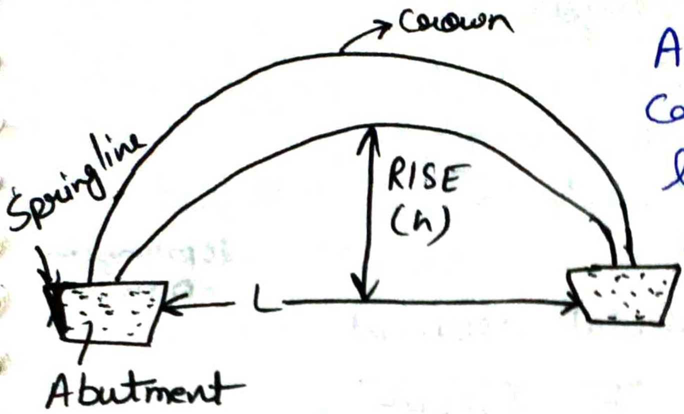
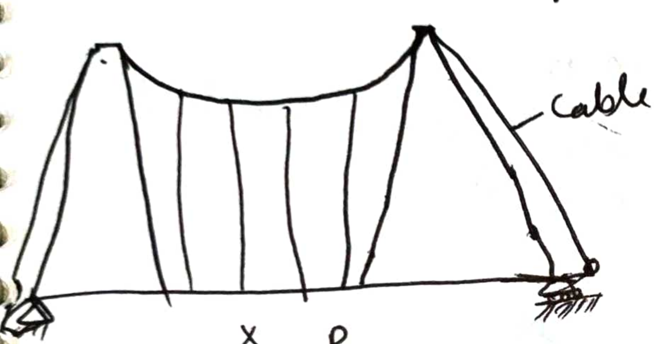
Arches

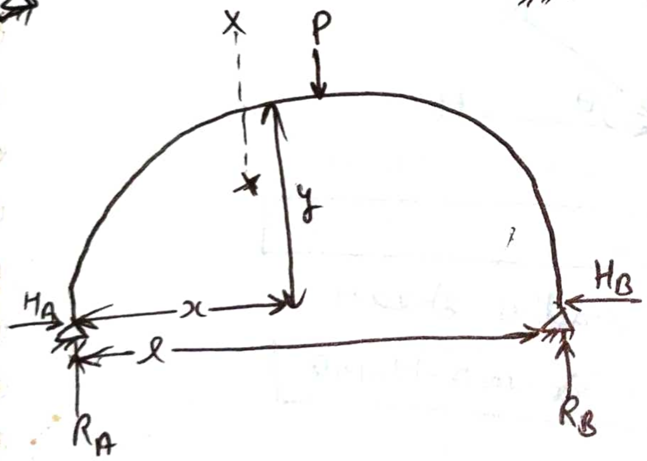


An arch acts like inverted cable it can be used as "reduce the BM" in long span structures. It is mainly subjected to compression but because of rigidity, it resists same BM and SF also.

Arch* $\longrightarrow$ BM$\downarrow$
Compression

Cables	Arches
• It is a tension member.	• It is a compression member
• It is a flexible member & it can change its shape under different loading	• It's a rigid member. Its shape can't be changed
• It has no resistance towards BM	• It is a comp. member but it resist some BM and SF





$M_x = R_A x - Hy$

$$\boxed{M_x = \frac{P}{2}x - Hy}$$

$R_A = \frac{P}{2}$

$R_B = \frac{P}{2}$

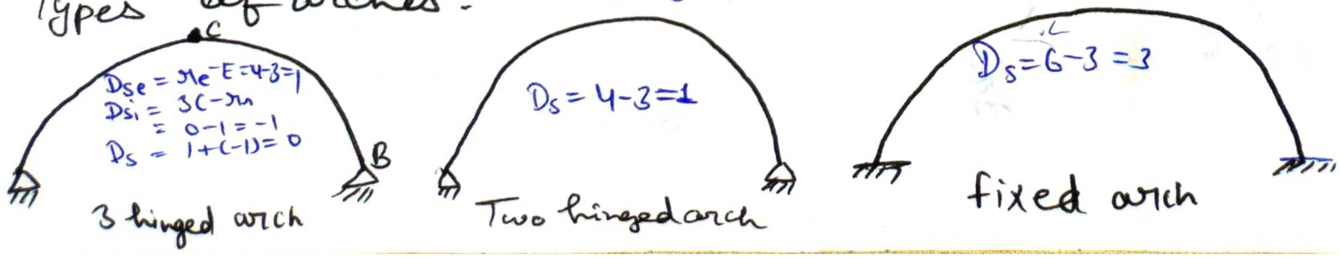
Advantages: (i) for long span structure, arches have less BM comparing with beam

(ii) Since in arch BM$\downarrow$, hence we can used lesser cross-sectional area members

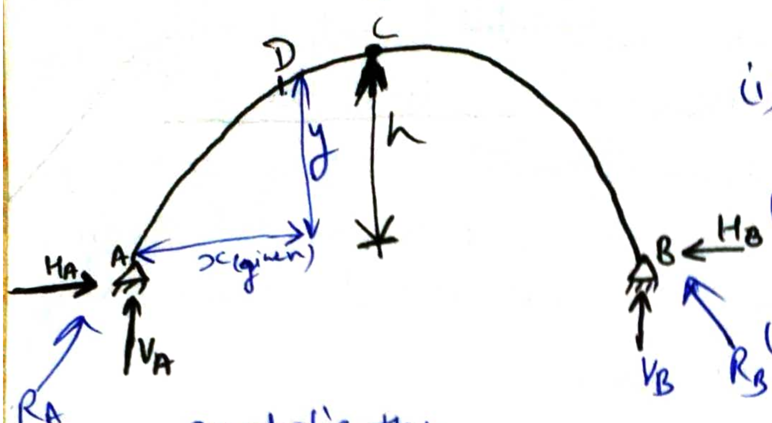
Disadvantages: (i) While constructing arches we need highly skilled people

(ii) We can't use it in residential areas

Types of arches :— Parabolic or circular arch



Three hinged Parabolic arch:-



parabolic the

Analysis

(i) Calculate verticle reactions V_A, V_B (using Eq)

(ii) Calculate Horizontal thrust H_A & H_B (Using $M_c = 0$) [at crown hinge or = 0 BM]

(iii) Calculate resultant forces

$$R = \sqrt{V^2 + H^2}$$

(iv) Cal^n y and evaluate the BM at given point

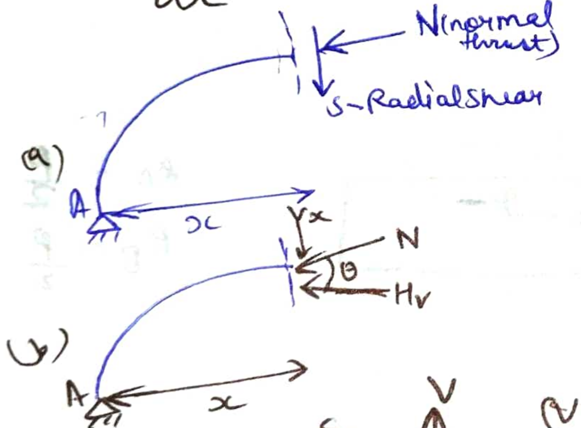
~~(v)~~ Cal^n θ

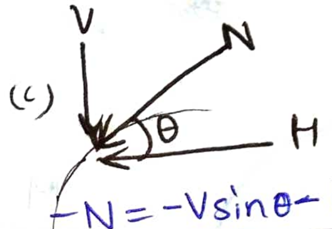
(V) find the normal thrust at the given point

Vertical distance at any x dist. for parabolic arch

$$\boxed{y = \frac{4h}{l^2} x(l-x)}$$

$$\tan\theta = \frac{dy}{dx}$$



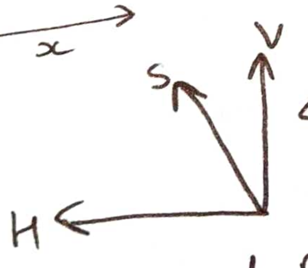


$$-N = -V\sin\theta - H\cos\theta$$

$$\boxed{N_x = V_x \sin\theta + H\cos\theta}$$

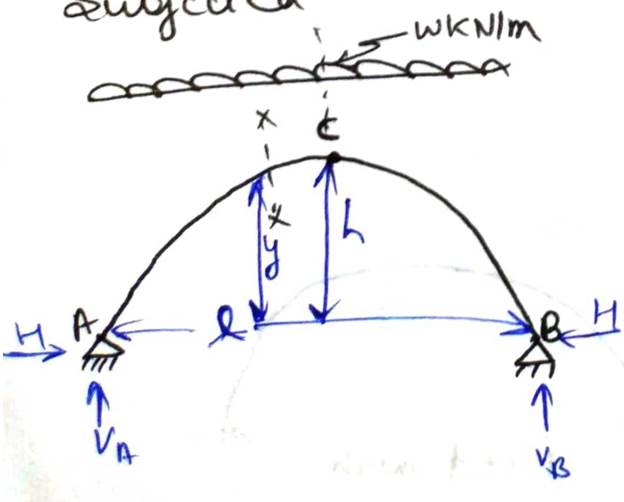
find radial shear

(vi) $$\boxed{S_x = V_x \cos\theta - H\sin\theta}$$



$V_x \rightarrow$ S.F at x

Analysis of 3 hinged Parabolic arch: Subjected to UDL over entire span



Step I V_A & V_B

$$\Sigma f_y = 0$$

$$V_A + V_B = wl$$

$$\Sigma M_A = 0, \quad V_B l - wl \times \frac{l}{2} = 0$$

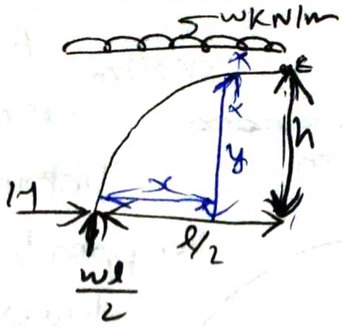
$$\boxed{V_B = \frac{wl}{2}}$$

$$\boxed{V_A = \frac{wl}{2}}$$

~~$\Sigma F_x = 0$~~

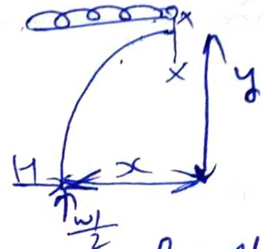
Step II $M_c = 0$

at leftside



$$\frac{wl \times l}{2 \times 2} - H \times h - \frac{wl}{2} \times \frac{l}{4} = 0$$

$$\boxed{H = \frac{wl^2}{8h}}$$



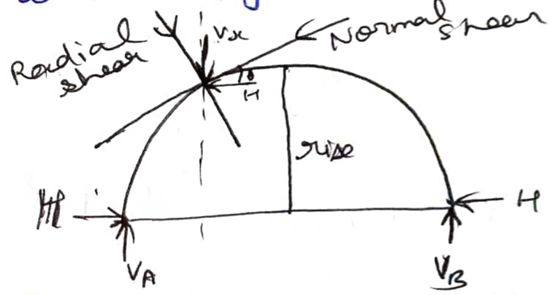
Step III → $BM_x = ?$

$$\frac{wl}{2} \times x - H \cdot y - \frac{wx^2}{2} = M_x$$

$$M_x = \left[\frac{wlx}{2} - \frac{wl^2}{8h} \frac{4h}{l^2} x(l-x) - \frac{wx^2}{2}\right] \rightarrow \text{putting value of } y$$

$M_x = 0$

Imp BM is zero in the entire span when subjected to U.D.L



Step IV → Radial Shear S_x

$$S_x = V_x \cos\theta - H \sin\theta$$

$$V_x = \frac{wl}{2} - wx$$

$$S_x = \left(\frac{wl}{2} - wx\right)\cos\theta - \frac{wl^2}{8h}\sin\theta$$

$$S_x = \cos\left[\left(\frac{wl}{2} - wx\right) - \frac{wl^2}{8h}\tan\theta\right]$$

$$S_x = \cos\theta\left[\left(\frac{wl}{2} - wx\right) - \frac{wl^2}{8h} \cdot \frac{4h}{l^2}(l-2x)\right]$$

$$S_x = \cos\theta\left[\frac{w(l-2x)}{2} - \frac{w(l-2x)}{2}\right]$$

$$S_x = 0$$

$$y = \frac{4h}{l^2} x(l-x)$$

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$$

$$\tan\theta = \frac{4h}{l^2}(l-2x)$$

Step V → $N_x = V\sin\theta + H\cos\theta$

$$= \left(\frac{wl}{2} - wx\right)\sin\theta - \frac{wl^2}{8h}\cos\theta$$

$$\cancel{= \cos\theta\left[\frac{wl}{2} - wx\right.}$$

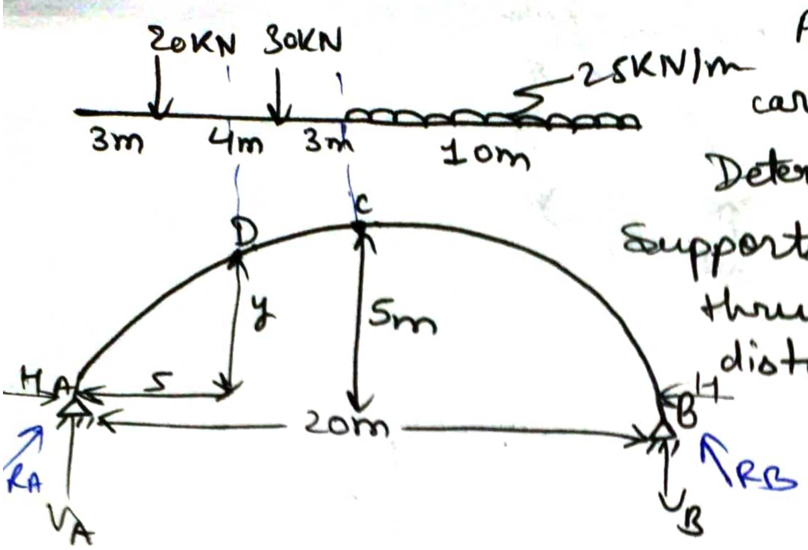
$$= \cos\theta\left[\left(\frac{wl}{2} - wx\right)\frac{4h}{l^2}(l-2x) + \frac{wl^2}{8h}\right]$$

$$= \cos\theta\left[\frac{wl}{2} \times \frac{4h}{l^2}(l-2x) - \frac{4xhw}{l^2}(l-2x) + \frac{wl^2}{8h}\right]$$

$$= \cos\theta\left[\frac{2wh(l-2x)}{l} - \frac{4xhw}{l^2}(l-2x) - \frac{wl^2}{8h}\right]$$

$$= 0$$

A parabolic 3-hinged arch carries load as shown in fig. Determine the resultant rxn at supports. find the BM, normal thrust and radial shear at distance 5m from A



Step I → V_A And V_B

$\Sigma F_y = 0, \quad V_A + V_B = 300$

$\Sigma M_A = 0$

$20 V_B - 250[5+10] - 30\times 7 - 20\times 3 = 0$

$\boxed{V_B = 201 KN}$

$\boxed{V_A = 99 KN}$

BM means kisi half part k left ya right k moment ko find karna h

Step II $H = ?$

$M_c = 0$

$V_A \times 10 - H\times 5 - 20\times 7 - 30\times 3 = 0$

$\boxed{H = 152 KN}$

Step III $R = ?$

$R_A = \sqrt{H_A^2 + V_A^2}$ $\qquad$ $R_B = \sqrt{H^2 + V_B^2}$

$= \sqrt{(152)^2 + (99)^2}$ $\qquad$ $= \sqrt{(152)^2 + (201)^2}$

$= 181.39 KN$ $\qquad$ $= 252 KN$

Step IV :- $BM_D = ?$

~~$M_D = 0$~~

$y = \frac{4h}{l^2}(l - x)$

$= \frac{4\times 5}{(20)^2}(20 - 5)$

$y = 3.75m$

$M_D = V_A \times 5 - Hy - 20\times 2$

$= 99\times 5 - 152\times 3.75 - 20\times 2$

$= -115 KN\text{-}m$

Step IV $N_x = V_x \sin\theta + H\cos\theta$

$V_D = SF_D = V_A - 20 = 79\ KN$

$N_D = 79\sin\theta + 152\cos\theta$

$= 79\sin(26.56) + 152\cos(26.56)$

$= 171.28\ KN$

$\tan\theta = \frac{dy}{dx} = \frac{4h}{l^2}(l-2x)$

$= \frac{4\times5}{(20)^2}(20-2\times5)$

$= 0.5$

$\theta = \tan^{-1}(0.5)$

$= 26.56^\circ$

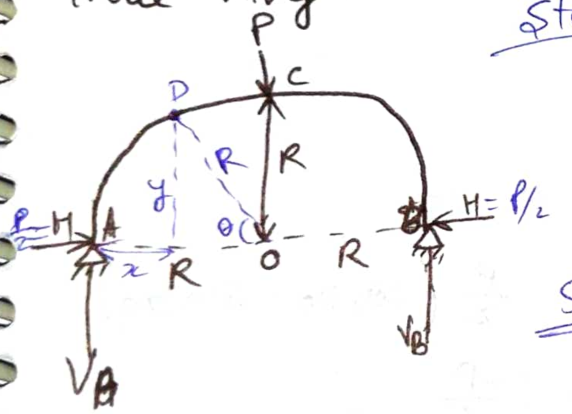
Step V $S_D = ?$

$S_D = V_D\cos\theta - H\sin\theta$

$= 79\times(26.50)^\circ + 152\,(26.50^\circ)$

$=$

Three hinged semicircular arch :-



Step I Verticle reactions

$\Sigma M_A = 0$, $V_B \cdot 2R - PR = 0$

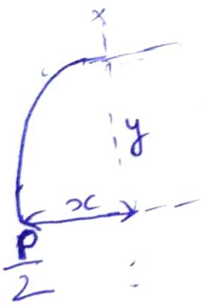
$\boxed{V_B = \frac{P}{2}}$

$F_y = 0$ $\boxed{V_A = \frac{P}{2}}$

Step II $M_c = 0$ (internal hinge)

$-RH + \frac{P}{2}\cdot R = 0$

$\boxed{H = \frac{P}{2}}$

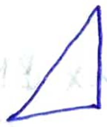


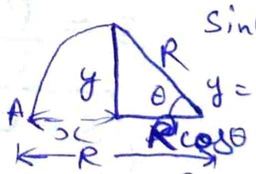
Step III

$M_x = 0$

$\frac{P}{2}x - Hy = M_x$

$M_x = \frac{P}{2}(x-y)$





$\sin\theta = \frac{y}{R}$

$y = R\sin\theta$

$\cos\theta = \frac{x'}{R}$

$x' = R\cos\theta$

$x = R - R\cos\theta$

$x = R(1-\cos\theta)$

$\boxed{x = R(1-\cos\theta)}$

$\boxed{y = R\sin\theta}$

Parabolic Arch :

$y = \frac{4h}{l^2}x(l-x)$

$M_x = \frac{P}{2}(x-y)$

$M_x = \frac{P}{2}[R(1-\cos\theta) - R\sin\theta]$

$\boxed{M_x = \frac{PR}{2}[1-\cos\theta-\sin\theta]}$

Step IV : $M_{max} = ?$

$M_x = \frac{PR}{2}(1-\cos\theta-\sin\theta)$

$\frac{dM_x}{d\theta} = 0 \Rightarrow \frac{PR}{2}[\sin\theta - \cos\theta] = 0$

$\sin\theta - \cos\theta = 0$

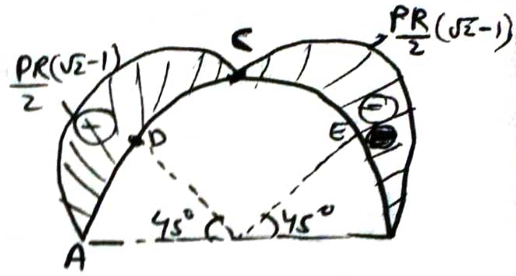
$\tan\theta = 1$

at $\theta = 45°$ then BM is Max.

$$M_{max} = \frac{PR}{2}\left(1-\cos 45 - \sin 45°\right)$$

$$M_{max} = -\frac{PR}{2}(\sqrt{2}-1)$$

BMD



Step V Radial Shear (S_x)

$S_x = V_x \cos\theta - H\sin\theta$

$V_x \rightarrow$ S.F at the Section at left side

$V_x = R_A = \frac{P}{2}$, $H = \frac{P}{2}$

$S_x = \frac{P}{2}\cos\theta - \frac{P}{2}(\sin\theta)$

$$\boxed{S_x = \frac{P}{2}(\cos\theta - \sin\theta)}$$

At A $\quad \theta = 0$

$S_A = \frac{P}{2}(\cos 0° - \sin 0°)$

$S_A = \frac{P}{2}$

At C $\quad \theta = 90 = \frac{\pi}{2}$

$S_C = \frac{P}{2}(\cos 90 - \sin 90)$

$S_C = -\frac{P}{2}$

At max BM $\quad \theta = 45°$

$S = \frac{P}{2}(\cos 45 - \sin 45)$

$\boxed{S = 0}$

Step VI Normal Thrust

$N_x = V_x \sin\theta + H\cos\theta$

$$\boxed{N_x = \frac{P}{2}(\sin\theta + \cos\theta)}$$

At A = 0

$N_A = \frac{P}{2}(\sin 0° + \cos 0°)$

$N_A = \frac{P}{2}$

At C $\quad \theta = 90$

$N_C = \frac{P}{2}(\sin 90 + \cos 90)$

$N_C = \frac{P}{2}$

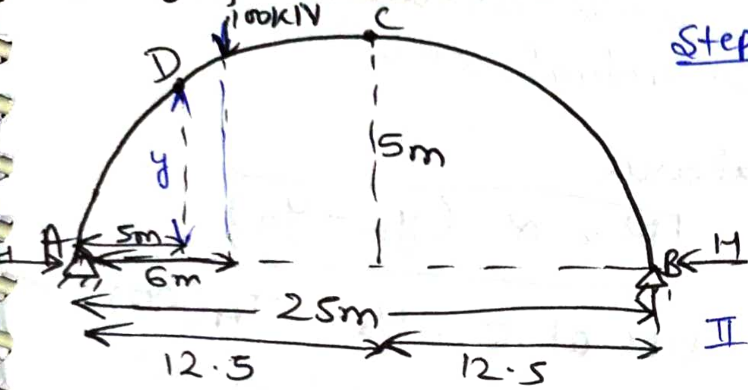
At max BM . $\theta = 45°$

$N = \frac{P}{2}(\sin 45° + \cos 45°)$

$= \frac{P}{2}(\cancel{0})$

$\boxed{N = 0}$

Ques. A symmetrical three hinged circular arch has a span of 25m and a rise of central hinge 5m. It carries a vertical load of 100KN at 6m from left hand find the BM, Radial shear and normal thrust ~~at~~ 5m from A



Step I : $\Sigma M_A = 0$

$+V_B \times 25 - 100 \times 6 = 0$

$V_B = \frac{600}{25} = 24\,KN$

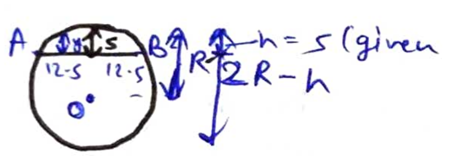
$\boxed{V_B = 24KN}$

$\boxed{V_A = 76KN}$

II : $M_C = 0$ (Right)

$24 \times 12.5 - H \times 5 = 0$

$\boxed{H = 60KN}$



Length of chord AB = 25

III $M_D = ?$

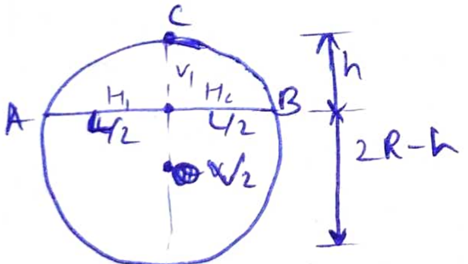
$M_D = 76 \times 5 - \cancel{60} y_D$

So By Property of circle

$h(2R-h) = \frac{L}{2} \times \frac{L}{2}$ $\{V_1 \times V_2 = H_1 \times H_2)$

$5(2 \times R - 5) = \frac{25}{2} \times \frac{25}{2}$

$\boxed{R = 18.125m}$



By Property of circle

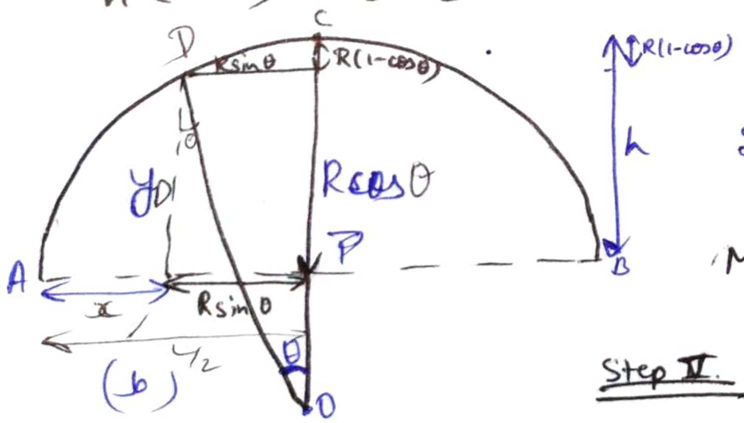
$\boxed{V_1 \times V_2 = H_1 \times H_2}$

