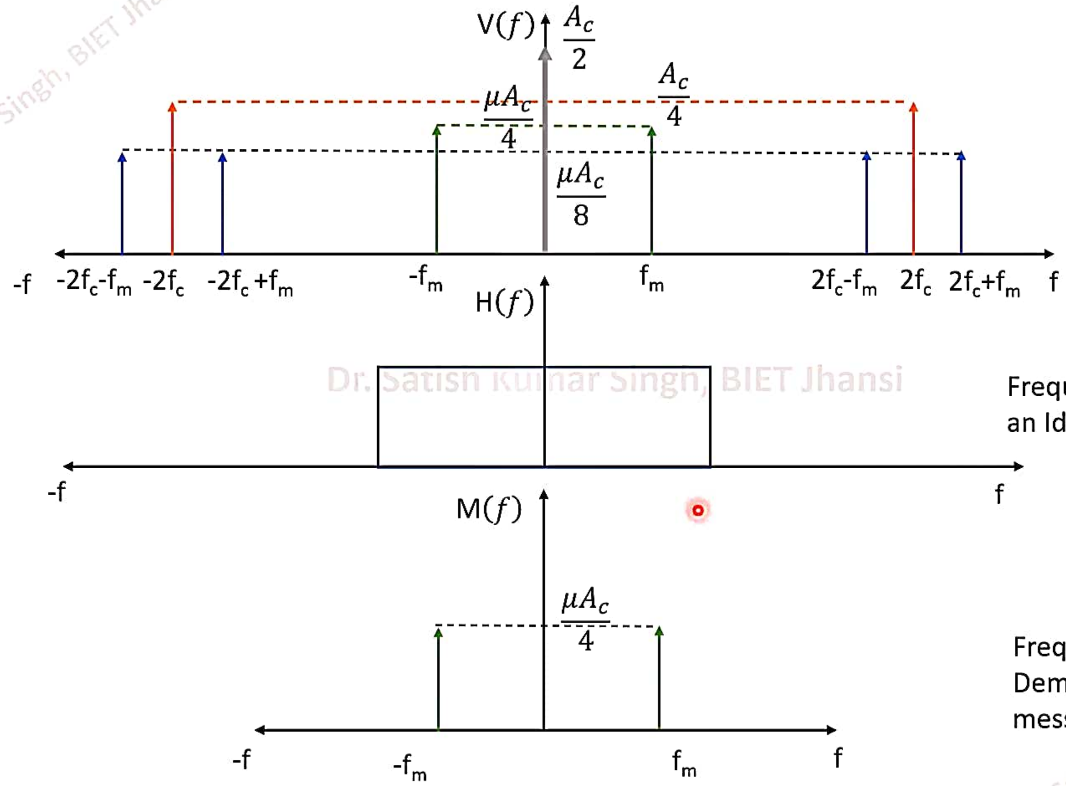

Frequency response of an Ideal LPF

Frequency spectrum of Demodulated Wave i.e. message signal

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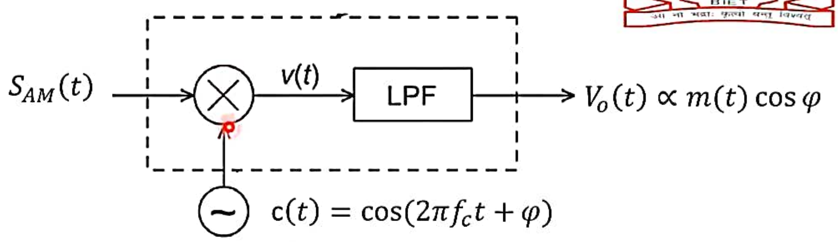
Quadrature Null Effect



$$V(t) = A_c[1 + \mu\cos 2\pi f_m t]\cos 2\pi f_c t \times \cos(2\pi f_c t + \varphi)$$

$$V(t) = [A_c\cos 2\pi f_c t \; + \tfrac{\mu A_c}{2}\cos 2\pi(f_c{+}f_m)t + \tfrac{\mu A_c}{2}\cos 2\pi(f_c{-}f_m)t] \times \cos(2\pi f_c t + \varphi)$$

$$V(t) = \frac{A_c}{2}[\cos(4\pi f_c + \varphi) + \cos\varphi] + \frac{\mu A_c}{4}\cos[2\pi(2f_c + f_m)t + \varphi] + \frac{\mu A_c}{4}\cos[2\pi(2f_c - f_m)t + \varphi]$$
$$+ \frac{\mu A_c}{4}\cos[2\pi f_m t - \varphi] + \frac{\mu A_c}{4}\cos[2\pi f_m t + \varphi]$$

After passing through LPF and blocking the d.c. component

$$V_o(t) = \frac{\mu A_c}{4}\cos[2\pi f_m t - \varphi] + \frac{\mu A_c}{4}\cos[2\pi f_m t + \varphi], \qquad V_o(t) = \frac{\mu A_c}{4}(2\cos[2\pi f_m t]\cos[\varphi])$$

$$V_o(t) = \frac{k_a A_c}{2}A_m\cos[2\pi f_m t]\cos[\varphi], \qquad V_o(t) \propto m(t)\cos\varphi, \qquad V_o(t) = 0,$$ (If φ=90 degree, then QNE occurs)

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Amplitude Modulation

Multitone Modulation

Standard time-domain expression for AM Signal (Conventional AM or DSB-LC)

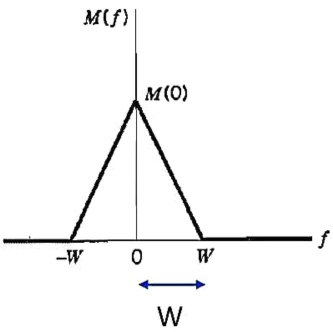
$$s_{AM}(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

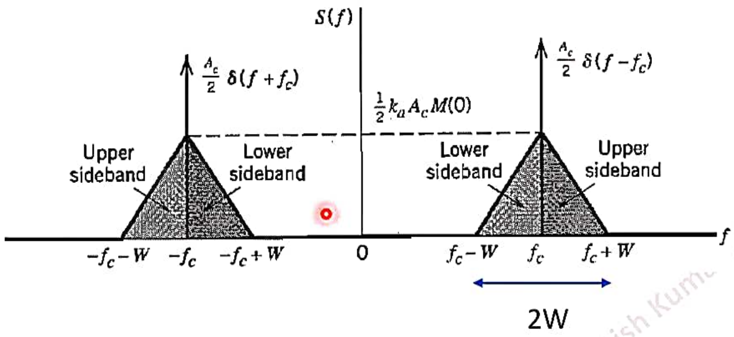
$$r(t) = m(t)c(t) \leftrightarrow R(f) = M(f) * C(f)$$

$$M(f) * \delta(f - f_c) = M(f - f_c)$$

Taking Fourier transform of the above expression

$$S_{AM}(f) = \frac{A_c}{2}\left(\delta(f - f_c) + \delta(f + f_c)\right) + \frac{k_a A_c}{2}\left(M(f - f_c) + M(f + f_c)\right)$$



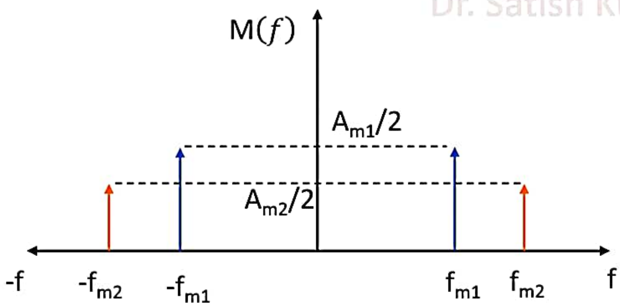


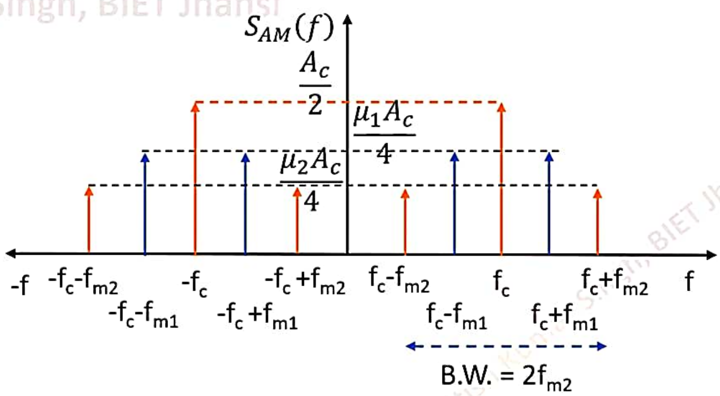
Amplitude Modulation

Let,
$$m(t) = A_{m_1}\cos 2\pi f_{m_1}t + A_{m_2}\cos 2\pi f_{m_2}t$$

$$s_{AM}(t) = A_c\left[1 + k_a\left(A_{m_1}\cos 2\pi f_{m_1}t + A_{m_2}\cos 2\pi f_{m_2}t\right)\right]\cos 2\pi f_c t$$

$$s_{AM}(t) = A_c\cos 2\pi f_c t + \frac{\mu_1 A_c}{2}\left[\cos 2\pi\left(f_c - f_{m_1}\right)t + \cos 2\pi\left(f_c + f_{m_1}\right)t\right] + \frac{\mu_2 A_c}{2}\left[\cos 2\pi\left(f_c - f_{m_2}\right)t + \cos 2\pi\left(f_c + f_{m_2}\right)t\right]$$





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Power Requirement of AM Signal for Multitone Modulation

$$s_{AM}(t) = A_c\cos 2\pi f_c t + \frac{\mu_1 A_c}{2}\left[\cos 2\pi\left(f_c - f_{m_1}\right)t + \cos 2\pi\left(f_c + f_{m_1}\right)t\right] + \frac{\mu_2 A_c}{2}\left[\cos 2\pi\left(f_c - f_{m_2}\right)t + \cos 2\pi\left(f_c + f_{m_2}\right)t\right]$$

$$P_T = \frac{A_c^2}{2} + \frac{\mu_1^2 A_c^2}{8} + \frac{\mu_1^2 A_c^2}{8} + \frac{\mu_2^2 A_c^2}{8} + \frac{\mu_2^2 A_c^2}{8}$$

$$P_T = \frac{A_c^2}{2}\left[1 + \frac{\mu_1^2 + \mu_2^2}{2}\right]$$

$$P_T = P_c\left[1 + \frac{\mu_t^2}{2}\right] \quad \text{where } \mu_t = \sqrt{\mu_1^2 + \mu_2^2}$$

$$I_T^2 R = I_c^2 R\left[1 + \frac{\mu_t^2}{2}\right]$$

$$I_T = I_c\sqrt{1 + \frac{\mu_t^2}{2}}$$

Modulation Efficiency

$$\eta = \frac{\text{Total Side Band Power}}{\text{Total Power}} = \frac{P_{SB}}{P_T}$$

$$\eta = \frac{P_c\,\mu^2/2}{P_c(1+\mu^2/2)} = \frac{\mu^2}{2+\mu^2}$$

μ	η
0.25	0.03
0.50	0.11
0.75	0.22
1.0	0.33

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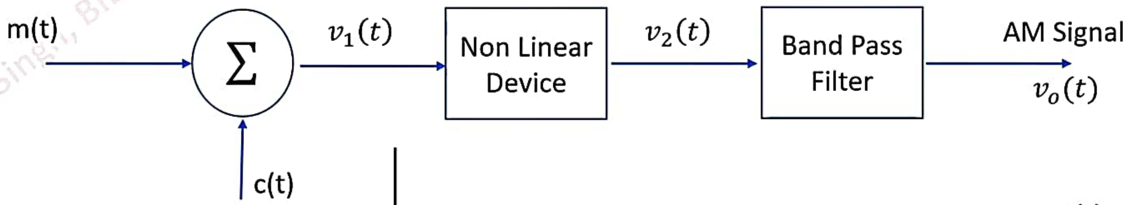
Generation of Amplitude Modulated Signal Using Square Law Modulator



$$v_1(t) = m(t) + A_c \cos 2\pi f_c t$$

$$v_2(t) = a v_1(t) + b v_1^2(t)$$

$$v_2(t) = am(t) + aA_c \cos 2\pi f_c t + b(m^2(t) + A_c^2 \cos^2 2\pi f_c t + 2A_c m(t) \cos 2\pi f_c t)$$

$$v_2(t) = aA_c\left(1 + \frac{2b}{a} m(t)\right)\cos 2\pi f_c t + am(t) + bm^2(t) + bA_c^2 \cos^2 2\pi f_c t$$