$h(2R-h) = \frac{L}{2} \times \frac{L}{2}$

from diagram (b)

$\boxed{y_D = h - R(1-\cos\theta)}$

$\boxed{R\sin\theta + x = \frac{L}{2}}$ or $\sin\theta = \frac{5}{R}$ → $\theta = \sin^{-1}\left(\frac{5}{R}\right)$



$18.125 \times \sin\theta + 5 = \frac{25}{2}$

$\theta = 24.4°$

So, $y_D = 5 - 18.125(1 - \cos 24.4)$

$= 3.375m$

$M_D = 76 \times 5 - 60 \times 3.375$

$= 177.5\,KNm$

Step IV. $N_D = V_x \sin\theta + H\cos\theta$

$= 76 \times 0.413 + 60 \times 0.910$

$= 86.048\,m$

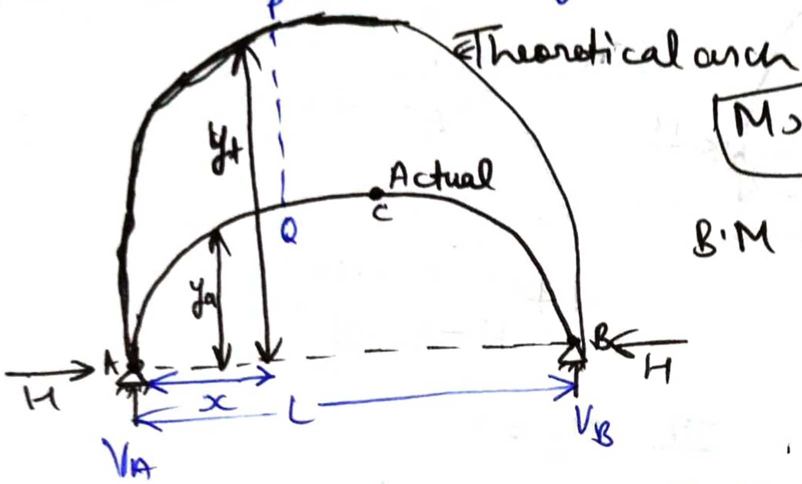
$S_D = V_x \cos\theta - H\sin\theta$

$= 76 \times 0.911 - 60 \times 0.413$

$= 44.456\,m$

~~Theo~~ Eddy's Theorem →

ET states that "The bending moment at any ~~point~~ section of an arch is propostional to the vertical intercept betⁿ the linear arch (or theoretical arch) and the centre line of the actual arch"



$$\boxed{M_x \propto (y_t - y_a)}$$

$B.M$ at $Q = M_x = T_H(PQ)$

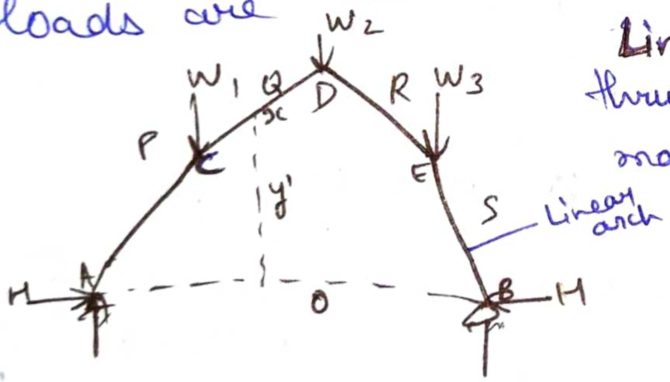
$= T\cos\theta\,(y_t - y_a)$

$= H[y_t - y_a]$

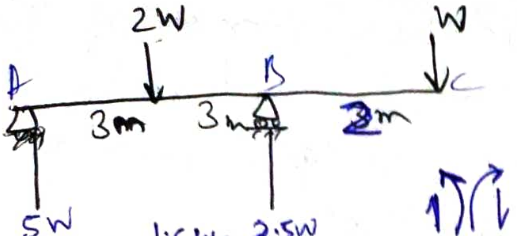
$\Rightarrow M_x \propto (y_t - y_a)$

$M_x = 0$ when $(y_t = y_a)$

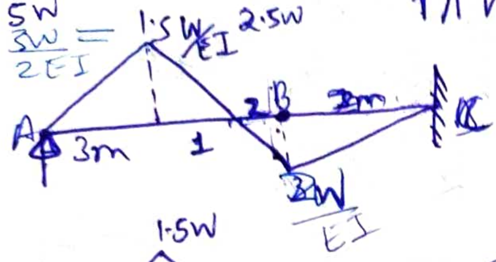
Linear arch or Theoretical arch: The arch which follows funicular polygon shape after application of series of loads are called as linear or theoretical arch



Linear arch is subjected to axial thrust only and there is no bending moment and shear force anywhere



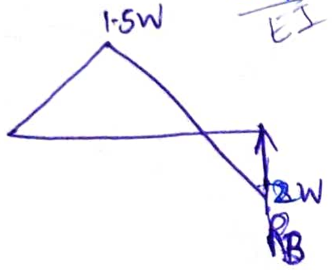
$2 \times \frac{63}{42} = \frac{3W}{2EI} = 1.5W$



$= 3 \times 1 \times \frac{1}{2} \, \omega \qquad 2 \times 3 \times \frac{1}{2} \quad \omega = 3\omega$

$= 1.5w$ – BMD (k/tarahn̄i kalna h)

↓+ ↑−



$\Sigma M_A = 0$

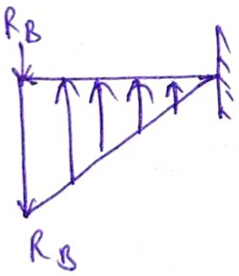
$\frac{1.5w \times 3 \times \frac{2}{3} \times 3}{EI} \downarrow + \frac{1.5w \times 1}{EI}\left(3 + \frac{1}{3} \times 1\right) \downarrow - \frac{3w \times 2}{EI} \times \left(4 + \frac{2}{3} \times 2\right) + R_B \times 6 \uparrow = 0$

$\frac{9w}{EI} + \frac{4.995}{EI} - \frac{31.92}{EI} + 6R_B = 0$

1..

$R_B = \frac{-1.5}{EI} - \frac{0.8325}{EI} + \frac{5.32}{EI}$

$R_B = \frac{\cancel{5.3w}}{\cancel{EI}} - \frac{3w}{EI}$



$M_c = R_B \times 3 - 3w \times \cancel{3} \times \frac{2}{3} \times 3 \rightarrow M_c = R_B \times 2 - 3w \times 2 \times \frac{2}{3}$

$\delta = -\frac{3w}{EI} \times 3 - 3w \times 6 \qquad = \frac{-6w}{EI} - \frac{4w}{EI}$

$= \frac{-10w}{EI}$ (downward)

$\cancel{R_c} = \quad \Delta_c = -\frac{9w}{EI} - \frac{18w}{EI}$

$\Delta_c = -\frac{27w}{EI}$

$\Delta_c = \frac{27w}{EI}$ (downward)

$\frac{27w}{\cancel{EI}} = \frac{91.67}{\cancel{EI}} \qquad \frac{-10w}{E} = \frac{91.67}{EI}$

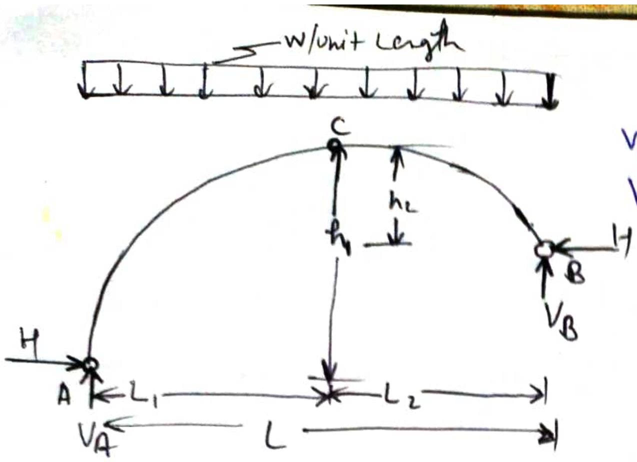
$w =$

$w = \frac{91.67}{27}$

$R_c = R_B - 3w \times 3$

$= 6$

$= 3.395$



Let L_1 be the distance of the vertex from the left hand abutment; With C as origin, the eq^n of parabola is

$$y = kx^2$$

for CA, therefore, $h_1 = kL_1^2$

$$k = \frac{h_1}{L_1^2}$$

for CB, we have, $h_2 = k(L-L_1)^2$

$$k = \frac{h_2}{(L-L_1)^2}$$

Equating the two, we get

$$\frac{h_1}{L_1^2} = \frac{h_2}{(L-L_1)^2}$$

$$L_1^2 h_2 = (L-L_1)^2 h_1$$

$$L_1\sqrt{h_2} = (L-L_1)\sqrt{h_1}$$

$$L_1\sqrt{h_2} + L_1\sqrt{h_1} = \sqrt{h_1}\,L$$

$$L_1 = \frac{L\sqrt{h_1}}{\sqrt{h_2}+\sqrt{h_1}}$$

Taking moment about C, from left side only

$$Hh_1 = V_A \frac{L\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} - \frac{W}{2}\frac{h_1L^2}{(\sqrt{h_1}+\sqrt{h_2})^2} \quad \text{---(i)}$$

By taking moment about B

$$H(h_1-h_2) + \frac{WL^2}{2} = V_A L$$

$$V_A = \frac{H(h_1-h_2)}{L} + \frac{WL}{2} \quad \text{---(ii)}$$

Substituting value of eq^n (ii)

$$Hh_1 = \left(\frac{H(h_1-h_2)}{L} + \frac{WL}{2}\right)\frac{L\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}} - \frac{W}{2}\frac{h_1L^2}{(\sqrt{h_1}+\sqrt{h_2})^2}$$

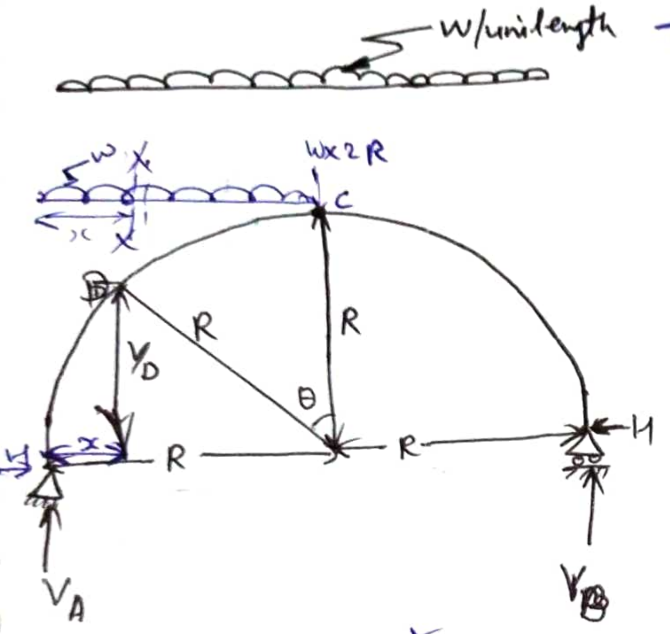
$$H\left[h_1 - \frac{(h_1-h_2)\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}\right] = \frac{WL^2\sqrt{h_1}}{2(\sqrt{h_1}+\sqrt{h_2})} - \frac{Wh_1L^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}$$

$$H\cancel{\sqrt{h_1h_2}} = \frac{WL^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}\cancel{\sqrt{h_1h_2}}$$

$$\boxed{H = WL^2/2(\sqrt{h_1}+\sqrt{h_2})^2}$$

#

Ques. A three hinged semi circular arch of radius 'R' carries a uniformly distributed load of w per unit run over the whole span. Find the horizontal thrust at each support and the location (from the crown) and magnitude of the maximum bending moment for the arch.



Step I Vertical reactions

$\Sigma M_A = 0$

$V_B \times 2R - w \times 2R \times R = 0$

$V_B = \dfrac{w \times 2R^2}{2R}$

$\boxed{V_B = wR}$ $\boxed{V_A = wR}$

Step II $M_C = 0$

$-HR + wR \times R - wR \times \dfrac{R}{2}$

$\boxed{H = \dfrac{wR}{2}}$

$\Rightarrow x = R - R\sin\theta$

$\Rightarrow$ ~~$h = R$~~

So, $\boxed{y_D = R\cos\theta}$

Step III $M_x = 0$

$M_x = -\dfrac{wx^2}{2} + \overset{V_A\, x}{wRx} - \overset{H}{\dfrac{wR}{2}} y_D$

$M_x = -\dfrac{w(R - R\sin\theta)^2}{2} + wR(R - R\sin\theta) - \dfrac{wR}{2}(h - R(1-\cos\theta))$

$M_x = -\dfrac{w(R - R\sin\theta)^2}{2} + wR(R - R\sin\theta) - \dfrac{wR}{2}(R\cos\theta)$

$= -\dfrac{wR^2}{2}[1 + \sin^2\theta - 2\sin\theta] + wR^2(1 - \sin\theta) - \dfrac{wR^2}{2}\cos\theta$

$R(R+1) = \frac{L}{2}\ \frac{L}{2}$

$M_x = \dfrac{wR^2}{2}[\sin\theta - 2\sin\cos\theta)$

$\dfrac{dM_x}{d\theta} = 0$

$\dfrac{d}{dx}\dfrac{wR^2}{2}[\sin\theta - \sin 2\theta] = 0$

$\dfrac{d}{dx}\dfrac{wR^2}{2}(\sin\theta - 2\sin\theta\cos\theta)$

$\dfrac{wR^2}{2}\sin\theta(1 - 2\cos\theta) = 0$

$\sin\theta(1 - 2\cos\theta) = 0$

$\cos\theta = 0.5$

when $\theta = 60°$

$\therefore x$ for max. moment point $= R(1-\cos 60°)$

$$M_{max} = \frac{wR^2}{2}(1-\cos 60° - \sin^2 60°)$$

$$= \frac{wR^2}{2}(1-\cos 60° - \sin^2 60°)$$

$$= -\frac{wR^2}{8}$$

Parabola eqn

$$x^2 = ay$$

$$a = \frac{x^2}{y}$$

Ex→ 7.7

Structural Analysis

Structure → Which is designed for load carrying or Load carrying capacity

Analysis → To find unknown Reaction or forces

Unknown Rxn → External Rxn → Moment, forces
→ Internal Rxn → Moment, forces

Planer structure → 2D Structure

Space structure → 3D structure

Determinate structure → Supports Rxn, 3 rxn or less than 3 supports

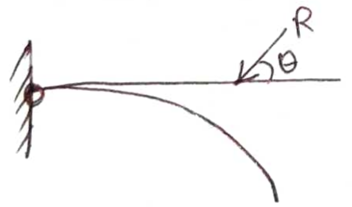
No. of Supports Rxn ≤ No. of Equilibrium Eqn

"The branch of Civil Engg. in which we find the Unknown Rxn (External rxn: at support & Internal Rxn at inside the material) is known as Structural Analysis."

Support Rxn & Degree of freedom →

2D-Structure

(a) Fixed Support (r = 3)



$\Delta y = 0 \Rightarrow R_y \neq 0$

$\Delta H = 0 \Rightarrow R_x \neq 0$

$\theta_A = 0 \Rightarrow M_R \neq 0$

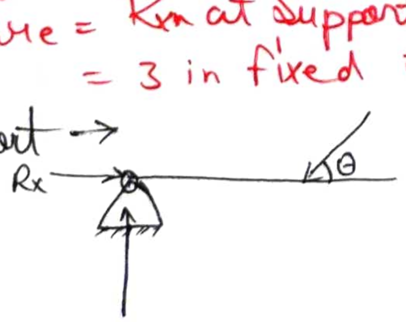
Rxn: when there is no displacement or restriction in displacement

Resistance in linear displacement – F_x, F_y, F_z

Angular displacement – M_x, M_y, M_z

r_e = Rxn at Supports = 3 in fixed support $\boxed{r_e = 3}$

(b) Hinge Support →



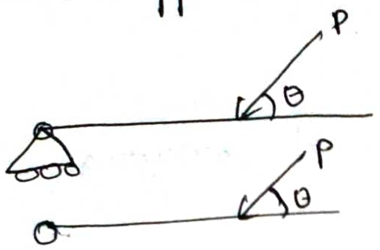
$\Delta x = 0 \Rightarrow R_x$ ✓ $R_x \neq 0$

$\Delta y = 0 \Rightarrow R_y$ ✓ $R_y \neq 0$

$M_R \neq 0$

$\boxed{r_e = 2}$

(c) Roller Support →



$\Delta x \neq 0 \Rightarrow R_x = 0$

$\Delta y = 0 \Rightarrow R_y \neq 0$

$\theta_A \neq 0 \Rightarrow M_R = 0$

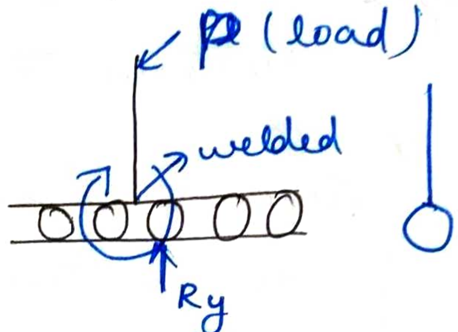
$\boxed{r_e = 1}$

(d) Guided Roller →

$\Delta x \neq 0$, $R_x = 0$

$\Delta y = 0$, $R_y \neq 0$

$\theta_A = 0$, $M_R \neq 0$



Note → Jis dirⁿ m displacement nhi hoga us pr rxn mil jayega

Degree of freedom → free to displace or move in any dirⁿ
(DOF)

Linear displacement → x, y, z

Angular displacement → x, y

Conclusion: (for 2 structure)

No. of Equilibrium Eqⁿ = r_e + DOF

$3 = r_e + \text{DOF}$

$\boxed{\text{DOF} = 3 - r_e}$ ↳ for 2D structure

V.V. Imp.

3D	6	4	3	1
2D	F	G	H	R
	3	2	2	1

	fixed	Guided	Hinged	Roller
r_e	3	2	2	1
DOF	0	1	1	2

DOF			
F	G	H	R
0	1	1	2

Number of Static Equilibrium = 3
SE

$\Sigma R_x = 0$

$\Sigma R_y = 0$

$\Sigma M = 0$

3 D-Structure = 6 E.E General

(a)

P_x, P_y, P_z

$\Delta x = 0 \Rightarrow R_x, \; R_x \neq 0$

$\Delta y = 0 \Rightarrow R_y, \; R_y \neq 0$

$\Delta z = 0 \Rightarrow R_z, \; R_z \neq 0$

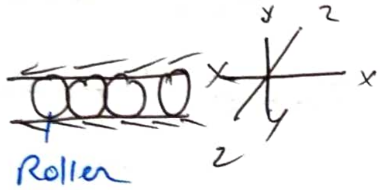
$\theta_x = 0 \Rightarrow M_{Rx} \neq 0$

$\theta_y = 0 \Rightarrow M_{Ry} \neq 0$

$\theta_z = 0 \Rightarrow M_{Rz} \neq 0$

$\boxed{r_e = 6}$

(b) Guided Roller →



$\Delta x \neq 0 \Rightarrow R_x = 0$

$\Delta y = 0 \Rightarrow R_y \neq 0$

$\Delta z \neq 0 \Rightarrow R_z = 0$

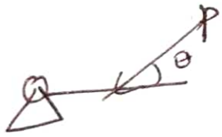
$\theta_z \neq 0 \Rightarrow \cancel{M_{Rz} \neq 0}$ $M_R = 0$

$\theta_y = 0 \Rightarrow M_R = 0$

$\theta_x = 0 \Rightarrow M_{Rx} = 0$

$\boxed{\begin{array}{l} DOF = 2 \\ r_e = 4 \end{array}}$

(c) Hinge Support →



$\Delta x = 0 \Rightarrow R_x \neq 0$

$\Delta y = 0 \Rightarrow R_y \neq 0$

$\Delta z = 0 \Rightarrow R_z \neq 0$

$\theta_x \neq 0 \Rightarrow M_{Rx} = 0$

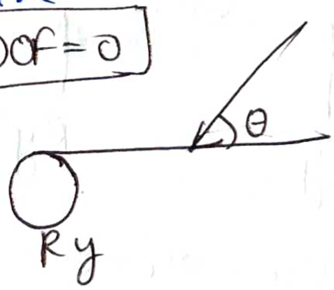
$\theta_y \neq 0 \Rightarrow M_{Ry} = 0$

$\theta_z \neq 0 \Rightarrow M_z = 0$

On Pin joint there is no Moment Rxn

$\boxed{r_e = 3}$ $\boxed{DOF = 0}$

(d) Roller Support



$\Delta x \neq 0 \Rightarrow R_x \neq 0$

~~$\Delta y \neq 0 \Rightarrow R_y \neq 0$~~

$\Delta x \neq 0 \Rightarrow R_x = 0$

$\Delta y = 0 \Rightarrow R_y \neq 0$

$\Delta z \neq 0 \Rightarrow R_z = 0$

Gives always Verticle rxn, No moment Rxn

$M_R = 0$

$\theta_x \neq 0, \; M_x = 0$

$\theta_y \neq 0, \; M_y = 0$

$\theta_z \neq 0, \; M_z = 0$

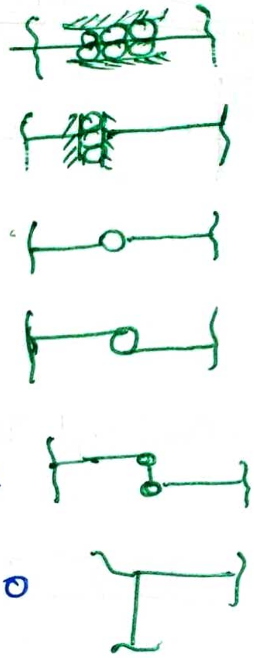
$\boxed{r_e = 1}$

	fixed	Guided	Hinge	Roller
r_e	6	4	3	1
DOF	0	2	3	5
	6 - 6	6 - 4	6 - 3	6 - 1

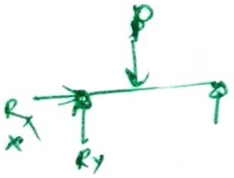
DOF = No. of Equilibrium Eqn − r_e

Internal Reaction

Double Roller	AF=0	SF	BM	$r_r = 1$	
Internal Slider	AF	SF=0	BM	$r_r = 1$	
Internal Hinge	AF	SF	BM=0	$r_r = 1$	
Internal Roller	AF=0	SF	BM=0	$r_r = 2$	
Internal link	AF	SF=0	BM=0	$r_r = 2$	
Rigid Joints	AF	SF	BM	$r_r = 0$	

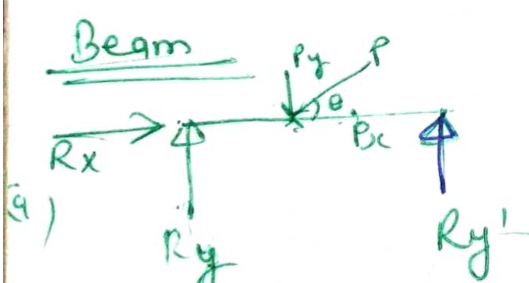


* In verticle loading



$\Sigma f_x = 0$ or horizontal rxn not consider
$R_H = 0$ not consider

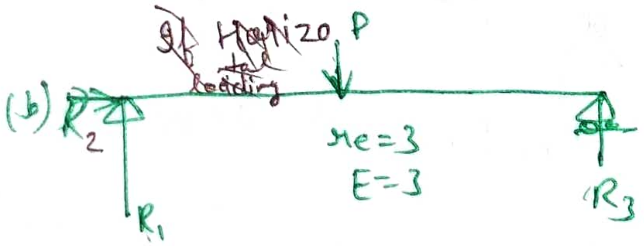
Beam



$r_e = 3$
$E = 3$

$D_{se} = r_e - E$
$= 3 - 3$
$= 0$

General loading (inclined loading)



$r_e = 3$
$E = 3$

~~$r_e = 3$, $E = 3, 4, 2$~~

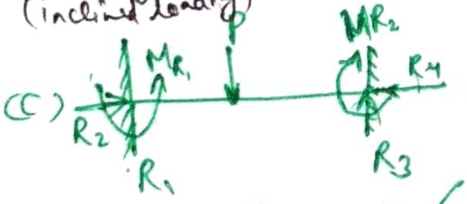
$r_e = 2$, $E = 2$ — $\Sigma f_y = 0$, $\Sigma M_2 = 0$

$D_{se} = 2 - 2$
$= 0$

* Due to verticle loading 'P' horizontal loading doesn't consider

r_e = external rxn

only R_H not consider
∴ $r_e = 2$
$E = 2$



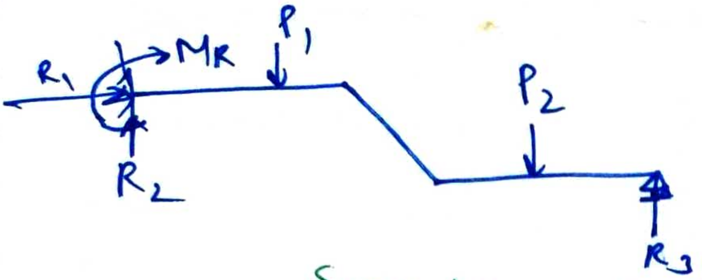
~~$r_e = 6$, $E = 3$~~
$D_{se} = 3 = r_e - E$

$r_e = 4$
$E = 2$ — $\Sigma f_y = 0$, $\Sigma M_2 = 0$

$D_{se} = r_e - E$
$= 4 - 2$
$= 2$

* In case of Beam, If loading is verticle & supports are at same level, then, No horizontal Rxn. Ex → (b)

for Truss & frame → Horizontal Rxn are always considered.