After passing through BPF

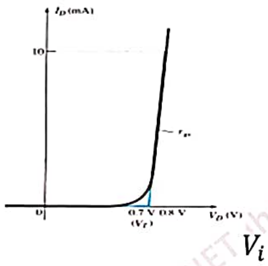
$$v_o(t) = aA_c\left(1 + \frac{2b}{a} m(t)\right)\cos 2\pi f_c t$$

$$v_o(t) = A_c'(1 + k_a A_m \cos 2\pi f_m t)\cos 2\pi f_c t$$

$$\mu = k_a A_m$$

$k_a = \frac{2b}{a}$ = Amplitude sensitivity of the square law modulator

$$v_o(t) = a v_i(t) + b v_i^2(t)$$



Considering $\quad m(t) = A_m \cos 2\pi f_m t$

$$m^2(t) = A_m^2 \cos^2 2\pi f_m t = A_m^2 \frac{(1 + \cos 2\pi(2f_m)t)}{2}$$

$$m^n(t) \propto \cos 2\pi\,(nf_m)t$$

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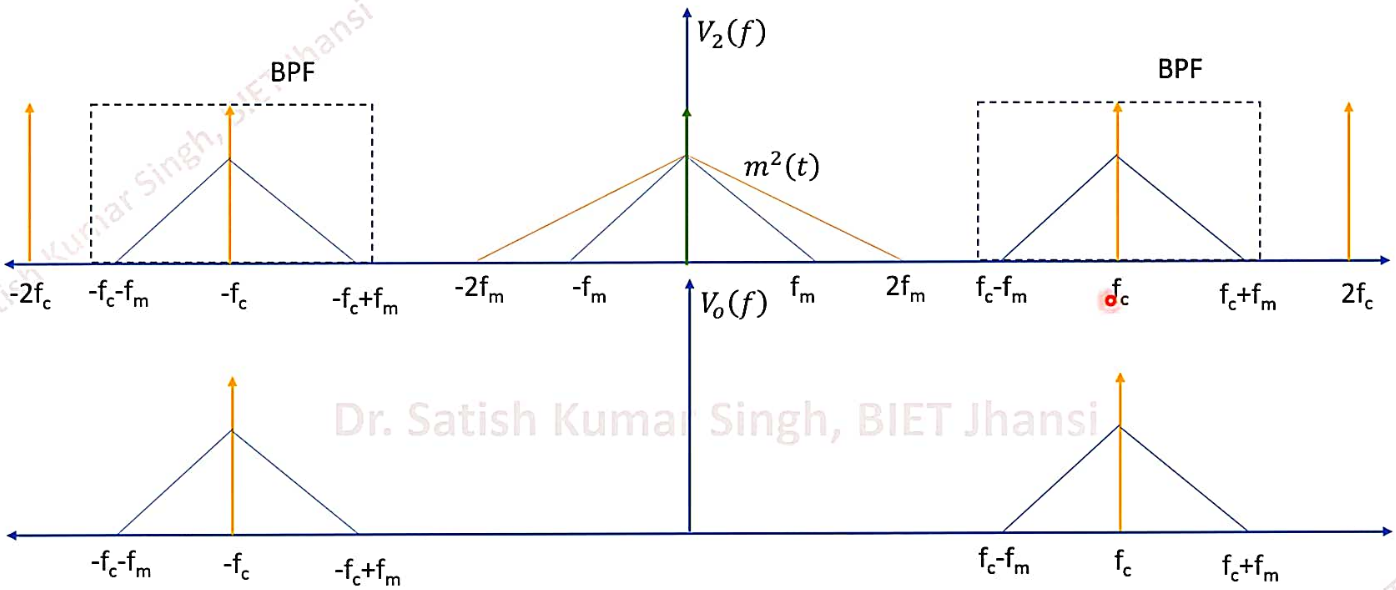

Condition for proper generation of AM wave using square law method is

$$f_c - f_m - 2f_m \geq 0$$

$$f_c \geq 3f_m$$

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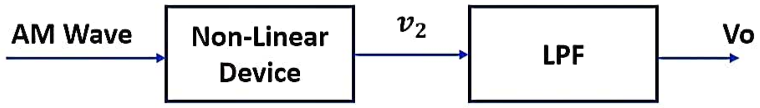
Demodulation of Amplitude Modulated Signal Using Square Law Demodulator

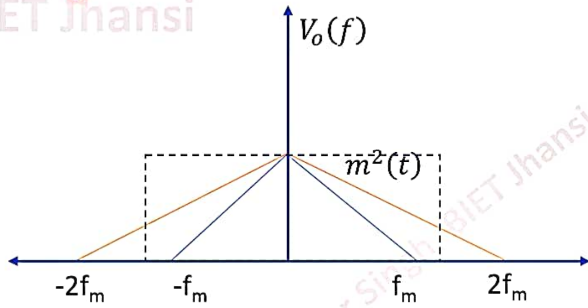


$$v_{AM} = A_c' \cos 2\pi f_c t + A_c' k_a m(t) \cos 2\pi f_c t \qquad v_2(t) = a v_{AM}(t) + b v_{AM}^2(t)$$

$$v_2(t) = aA_c' \cos 2\pi f_c t + aA_c' k_a m(t) \cos 2\pi f_c t + bA_c'^2 \cos^2 2\pi f_c t + bA_c'^2 k_a^2 m^2(t) \cos^2 2\pi f_c t + 2bA_c'^2 k_a m(t) \cos^2 2\pi f_c t$$

The output of LPF would be

$$v_o = \underbrace{bA_c'^2 k_a m(t)}_{\text{Signal component}} + \underbrace{\frac{b}{2} A_c'^2 k_a^2 m^2(t)}_{\text{Noise component}}$$



So the demodulation using square law demodulator is possible if

$$\frac{\text{Signal component}}{\text{Noise component}} = \frac{bA_c'^2 k_a m(t)}{\frac{b}{2} A_c'^2 k_a^2 m^2(t)} = \frac{2}{k_a m(t)} \gg 1$$

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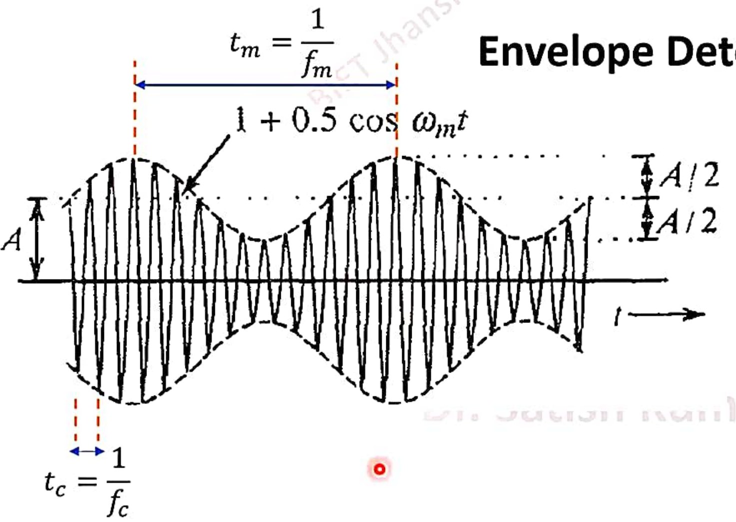
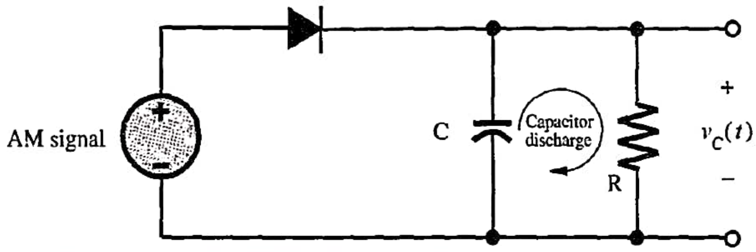
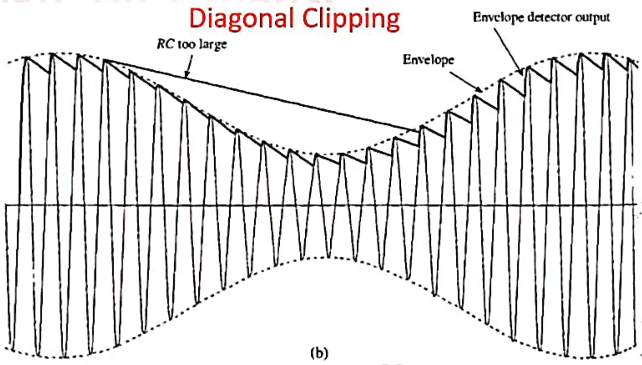
Envelope Detection of AM Signal







- Charging time constant (R_sC) should be as small as possible. Ideally, it should be zero.
- Discharging time constant (RC) should be large ($RC \gg \frac{1}{f_c}$).

$$\frac{1}{f_c} \ll RC \ll \frac{1}{f_m}$$

Upper Limit of RC-time constant to ensure the Envelope Detection

Voltage across the capacitor

$v_c(t) = E_1 e^{-t/RC}$, E_1 is the peak voltage at any time

The first two terms of exponential series

$$v_c(t) = E_1\left(1 - \frac{t}{RC}\right) \qquad .....(1)$$

Slope of the discharge

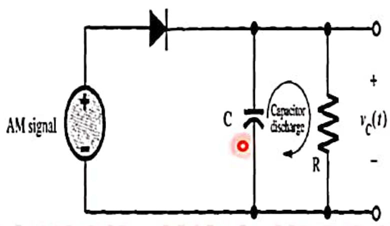
$$\frac{dv_c(t)}{dt} = -\frac{E_1}{RC}$$

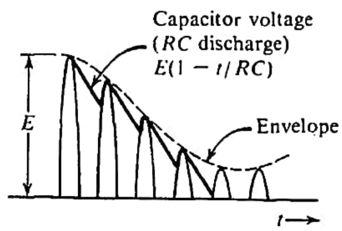
To ensure envelope detection

$$\left|\frac{dv_c(t)}{dt}\right| = \frac{E_1}{RC} \geq \left|\frac{dE(t)}{dt}\right| \qquad .....(2)$$

Slope of the discharge **Slope of the envelope**

$$E(t) = A_c(1 + \mu \cos 2\pi f_m t)$$





$$\frac{dE(t)}{dt} = -\mu A_c \omega_m \sin \omega_m t$$

$$\frac{A_c(1 + \mu \cos 2\pi f_m t)}{RC} \geq \mu A_c \omega_m \sin \omega_m t \qquad \text{For all 't'}$$

$$\boxed{RC \leq \frac{(1 + \mu \cos 2\pi f_m t)}{\mu \omega_m \sin \omega_m t}} \qquad .....(3)$$

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Upper Limit of RC-time constant to ensure the Envelope Detection

Maximum possible value of RC to ensure envelope detection

This will happen when RHS of Eq. (3) will be minimum

Finding the minima

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{1+\mu\cos 2\pi f_m t}{\mu\omega_m \sin\omega_m t}\right)=0$$

$$\Rightarrow \quad \frac{-\mu\omega_m \sin\omega_m t\,.\,\mu\omega_m \sin\omega_m t-\mu\omega_m^2\cos\omega_m t\,(1+\mu\cos 2\pi f_m t)}{\mu^2\omega_m^2\sin^2\omega_m t}=0$$

$$\Rightarrow \quad -\mu^2\omega_m^2\sin^2\omega_m t-\mu\omega_m^2\cos\omega_m t-\mu^2\omega_m^2\cos^2\omega_m t=0$$

$$\Rightarrow \quad \mu^2\omega_m^2+\mu\omega_m^2\cos\omega_m t=0$$

$$\Rightarrow \quad \cos\omega_m t=-\mu \qquad .....(4)$$

$$\Rightarrow \quad \cos^2\omega_m t=\mu^2$$

$$\Rightarrow \quad 1-\sin^2\omega_m t=\mu^2$$

$$\Rightarrow \quad \sin\omega_m t=\sqrt{1-\mu^2} \qquad .....(5)$$

Putting Eq. (4), (5) into eq. (3)

$$\Rightarrow \quad RC\le\frac{(1-\mu^2)}{\mu\omega_m\sqrt{1-\mu^2}}$$

$$\Rightarrow \quad \boxed{RC\le\frac{1}{\omega_m}\frac{\sqrt{1-\mu^2}}{\mu}} \qquad .....(6)$$

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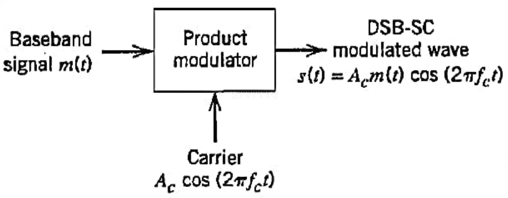
Double Side Band-Supressed Carrier (DSB-SC) Modulation Scheme

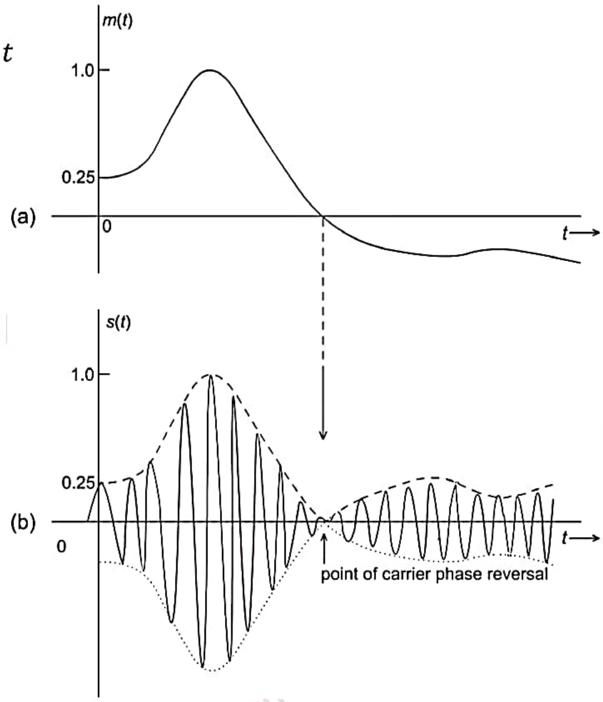
Standard time-domain expression for conventional AM Signal

$$s_{AM}(t) = A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t = A_c(1 + \mu \cos 2\pi f_m t) \cos 2\pi f_c t$$

Time-Domain Expression for DSB-SC

$$S_{DSB-SC}(t) = A_c m(t) \cos 2\pi f_c t$$





Single-Tone Modulation

$$m(t) = A_m \cos 2\pi f_m t$$

$$S_{DSB-SC}(t) = A_c A_m \cos 2\pi f_m t \cos 2\pi f_c t$$

$$S_{DSB-SC}(t) = \frac{A_m A_c}{2} \cos 2\pi(f_c + f_m)t + \frac{A_m A_c}{2} \cos 2\pi(f_c - f_m)t$$

$$S_{DSB-SC}(f) = \frac{A_m A_c}{4}\left(\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m)\right) + \frac{A_m A_c}{4}\left(\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m)\right)$$

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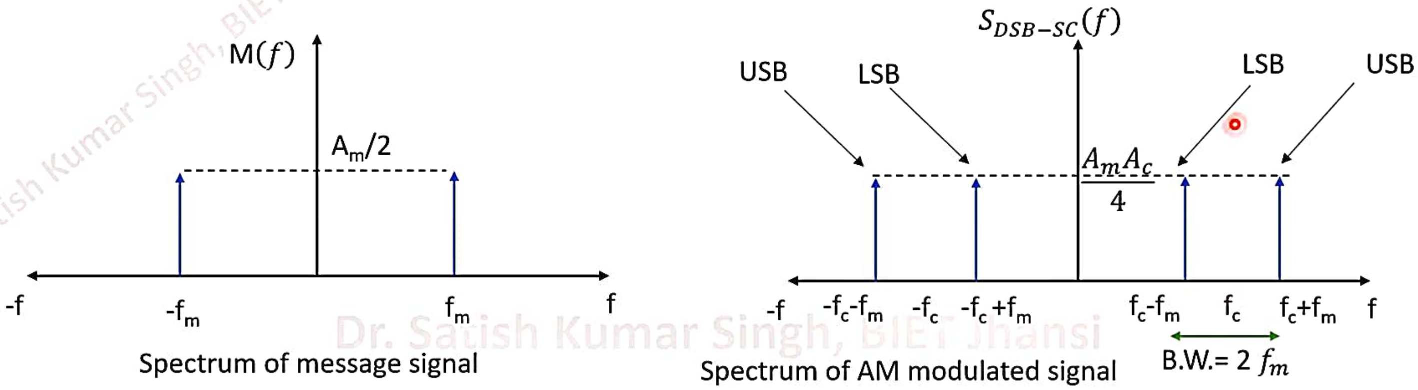
Double Side Band-Supressed Carrier (DSB-SC) Modulation Scheme



$$S_{DSB-SC}(t) = \frac{A_m A_c}{2}\cos 2\pi(f_c+f_m)t + \frac{A_m A_c}{2}\cos 2\pi(f_c-f_m)t$$

$$P_T = P_{SB}$$

$$P_T = P_{USB} + P_{LSB}$$

$$P_T = \frac{A_m^2 A_c^2}{8} + \frac{A_m^2 A_c^2}{8} = \frac{A_m^2 A_c^2}{4}$$

Modulation Efficiency

$$\eta = \frac{P_{SB}}{P_T} = 100\ \%$$

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DSB-SC Modulation Scheme for Multi-tone Signal

Time-Domain Expression

$$S_{DSB-SC}(t) = A_c m(t)\cos 2\pi f_c t$$

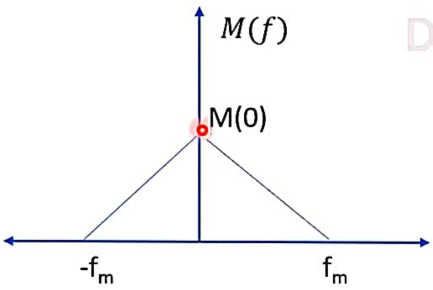
Taking Fourier transform and using modulation property, we can have

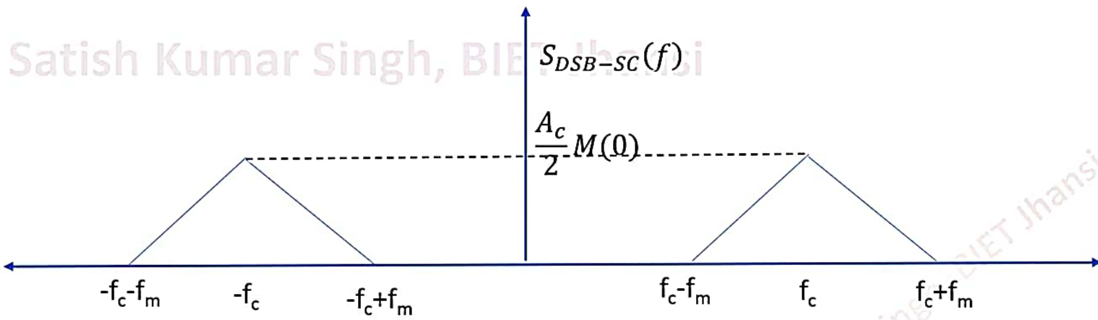
$$S_{DSB-SC}(f) = \frac{A_c}{2}[M(f-f_c) + M(f+f_c)]$$

Synchronous Demodulation

$$V_o(t) = A_c m(t)\cos 2\pi f_c t \times \cos 2\pi f_c t$$
$$= \frac{A_c}{2}(m(t) + m(t)\cos 4\pi f_c t)$$

$$V_o(f) = \frac{A_c}{2}M(f) + \frac{A_c}{2}(M(f-2f_c) + M(f+2f_c))$$



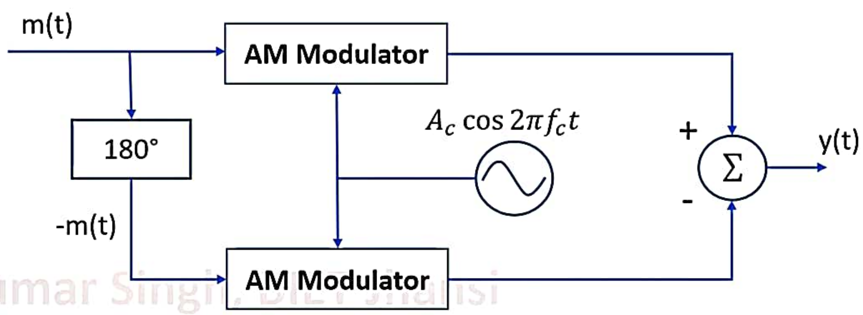


Generation of DSB-SC Modulation

1. Balanced Modulator

$$y(t) = A_c(1 + k_a m(t)) \cos 2\pi f_c t - A_c(1 - k_a m(t)) \cos 2\pi f_c t$$

$$y(t) = 2A_c k_a m(t) \cos 2\pi f_c t$$



Numerical Problems (Communication Engg.)

1. A carrier $c(t) = 5\cos(2\pi 10^6 t)$, and message signal $m(t) = \cos(8\pi 10^3 t)$ are used to generate AM signal with modulation index is 0.5.

(i) Sketch the spectrum and determine the bandwidth and power.

(ii) Determine the quantity $\frac{P_{SB}}{P_c}$.

2. An AM signal represented in time domain as
   $s(t) = 4\cos(1800\pi t) + 10\cos(2000\pi t) + 4\cos(2200\pi t)$
   Sketch the spectrum and calculate: bandwidth, efficiency, and power of $s(t)$.

Numerical Problems (Communication Engg.)

3. An AM transmitter radiates 50 watts when the carrier is modulated by a sinusoidal signal with modulation index of 0.707.

(i) Determine modulation efficiency, carrier power and side band power.

(ii) Determined the peak amplitude of the carrier before modulation and after modulation.

4. The peak amplitude of an AM signal varies from 2 Volts to 10 Volts. Determine the modulation index, carrier power and side band power.

5. An AM signal is represented in time domain as

$$s(t) = 10(1 + 0.4\cos(2\pi 10^4 t)\cos(2\pi 10^6 t)$$

Determine the bandwidth, efficiency and total power.

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Numerical Problems (Communication Engg.)

6. The antenna current of AM transmitter is 8 A, when carrier is transmitted alone. The antenna current is increased to 8.5 A when the carrier is modulated. Determine the modulation index and modulation efficiency.

7. A carrier signal $c(t) = 20\cos(2\pi 10^6 t)$ is modulated by a message signal $m(t) = 5\cos(8\pi 10^3 t)$ to generate a DSB-SC signal. Sketch the spectrum and determine bandwidth power and modulation efficiency.

8. Consider a signal $x(t) = A_c \cos 2\pi f_c t + K \sin 2\pi f_m t$ passed through a square law device with output $y(t)$ corresponding to input $x(t)$ given as $y(t) = x^2(t)$. The output is passed through a bandpass filter with centre frequency f_c. Find the expression of DSB-SC signal generated.

9. The output voltage $v_2(t)$ of the non-linear device can be expressed in terms of the input voltage $v_1(t)$ as $v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$, where a_1, a_2 are known constants. Consider an input voltage signal $v_1(t)$ defined as $v_1(t) = A_c \cos 2\pi f_c t + m(t)$, where m(t) is message signal of bandwidth W and $A_c \cos 2\pi f_c t$ is the carrier wave. The output $v_2(t)$ is fed as input to a filter tuned to f_c. What is the amplitude sensitivity k_a of the AM signal thus generated with carrier frequency f_c?

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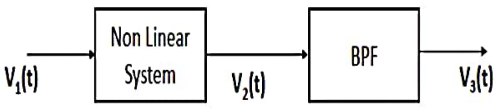
Numerical Problems (Communication Engg.)