Supports are not at Same level

Structure

- Determinate
  (In structure reaction SF, BM horizontal thrust calculate by help of equilibrium condition)
- Indeterminate
  (Not calculated by equilibrium condition)
  - Static Indeterminancy (D_s)
    (Related to force)
    - External Indeterminancy (D_{se})
      (Related to external support & Rxn)
    - Internal indeterminancy (D_{si})
      (Related to member & their arrangement)
      (Depends upon geometry of body.
      ex: ~~Beam~~, truss, frame)
  - Kinematic indeterminancy (D_k) (Related to displacement depend on degree of freedom)

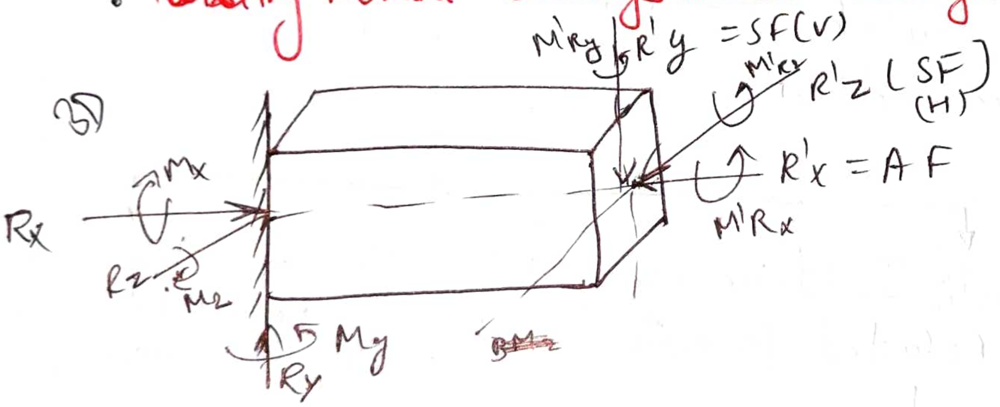
Internal Reaction

In structure → Beam, frame, Truss

Internal Reaction →
- Axial forces (AF)
- Shear force (SF)
- Bending Moment (B·M)
- Twisting Moment (TM)

Imp:
- B·M always about Transverse Axis
- Twisting Moment always about Longitudinal Axis

3D



3D-structure:

A.F = X-dir^n always $\perp^r$ to Transverse plane

SF = → y dir^n (SF$_V$), → z dir^n (SF$_H$) } lies always in Transverse plane

BM = → y Axis about ⇒ B·M$_y$, → z Axis about ⇒ BM$_z$ } always about Transverse Axis

T.M = — about x-axis always about polar axis

2D → AF, BMz, SFy SF(V)

Structure

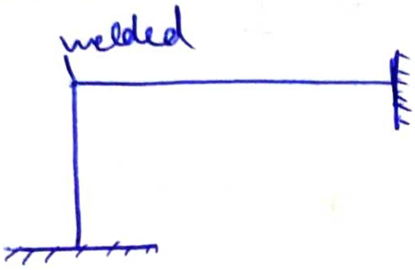
(a) Rigid jointed structure
frame & Beam

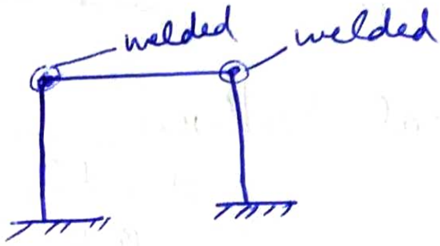
(b) Pin jointed Structure
Truss – AF

(2D)

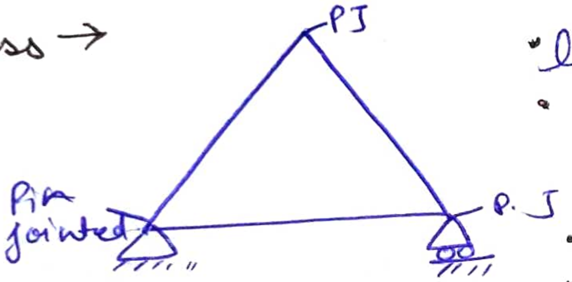
frame →

A.F
S.F
B.M





Truss →



BM = 0
SF = 0
Axial force ✓

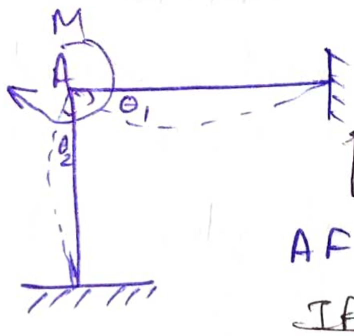
- load is applied on pin joint
- Every member should prismatic & homogenous, isotropic
- Material should obeys Hooks law
- Straight member
- Truss is designed for Axial forces

Imp

Internal Joint:

(a) Rigid internal Joints → (welded Joints). (in frame)

$\boxed{\theta_1 = \theta_2}$



Imp.

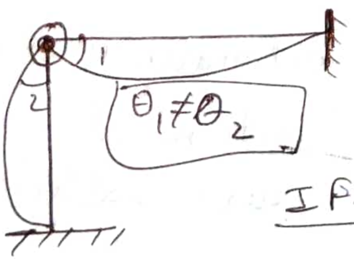
$\boxed{\theta_1 = \theta_2}$

AF, B.M, SF

IF = 3

(b) Pin Jointed →

$\boxed{\theta_1 \neq \theta_2}$



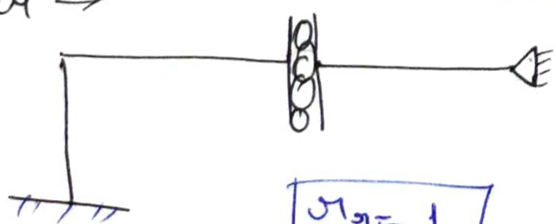
$\boxed{\theta_1 \neq \theta_2}$

IF = 2

B M = 0
SF ✓
AF ✓

r_r = Reaction Released = 1

Internal Slider →

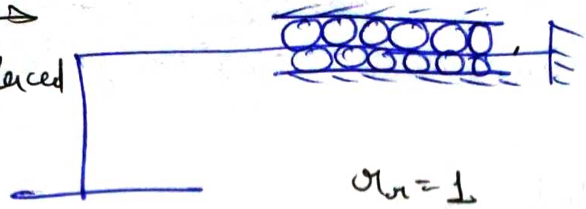


2D structure

IF
BM ✓
AF ✓
S.F = 0

$\boxed{r_r = 1}$

* Double Roller →



AF = 0 (Displaced

SF ✓

BM ✓

$r_r = 1$

Internal Roller

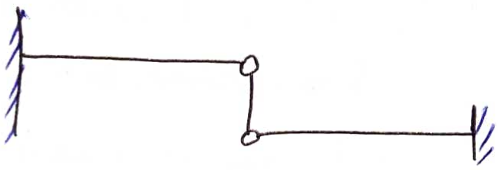
2D Structure

AF = 0
BF = 0 } = 2

SF ✓

$r_r = 2$

Link (Internal)



SF = 0
BM = 0
AF = ✓

$\boxed{r_r = 2}$

imp. Internal Connection :

Rigid Connection welded ⇒ $\boxed{r_r = 0}$ learn
Internal Roller ⇒ $\boxed{r_r = 2}$

Internal link, In. Hinge, Double roller, Internal slider

$\boxed{r_r = 1}$

$r < e$ externally unstable
$r = e$ determinate
$r > e$ externally indeterminate

Inderterminancy

If Unknown reactions > No. of Static Equilibrium Eq^n

∴ more that 3 rxn

Degree of Indeterminancy ⇒ $\boxed{n = r_e - E}$

unknown – Equilibrium

$\boxed{D_{se} = r_e - E}$

Indeterminancy

- Static inderminancy (D_s)
  - External (D_{se}) → Related to external unknown rxns at Supports
  - Internal (D_{si}) Related to Internal force (AF, SF, BM) depends upon type of str. & Geometry of str.
- Kinamatic ~~Static~~ Indeterminancy (D_k)

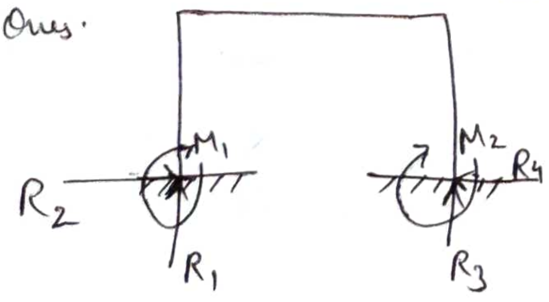
↓

$D_{se} = U_{ext} - E$ or $= r_e - E$ } 2D str = $D_{se} = r_e - 3$; 3D str = $D_{se} = r_e - 6$

U = unknown $U_{ext} = r_e$

Calculate Dse →

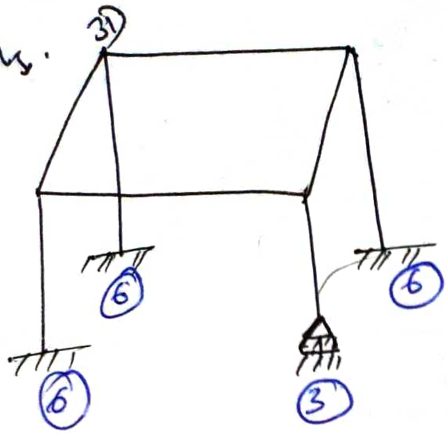
Ques.



2D frame

$D_{se} = r_e - 3$
$= 6 - 3$
$= 3$

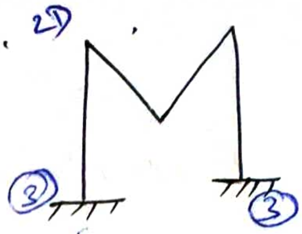
Ans. 3D



F G H R
6 4 3 1

$D_{se} = r_e - 6$
$= 21 - 6$
$= 15$

Ans. 2D



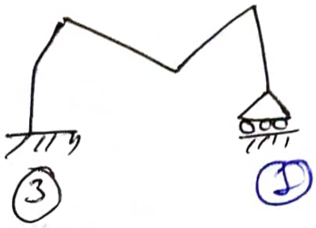
F G H R
3 2 2 1

$D_{se} = r_e - 3$
$= 6 - 3$

Ans 2D

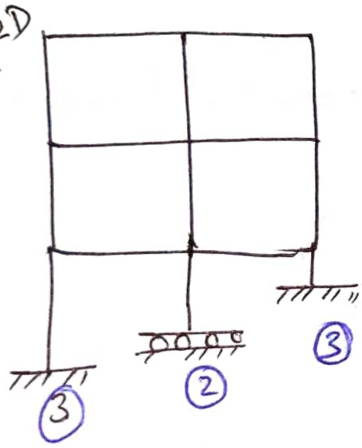
$D_{se} = 0$

Ans.



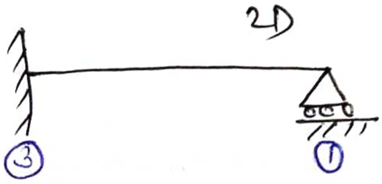
$D_{se} = 1$

2D
Ans.



$D_{se} = 8 - 3$
$= 5$

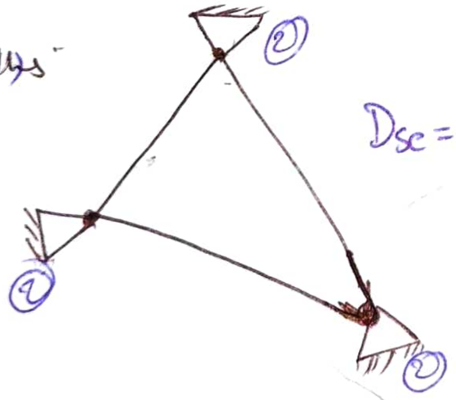
2D
Ans.



$D_{se} = 1$

1 eq^n needs

Ans.

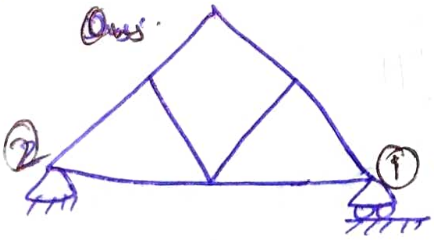


$D_{se} = 3$

Ans.

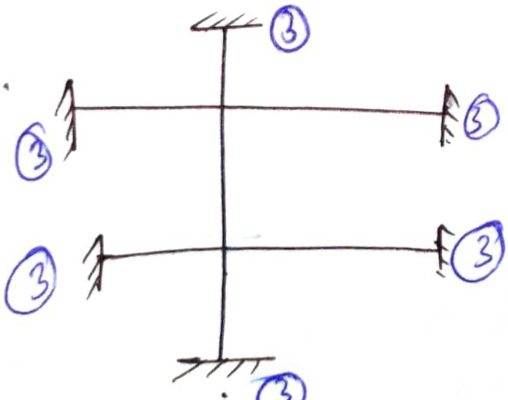
$D_{se} = 0$

Ans.



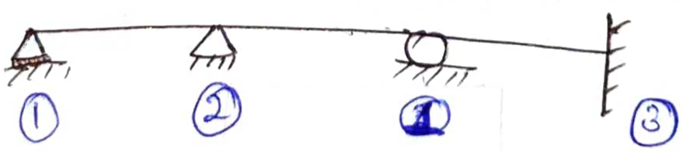
$- D_{se} = 0$

Ans.



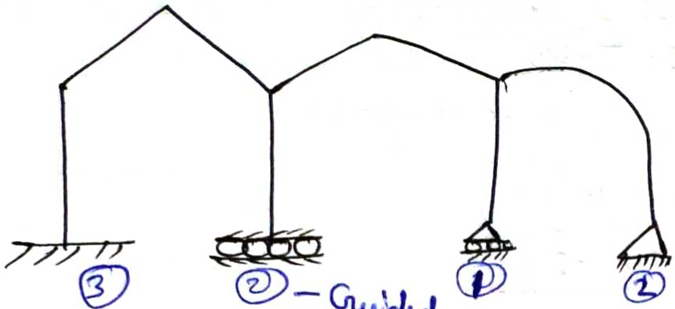
$D_{se} = 18 - 3$
$= 15$

Ans.



$D_{se} = 7 - 3$
$= 4$

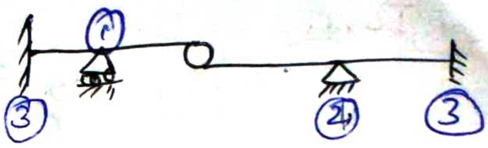
Ans.



$D_{se} = 8 - 3$

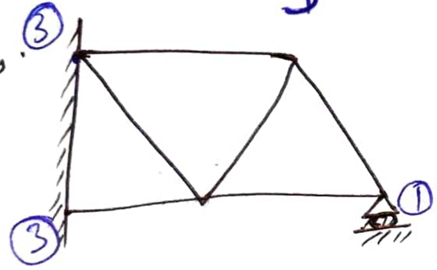
5

Ans.



$D_{se} = 9 - 3$
$= 6$

Ans.



$D_{se} = 7 - 3$
$= 4$

⇒ Internal static Indeterminancy → It is in pin jointed frames & Rigid structures

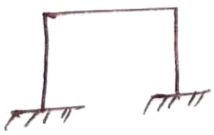
→ Truss (Axial force)

$\to D_{si} = m - (2j - 3) \to 2D$

$\to D_{si} = m - (3j - 6) \to 3D$

frames - cut method

→ frame (AF BM, SF)



open $D_{si} = 0$ always

for 2D

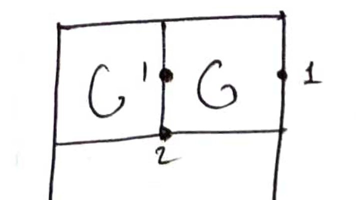
$$\boxed{D_{si} = 3C - r_r}$$

where C = no. of close loop

r_r = rxn releases.

$$\boxed{r_r = \Sigma (m' - 1)}$$

m' = connected member



$r_r = 2 + 1 + 1$
$= 4$

$D_{si} = 3C - r_r$
$= 3 \times 2 - 4 = 2$

for 3D $D_{si} = 6C - r_r$

$r_r = \Sigma 3 (m' - 1)$

~~$D = D_{se} + D_{si}$~~

for Truss

2D: $D_s = D_{se} - D_{si}$

$= r_e - 3 + m - (2j - 3)$

$= r_e + m - 2j$

$\boxed{D_s = m + r_e - 2j}$

3D: $D = D_{se} + D_{si}$

$= r_6 - 6 + m - (3j - 6)$

$\boxed{D_s = m + r_e - 3j}$

for frame:

$D_s = U_{total} - E$

$\boxed{D_s = U_{ext} + U_{int} - E}$

$\boxed{D_s = r_e + 3m - 3j}$ — F_y, F_x, M

$U_{ext} = r_e$

$U_{int} = 3m$

AF, BM, SF

$\boxed{D_s = r_e + 3(m - j)}$

for Truss:-

$D_s = U_{Total} - E_{Total}$

$= U_{ext} + U_{int} - E$

$\boxed{D_s = (r_e + m) - 2j}$

AF=1

$\Sigma f_y = 0$

$\Sigma f_x = 0$

$m = 2j - 3$ = Perfect truss

$m \neq 2j - 3$ = Imperfect

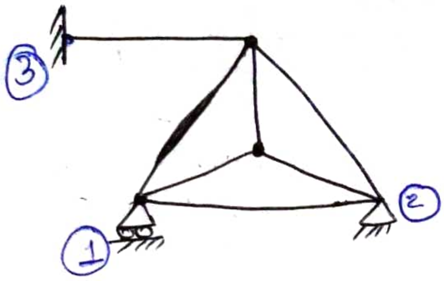
$m < 2j - 3 \Rightarrow$ determinate ; deficient

$m > 2j - 3 \Rightarrow$ indeterm. ; overstiff

Truss (2D)

$$D_s = (m + r_e) - 2j$$
$$D_{se} = r_e - 3$$
$$D_{si} = m - (2j - 3)$$

(i)

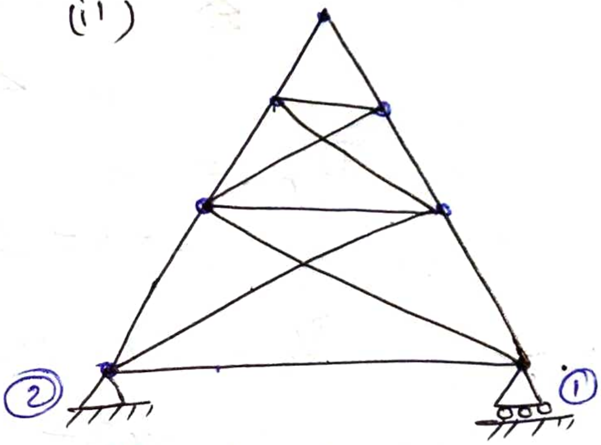


$D_s = (m + r_e) - 2j$
$= (7+6) - 2\times5$
$= 13 - 10$
$= 3$

$D_{se} = 6 - 3 = 3$

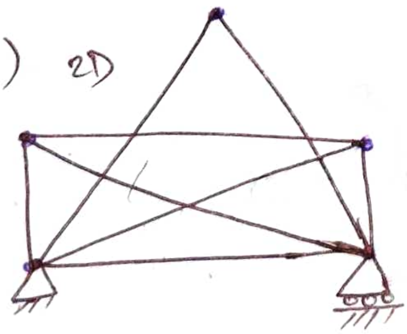
$D_{si} = 7 - (2\times5 - 3)$
$= 7 - 7 = 0$

(ii)



$D_s = (13+3) - 2\times7$
$= 16 - 14$
$= 2$

(iii) 2D



$D_s = (8+3) - 2\times5$
$= 11 - 10$
$= 1$

frame:

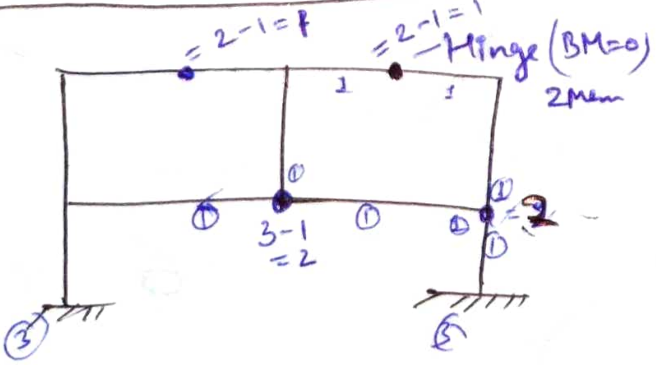
(2D) $D_{se} = r_e - 3$

$D_{si} = 3C - r_r$

$r_r = \Sigma(m' - 1)$

when no rxn release $\longleftarrow$ $D_s = r_e + 3(m - j)$

(i)

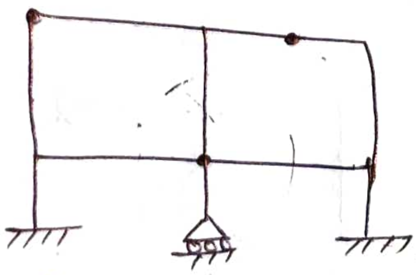


$D_{se} = r_e - 3$
$= 6 - 3$
$= 3$

$r_r = \Sigma(m' - 1)$
$= 1 + 1 + 2 + 2$
$= 6$

$D_{si} = 3C - r_r$
$= 3\times2 - 6$
$= 0$

(ii)



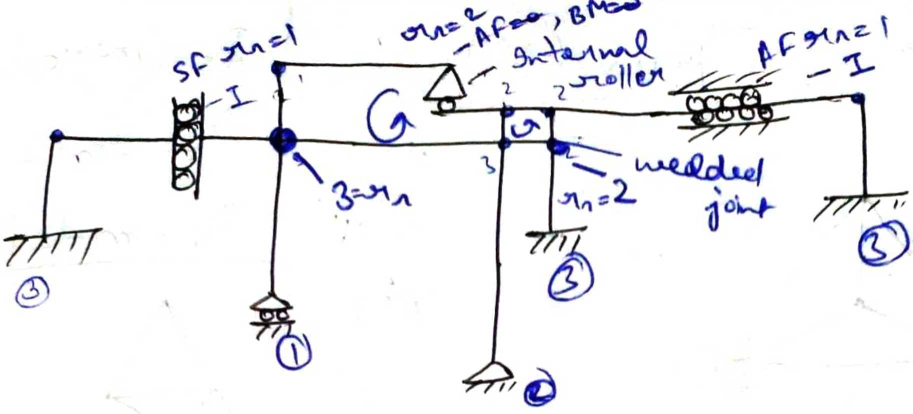
$r_r = 1+3+1$
$= 5$

$D_{se} = 7 - 3 = 4$

$D_{si} = 3\times2 - 5$
$= 1$

$D_s = D_{se} + D_{si}$
$= 4 + 1$
$= 5$

(iii)