10. A carrier signal $c(t) = 20\cos(2\pi 10^6 t)$ is modulated by a message signal $m(t) = 5\cos(8\pi 10^3 t) + 2\cos(2\pi 10^4 t)$ to generate a DSB-SC signal. Sketch the spectrum and determine bandwidth, power and modulation efficiency.
11. Obtain the expression of the signal $V_3(t)$ in the shown figure. Where $V_1(t) = 10\cos(2000\pi t) + 4\cos(200\pi t)$, $V_2(t) = V_1(t) + 0.1V_1^2(t)$. The BPF is having unity gain with passband from 800 Hz to 1200 Hz.



12. A carrier signal of 1 V amplitude sinusoid and a sinusoidal modulating signal of 0.5 V connected in series are applied to a square law modulator of characteristics

$$i_o = 10 + K_1 V_i + K_2 V_i^2 \text{ mA},$$

where V_i is the input in volts and $K_1 = 2\ mA/V$, $K_2 = 0.2\ mA/V^2$. Considering only the frequency components of the AM signal determine the modulation index.

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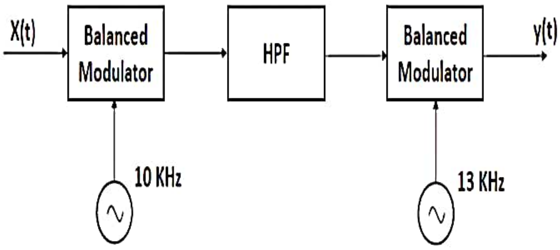
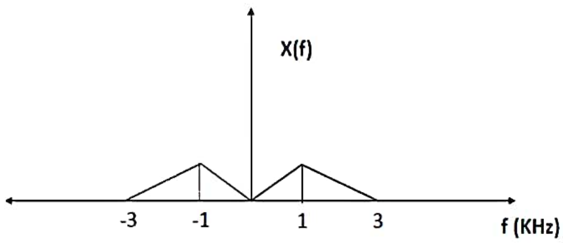
Numerical Problems (Communication Engg.)

13. A DSB signal is to be generated with a carrier frequency of 1 MHz using non-linear device having characteristics $V_o = aV_i + bV_i^3$, where a, b are constants and $V_i = m(t) + \cos(2\pi f_1 t)$. Output of the non-linear device can be filtered by an appropriate band pass filter. Determine the value of f_1 such that the carrier frequency of the DSB signal is 1 MHz.
14. Consider the system shown in the figure. Let *X(f)*, *Y(f)* represents the Fourier transform of *x(t)* and *y(t)*, respectively. The ideal high pass filter has a cut off frequency of 10 KHz. Find the spectrum of *Y(f)*.





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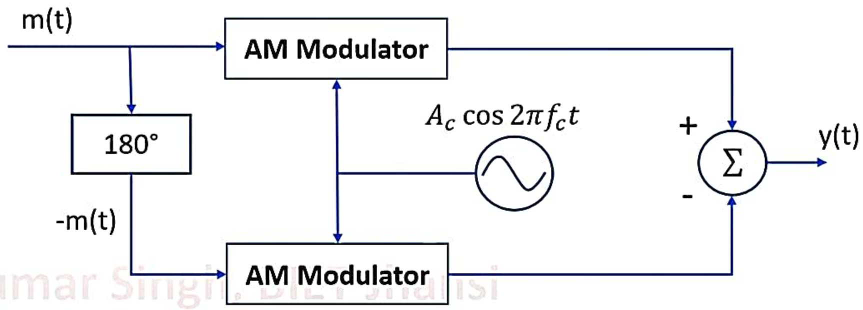
Numerical Problems (Communication Engg.)

15. A 100 MHz carrier of 1 V amplitude and a 1 MHz modulating signal of 1 V are applied to a balanced modulator. The output of the balanced modulator is passed to an ideal high pass filter with cut off frequency of 100 MHz signal. The output of the filter is added with 100 MHz signal of 1 V amplitude and 90° phase shift. Find the envelope of the resultant signal.
16. A 1 MHz sinusoidal carrier is amplitude modulated by a symmetrical square wave of time period 100 µsec. Sketch the spectrum of modulated signal.

Generation of DSB-SC Modulation

1. Balanced Modulator

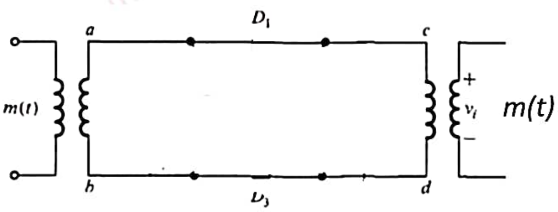


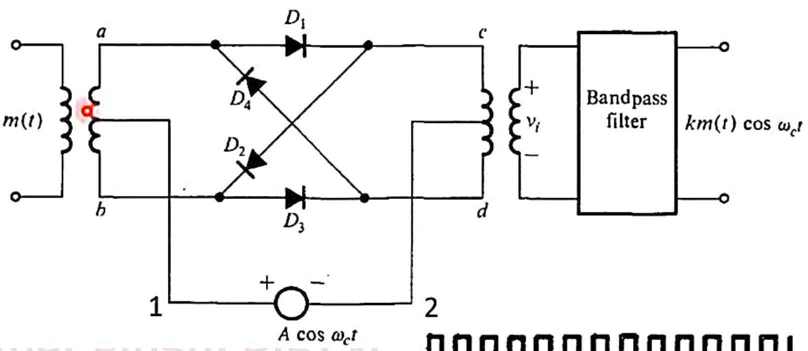
$$y(t) = A_c(1 + k_a m(t)) \cos 2\pi f_c t - A_c(1 - k_a m(t)) \cos 2\pi f_c t$$

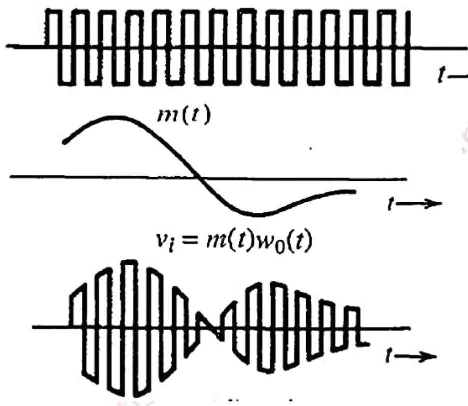
$$y(t) = 2A_c k_a m(t) \cos 2\pi f_c t$$

DSB-SC Signal Generation using Ring Modulator

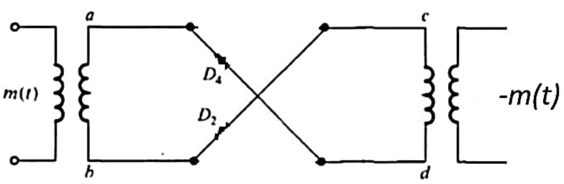
Case 1: When terminal 1 is more positive than terminal 2 then D1 and D3 will be forward biased and D2 and D4 will be reverse biased.







Case 2: When terminal 2 is more positive than terminal 1 then D1 and D3 will be reverse biased and D2 and D4 will be forward biased.



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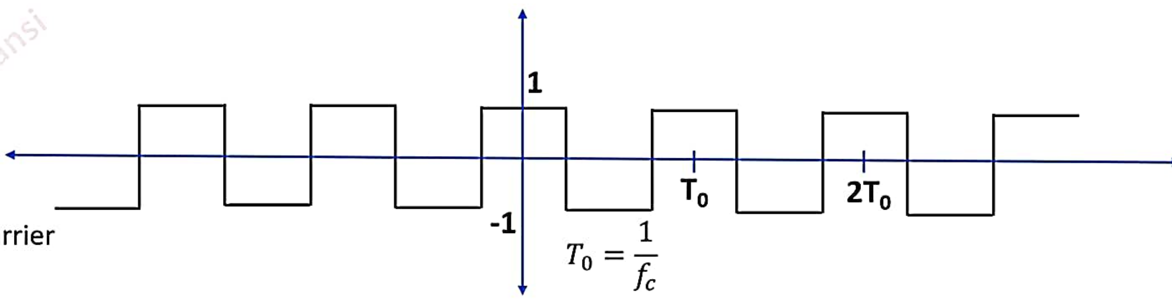
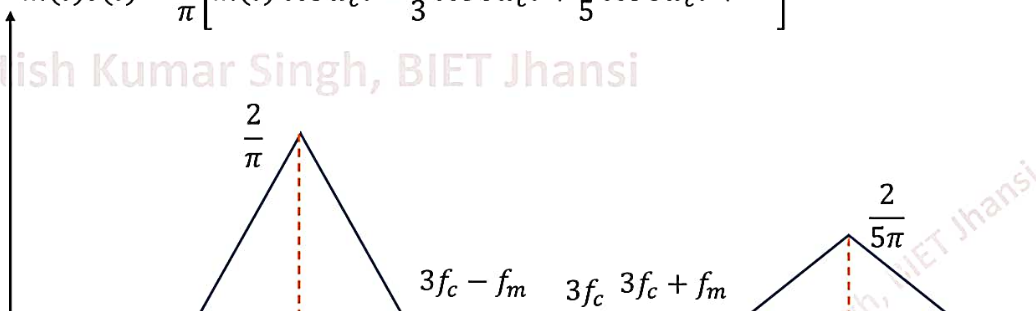


For a symmetrical square wave carrier trigonometric Fourier series is

$$c(t) = \frac{4}{\pi} \sum_{n=1,3,5\ldots} \frac{(-1)^{\frac{n-1}{2}}}{n} \cos n\omega_c t$$

$$m(t)c(t) = \frac{4}{\pi}\left[m(t)\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t + \cdots\right]$$



After passing the $m(t)c(t)$ through an appropriate BPF, we can get

$$S_{DSB-SC}(t) = \frac{4}{\pi} m(t) \cos\omega_c t$$

Other Method

DSB-SC Modulation Scheme for Multi-tone Signal

Time-Domain Expression

$$S_{DSB-SC}(t) = A_c m(t) \cos 2\pi f_c t$$

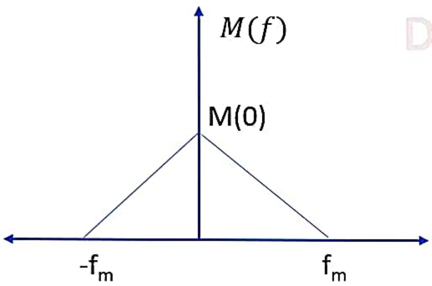
Taking Fourier transform and using modulation property, we can have

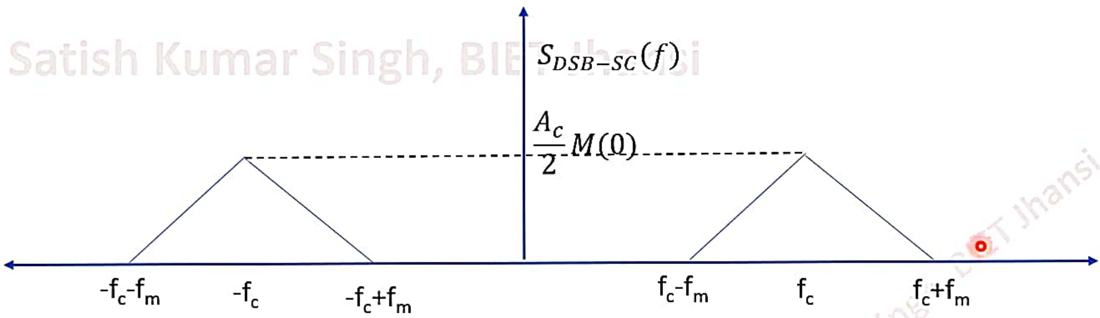
$$S_{DSB-SC}(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$

Synchronous Demodulation

$$V_o(t) = A_c m(t) \cos 2\pi f_c t \times \cos 2\pi f_c t$$
$$= \frac{A_c}{2}(m(t) + m(t) \cos 4\pi f_c t)$$

$$V_o(f) = \frac{A_c}{2} M(f) + \frac{A_c}{2}(M(f - 2f_c) + M(f + 2f_c))$$





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Single Side-Band-Suppressed Carrier (SSB-SC)

Time-domain expression of DSB-SC Signal

$$S_{DSB-SC}(t) = \frac{A_m A_c}{2}\cos 2\pi(f_c + f_m)t + \frac{A_m A_c}{2}\cos 2\pi(f_c - f_m)t$$

Time-domain expression of SSB-SC Signal

$$S_{SSB-USB}(t) = \frac{A_m A_c}{2}\cos 2\pi(f_c + f_m)t$$

$$S_{SSB-LSB}(t) = \frac{A_m A_c}{2}\cos 2\pi(f_c - f_m)t$$

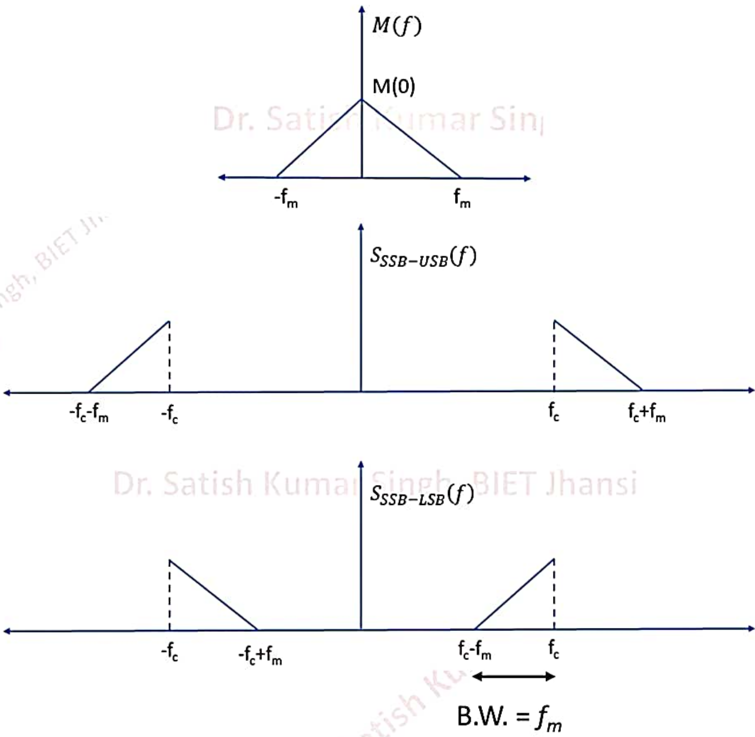
$$S_{SSB}(t) = \frac{A_m A_c}{2}\cos 2\pi(f_c \pm f_m)t$$

$$S_{SSB}(t) = \frac{A_m A_c}{2}\cos 2\pi f_c t \cos 2\pi f_m t \mp \frac{A_m A_c}{2}\sin 2\pi f_c t \sin 2\pi f_m t$$

$$\boxed{S_{SSB}(t) = \frac{A_c}{2} m(t)\cos 2\pi f_c t \mp \frac{A_c}{2}\widehat{m(t)}\sin 2\pi f_c t}$$

where $\widehat{m(t)}$ is the Hilbert transform of $m(t)$

'-' sign represents USB modulated
'+' sign represents LSB modulated



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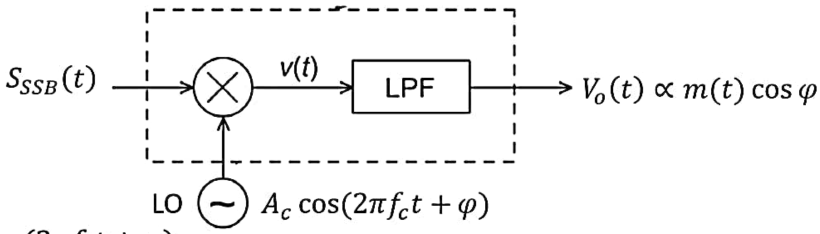
Demodulation of SSB-SC Signal

$$S_{SSB}(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t \mp \frac{A_c}{2} \widehat{m(t)} \sin 2\pi f_c t$$



$$v(t) = \left(\frac{A_c}{2} m(t) \cos 2\pi f_c t \mp \frac{A_c}{2} \widehat{m(t)} \sin 2\pi f_c t\right) \times A_c \cos(2\pi f_c t + \varphi)$$

$$v(t) = \frac{A_c^2}{4} m(t)(\cos(4\pi f_c t + \varphi) + \cos\varphi) \mp \frac{A_c^2}{4} \widehat{m(t)}(\sin(4\pi f_c t + \varphi) - \sin(-\varphi))$$

After passing through LPF

$$v_o(t) = \frac{A_c^2}{4} m(t) \cos\varphi \mp \frac{A_c^2}{4} \widehat{m(t)} \sin(\varphi)$$

If $\varphi = 0°$, $v_o(t) = \frac{A_c^2}{4} m(t)$

If $\varphi = 90°$, $v_o(t) = \frac{A_c^2}{4} \widehat{m(t)}$

No Quadrature Null Effect

Power Requirement

$$S_{SSB}(t) = \frac{A_m A_c}{2} \cos 2\pi(f_c \pm f_m)t \qquad P_T = P_{SB} \qquad P_T = \frac{A_m^2 A_c^2}{8}$$

Advantages:

- Low Bandwidth,
- Less Power Requirement
- No Quadrature Null Effect

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Power Saving (PS)

AM $\longrightarrow$ DSB-SC $\longrightarrow$ SSB-SC

Power Requirement of AM Signal

$$P_T = P_C + P_{USB} + P_{LSB}$$

$$P_T = P_C\left(1 + \frac{\mu^2}{2}\right)$$

Power Requirement of DSB-SC Signal

$$P_T = P_{USB} + P_{LSB}$$

$$P_T = \frac{\mu^2 A_c^2}{8} + \frac{\mu^2 A_c^2}{8} = \frac{\mu^2 A_c^2}{4}$$

Power Requirement of SSB-SC Signal

$$P_T = P_{USB} \text{ or } P_{LSB}$$

$$P_T = \frac{\mu^2 A_c^2}{8}$$

Power saved in DSB-SC w.r.t. AM

$$\%\,\text{PS} = \frac{\text{Power Saved}}{\text{Power used in AM}} \times 100\,\%$$

$$\text{PS} = \frac{A_c^2}{2} + \frac{A_c^2\mu^2}{4} - \frac{A_c^2\mu^2}{4} = \frac{A_c^2}{2}$$

$$\%\,\text{PS} = \frac{A_c^2/2}{\frac{A_c^2}{2}\left(1 + \frac{\mu^2}{2}\right)} \times 100\,\%$$

$$\%\,\text{PS} = \frac{2}{(2 + \mu^2)} \times 100\,\%$$

Power saved in SSB-SC w.r.t. AM

$$\text{PS} = \frac{A_c^2}{2} + \frac{A_c^2\mu^2}{4} - \frac{A_c^2\mu^2}{8}$$

$$\text{PS} = \frac{A_c^2}{2}\left(1 + \frac{\mu^2}{4}\right)$$

$$\%\,\text{PS} = \frac{\frac{A_c^2}{2}\left(1 + \frac{\mu^2}{4}\right)}{\frac{A_c^2}{2}\left(1 + \frac{\mu^2}{2}\right)} \times 100\,\%$$

$$\%\,\text{PS} = \frac{\left(1 + \frac{\mu^2}{4}\right)}{\left(1 + \frac{\mu^2}{2}\right)} \times 100\,\%$$

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$$S_{SSB}(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t \mp \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

$$A \cos 2\pi f_c t + B \sin 2\pi f_c t$$

Have the complex envelope $(A + jB)$

and phase $\varphi = \tan^{-1}\left(\frac{B}{A}\right)$

$$A \cos 2\pi f_c t + B \sin 2\pi f_c t = \sqrt{A^2 + B^2} \cos[2\pi f_c t + \varphi]$$

In-phase component — Quadrature-phase component

Canonical representation of narrowband bandpass noise process

$$n(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t$$

In-phase component — Quadrature-phase component

Pre-envelope :

For any narrowband bandpass noise process signal *n(t)*

$$n_{pe}(t) = n(t) + j\hat{n}(t) \qquad \text{..... (a)}$$

Complex envelope :

$$n_{ce}(t) = n_{pe}(t) e^{-j2\pi f_c t} \qquad \text{..... (b)}$$

$$n_{ce}(t) = (n(t) + j\hat{n}(t))(\cos 2\pi f_c t - j \sin 2\pi f_c t)$$

$$n_{ce}(t) = n(t) \cos 2\pi f_c t + \hat{n}(t) \sin 2\pi f_c t + j(\hat{n}(t) \cos 2\pi f_c t - n(t) \sin 2\pi f_c t)$$

$$n_{ce}(t) = n_c(t) + j n_s(t)$$

where

$$n_c(t) = n(t) \cos 2\pi f_c t + \hat{n}(t) \sin 2\pi f_c t$$

$$n_s(t) = \hat{n}(t) \cos 2\pi f_c t - n(t) \sin 2\pi f_c t$$

We have

$$n(t) = \frac{n_{pe}(t) + n_{pe}^*(t)}{2} \qquad \text{..... (c)}$$

SSB-SC Modulation as a Hybrid AM and Phase Modulation

from Eq. (b)

$$n_{pe}(t) = n_{ce}(t)e^{j2\pi f_c t}$$

$$\Rightarrow \; n_{pe}^*(t) = n_{ce}^*(t)e^{-j2\pi f_c t}$$

So from eqn (c)

$$n(t) = \frac{1}{2}\{n_{ce}(t)e^{j2\pi f_c t} + n_{ce}^*(t)e^{-j2\pi f_c t}\}$$

$$n(t) = \frac{(n_{ce}(t) + n_{ce}^*(t))}{2}\cos 2\pi f_c t - \left[-\frac{j}{2}(n_{ce}(t) - n_{ce}^*(t))\right]\sin 2\pi f_c t$$

$$n(t) = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t \quad \text{..... (d)}$$

$x_c(t)$ = Real part of complex envelope $x_{ce}(t)$

$x_s(t)$ = Imaginary part of complex envelope $x_{ce}(t)$

So, the complex envelope is

$$n_{ce}(t) = n_c(t) + jn_s(t)$$

Envelope and Phase Representation or Polar Representation

Therefore, $n_{ce}(t)$ can be expressed in polar form

$$n_{ce}(t) = A(t)e^{j\varphi(t)}$$

where,

$$A(t) = \sqrt{n_c^2(t) + n_s^2(t)} \qquad \varphi(\mathrm{t}) = \tan^{-1}\left(\frac{n_s(t)}{n_c(t)}\right)$$

again

$$n(t) = Re[x_{pe}(t)]$$

$$n(t) = Re[x_{ce}(t)e^{j2\pi f_c t}]$$

$$n(t) = Re[A(t)e^{j\varphi(t)}e^{j2\pi f_c t}]$$

$$n(t) = A(t)\cos[2\pi f_c t + \varphi(t)] \quad \text{..... (e)}$$

SSB-SC Modulation as a Hybrid AM and Phase Modulation

from Eq. (b)

$$n_{pe}(t) = n_{ce}(t)e^{j2\pi f_c t}$$

$$\Rightarrow \; n_{pe}^*(t) = n_{ce}^*(t)e^{-j2\pi f_c t}$$

So from eqn (c)

$$n(t) = \frac{1}{2}\{n_{ce}(t)e^{j2\pi f_c t} + n_{ce}^*(t)e^{-j2\pi f_c t}\}$$

$$n(t) = \frac{(n_{ce}(t) + n_{ce}^*(t))}{2}\cos 2\pi f_c t - \left[-\frac{j}{2}(n_{ce}(t) - n_{ce}^*(t)\right]\sin 2\pi f_c t$$

$$n(t) = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t \qquad \text{..... (d)}$$

$x_c(t)$ = Real part of complex envelope $x_{ce}(t)$

$x_s(t)$ = Imaginary part of complex envelope $x_{ce}(t)$

So, the complex envelope is

$$n_{ce}(t) = n_c(t) + jn_s(t)$$

Envelope and Phase Representation or Polar Representation

Therefore, $n_{ce}(t)$ can be expressed in polar form

$$n_{ce}(t) = A(t)e^{j\varphi(t)}$$

where,

$$A(t) = \sqrt{n_c^2(t) + n_s^2(t)} \qquad \varphi(t) = \tan^{-1}\left(\frac{n_s(t)}{n_c(t)}\right)$$

again

$$n(t) = Re[x_{pe}(t)]$$

$$n(t) = Re[x_{ce}(t)e^{j2\pi f_c t}]$$

$$n(t) = Re[A(t)e^{j\varphi(t)}e^{j2\pi f_c t}]$$

$$n(t) = A(t)\cos[2\pi f_c t + \varphi(t)] \qquad \text{..... (e)}$$

SSB-SC Modulation as a Hybrid AM and Phase Modulation

from Eq. (b)

$$n_{pe}(t) = n_{ce}(t)e^{j2\pi f_c t}$$

$$\Rightarrow \quad n_{pe}^*(t) = n_{ce}^*(t)e^{-j2\pi f_c t}$$

So from eqn (c)

$$n(t) = \frac{1}{2}\{n_{ce}(t)e^{j2\pi f_c t} + n_{ce}^*(t)e^{-j2\pi f_c t}\}$$

$$n(t) = \frac{(n_{ce}(t) + n_{ce}^*(t))}{2}\cos 2\pi f_c t - \left[-\frac{j}{2}(n_{ce}(t) - n_{ce}^*(t)\right]\sin 2\pi f_c t$$

$$n(t) = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t \quad \text{..... (d)}$$