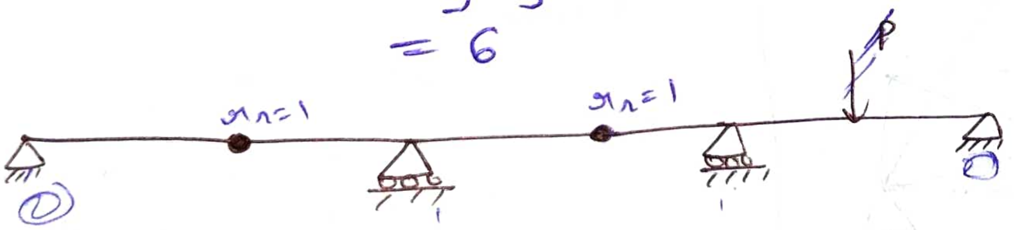


$$D_{se} = r_e - 3$$
$$= 12 - 3 = 9$$

$$D_{si} = 3C - r_r$$
$$= 3\times 2 - 9 - 5+1+2+1$$
$$= -3$$

$$D_s = D_{si} + D_{se}$$
$$= 9 - 3$$
$$= 6$$

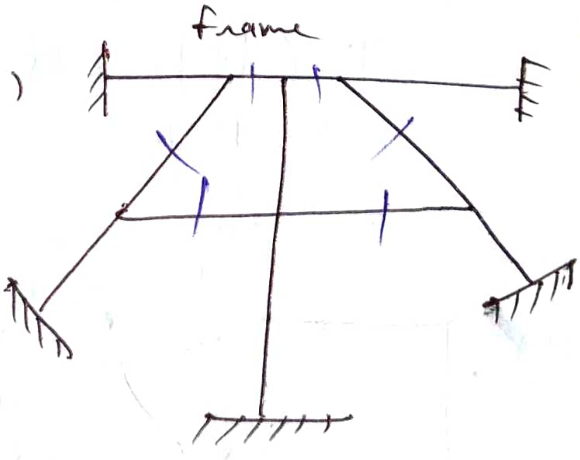
(iv)



$$D_{se} = 6 - 3 = 3$$
$$D_{si} = 3\times 0 - 2$$
$$= -2$$
$$D_s = 3 - 2 = 1$$

frame

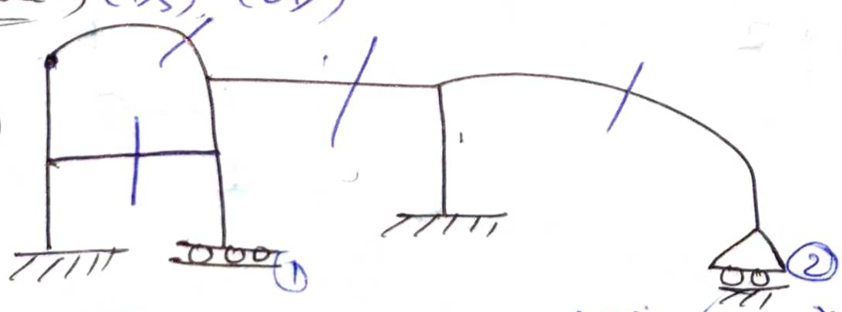
(ii)



$$D_s = 3\times 6 = 18$$

frame => (D_s) (2D)

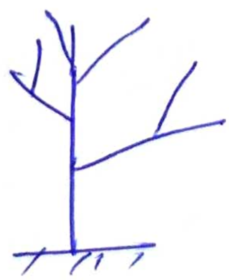
(i)



$$D_s = 3 * \text{cut} - x^{\text{-supp}}$$

$x = 2+1$

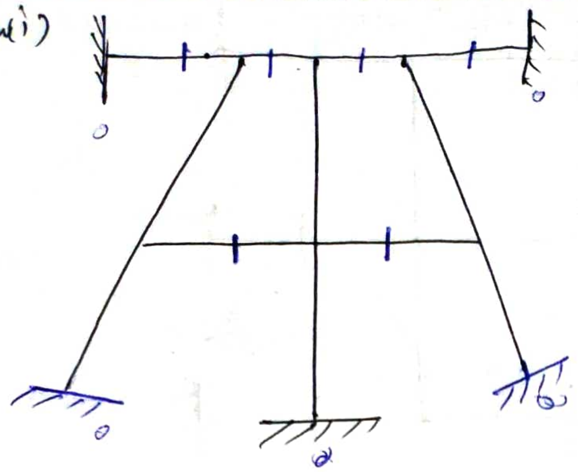
$$= 3 * 4 - 3$$
$$= 9$$



(iii)

$= D_s = 3\times 2 = 6$

Que i)

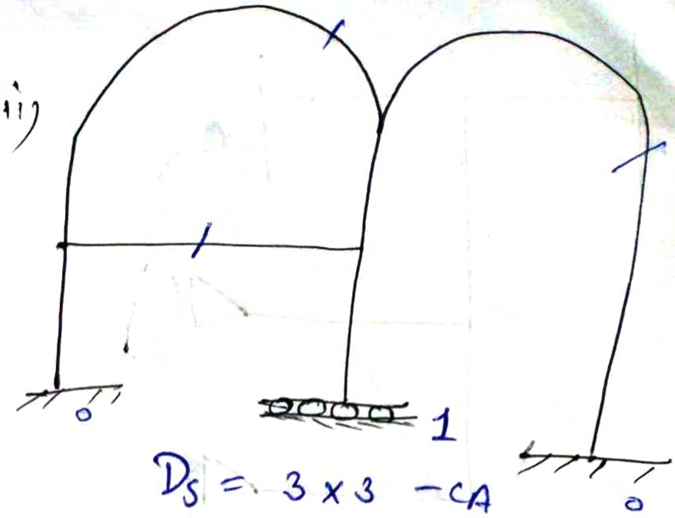


Short Trick:

$D_s = 3 \times Cut - C.A$

$= 3 \times 6 - 0 = 18$

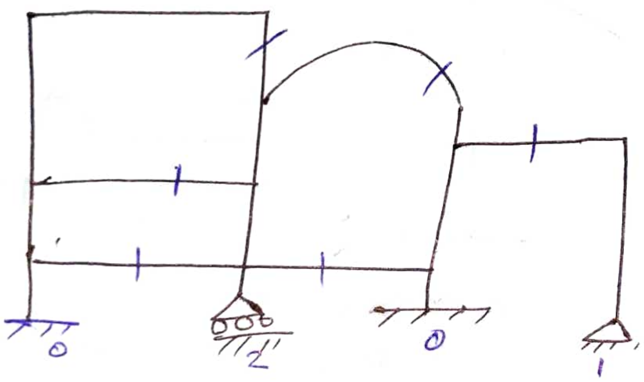
(iii)



$D_s = 3 \times 3 - CA$

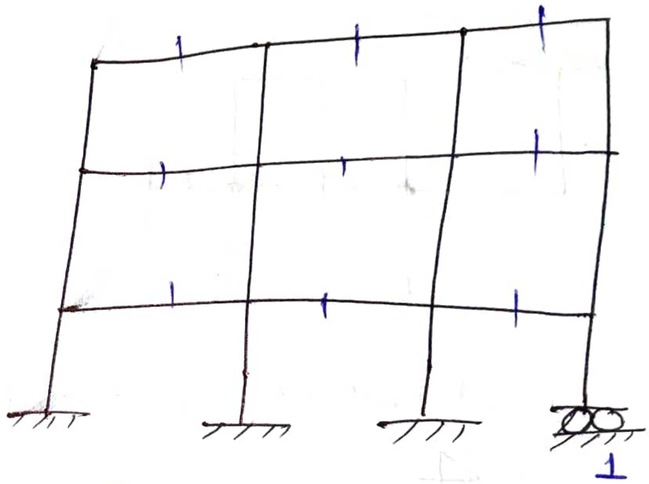
$= 3 \times 3 - 1$

$= 8$



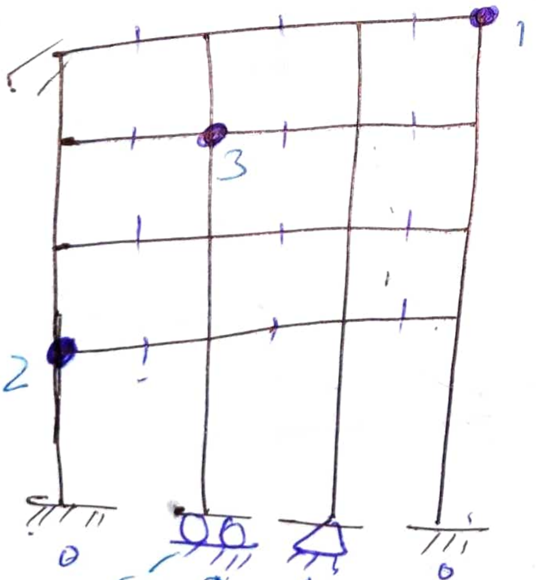
$D_s = 3 \times 6 - 3$

$= 15$



$D_s = 3 \times 9 - 1$

$= 26$



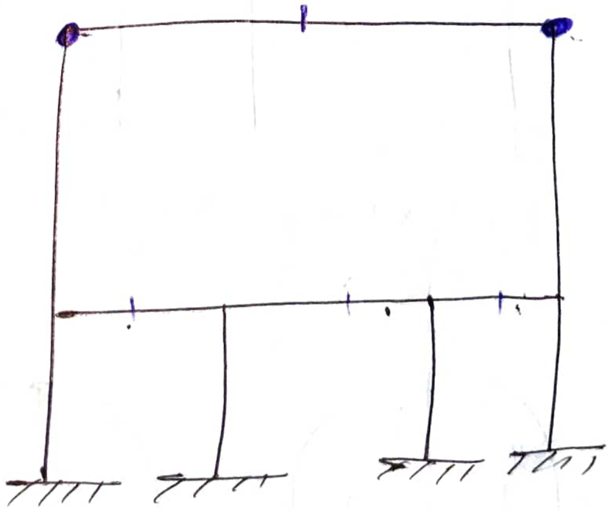
$r_r = \Sigma(m' - 1)$

$= 1 + 3 + 2$

$= 6$

$D_s = 3 \times 12 - (6 + 1 + 1)$

$= 28$

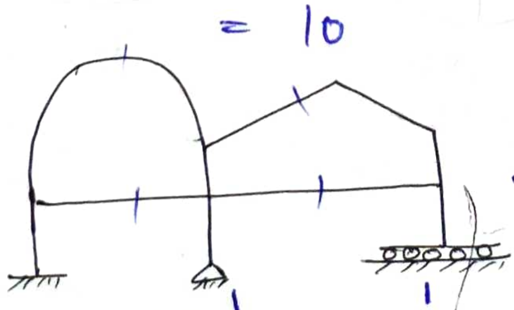


$3 \times cut - (2R + H + Pin_j(j-1))$

$3 \times 12 - (2 + 3$

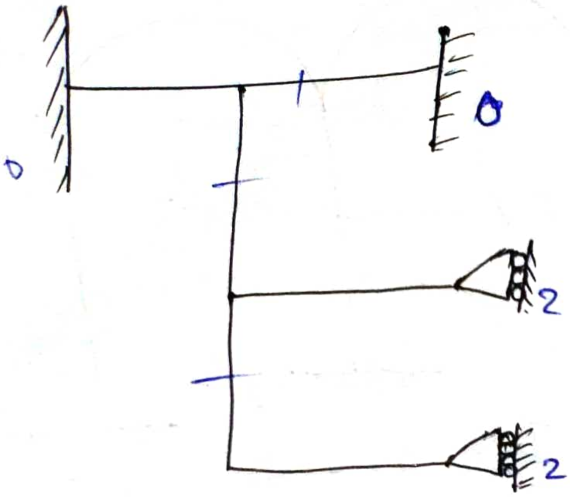
$D_s = 3 \times 4 - (2)$

$= 10$

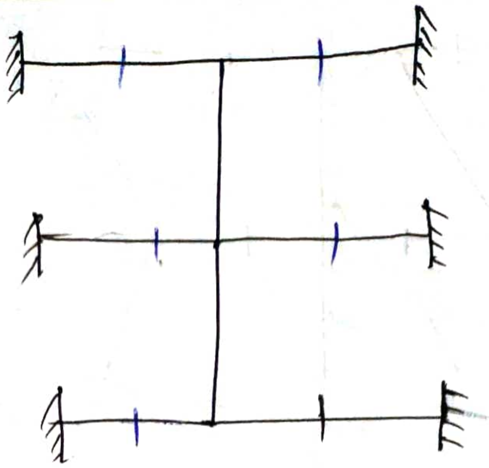


$D_s = 3 \times 4 - 2$

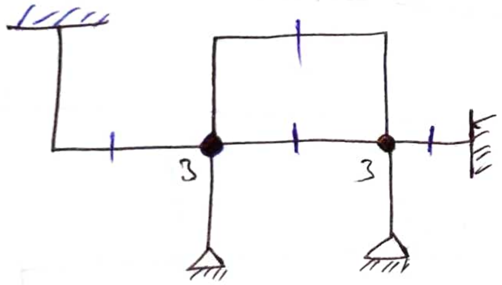
$= 10$



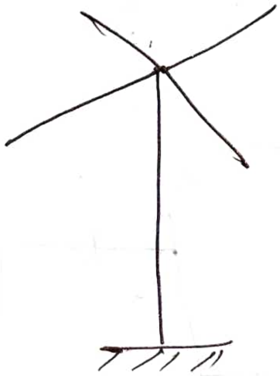
$D_s = 3\times 3 - 4$
$= 5$



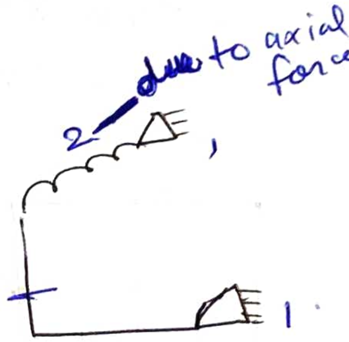
$D_s = 3\times 6 - 0$
$= 15$



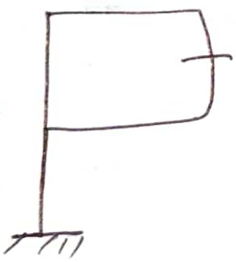
$D_s = 3\times 4 - (2+3+3)$
$= 4$



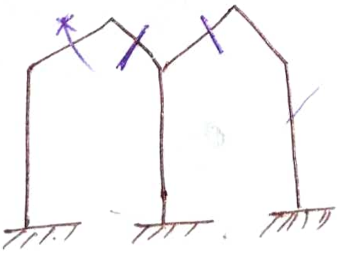
$D_s = 3\times 0 - 0$
$= 0$



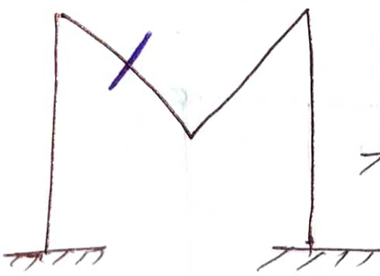
$D_s = 3\times 1 - 3$
$= 0$



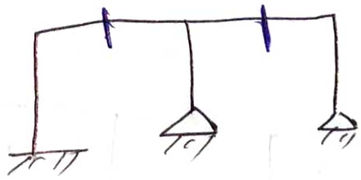
$D_s = 3\times 1 - 0$
$= 3$



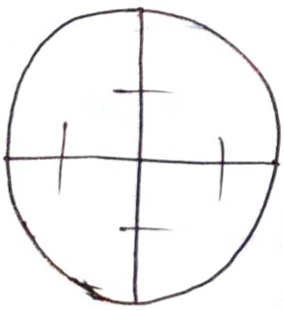
$D_s = 3\times 2$
$= 6$



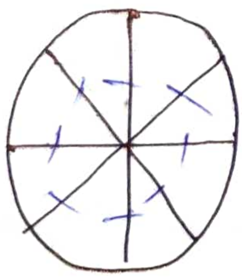
$D_s = 3\times 1$
$= 3$



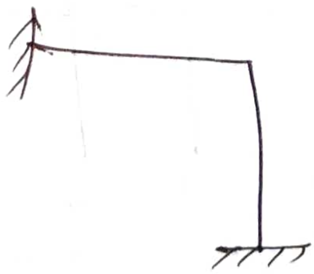
$D_s = 3\times 2 - 2$
$= 4$



$D_s = 3\times 4$
$= 12$



$D_s = 3\times 8$
$= 24$

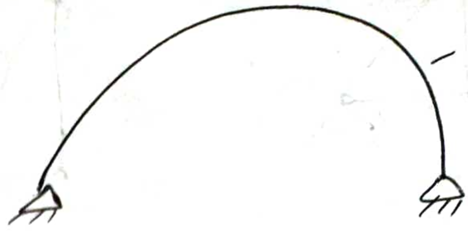


$D_s = 3\times 1$
$= 3$

Q external Support K bich m cut kerna hai

-3 Hinged Arch

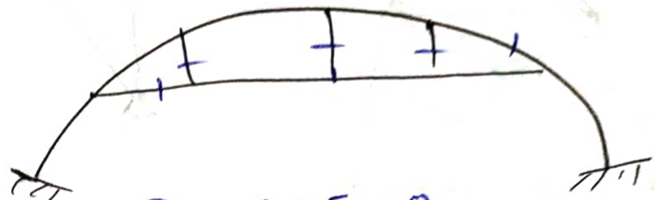
$D_s = 3 \times 1 - 3$
$= 0$



-2 hinged Arch

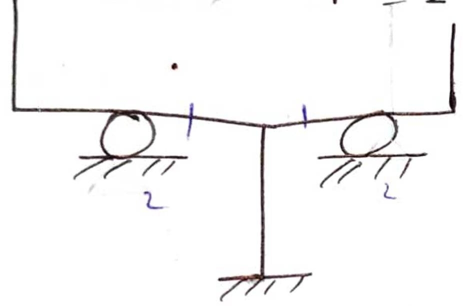
$D_s = 3 \times 1 - 2$
$= 1$

$D_s = 3 \times 3 - 7$
$= 2$

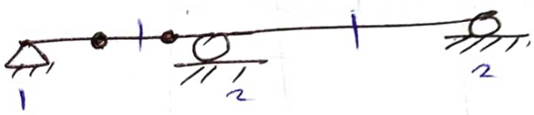


$D_s = 3 \times 5 - 0$
$= 15$

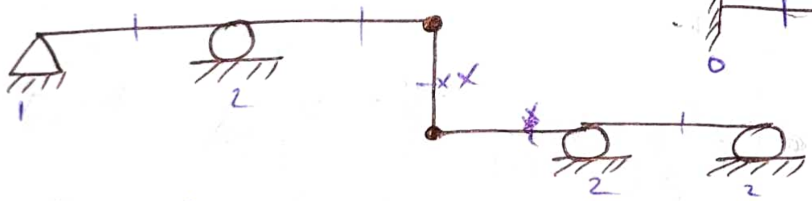
$D_s = N - E - \Sigma I_{n\,sup} = 6 - 3 - 1 = 2$



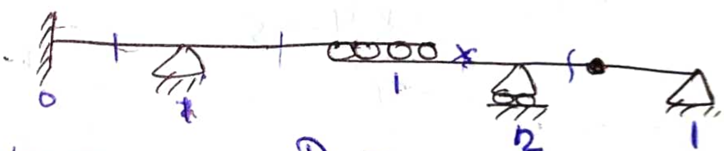
$D_s = 3 \times 2 - 4 = 2$



$D_s = 3 \times 2 - 7$
$= -1$

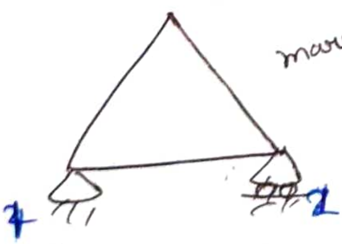


$D_s = 8 \times 3 - 9$
$= 0$



$D_s = 3 \times 3 - 6$
$= 3$
$= 3$

Truss : $D_s = (r_e - 3) +$ No of Diagonal Pannel

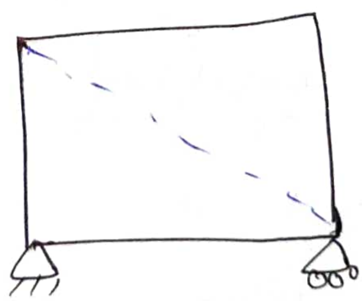


mare 2 jan

$D_s = 3 + 3 - 2 \times 3$
$= 0$

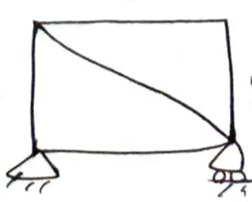
$D_s = -1$
Trick

$D_s = 2 + 3 - 2 \times 3$
$= 5 - 6$
$= -1$



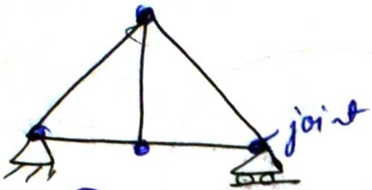
$D_s = -1$

$D_s = +1$

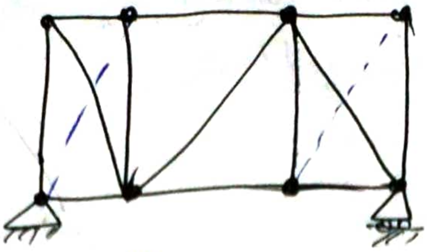


$D_s = 0$

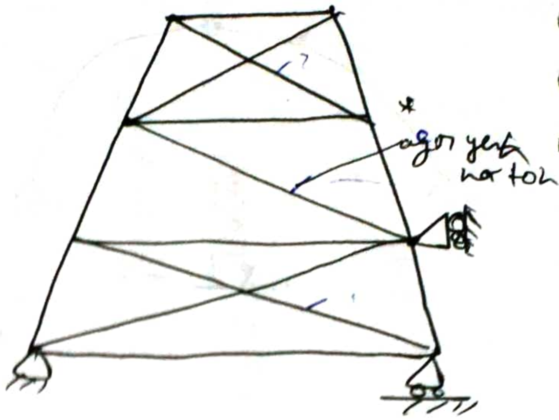
Trick = △ = $D_s = 0$
Complete △



$D_s = 0$

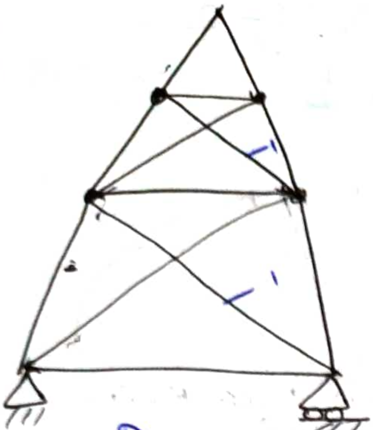


$D_s = +2$

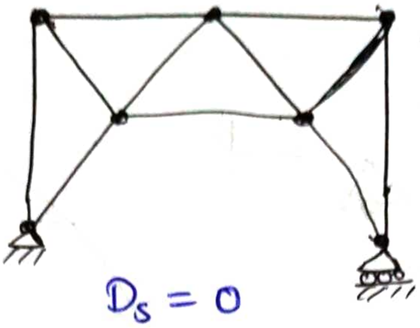


$D_s = +2$

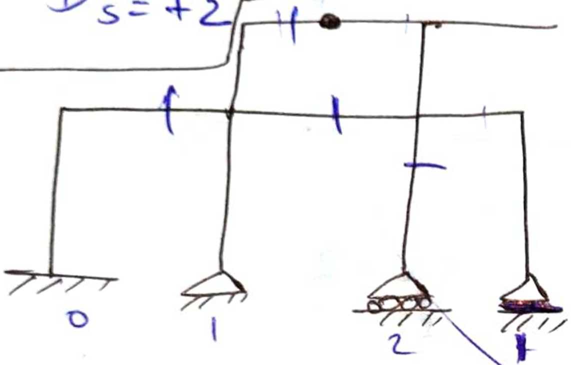
$* \; D_s = \cancel{-1} + 1 + \cancel{1}$
$= +1$



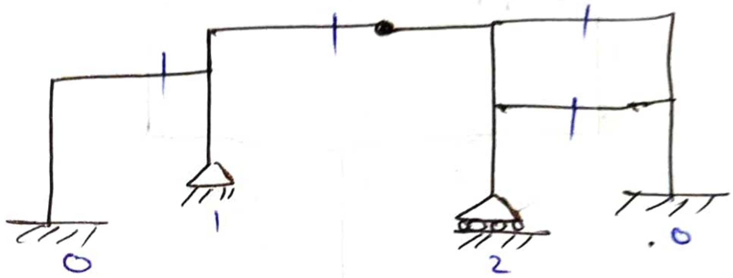
$D_s = +2$



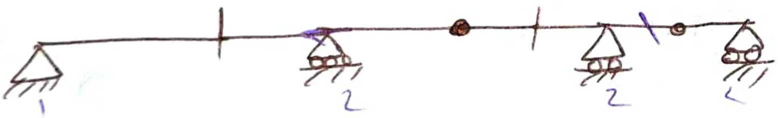
$D_s = 0$



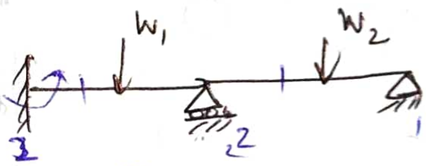
$D_s = 3 \times 4 - 5$
$= 7$



$D_s = 3 \times 4 - 4$
$= 8$

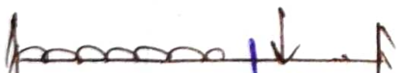


$D_s = 3 \times 3 - 9$
$= 0$



$D_s = 3 \times 2 - 3$
$= 3$

$D_{se} = r_e - 2$
$= 4 - 2$
$= 2$

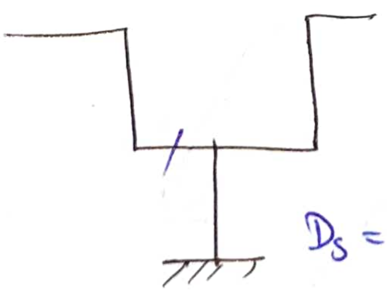


$m_t = 1, \; j = 2$

$D_s = 3 \times 1 - 0$

$D_{se} = r_e - 2$

D



$D_s = 0$

Unstability of Structure

External Unstability

- It deals with supports. If the str. can't balance applied load, str. is externally Unstable.