$x_c(t)$ = Real part of complex envelope $x_{ce}(t)$

$x_s(t)$ = Imaginary part of complex envelope $x_{ce}(t)$

So, the complex envelope is

$$n_{ce}(t) = n_c(t) + jn_s(t)$$

Envelope and Phase Representation or Polar Representation

Therefore, $n_{ce}(t)$ can be expressed in polar form

$$n_{ce}(t) = A(t)e^{j\varphi(t)}$$

where,

$$A(t) = \sqrt{n_c^2(t) + n_s^2(t)} \qquad \varphi(\text{t}) = \tan^{-1}\left(\frac{n_s(t)}{n_c(t)}\right)$$

again

$$n(t) = Re[x_{pe}(t)]$$

$$n(t) = Re[x_{ce}(t)e^{j2\pi f_c t}]$$

$$n(t) = Re[A(t)e^{j\varphi(t)}e^{j2\pi f_c t}]$$

$$n(t) = A(t)\cos[2\pi f_c t + \varphi(t)] \quad \text{..... (e)}$$

SSB-SC Modulation as a Hybrid AM and Phase Modulation

For SSB-USB modulation

$$S_{SSB-USB}(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t - \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

$$n_c(t) = \frac{A_c}{2} m(t) \qquad n_c(t) = \frac{A_c}{2} \hat{m}(t)$$

Complex envelope

$$n_{ce}(t) = \frac{A_c}{2} m(t) + j \frac{A_c}{2} \hat{m}(t)$$

$$A(t) = \frac{A_c}{2} \sqrt{m^2(t) + \hat{m}^2(t)}$$

$$\varphi(t) = \tan^{-1}\left(\frac{\hat{m}(t)}{m(t)}\right)$$

$$S_{SSB-USB}(t) = A(t) \cos[2\pi f_c t + \varphi(t)]$$

For SSB-LSB modulation

$$S_{SSB-LSB}(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

$$n_c(t) = \frac{A_c}{2} m(t) \qquad n_c(t) = -\frac{A_c}{2} \hat{m}(t)$$

Complex envelope

$$n_{ce}(t) = \frac{A_c}{2} m(t) - j \frac{A_c}{2} \hat{m}(t)$$

$$A(t) = \frac{A_c}{2} \sqrt{m^2(t) + \hat{m}^2(t)}$$

$$\varphi(t) = \tan^{-1}\left(\frac{-\hat{m}(t)}{m(t)}\right)$$

$$S_{SSB-LSB}(t) = A(t) \cos[2\pi f_c t + \varphi(t)]$$

Note: SSB-SC modulation is having both amplitude and phase variation with message signal, therefore, it comes under hybrid amplitude and phase modulation.

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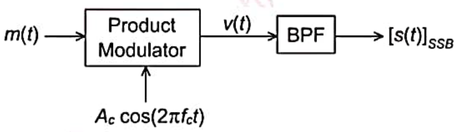
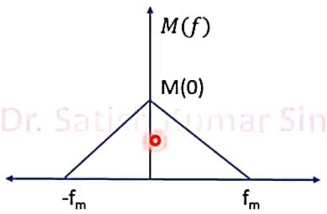
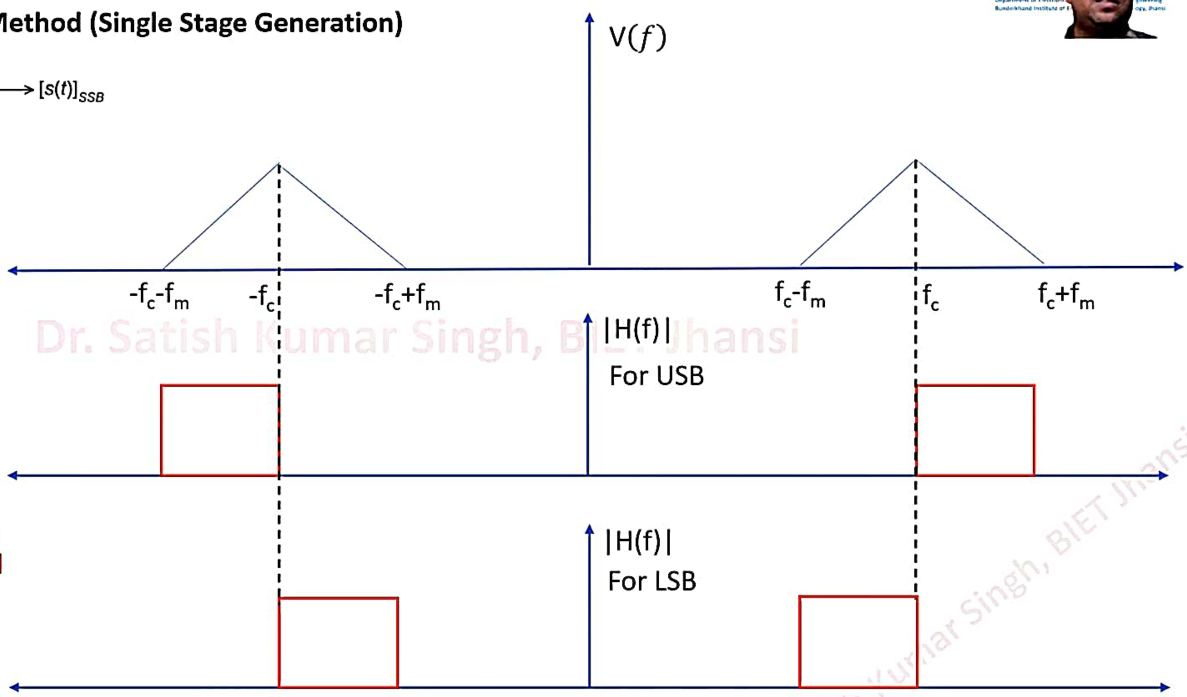
Generation of SSB-SC Signal

1. Frequency Discrimination Method (Single Stage Generation)







- If no spectral null around the dc then ideal filter is required for SSB generation using FD method.

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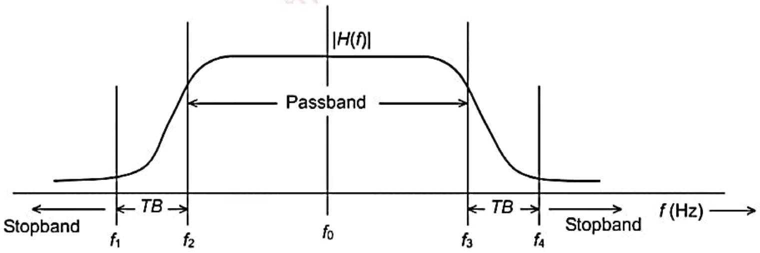
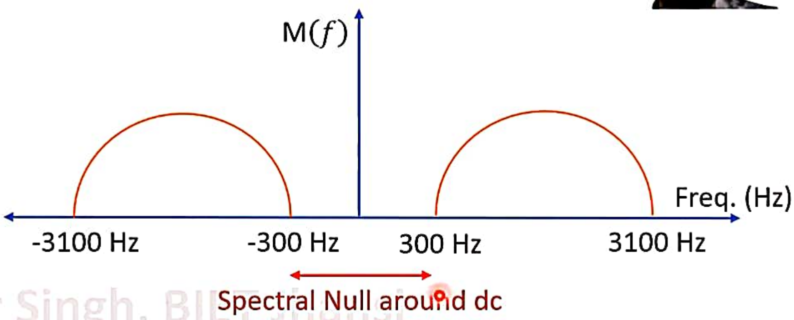
Generation of SSB-SC Signal

Practical Filter Frequency Response





Rule of the thumb for filter design:

TB of the filter should be greater than 1% of center frequency of the bandpass filter.

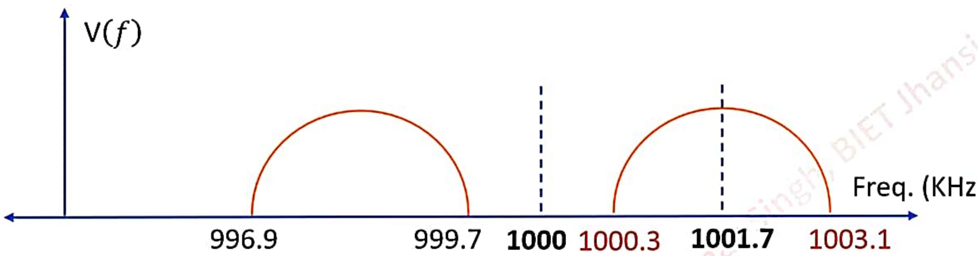
Carrier frequency

f_c = 1 MHz

Center frequency of BPF for USB extraction is

f_0 = 1001.7 KHz

Req. Min. Transition Band (T.B.) = 10.017 KHz



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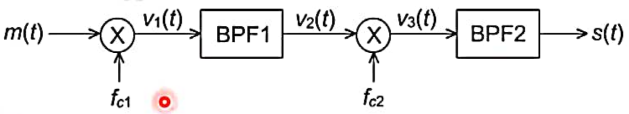
Generation of SSB-SC Signal

1. Frequency Discrimination Method (Two Stage Generation)



Carrier frequency

f_{c1} = 50 kHz

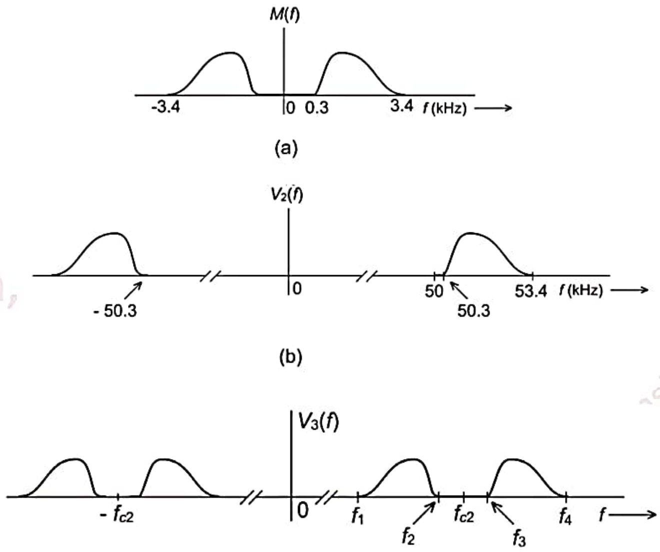
Center frequency of BPF1 for USB extraction is

f_{01} = 51.55 kHz

Req. Min. Transition Band (T.B.) for BPF1

T.B. = 515 Hz

which is less than 600 Hz sideband separation of message signal, therefore, one of the sideband can be filtered out easily



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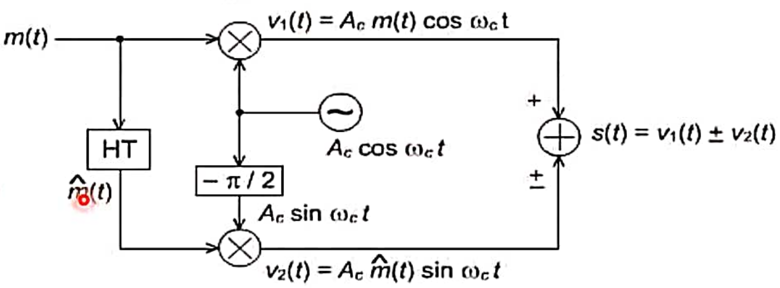
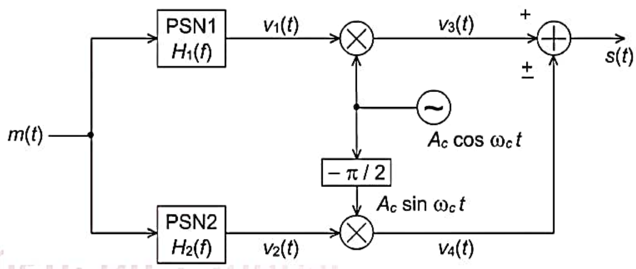
Generation of SSB-SC Signal