⇓

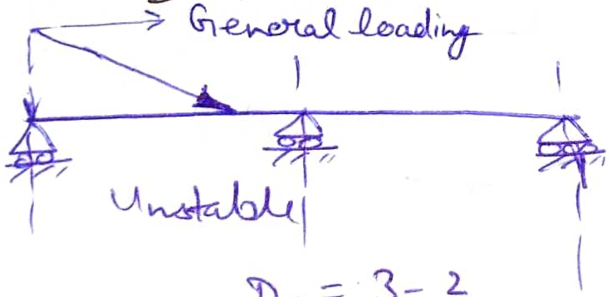
Due to Supports

Geometrical Unstability

Static Unstability

for Stability → Rxn should be → Non Parallel, → Non-concurrent, → Non-Trivial

General loading

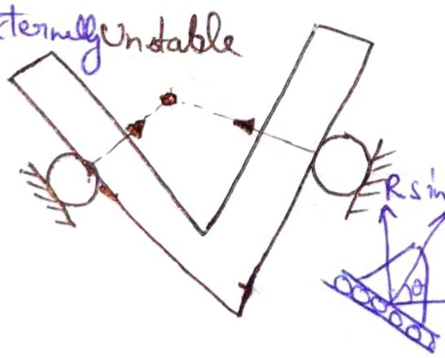


Unstable

$D_s = 3 - 2$
$= 1$

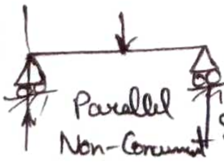
Indeterminant

Externally Unstable

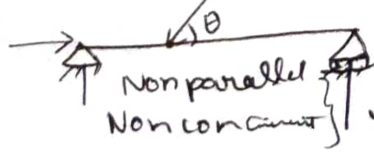


R sinθ, R, θ, R cosθ

Ustable

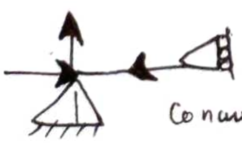


Parallel Non-Concurrent Stable



θ

Non parallel Non concurrent Stable



Concurrent Unstable

Internal Unstability

- It referes to mechanism which means either rigid body translation rigid body rotation

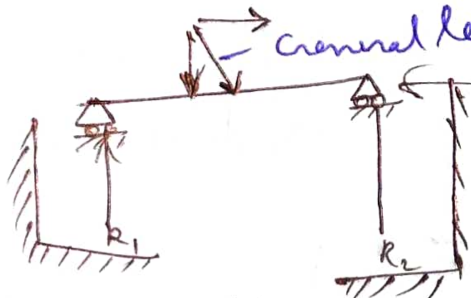
Rigid Rotation

Rigid body Translation

Rigid body Rotation

Unstable Vertically Translation

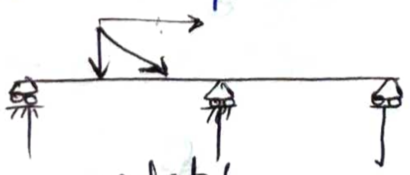
General loading



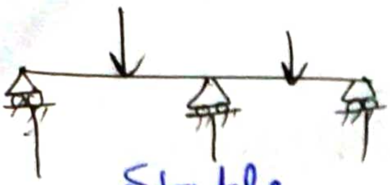
R_1 R_2

Stable

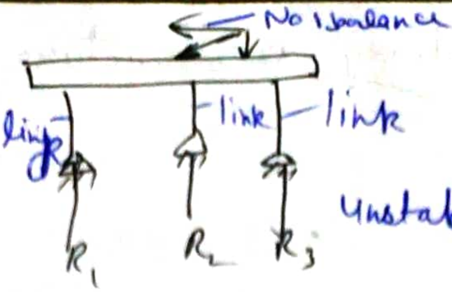
Geometrical Unstability → for General loading, if the total no. of supports rxn are more than or equal to 3 & all rxn are either concurrent or parallel then the structure is geometrically Unstable.
Also Non-Coplanar → for 3D str.



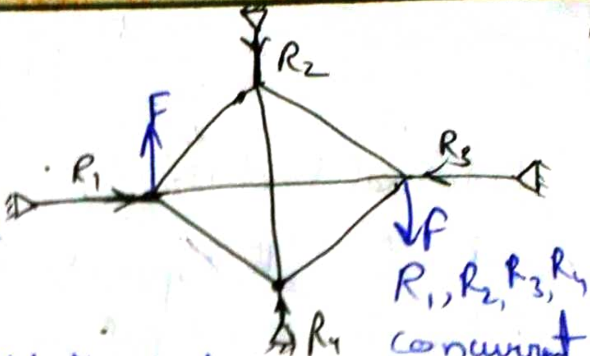
unstable



Stable



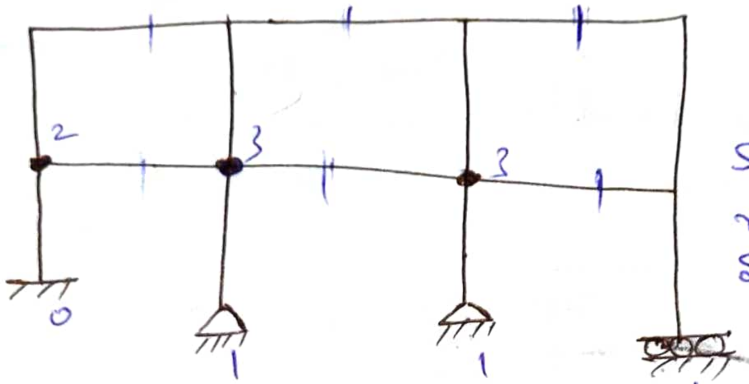
Unstable



R_1, R_2, R_3, R_4 concurrent unstable

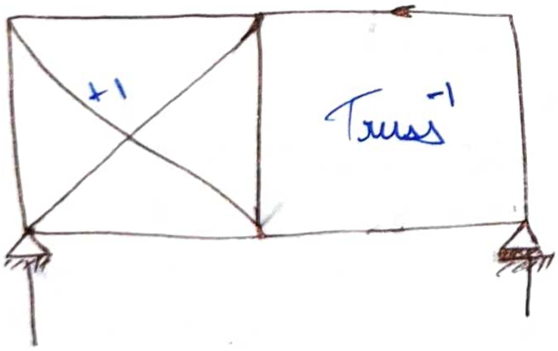
Static Unstability → For General loading. If the total no. of supports rxns are two, then strn. is statically unstable

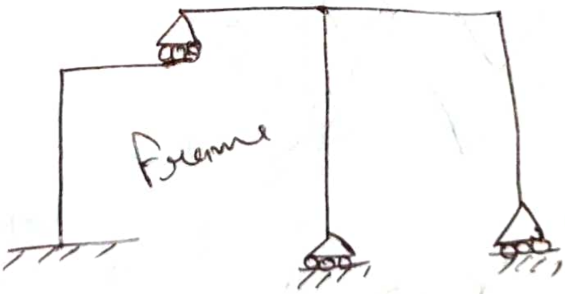
Internally Unstable →



Continuous form forms unstable structure provided middle hinge not supported

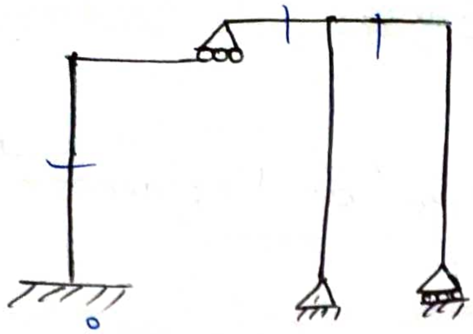
$D_s = 3 \times 6 - 11$
$= 7$ $D_s > 0$ stable
Indeterminate





$D_s = D_{se} + D_{si}$
$= (r_e - 3) + m - (2j - 3)$
$= 0 + 1 - 1$
$= 0$
Stable

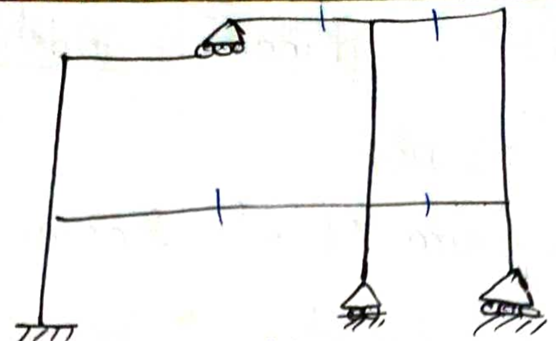
$D_s = 3 \times 2 - 6$
$= 0$
Unstable, more translation



Stable

$D_s = 3 \times 3 - 8$

$= 1$



Stable

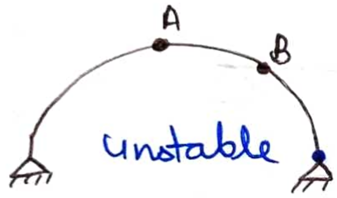
$D_s = 3 \times 4 - 6$

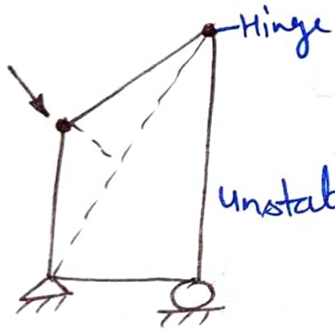
$= 6$ Indeterminate

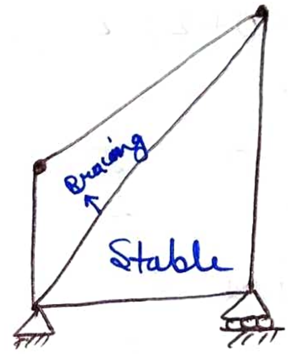
Unstable

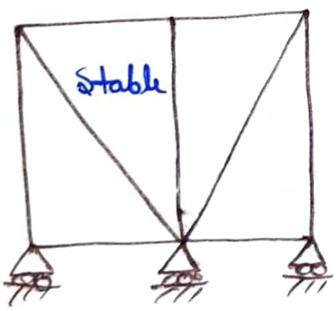
Due to one part of str. move w.r.t other part.

due to presence of 3-collinear hinge









— for Truss

if $D_{si} > 0$, $m > (2j - 3)$, over stiffness — internal (stable) indeterminacy

if $D_{si} = 0$, $m = 2j - 3$, Perfect Truss — Determinate (Stable)

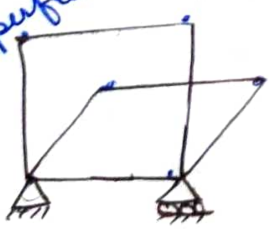
if $D_{si} < 0$, $m < 2j - 3$, Imperfect Truss — unstable

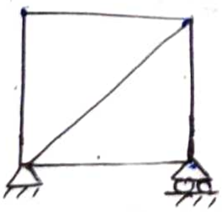
$D_s = m + r_e - 2j$

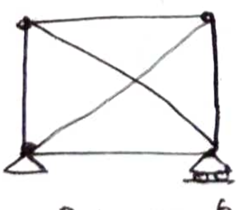
$D_{si} = m - (2j - 3)$

$D_{se} = r_e - 3$

Imperfect Trust







$D_{si} = m - 2j - 3$

$= 7 - (2 \times 6 - 3)$

$= 7 - 9$

$= -2$ (unstable)

$D_{si} = 5 - (2 \times 4 - 3)$

$= 5 - 5$

$= 0$ Stable

Determinant, Perfect

$D_{si} = m - (2j - 3)$

$= 1$

Over stiff

Stable

interde.

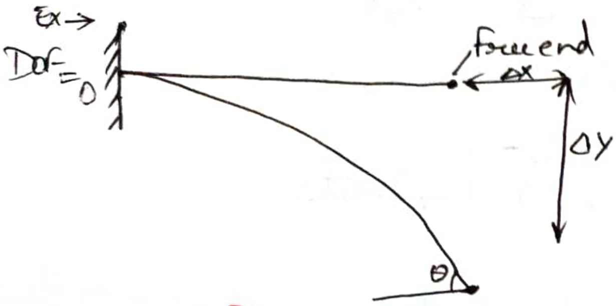
Unstable

Kinematic indeterminancy

$D_K = \Sigma DOF$

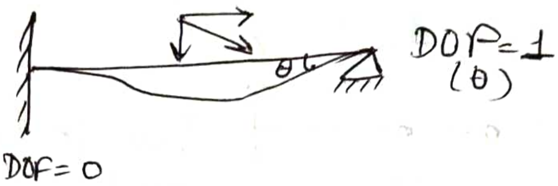
= Sum of all linear & Angular displacement (Independent)

or Sum of unrestrained displacement at all joints



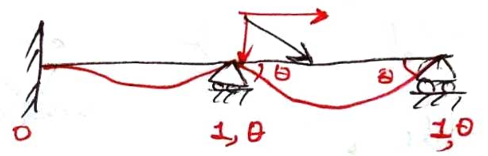
$D_K = \Sigma DOF$

$= 0 + 3 = 3$

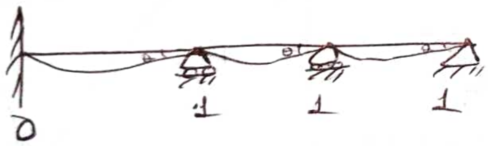


$D_K = 0 + 1 = 1$

$D_K = 0 + 2 = 2$

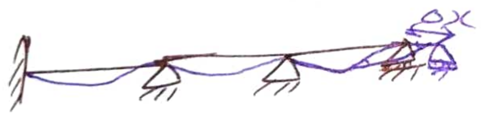


$D_K = 0 + 2 + 2 = 4$

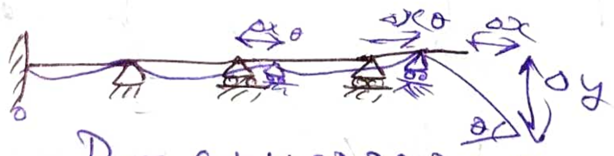


$D_K = 1 + 1 + 1 = 3$

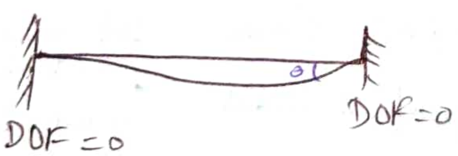
$D_K = 0 + 1 + 0 = 1$



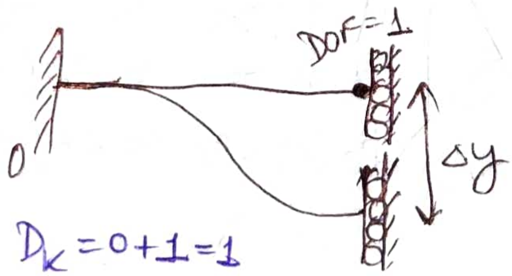
$D_K = 0 + 1 + 1 + 2 = 4$



$D_K = 0 + 1 + 2 + 2 + 3 = 8$



$D_K = 0 + 0 = 1$



$D_K = 0 + 1 = 1$

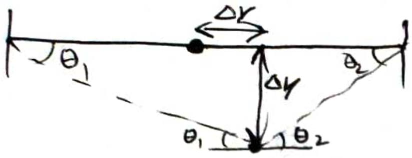
formula to Calculate D_K:

Rigid frame str. (2D)

$$\boxed{DOF = 3N_j - 3\Sigma F - 2\Sigma h - \Sigma R}$$

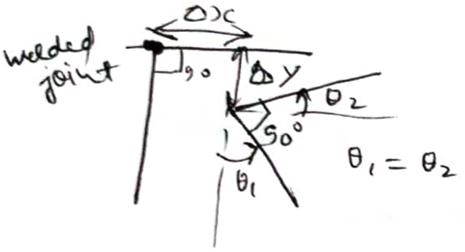
Internal Hinge → $\theta_1, \theta_2, \Delta x, \Delta y \Rightarrow$ 4 DOF

Trick
$DOF = 3 + r_r$
$= 3 + 1$
$= 4$

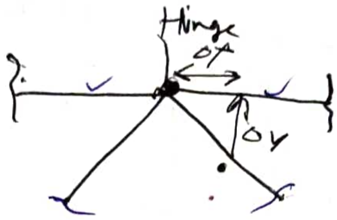


1-Horizontal
1-Verticle

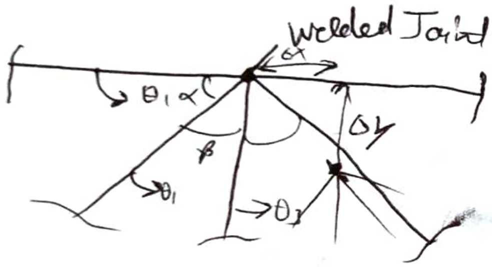
2. fixed Joint



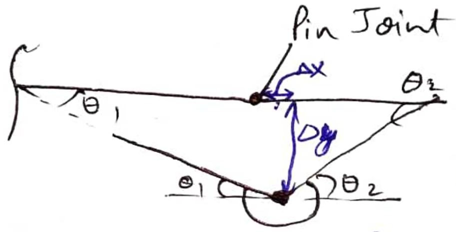
DOF = 3
Trick $= 3 + r_r$
$= 3 + 0$
$= 3$



$DOF = 3 + (4-1)$
$= 6$



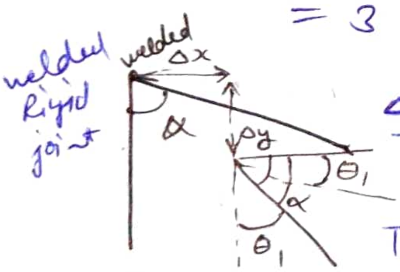
Trick $= 3 + r_r$
$= 3 + 0$
$= 3$



$\Delta x, \Delta y$
θ_1, θ_2

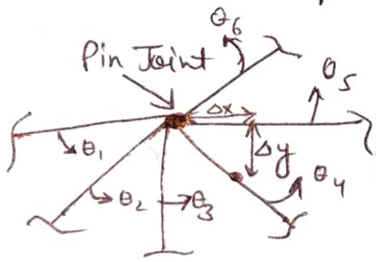
θ_1 and θ_2 are independent

Trick $= 3 + r_r$
$= 3 + 1$
$= 4$



$\Delta x, \Delta y, \theta$
↓
3 DOF

Trick
$3 + r_r$
$3 + 0$
3

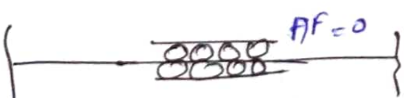


$DOF = 3 + \Sigma(m'-1)$
$= 3 + 5$
$= 8$

$\theta_1, \theta_2, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \Delta x, \Delta y$

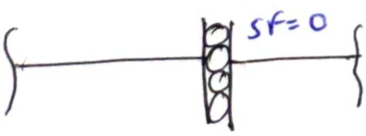
If joint is Rigid $\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = \theta_6 = \theta$
3 D.O.F
$\Delta x, \Delta y$

③ Horizontal Shear Release :



$D.O.F = 3 + r_r = 3 + 1 = 4$

④ verticle shear release



$\Delta x, \Delta y_1, \Delta y_2$
θ_1

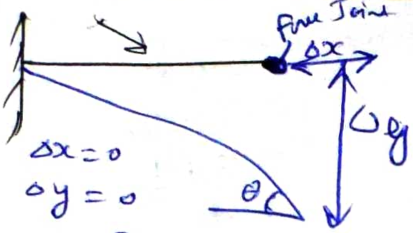
$DOF = 3 + 1$
$= 4$

$$\boxed{D_K = 3J - r_e + A - m'}$$

A = additional DOF due to internal Hinge

m' = No. of mexterable

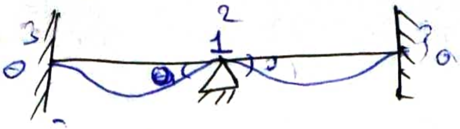
Ans.



$\Delta x = 0$
$\Delta y = 0$
$\theta = 0$

$D_K = 0 + 3$
$= 3$

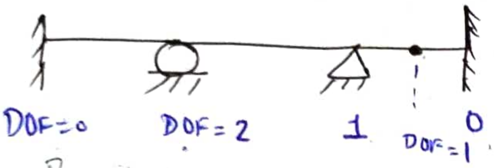
$D_K = 3 \times 2 - 3 = 3$



$D_K = 0 + 1 = 1$

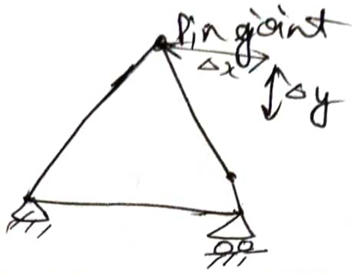
$D_K = 3 \times 3 - \cancel{8}$
$= 1$

F GHR
0 1 1 2



$D_K = 4$

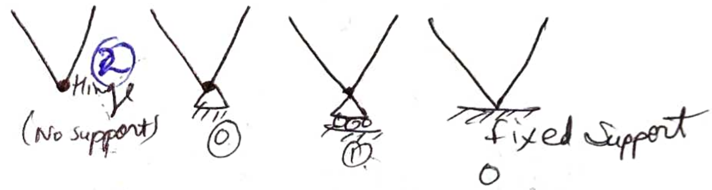
$D_K = 3 \times 5 - 9 + 1$
$= 4$

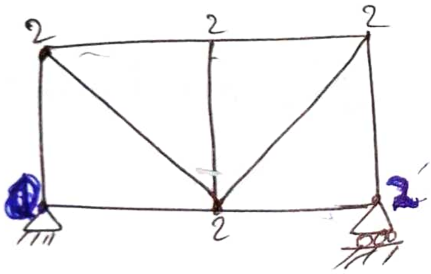


In the case of Truss Joint.
→ 2 DOF ($\Delta x, \Delta y$)
$D_K = 0 + 2 + 1 = 3$

For DOF of Truss joint

1

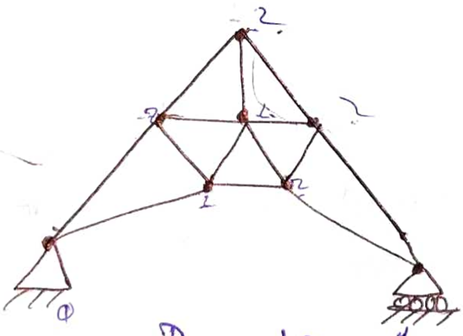




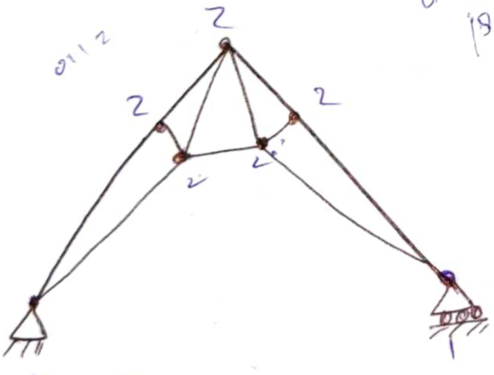
$D_K = 9$

9 - 1

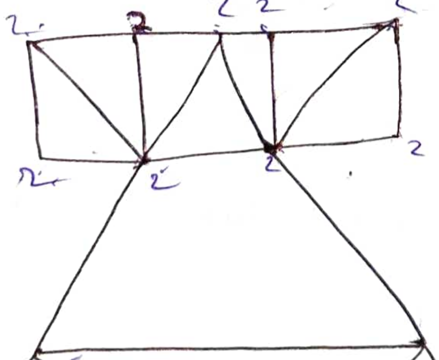
6×3 - 2 - 1
18 - 3



$D_K = 13$

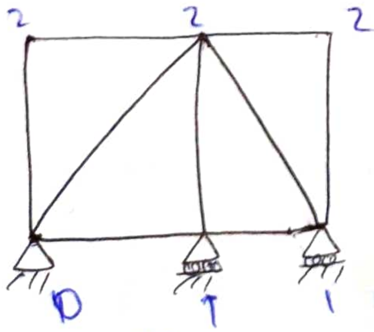


$D_K = 11$

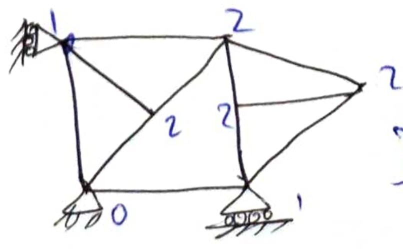


$D_K = 18$

2J - ex
- R

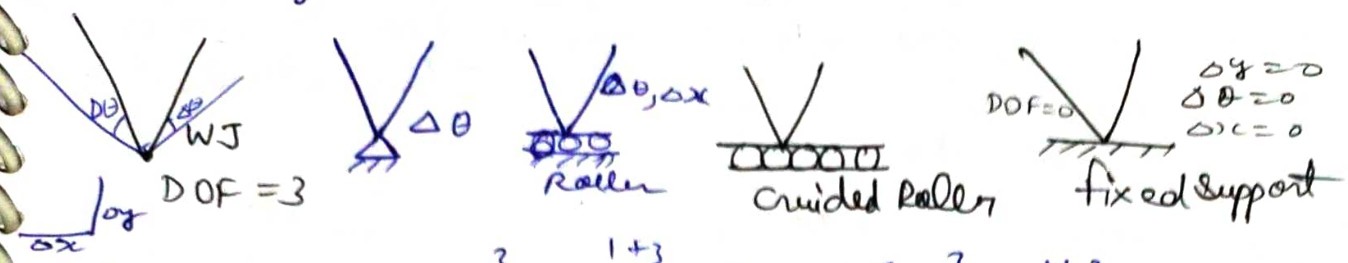


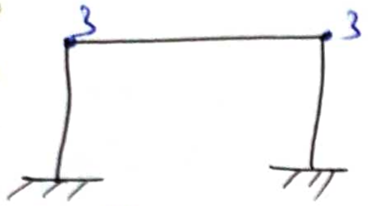
$D_K = 8$



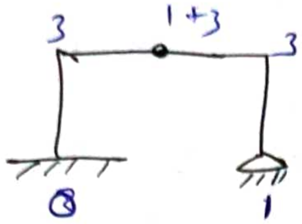
$D_K = 10$

frame- Rigid Jointed structure ($\Delta x, \Delta y, \Delta\theta$)

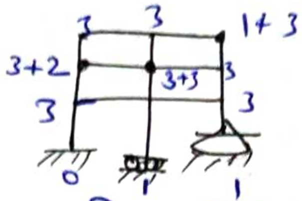




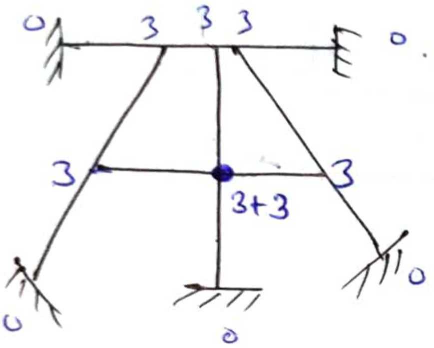
$D_K = 6$



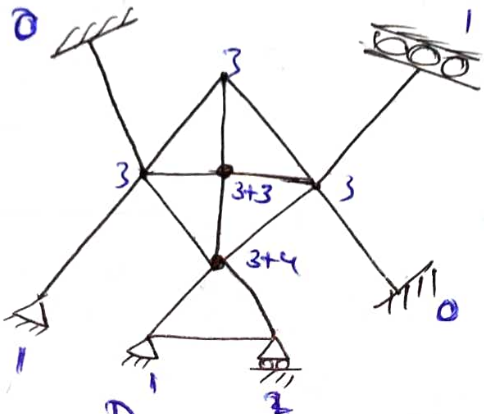
$D_K = 10$



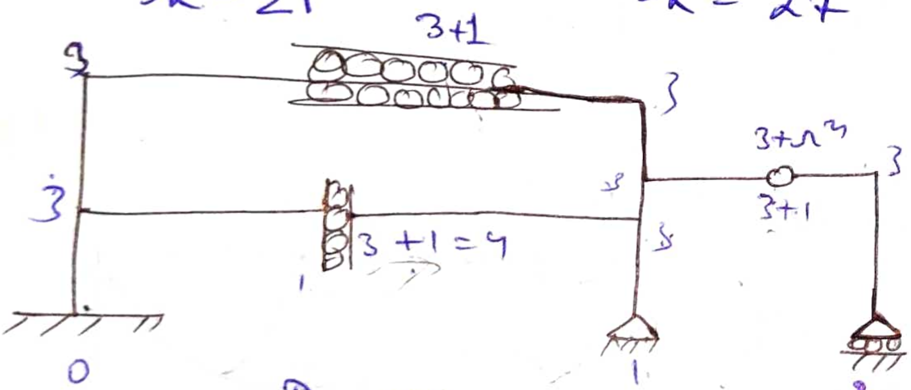
$D_K = 35$



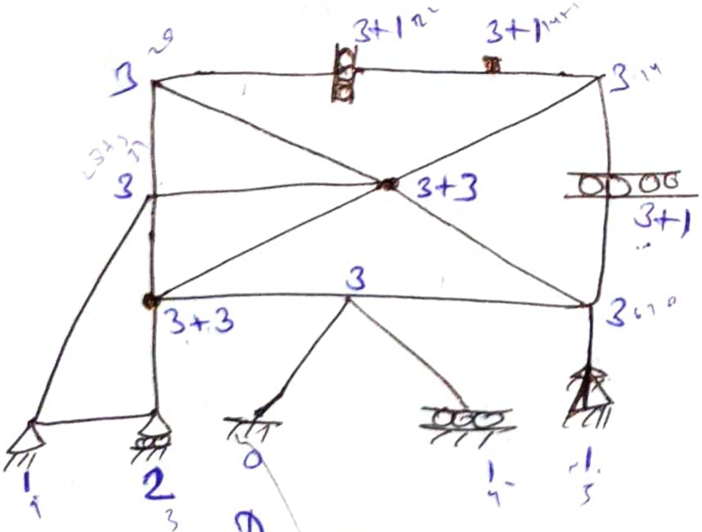
$D_K = 21$



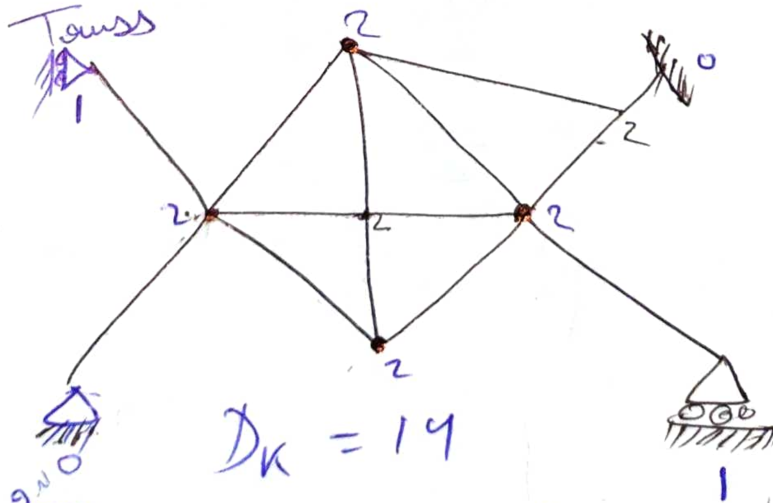
$D_K = 27$



$D_K = 30$



$D_K = 37$



$D_K = 14$

Imp.

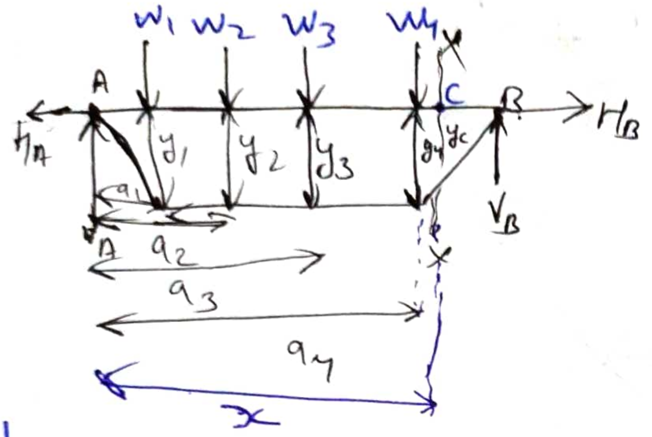
No internal joint in truss

Cables

Cables are used as support as well as for transmission of loads from one member to the other. Cable stands for a flexible tension member

No Rigidity, No moment, No shear,

Tension balance the place of moment



$$M_c = V_A x - W_1(x-a_1) - W_2(x-a_2) - W_3(x-a_3) - W_4(x-a_4) - Hy_c$$

$Hy_c =$ Beam Moment at C

$$\boxed{y_c = \frac{B \cdot M \text{ of } (M_c)}{H}}$$

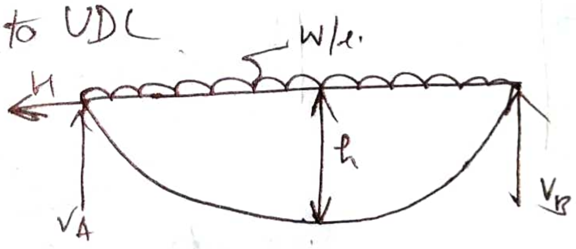
$$y_1 = \frac{M_1}{H},\ y_2 = \frac{M_2}{H},\ y_3 = \frac{M_3}{H}$$

$$\boxed{T_A = \sqrt{V_A^2 + H_A^2}}$$

Tension ↑ = V ↑

Supports ↑ tension more

when cable subjected to UDL



$$M_C = V_A\frac{l}{2} - \frac{Wl}{2}\times\frac{l}{4} - Hh$$

$$0 = \frac{Wl}{2}\times\frac{l}{2} - \frac{wl^2}{8} - Hh$$

$$V_A = V_B = \frac{wl}{2}$$

$$Hh = \frac{wl^2}{4} - \frac{wl^2}{8}$$

$$\boxed{H = \frac{wl^2}{8h}}$$ Expression of horizontal rxn

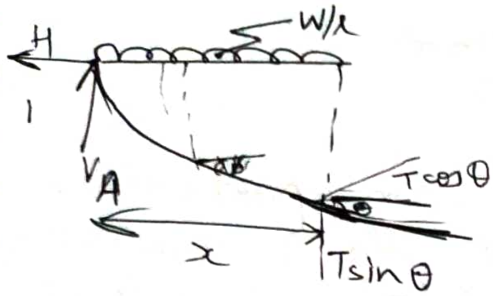
$$T = \sqrt{V_A^2 + H^2}$$

$$= \sqrt{\left(\frac{wl}{2}\right)^2 + \left(\frac{wl^2}{8h}\right)^2}$$

{Tmax. at supports
Tmin at centre}

Tmax at Supports:

Shape of the Cable



$T\cos\theta = H$

$T\sin\theta = V_A - Wx$

$= \frac{wl}{2} - wx$

$\tan\theta = \frac{\frac{wl}{2} - wx}{H}$

$\frac{dy}{dx} = \frac{\left(\frac{wl}{2} - wx\right)}{\frac{wl^2}{8h}} H$

Integrated

$y = \frac{\frac{wlx}{2} - \frac{wx^2}{2}}{\frac{wl^2}{8h}}$

$y = \frac{4hx(l-x)}{l^2}$

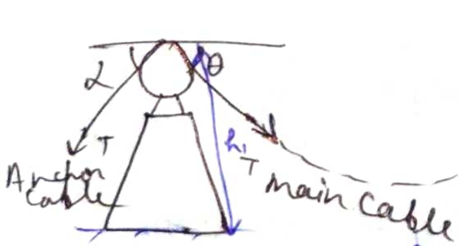
* Length of the Cable (Both ends at the same level)

$L = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ $y = 4h$ $L = \sqrt{1 + \frac{dy}{dx}}$

$$\boxed{L = l + \frac{8h^2}{3l}}$$

or $L = l\left(1 + \frac{8}{3}\frac{h^2}{l^2}\right)$

Guided Pulley Support →



Horizontal force $= T\cos\theta - T\cos\alpha$

$= T(\cos\theta - \cos\alpha)$

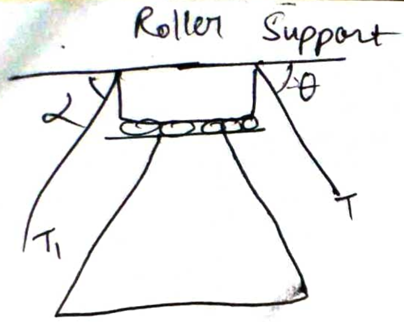
$BM = T(\cos\theta - \cos\alpha) \times h_1$

verticle force on abutment or Tower

$= T\sin\theta + T\sin\alpha$

$= T(\sin\theta + \sin\alpha)$

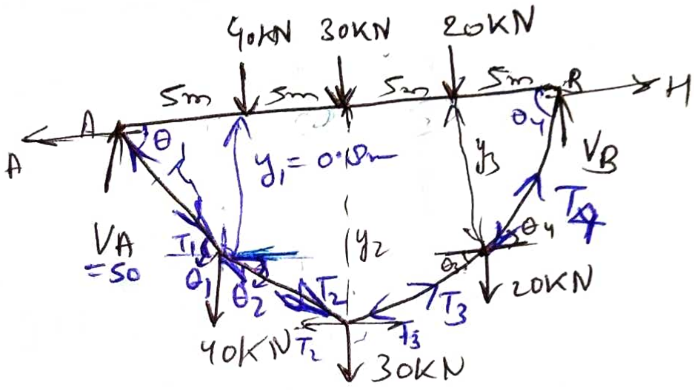
Roller Support



$T\cos\theta = T_1\cos\alpha$

$V = T\sin\theta + T_1\sin\alpha$

Ques. A light cable is supported two points 20m apart which are at the same level the cable supports the 3 concentrated loads as shown in the figure Deflection at 1st Point is found to be 0.8m depending the tension in the simple segments and to length of the cable



$\Sigma M_A = 0$

$M_A = 40\times5 + 30\times10 + 20\times15 - V_B\times20$

$0 = 800 - V_B\times20$

$V_B = 40KN$ $\quad V_A = 50KN$

$y_1 = \frac{M_{BM_1}}{H} = \frac{50\times5}{H}$

y

$Hy_1 = 250 KN$

$H = \frac{250}{0.8} = 312.5KN$

$\tan\theta_1 = \frac{y_1}{5} =$

$\theta_1 = 90.9°$

$y_2 = \frac{M_{B_2}}{H}$

$\tan\theta_2 = \frac{y_2 - y_1}{5} = 1.833°$

$\theta_2 = 1.833°$

$y_2 = \frac{50\times10 - 40\times5}{312.5}$

$= .96m$

$\tan\theta_3 = \frac{y_2 - y_3}{5} = 3.662°$

$\theta_3 = 3.662°$

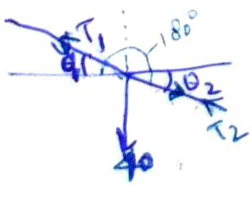
$y_3 = \frac{50\times15 - \cancel{40\times10} - 30\times5}{312.5}$

$\tan\theta_4 = \frac{y_3}{5} = \frac{.64}{5}$

$= 0.64$

$\theta_4 = 7.294°$

Lami Theorem

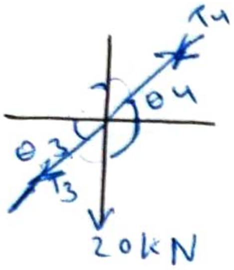


$\frac{T_1}{\sin(90-\theta_2)} = \frac{T_2}{\sin(90+\theta_1)} = \frac{40}{\sin(180-\theta_1+\theta_2)}$

$$T_1 = \frac{40 \sin(90 - 1.833)}{\sin(180 - 9.09 + 1.833)}$$

$$= 316.49\,KN$$

$$T_2 = \frac{40 \times \sin(90 + 9.09)}{\sin(180 - 9.09 + 1.833)} = 312.68\,KN$$

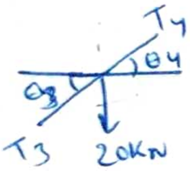


$$T_3 = \frac{20(\sin(90 + \theta_4))}{\sin(180 + \theta_3 - \theta_4)}$$

$$\frac{T_3}{\sin(90+\theta_4)} = \frac{T_4}{\sin(90-\theta_3)} = \frac{20}{(180-\theta_4+\theta_3)}$$

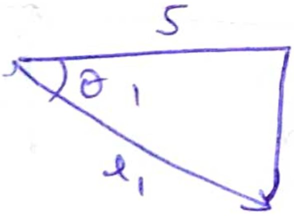
$$= \frac{20 \sin(90° + 7.294°)}{\sin(180 + 3.662 - 7.294°)}$$

$$= 318\,KN$$



$$T_4 = \frac{20 \sin(90 - 3.662)}{\sin(180 + 3.662 - 7.294°)}$$

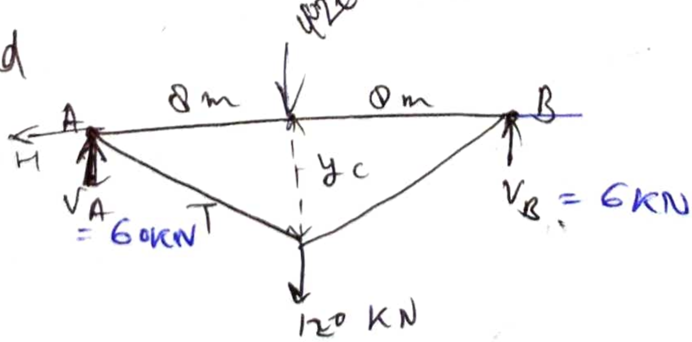
$$T_4 = 315.072\,KN$$



$$l = 5\sec\theta_1 + 5\sec\theta_2 + 5\sec\theta_3 + 5\sec\theta_4$$

$$= 20.117\,m$$

Ques. A light cable 18 m long is supported at two ends at the same level the supports are 16m apart the cable supports 120N load dividing the distan into two equal parts find the shape of the cable and



$$L = 18 = 2 \times \sqrt{\left(\frac{l}{2}\right)^2 + y_c^2}$$

$$y_c = 4.123\,m$$

$M_C = 0$

$V_A \times 8 - H \cdot y_c = 0$

$60 \times 8 - H \cdot y_c =$

$H = 116.42$

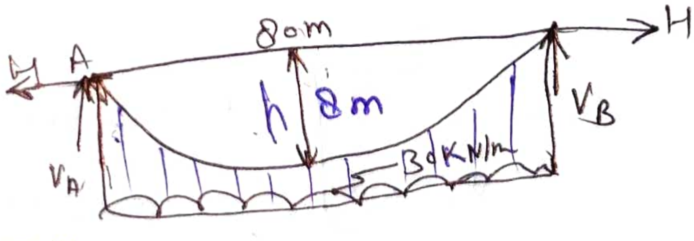
$T = \sqrt{V_A^2 + H^2}$

$= \sqrt{60^2 + (116.42)^2}$

$= 130.97 KN$

Ques. A bridge cable is suspended from towers 80m apart and carries a load of 30KN/m on entire span. If the max sag is 8m calculate max tension in the cable. If the cables is supported by saddles which are stayed by wires inclined 30° to the horizontal. Determine the forces acting on the towers. If the same inclination of back stay passes over pulley determine the forces on the tower.

$\frac{wl}{2} = V_A = V_B = \frac{30 \times 80}{2} = 1200K$

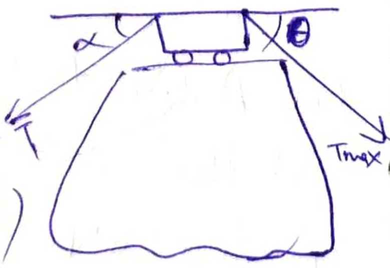


$H = \frac{wl^2}{8h} = \frac{30 \times 80^2}{8 \times 8} = 3000KN$

Max tension occurs at Support

$T_{max} = \sqrt{V^2 + H^2} = \sqrt{(1200)^2 + (3000)^2}$

$= 3231.1 KN$

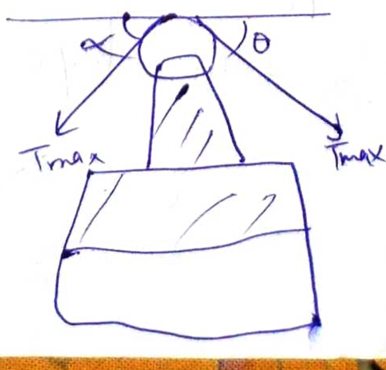


$H = T_{max} \cos\theta$

$\theta = \cos^{-1}\left(\frac{H}{T}\right)$

$= \cos^{-1}\left(\frac{3000}{3231.1}\right)$

$= 21.80°$



(i) If the cable is supported by saddle the anchor cable tension T_1 is given by,

$$T_1 \cos\alpha = T_{max} \cos\theta$$

$$T_1 \times \cos 30^\circ = 3231.10 \times \cos 21.80$$

$$T_1 = 3464.1 KN$$

There is no horizontal force on the tower. The verticle force on the tower is given by

$$= T_1 \sin\alpha + T_{max} \sin\theta$$

$$= 3464.1 \sin 30^\circ + 3231.1 \sin 21.80^\circ$$

$$= 2931.90 KN$$

(ii) If the cable is on pulley, the verticle force on tower is

$$= T_{max} (\sin\alpha + \sin\theta)$$

$$= 3231.1 (\sin 30^\circ + \sin 21.80^\circ)$$

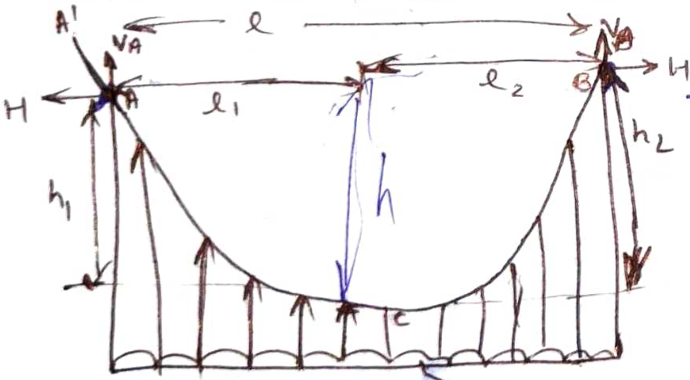
$$= 2815.40 KN$$

Horizontal force on the tower $= T_{max} (\cos\theta - \cos\alpha)$

$$= 3231.1 (\cos 21.8^\circ - \cos 30^\circ)$$

$$= 201.02 KN$$

* Cable with ends at diff^n levels:

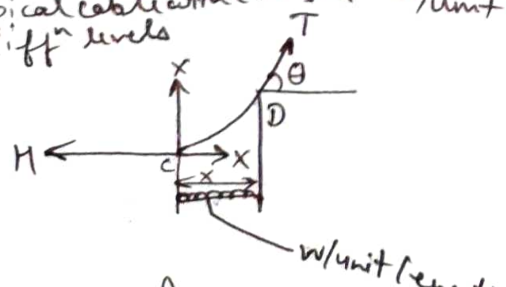


Typical cable with ends at diff^n levels

$T\cos\theta = H$

$T\sin\theta = Wx$

$\tan\theta = \frac{Wx}{H}$



free body diagram of cable

$x = 0, y = 0, C_1 = 0$

$\frac{dy}{dx} = \frac{Wx}{H}$ or $dy = \frac{Wx\,dx}{H}$

$y = \frac{Wx^2}{2H} + C_1$

$$\boxed{y = \frac{Wx^2}{2H}}$$

$x = l_1$ then $h_1 = \dfrac{w l_1^2}{2H}$

$y = h_1$

$x = l_2$

$y = h_2$

$$\frac{h_1}{h_2} = \frac{l_1^2}{l_2^2}$$

$$\frac{l_1}{l_2} = \frac{\sqrt{h_1}}{\sqrt{h_2}}$$

$$\frac{l_1}{l_1+l_2} = \frac{\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}$$

$$\frac{l_2}{l_1+l_2} = \frac{\sqrt{h_2}}{\sqrt{h_2}+\sqrt{h_1}}$$

$$l_1 + l_2 = l$$

$$\boxed{l_1 = \frac{l\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}} \text{ or } \boxed{l_2 = \frac{l\sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}}$$

$$L = l + \frac{2}{3}\frac{h_1^2}{l_1} + \frac{2}{3}\frac{h_2^2}{l_2}$$

$$\Sigma M_c = 0$$

$$V_A \cdot l_1 - H h_1 - \frac{w l_1^2}{2} = 0$$

$$V_A = \frac{w l_1}{2} + \frac{H h_1}{l_1}$$

$$V_B = \frac{w l_2}{2} + \frac{H h_2}{l_2}$$

$$V_A + V_B = \frac{w(l_1+l_2)}{2} + \frac{H h_1}{l_1} + \frac{H h_2}{l_2}$$

$$wl = \frac{wl}{2} + H\left(\frac{h_1}{l_1} + \frac{h_2}{l_2}\right)$$

$$H = \frac{wl}{2\left(\frac{h_1}{l_1} + \frac{h_2}{l_2}\right)}$$

$$H = \frac{wl}{2\left[\dfrac{h_1}{\frac{l \times \sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}} + \dfrac{h_2}{\frac{l \times \sqrt{h_2}}{\sqrt{h_1}+\sqrt{h_2}}}\right]}$$

$$= \frac{wl^2}{2}\left[\left[\sqrt{h_1}(\sqrt{h_1}+\sqrt{h_2}) + \sqrt{h_2}(\sqrt{h_1}+\sqrt{h_2})\right)\right.$$

$$\boxed{H = \frac{wl^2}{2(\sqrt{h_1}+\sqrt{h_2})^2}}$$

Length of Cable ACB = $\frac{1}{2}$ sum of length of ACB' & A'CB

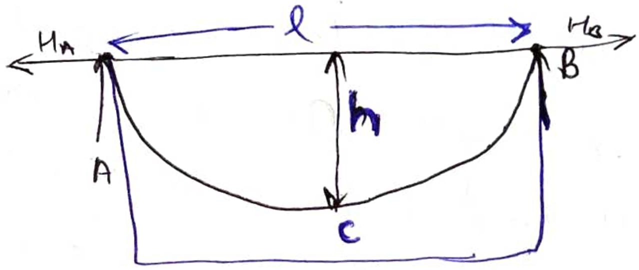
$$= \frac{1}{2} \times \left[2l_1 + \frac{8}{3} \times \frac{h_1^2}{2l_1} + 2l_2 + \frac{8}{3} \times \frac{h_2^2}{2l_2}\right]$$

$$= l_1 + l_2 + \frac{2}{3} \times \frac{h_1^2}{l_1} + \frac{2}{3} \times \frac{h_2^2}{l_2}$$

$$\boxed{L = l + \frac{2}{3} \times \frac{h_1^2}{l_1} + \frac{2}{3} \times \frac{h_2^2}{l_2}}$$

Effect of Temperature on Cable:

Let temperature



$L = l + \frac{8}{3} \times \frac{h^2}{l}$

$$\frac{dL}{dh} = \frac{8}{3} \times \frac{2h}{l} \times \delta h$$

$$\delta L = \frac{16}{3} \frac{h}{l} \times \delta h \quad --- (i)$$

If α is the coefficient of thermal expansion

$$\Rightarrow \delta L = L \cdot \alpha \cdot t$$

$$= \alpha t \left[l + \frac{8h^2}{3l}\right]$$

$\delta L = l \cdot \alpha \cdot t \quad --- (ii)$

from eq^n (i) & (ii)

$$l \alpha t = \frac{16}{3} \frac{h}{l} \delta h$$

$\delta h = \frac{3}{16} \times \frac{l^2 \alpha t}{h}$

The above expression gives increase in dip due to the rise in temperature.