2. Phase Discrimination Method



Alternate Arrangement for PD Method



$$H_1(f) = e^{j\theta_1(f)} \qquad H_2(f) = e^{j\theta_2(f)} \qquad \theta_1(f) - \theta_2(f) = \frac{\pi}{2}$$

$$\theta_1(f)|_{f=f_m} = \theta_1 \qquad \theta_2(f)|_{f=f_m} = \theta_2 \qquad \theta_2 = \frac{\pi}{2} + \theta_1$$

$$V_1(t) = A_m \cos(\omega_m t + \theta_1)$$

$$V_2(t) = A_m \cos(\omega_m t + \theta_2)$$

$$V_3(t) = A_m A_c \cos(\omega_m t + \theta_1)\cos(\omega_c t)$$

$$V_4(t) = A_m A_c \cos(\omega_m t + \theta_2)\sin(\omega_c t)$$

$$V_4(t) = A_m A_c \cos(\omega_m t + \theta_1 + \frac{\pi}{2})\sin(\omega_c t)$$

$$V_4(t) = -A_m A_c \sin(\omega_m t + \theta_1)\sin(\omega_c t)$$

$$V_3(t) + V_4(t) = A_m A_c \cos[(\omega_c + \omega_m)t + \theta_1]$$

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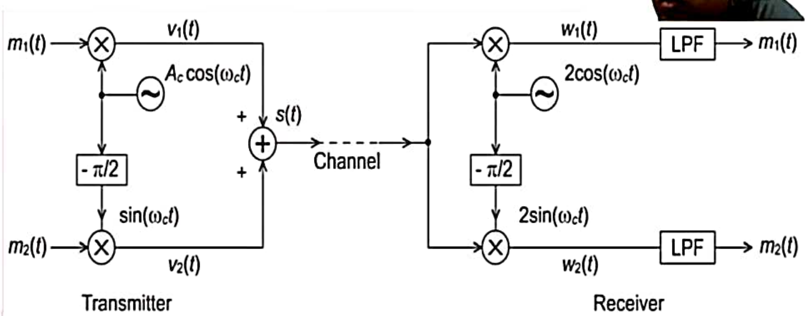
Quadrature Carrier Multiplexing (QCM)/Quadrature Amplitude Modulation (QA

- It is possible to send two DSB-SC signal within 2W bandwidth by quadrature amplitude modulation (QAM).



$$s(t) = m_1(t) A_c \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t$$

If the local carrier has frequency and phase offset then

$$2\cos[2\pi(f_c + \Delta f)t + \varphi]$$

Output of the upper branch is

$$A_c\{m_1(t)\cos[2\pi(\Delta f)t + \varphi] - m_2(t)\sin[2\pi(\Delta f)t + \varphi]\}$$

Output of the lower branch is

$$A_c\{m_2(t)\cos[2\pi(\Delta f)t + \varphi] + m_1(t)\sin[2\pi(\Delta f)t + \varphi]\}$$

If frequency and phase offset exist then $m_1(t)$ will interfere $m_2(t)$ and vice-versa, this is called as cochannel interference.

QAM is used in color TV for multiplexing the chrominance signals.

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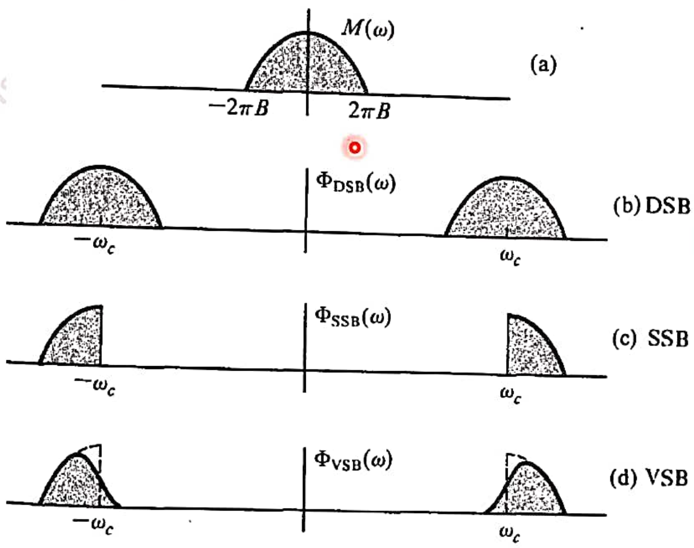
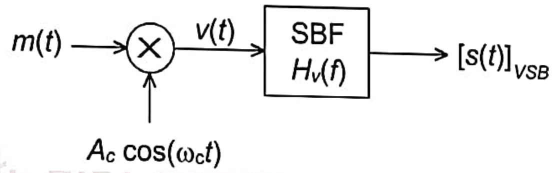
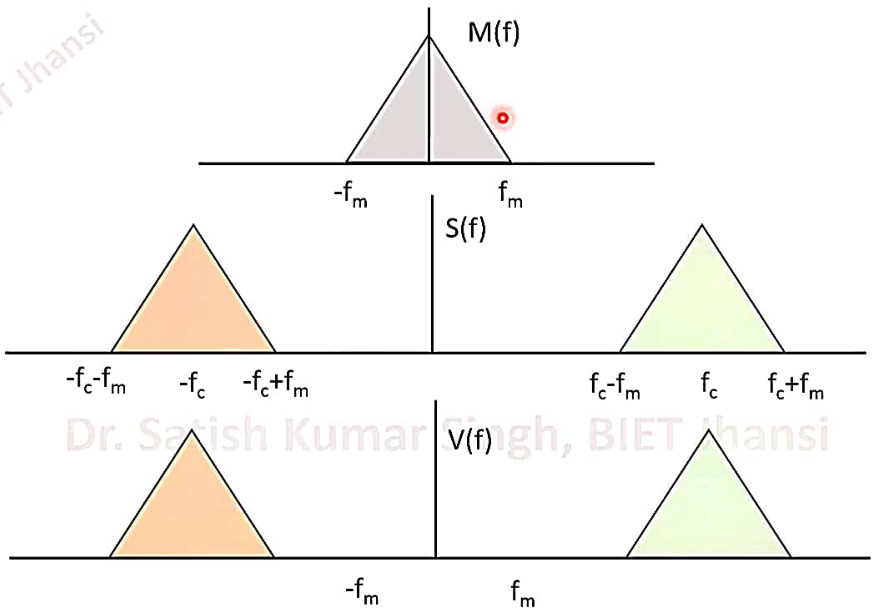
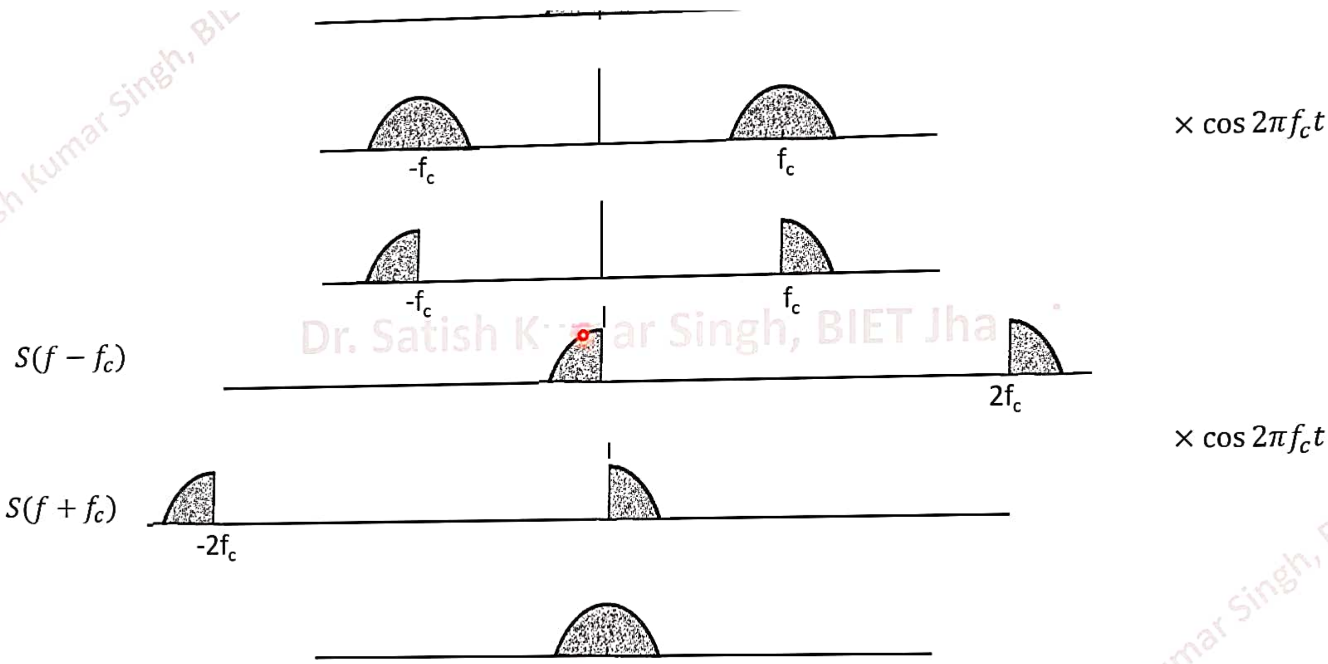
... is added to the wanted sideband.






M(f)
-f_m
f_m
S(f)
-f_c-f_m
-f_c
-f_c+f_m
f_c-f_m
f_c
f_c+f_m
V(f)
-f_m
f_m



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Side Band Filter Characteristics

- Overlap of the spectrum should be such that the resultant spectrum must be undistorted message
- For undistorted message spectrum the sideband filter should have the following two characteristics

1. $H_V(f - f_c) + H_V(f + f_c) = 1, \qquad -W < f < W$
2. Phase response $\theta_V(f)$ should be linear function of frequency

$$\theta_V(f_c) = -2\pi f t_d$$

where t_d represents the slope of the phase characteristics

and also $\quad \theta_V(f_c) = -\theta_V(-f_c) = -2\pi m \quad$ m is an integer

The filter characteristics satisfying the above two conditions is

$$H_V(f) = |H_V(f)|e^{j\theta_V(f)} \qquad f_l \le |f| \le f_c + W$$

Synchronous demodulation of VSB modulated wave

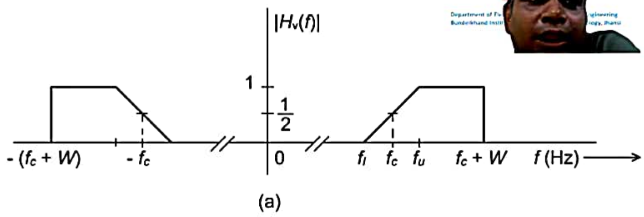
$$v_o(t) = v(t) * h(t)$$

$$V_0(f) = K_1 M(f)[H_V(f - f_c) + H_V(f + f_c)], \qquad \text{for } |f| \le W$$

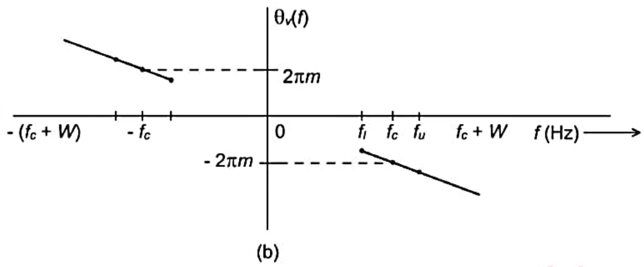
If $\quad [H_V(f - f_c) + H_V(f + f_c)] = K_2 e^{-j2\pi f t_d}, \qquad |f| \le W$

where K_2 is a constant then

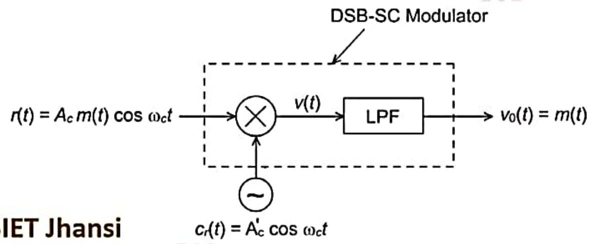
$$v_o(t) = Km(t - t_d), \qquad \text{where } K = K_1 K_2$$