Taking moment about lowest point C. we get

$Hh = M_c$ when M_c is the beam Moment at C

$$H = \frac{M_c}{h}$$

$$\frac{dH}{dh} = -\frac{M_c}{h^2} = -\left(\frac{M_c}{h}\right)\frac{1}{h}$$

$$= -\frac{H}{h}$$

$$\frac{\delta H}{H} = -\frac{1}{h} \times \delta h$$

Substituting the value of δh from eq^n

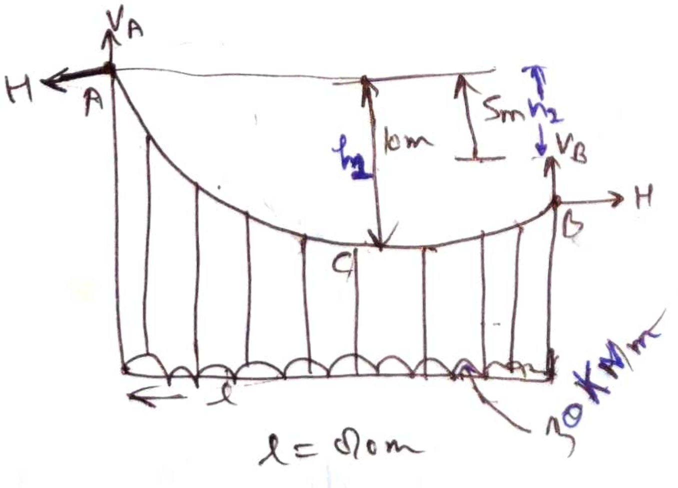
$$\boxed{\frac{\delta H}{H} = -\frac{3}{16} \times \frac{l^2}{h^2} \times \alpha t}$$

Increase in temp. reduces the horizontal thrust

Ques: A cable is suspended from the points A and B which are 80m apart horizontal & ~~at~~ 1 in diff^n level vertically B an the, lowest point in the point A be 5 m below A cable is subjected to UDL of the table is 10m ~~below A~~ horizontal. ~~& verticle rxn~~ Determine 30KN/m over the the hori & verticle rxn at each end and also the max. tension in the cable.

$h_1 = 10m$ $\quad w = 30 KN/m$

$h_2 = 5m$ $\quad l = 80m$



$$\frac{h_1}{h_2} = \frac{l_1^2}{l_2^2}$$

$$l_1 = \frac{l \cdot \sqrt{h_1}}{\sqrt{h_1} + \sqrt{h_2}}$$

$$\frac{l_1}{l_2} = \frac{\sqrt{h_1}}{\sqrt{h_2}}$$

$$= \sqrt{\frac{10}{5}} = \sqrt{2}$$

$$l_1 = \sqrt{2}\, l_2$$

~~$l_2 = 80m$~~

$$l_1 + l_2 = 80\,m$$

$l_2(\sqrt{2}+1) = 80$

$l_2 = 33.137 m$

$l_1 = 80 - 33.137$

$= 46.863 m$

$H \times 10 - \frac{w l_1^2}{2} = 0$

$H = wl$

$$H = \frac{w l^2}{10 \times 2} = \frac{30 \times 80^2}{20(\sqrt{10}+\sqrt{5})^2} = 32\cancel{9}4.2 KN$$

$$\text{or} = \frac{30 \times (46.863)^2}{20} = 3294.2 KN$$

$V_A = w l_1 = 30 \times 46.863 = 1405.89 KN$

$V_B = w l_2 = 30 \times 33.137 = 994.11 KN$

$V_A > V_B$,

$$T_{max} = \sqrt{V_A^2 + H^2} = \sqrt{(1405.89)^2 + (3294.2)^2}$$
$$= 3581.66 \ KN$$

Ques A cable of span 80 m & dip 6 m is subjected to a rise in temp of 20°c. If the $\alpha = 12 \times 10^{-6}/°c$, determine the ince in the dip of the cable. What are the changes in rxn and max tension, if the able carries a load of 15KN/m?

$w = 15 KN/m$ $l = 80 m$, $h = 6 m$, $t = 20°c$, $\alpha = 12 \times 10^{-6}/°c$

$$\delta h = \frac{3 l^2 \alpha t}{16 h} = \frac{3 \times 80^2 \times 12 \times 10^{-6} \times 20}{16 \times 6} = 0.048 m$$

If cable carries a udl 15KN/m, then

$$H h = \frac{w l^2}{8} =$$

$$H = \frac{15 \times 80^2}{6 \times 8} = 2000 KN$$

$$\frac{\delta H}{H} = -\frac{3}{16} \times \frac{l^2}{h^2} \times \alpha t$$

$$\frac{\delta H}{2000} = -\frac{3}{16} \times \frac{80^2}{6^2} \times 12 \times 10^{-6} \times 20$$

$$\delta H = -16 \cancel{kN}$$

$$H = 2000 - 16\ kN = 1984\ kN$$

$$V_A = \frac{W l}{2} = \frac{15 \times 80}{2} = 600\ kN$$

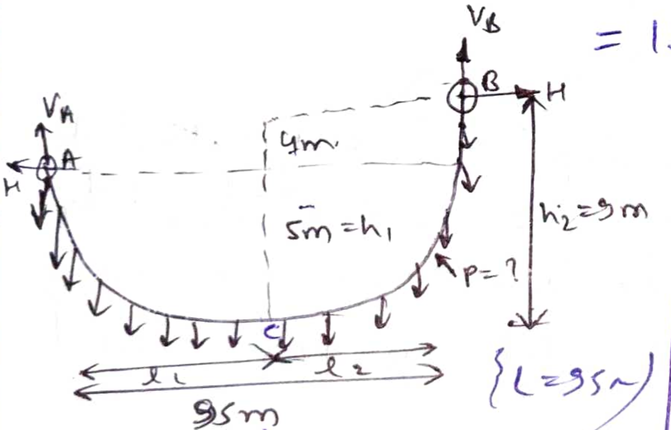
$$T = \sqrt{(2000)^2 + 600^2}$$
$$= 2088.06\ kN$$

After rise in temp.

$$T = \sqrt{1984^2 + 600^2} = 2072.74\ kN$$

Change in max tension

$$= 2088.06 - 2072.74$$
$$= 15.32\ kN$$



To find length:

$$\frac{l_1^2}{l_2^2} = \frac{h_1}{h_2}$$

$$l_1 = \sqrt{\frac{h_1}{h_2}} * l_2$$

$$l_1 = \sqrt{\frac{5}{9}} * l_2$$

$$l_1 = 0.7453 l_2$$

$$L = l_1 + l_2 \qquad l = l_1 + l_2 = 95$$

$$95 = 0.7453 l_2$$

$$\boxed{l_2 = 54.43m}$$

$$95 = l_1 + 54.43$$

$$\boxed{l_1 = 40.57m}$$

$$V_A = w \times l_1 = P \times 40.57$$
$$= 40.57 PN$$

$$V_B = 54.43\ PN$$

$$H = \frac{w l_2^2}{2h_2} = \frac{P(54.43)^2}{2 \times 9} = 164.65P$$

$$T_{max} = T_B = \sqrt{V_B^2 + H^2}$$

$$T_{max} = \sqrt{(54.43P)^2 + (40.57P)^2}$$

$$T_{max} = 173.413P\ kN$$

$$\sigma = \frac{T}{A}$$

$$T = \sigma \times A$$

$$173.413P = 600 \times 3500$$

$$P = 12109 N/m$$

$$\boxed{P = 12.10 kN/m}$$

—: Trusses :—

This is a frame work in which slender member are joined by then ends with bolts or welding.

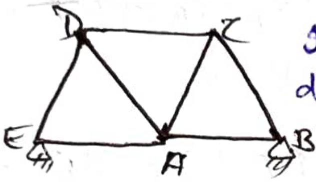
Types of Trusses :

compression member are always thicker than tension.

(a) Roof Trusses
(b) Bridge Trusses

Classification of Trusses

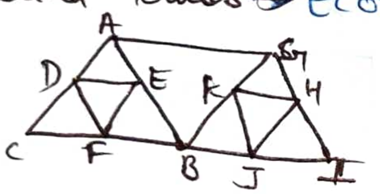
(a) Simple Truss →

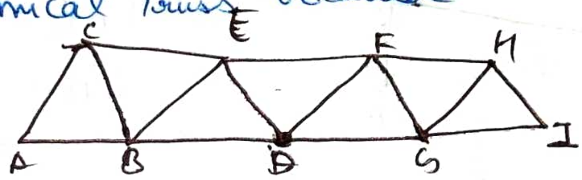


It is imp. to realize that simple trusses do not have to consist entirely of Δ. A simple truss is constructed by starting with a basic Δ element

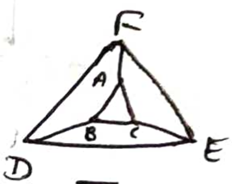
(b) Compound Truss → Economical Truss because

Type 1

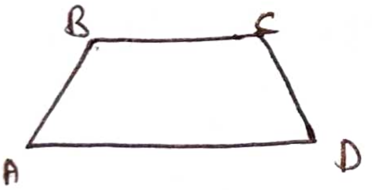


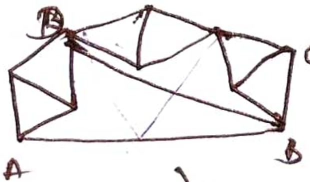


Type-I CT



Type-2 CT

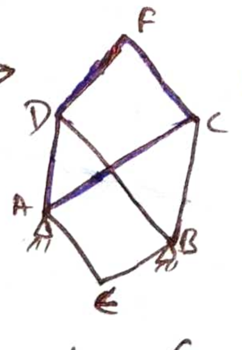




A CT is formed by connecting two or more ST together.

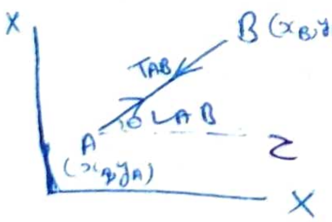
secondary truss.

(c) Complex Truss →



A CT is one that can't be classified as being either simple or compound.
{ sec stability

Method of Tension Coefficient



$$t_{AB} = \frac{T_{AB}}{L_{AB}}$$

$$T_{AB}\cos\theta = T_{AB}\cdot\frac{AC}{AB}$$

$$= \frac{T_{AB}}{L_{AB}}(x_B - x_A)$$

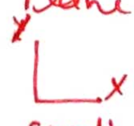
$$T_{AB}\cos\theta = t_{AB}(x_B - x_A)$$

$$F_V = T_{AB}\sin\theta$$

$$= \frac{T_{AB}}{L_{AB}} BC = t_{AB}(y_B - y_A)$$

Truss

Plane — Space



Coordinates
(x, y) — (x, y, z)

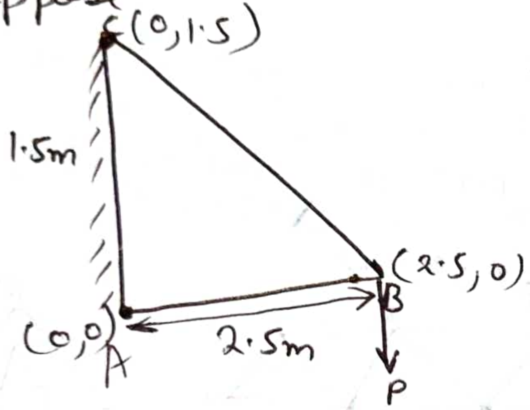
$\Sigma H = 0$

$$\boxed{\Sigma t_{ij}(x_j - x_i) + X_P = 0}$$

$$t_{AB}(X_B - X_A) + t_{AC}(x_C - X_A) + t_{AD}(x_D - X_A) + t_{AE}(X_E - X_A) + H = 0$$

$$\Sigma V = 0; \boxed{\Sigma t_{ij}(y_j - y_i) - y_P = 0}$$

Ques. Suppose



$\Sigma H = 0$

$$t_{BA}(x_A - x_B) + t_{BC}(x_C - X_B) = 0$$

$\Sigma V = 0$

$$t_{BA}(y_A - y_B) - P = 0$$

$$L_{AB} = 2.5$$

$$L_{AC} = \sqrt{(2.5)^2 + (1.5)^2}$$
$$= 2.9\,m$$

$$t_{BA}(0 - 2.5) + t_{BC}(0 \cdot 2.5) = 0$$

$$-2.5\,t_{BA} - 2.5\,t_{BC} = 0$$

$$t_{BA} = -t_{BC} \quad \text{---(ii)}$$

$$t_{BA}(y_A - y_B) + t_{BC}(y_C - y_B) = P$$

$$t_{BA}(0 - 0) + t_{BC}(1.5 - 0) = P$$

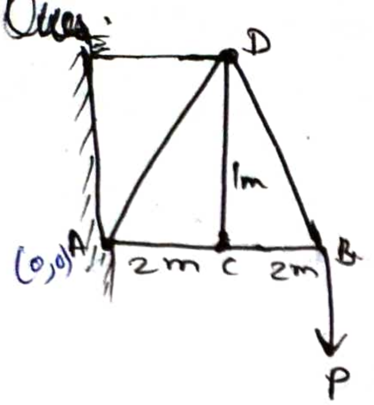
$$1.5\,t_{BC} = P \quad \text{---(iii)}$$

$$t_{BC} = \frac{P}{1.5} = -0.666\,P$$

$$T_{AB} = t_{BA} \times L_{AB}$$
$$= -0.666\,P \times 2.5$$
$$= -1.66\,P \text{ (compression)}$$

$$T_{BC} = t_{BC} \times L_{BC}$$
$$= 0.66\,P \times 2.9$$
$$= 1.9\,P \text{ (Tension)}$$

Ques:



Taking Joint A

Joint A [AD, AB] [H_A, R_A]

Members forces

$\Sigma F_x = 0$

$t_{AD}(x_D - x_A) + t_{AB}(x_B - x_A) + X_A = 0$ (force action)

$t_{AD}(2-0) + t_{AB}(2-0) - 36 = 0$

$2t_{AD} + 2t_{AB} = 36$

$\boxed{t_{AD} + t_{AB} = 18}$ — (ii)

Joint C [CB, CD] [0, R_c]

x-axis

$t_{CB}(x_B - x_c) + t_{CD}(x_D - x_c) + X_c = 0$

$t_{CB}(2-4) + t_{CD}(2-4) + 0 = 0$

$\boxed{t_{CD} = 27 \text{ KN/m}}$

t = tension coefficient

$T = (t \times L)$ KN

$\Sigma F_y = 0$

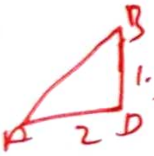
$t_{AD}(y_D - y_A) + t_{AB}(y_B - y_A) + Y_A = 0$

$t_{AD}(0-0) + t_{AB}(1.5-0) + 13.5 = 0$

~~$1.5 t_{AB} = 1.5$~~

$1.5 t_{AB} = -13.5$

$t_{AB} = -9$ KN/m



$AB = \sqrt{2^2 + 1.5^2}$

$= 2.5$ m

Put in eq^n (ii)

$t_{AD} = 27$ KN/m

Member	t(kN/m)	l(m)	T(KN)
AB	-9	2.5	-22.5
BC	-27	2.5	-67.5
BD	36	1.5	54
AD	27	2	54
DC	27	2	54

Joint B. [BA, BD, BC] [36, 0]

$\Sigma F_x = 0$

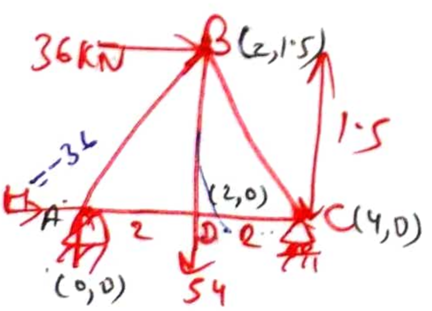
$t_{BA}(x_A - x_B) + t_{BC}(x_D - x_B)$ ~~$+ 36 = 0$~~

$+ t_{BC}(x_B - x_c) + 36 = 0$

$-2t_{BA} + 2t_{BC} = -36$

$-2 \times -9 + 2t_{BC} = -36$

$\boxed{t_{BC} = -27 \text{ KN/m}}$



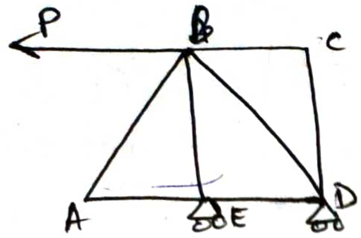
$\Sigma F_y = 0$

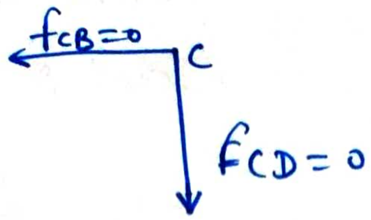
$t_{BA}(y_A - y_B) + t_{BC}(y_D - y_c) + t_{BC}(y_B - y_c) + Y_B = 0$

$t_{BD} = -t_{BA} - t_{BC}$

$t_{BD} = 36$ KN/m

Zero frame Members in a Truss





Case-I

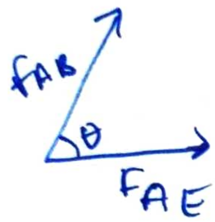
$\Sigma f_y = 0$

$f_{AB} \sin\theta = 0$

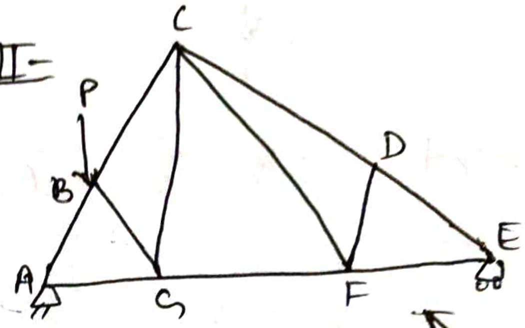
$\sin\theta \neq 0$

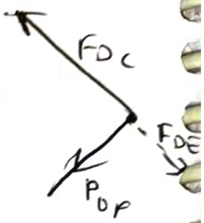
$f_{AB} = 0$

$\Sigma f_x = 0, \; f_{AE} = 0$



Case II-

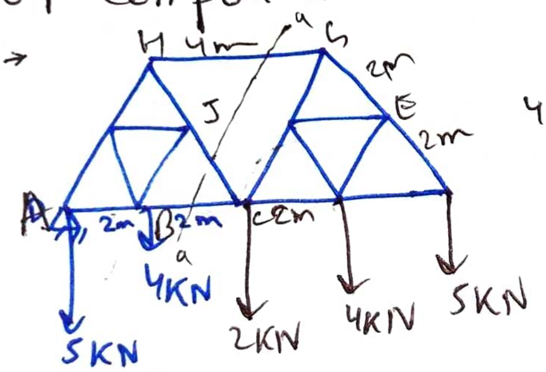


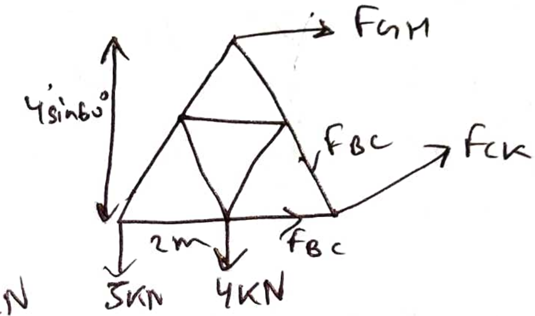


Analysis of Compound Trusses →

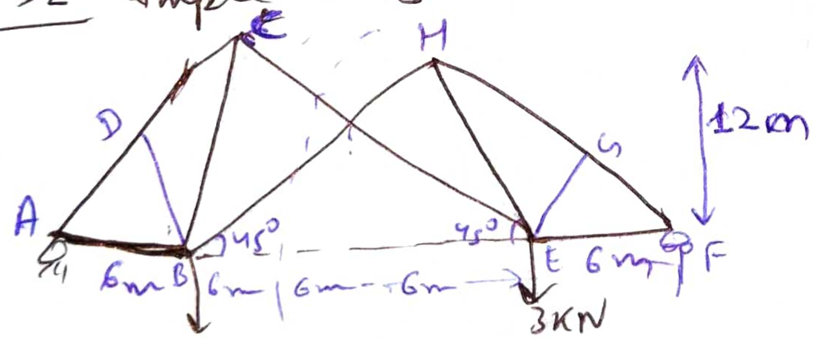
Type-1

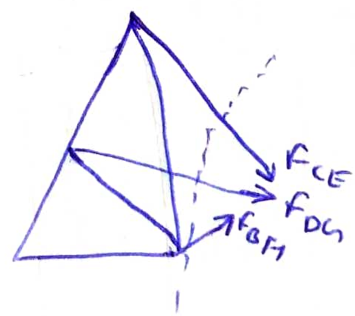
Ex →



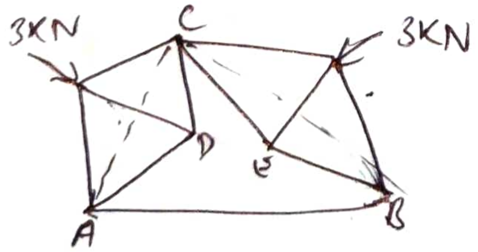


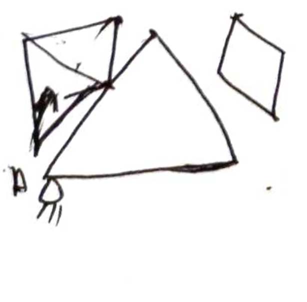
Type→2 Simple Truss are connected by three bars





Type → 3

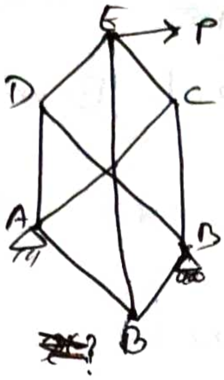


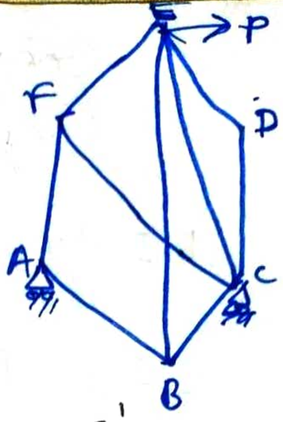


Complex Trusses:—

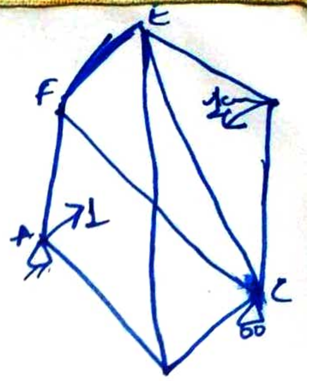
Procedure Analysis →

Method of Substitute Method





S_i'



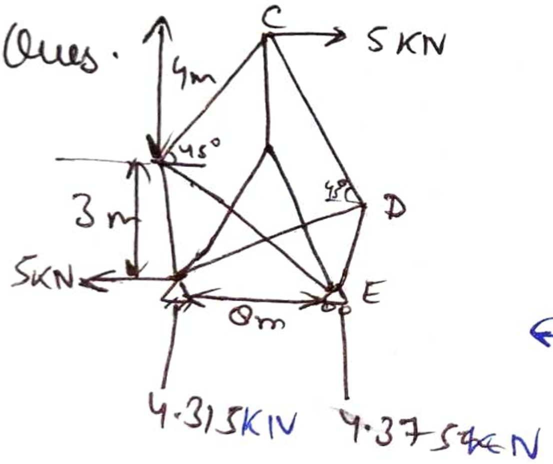
S_i

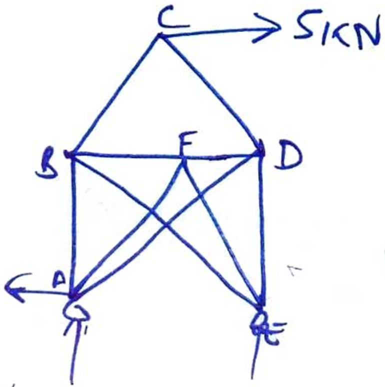
$$S_i = S_i' + x\,S_i$$

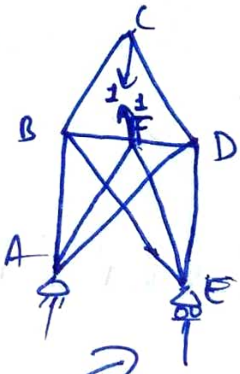
$$x = ?$$

$$x = \frac{S'_{EC}}{S_{EC}}$$

Ques.