(a)



(b)



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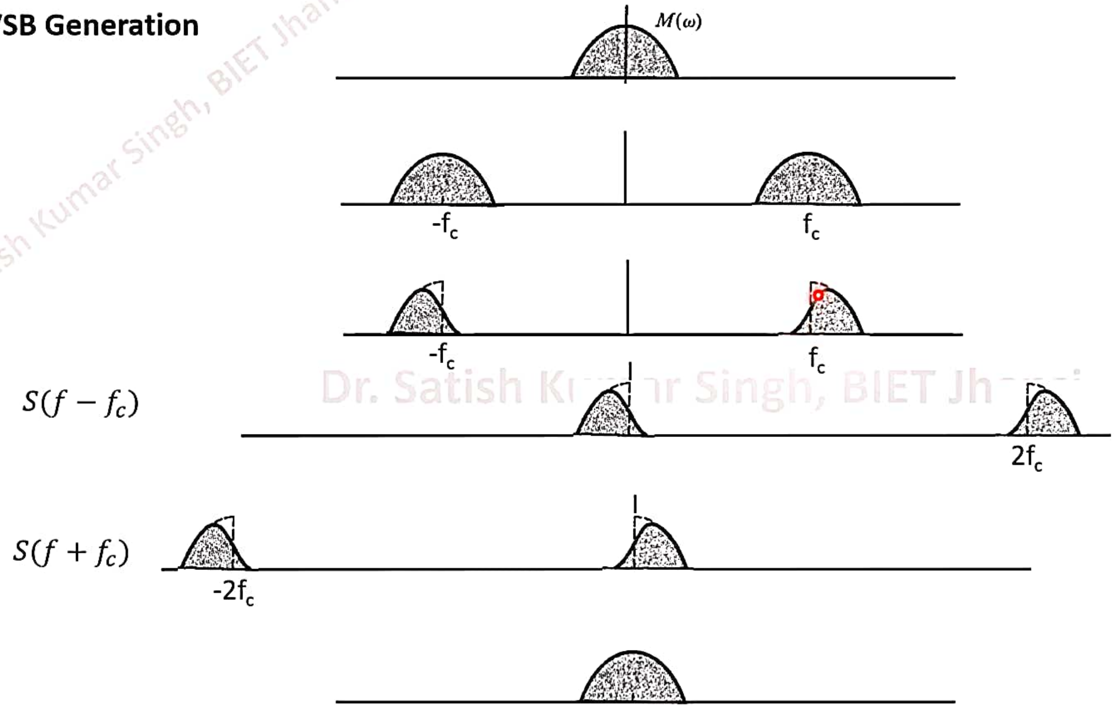
VSB Generation



$\times \cos 2\pi f_c t$

$\times \cos 2\pi f_c t$

Overlap of the spectrum

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Side Band Filter Characteristics

Let

$$H_{1,V}(f) = |H_{1,V}(f)|e^{j\theta_{1,V}(f)} = H_V(f - f_c), \qquad -W \leq f \leq f_V$$

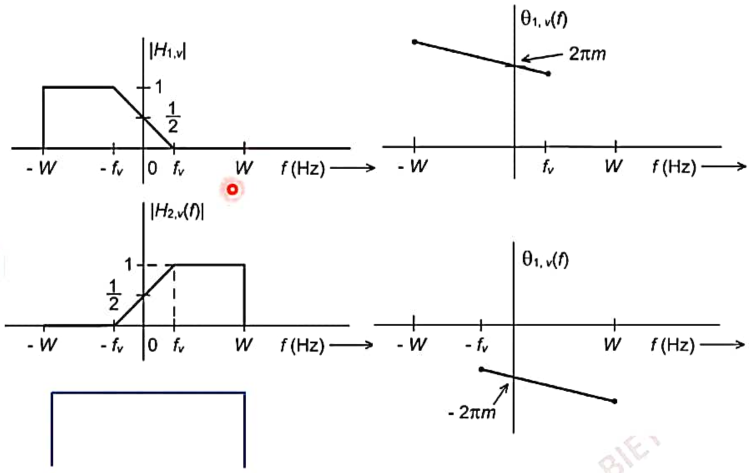
and

$$H_{2,V}(f) = |H_{2,V}(f)|e^{j\theta_{2,V}(f)} = H_V(f + f_c), \qquad -f_V \leq f \leq W$$

$$|H_{1,V}(f)| = 0, \qquad \text{for} \quad f_V \leq f \leq W \quad \text{and} \quad e^{\pm j2\pi m} = 1$$

We can write

$$H_V(f - f_c) = |H_{1,V}(f)|e^{-j2\pi f t_d}, \qquad |f| \leq W$$



Similarly,

$$H_V(f + f_c) = |H_{2,V}(f)|e^{-j2\pi f t_d}, \qquad |f| \leq W$$

$$H_V(f - f_c) + H_V(f + f_c) = \left[|H_{1,V}(f)| + |H_{2,V}(f)|\right]e^{-j2\pi f t_d}, \qquad |f| \leq W$$

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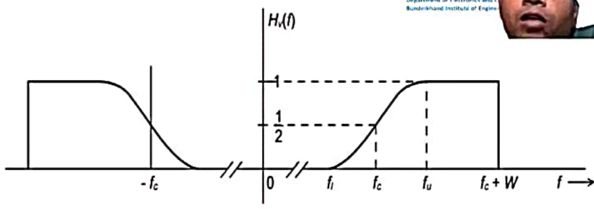
Time-Domain Expression for VSB Modulation



Let $h_v(t)$ denote the impulse response of the sideband filter, $H_v(f)$.

$$s(t)_{VSB} = \int_{-\infty}^{\infty} h_v(\tau) v(t-\tau) d\tau$$

We have $v(t) = A_c m(t) \cos \omega_c t$ For convenience $A_c = 1$

Then, $$s(t)_{VSB} = \int_{-\infty}^{\infty} h_v(\tau) m(t-\tau) \cos[\omega_c(t-\tau)]\, d\tau$$

$$s(t)_{VSB} = \int_{-\infty}^{\infty} h_v(\tau) m(t-\tau)[\cos(\omega_c t)\cos(\omega_c \tau) + \sin(\omega_c t)\sin(\omega_c \tau)] d\tau$$

$$s(t)_{VSB} = \left[\int_{-\infty}^{\infty} h_v(\tau) m(t-\tau) \cos(\omega_c \tau)\, d\tau\right] \cos(\omega_c t) - \left[-\int_{-\infty}^{\infty} h_v(\tau) m(t-\tau) \sin(\omega_c \tau)\, d\tau\right] \sin(\omega_c t)$$

The above equation is similar to canonical form of narrowband signal

Let, $$m_c(t) = \left[\int_{-\infty}^{\infty} h_v(\tau) m(t-\tau) \cos(\omega_c \tau)\, d\tau\right] = \textbf{in-phase component}$$

$m_s(t) = \left[-\int_{-\infty}^{\infty} h_v(\tau) m(t-\tau) \sin(\omega_c \tau)\, d\tau\right]$ =**quadrature-phase component**

Then, $s(t)_{VSB} = m_c(t) \cos(\omega_c t) - m_s(t) \sin(\omega_c t)$ **Canonical representation of VSB signal.**

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Let,

$$h_i(t) = h_v(t)\cos(\omega_c t) \qquad h_q(t) = -h_v(t)\sin(\omega_c t)$$

Then,

$$m_c(t) = m(t) * h_i(t) \qquad m_s(t) = m(t) * h_q(t)$$

Taking FT of above equation, we have

$$M_c(f) = M(f)H_i(f) \qquad M_s(f) = M(f)H_q(f)$$

If $\quad M(f) = 0$ for $|f| \leq W$

Then, $M_c(f)$ and $M_s(f)$ are also bandlimited to W.

We have $\quad h_i(t) = h_v(t)\cos(\omega_c t)$

$$\Rightarrow \quad H_i(f) = \frac{H_v(f - f_c) + H_v(f + f_c)}{2}$$

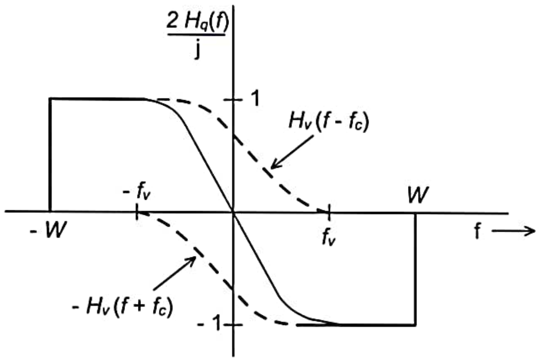
Since,

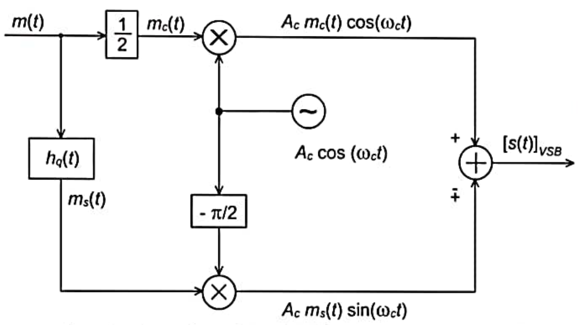
$$H_V(f - f_c) + H_V(f + f_c) = 1, \qquad -W < f < W$$

Hence, $\quad m_c(t) = \dfrac{m(t)}{2}$

Similarly, $\quad H_q(f) = \dfrac{H_v(f - f_c) - H_v(f + f_c)}{2j}$

$$\frac{2H_q(f)}{j} = H_v(f - f_c) - H_v(f + f_c)$$





Phase discrimination Method of VSB Generation

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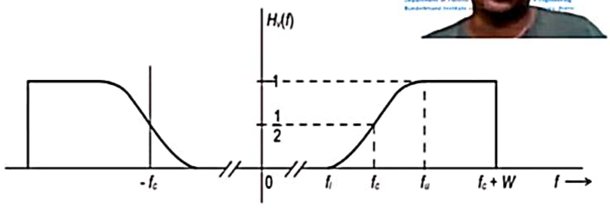
Time-Domain Expression for VSB Modulation

Let $h_v(t)$ denote the impulse response of the sideband filter, $H_v(f)$.



$$s(t)_{VSB} = \int_{-\infty}^{\infty} h_v(\tau) v(t-\tau) d\tau$$

We have $v(t) = A_c m(t)\cos\omega_c t$ For convenience $A_c = 1$

Then, $$s(t)_{VSB} = \int_{-\infty}^{\infty} h_v(\tau) m(t-\tau)\cos[\omega_c(t-\tau)]\, d\tau$$

$$s(t)_{VSB} = \int_{-\infty}^{\infty} h_v(\tau) m(t-\tau)[\cos(\omega_c t)\cos(\omega_c\tau) + \sin(\omega_c t)\sin(\omega_c\tau)] d\tau$$

$$s(t)_{VSB} = \left[\int_{-\infty}^{\infty} h_v(\tau) m(t-\tau)\cos(\omega_c\tau)\, d\tau\right]\cos(\omega_c t) - \left[-\int_{-\infty}^{\infty} h_v(\tau) m(t-\tau)\sin(\omega_c\tau)\, d\tau\right]\sin(\omega_c t)$$

The above equation is similar to canonical form of narrowband signal

Let, $$m_c(t) = \left[\int_{-\infty}^{\infty} h_v(\tau) m(t-\tau)\cos(\omega_c\tau)\, d\tau\right] = \textbf{in} - \textbf{phase component}$$

$$m_s(t) = \left[-\int_{-\infty}^{\infty} h_v(\tau) m(t-\tau)\sin(\omega_c\tau)\, d\tau\right]$$ =**quadrature-phase component**

Then, $s(t)_{VSB} = m_c(t)\cos(\omega_c t) - m_s(t)\sin(\omega_c t)$ **Canonical representation of VSB signal.**

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