$$S_{BD} = S'_{BD} + x\,S_{BD}$$

$$\Rightarrow -2.50 + x(1.167) = 0$$

$$\boxed{x = 2.143}$$

	S_i'	S_i	$x\,S_i$	$S_i = S_i' + xS_i$
CB	3.54	−0.707	−1.59	2.02
CD	−3.54	−0.707	−1.52	−5.05
FA	0	+0.833	+1.79	+1.79
FE	0	+0.833	−1.53	+1.79
EB	0	−0.712	−0.536	−1.53
ED	−4.38	−2.50	−1.53	−4.31
DA	5.34	−0.712	−0.536	+3.51
DB	−2.50	1.167	2.5	−0
BA	+2.50	6.750	−0.536	1.96

$$S_{\cdot} = S' + \cancel{x}\,S_B$$

$$\Rightarrow 2.5 + x(1.167) = 0$$

$$x = 2.143$$

~~Conjugat~~

Conjugate Beam Method

$$EI\frac{dy}{dx} = \int M\,dx$$

$$EIy = \iint M\,dx$$

Only find out the slope & deflection

$$\theta = \int \frac{M\,dx}{EE}$$

(a) Draw the BMD

(b) divide by EI

(c) Convert in Conjugate Beam

$$M = \int s\,dx \cdot \frac{d^2y}{dx^2}$$

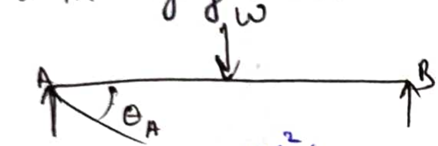
$$\frac{dM}{dx} = S$$

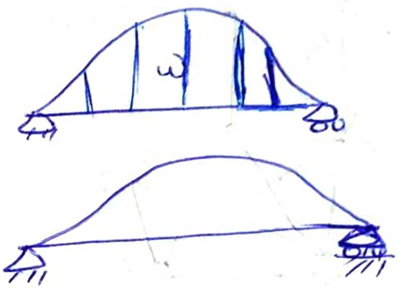
$$\frac{dS}{dx} = w$$

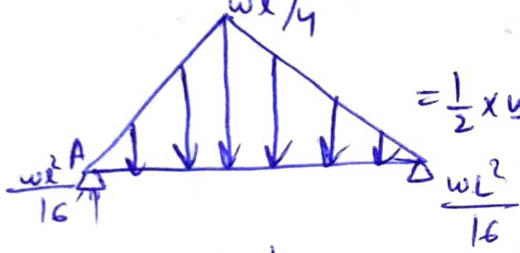
$$\frac{d^2M}{dx^2} = w$$

$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$

Ex →



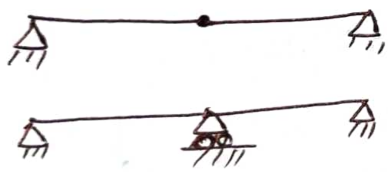




$$= \frac{1}{2} \times \frac{wL}{4} \times L = \frac{wl^2}{8}$$

$$\theta_A = -V_A'$$

Ex →



internal hinge becomes simple support

Ex →

$\theta = 0$

$y_A = 0$

$S = 0$

$M = 0$

(free end)

θ_B

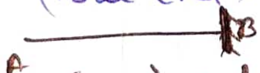
B

y_B

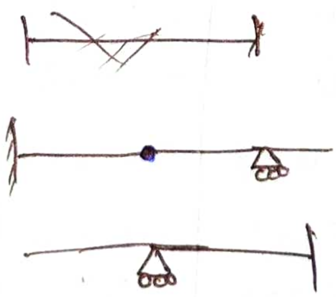
S_B

M_B

(fixed support)



Conjugate beam



Ex →

$\theta = ?$

$y = 0$

S

$M = 0$

internal hinge becomes internal support

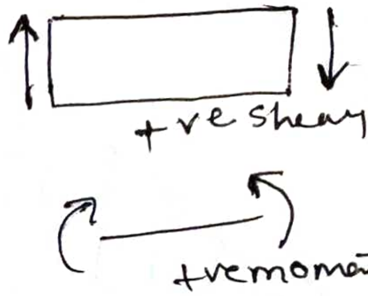
Internal roller becomes internal hinge

-: Conjugate Beam Method :-

Mathematical similarity with following equation :-

$$\frac{dy}{dx} = -w \qquad \frac{d^2M}{dx^2} = -w$$

$$\frac{d\theta}{dx} = \frac{M}{EI} \qquad \frac{d^2y}{dx^2} = \frac{M}{EI}$$



On intregation,

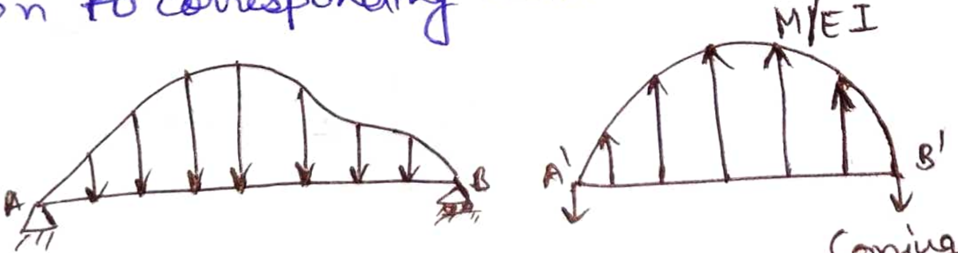
$$V = \int -w\,dx \qquad M = \int(\int -w\,dx)\,dx$$

$$\theta = \int (M/EI)\,dx \qquad y = \int(\int M/EI\,dx)\,dx$$

Shear compares with slope

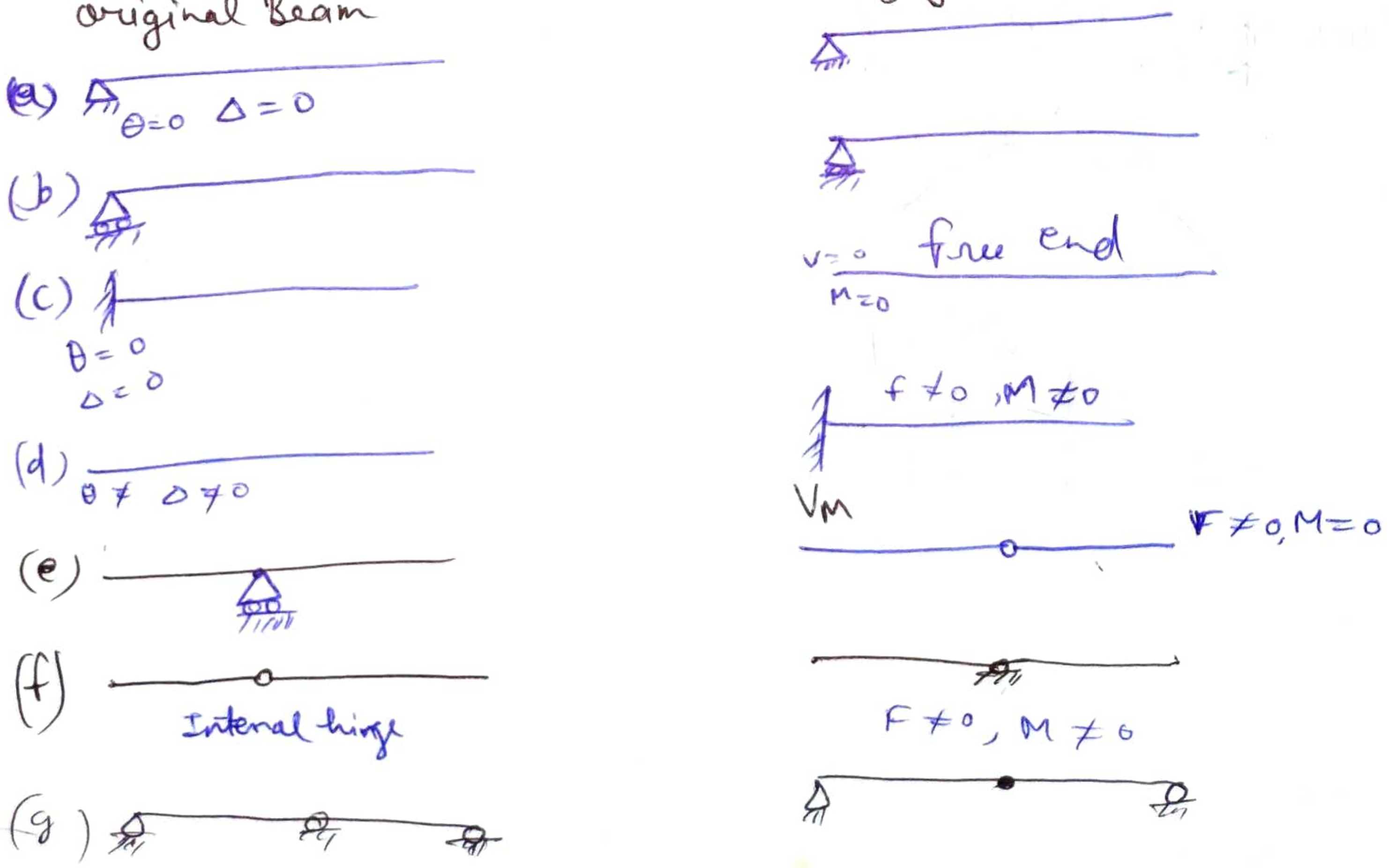
Moment compares with deflection

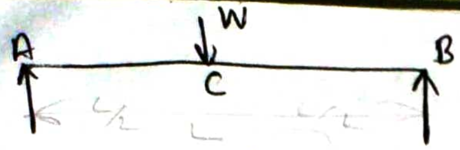
Consider a beam slope of real beam (M/EI) derived from this beam called conjugate beam in which shear at any point is numerically equal to slope at real beam. Similarly moment at point in conjugate beam is numerically equal to deflection to corresponding beam.



original Beam

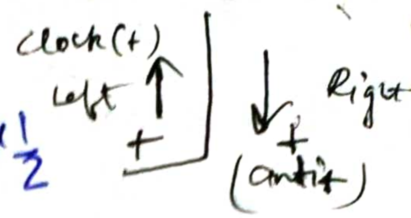
Conjugate Beam



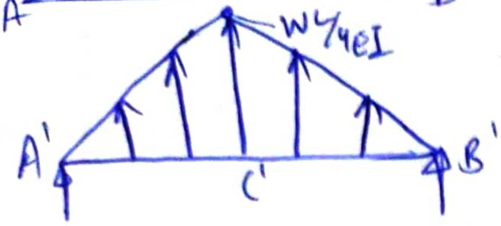


EI constant

θ_A, θ_B & centre deflection



$$R_A' = \frac{1}{2} \times \frac{WL}{4EI} \times L \times \frac{1}{2}$$



$$R_A' = \frac{WL^2}{16EI} \downarrow$$

$$\theta_A = V_A' = -R_A' = -\frac{WL^2}{16EI} \text{ radian (clockwise)}$$

Real Beam

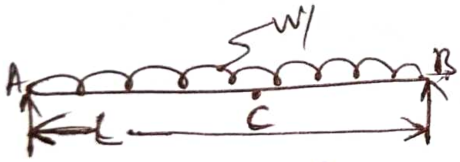
$$\theta_B = V_B' = R_B' = \frac{WL^2}{16EI} \text{ anticlockwise}$$

$$\Delta_C = M_C'$$

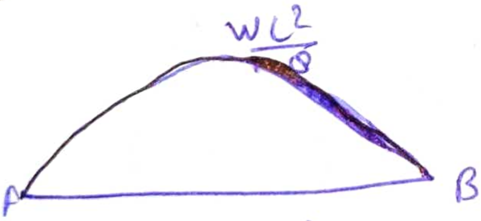
$$= -R_A' \times \frac{L}{2} + \frac{1}{2}\frac{WL}{4EI} \times \frac{L}{2} \times \left(\frac{1}{3}\right)\left(\frac{L}{2}\right)$$

$$= \frac{WL^3}{36EI} + \frac{WL^3}{96EI}$$

Ques.



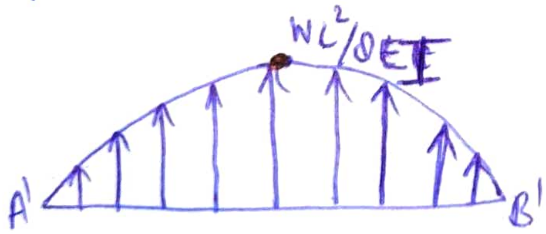
$$R_A' = R_B' = \frac{2}{3} \times \frac{wL^2}{8EI} \times \frac{L}{2}$$

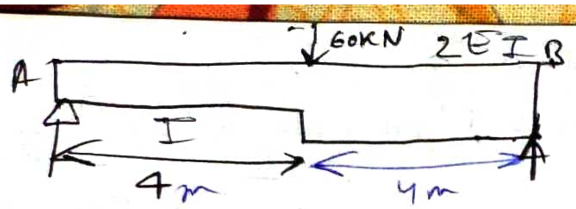


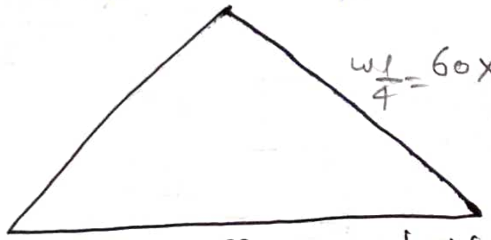
$$= \frac{wL^3}{24EI}$$

θ_C = S.F at C in conjugate beam

$$= \frac{wL^3}{24EI}$$







$\frac{wl}{4} = \frac{60\times 8}{4} = 120$

$120/EI = \frac{1}{2}\times 60\times 4/EI$

$60/EI = \frac{1}{2}\times 60\times 4 / 2EI$

$\frac{1}{2}\times 4\times 60$

120

$M_B = 0$

$\frac{1}{2}\times\frac{120}{EI}\times 4 = 8 \cdot \frac{2}{3}\times 8$

$R_A' \times 8 = \frac{240}{EI}\times 5.33 + \frac{120}{EI}\times\frac{8}{3}$

$R_A' = \frac{200}{EI}$

$R_B' = \frac{240}{EI} + \frac{120}{E} - \frac{200}{EI}$

$= \frac{160}{EI}$ units

Load bet^n A & C

$\frac{1}{2}\times\frac{120}{EI}\times 4 = \frac{240}{EI}$

& its COG (CG) is at

$\left(4+\frac{4}{3}\right) = 5.3$ m from B

Load bet^n C & B is

$\frac{1}{2}\times\frac{60}{EI}\times 4 = \frac{120}{EI}$

& its CG is at a distance

$\left(\frac{2}{3}\times 4\right) = \frac{8}{3}$ m from B

θ_A = Shearforce (S.F.) = A = $R_A' = \frac{200}{EI}$ clockwise

θ_B = S.F at B = $-R_B' = -\frac{160}{EI}$

$= \frac{160}{EI}$ radians, anticlockwise

θ_C = S.F. at C {from A to C}

$= \left(\frac{200}{EI}\right) - \left(\frac{240}{EI}\right) = \frac{-40}{EI}$ units

$= \frac{40}{EI}$ radians, anticlockwise

$R_A' + R_B' = \frac{240}{EI} + \frac{120}{EI}$

$R_B' = \frac{240}{EI} + \frac{120}{EI} - \frac{200}{EI}$

$R_B' = \frac{160}{EI}$

Δ_C = Moment at C

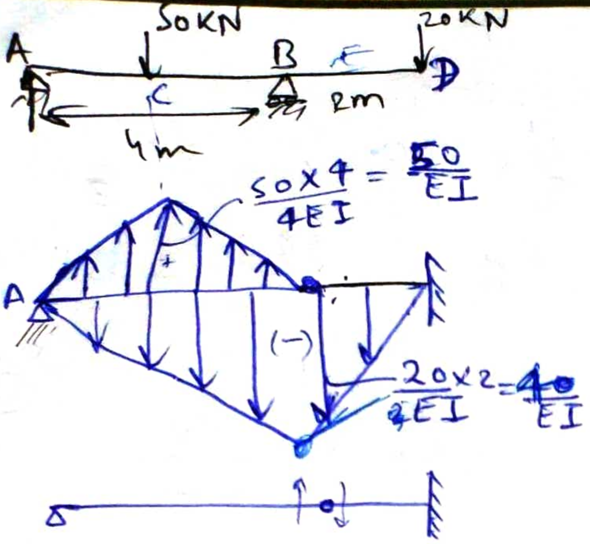
$= \left(\frac{200}{EI}\times 4\right) - \left(\frac{240}{EI}\times\frac{4}{3}\right)$

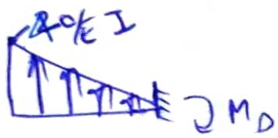
$= \frac{480}{EI}$, downward

Area of Δ × distance from the point acting x

deflec- moment

rotation - S.F θ





$$\Sigma M_B = 0$$

$$R_A \times 4 + \frac{1}{2} \times 4 \times \frac{40}{EI} \times \left(\frac{1}{3} \times 4\right) - \frac{1}{2} \times 4 \times \frac{50}{EI}(2) = 0$$

$$4R_A + \frac{160}{3EI} - \frac{200}{EI} = 0$$

$$R_A = \frac{200}{4EI} - \frac{40}{3EI}$$

$$\Rightarrow R_A = \frac{440}{12EI}$$

$$R_A + R_B + \frac{1}{2} \times 4 \times \frac{20}{EI} - \frac{1}{2} \times 4 \times \frac{50}{EI} =$$

$$R_B = +\frac{100}{EI} - \frac{440}{12EI} - \frac{40}{EI}$$

$$\Rightarrow R_B = \frac{280}{12EI}$$

$$R_D + \frac{1}{2} \times 2 \times \frac{20}{EI} - \frac{280}{EI} = 0$$

$$R_D = \frac{280}{EI} - \frac{20}{EI}$$

$$\Rightarrow \boxed{R_D = \frac{260}{EI}}$$

$$M_D = R_B \times 2 - \frac{1}{2} \times 2 \times \frac{20}{EI} \times \left(\frac{4}{3}\right)$$

$$M_D = \frac{360}{12EI} - \frac{80}{3EI}$$

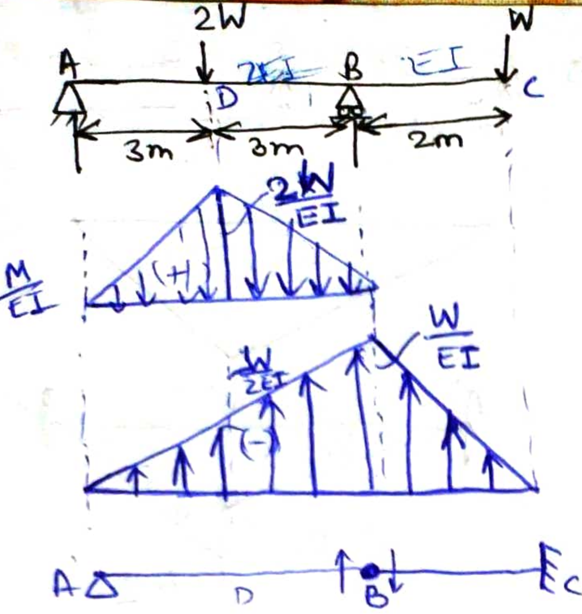
$$\Rightarrow M_D = \frac{10}{3EI}$$

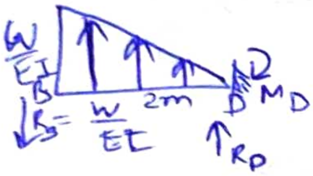
$$M_C = R_A \times 2 - \frac{1}{2} \times 2 \times \frac{50}{EI} \times \left(\frac{1}{3} \times 2\right) + \frac{1}{2} \times 2 \times \frac{10}{EI} \times \frac{1}{3} \times 2$$

$$= \frac{880}{12EI} - \frac{100}{3EI} + \frac{20}{3EI}$$

$$= \frac{880 - 400 + 80}{12EI}$$

$$\Rightarrow M_C = \frac{46.7}{EI}$$





$\Sigma M_B = 0$

$R_A \times 6 + \frac{1}{2} \times 6 \times \frac{W}{EI}\left(\frac{1}{3}\times 6\right) - \frac{1}{2}\times 6 \times \frac{2W}{EI} \times 3 = 0$

$6R_A - \frac{6W}{EI} - \frac{18W}{EI} = 0$

$R_A = \frac{24W}{6EI} = \frac{4W}{EI}$

$R_A + R_B + \frac{1}{2}\times 6 \times \frac{W}{EI} - \frac{1}{2}\times 6 \times \frac{2W}{EI} = 0$

$\frac{4W}{EI} + R_B + \frac{3W}{EI} - \frac{6W}{EI} = 0$

$R_B = \frac{W}{EI}$

$R_C + \frac{1}{2}\times 2 \times \frac{W}{EI} - \frac{W}{EI} = 0$

$R_C = 0$

$M_C = R_B \times 2 - \frac{1}{2}\times 2 \times \frac{W}{EI} \times \left(\frac{2}{3}\times 2\right)$

$M_C = \frac{2W}{EI} - \frac{4W}{3EI}$

$M_C = \frac{2W}{3EI}$

$M_D = R_A \times 3 - \frac{1}{2}\times 3 \times \frac{2W}{EI}\left(\frac{1}{3}\times 3\right) + \frac{1}{2}\times 3 \times \frac{W}{2EI} \times \left(\frac{1}{3}\times 3\right)$

$= \frac{12W}{EI} - \frac{3W}{EI} + \frac{3W}{4EI}$

$= \frac{39W}{4EI}$

Slope At

A $= R_A = \frac{4W}{EI}$, $R_C = 0$

Deflection

$M_C = \frac{2W}{3EI} = \frac{0.7W}{EI}$

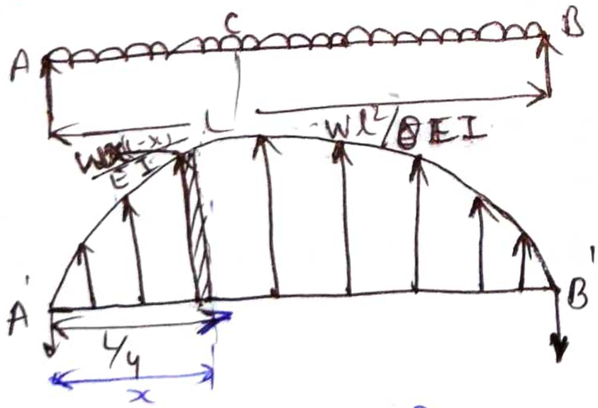
$M_D = \frac{39W}{4EI} = 9.75 \times \frac{W}{EI}$

$M_C = W \times 2$

$- 3 \times W -$

Imp.

Ques.



Total load $= \frac{2}{3} \times \frac{wl^2}{8EI} \times L$

$= \frac{wL^3}{12EI}$

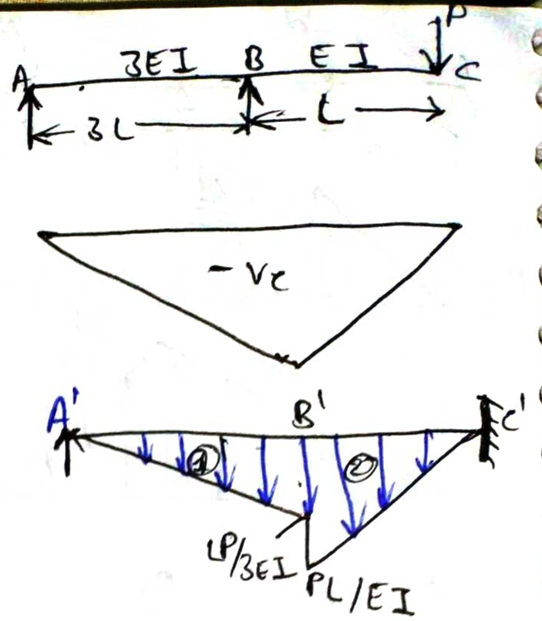
$R_A' = R_B' = \frac{1}{2}\,\frac{wL^3}{12EI} = \frac{wL^3}{24EI}$

$$\theta_c = F_c'$$
$$= -R_A' + \int_0^{L/4} \frac{w x(L-x)}{EI} dx$$
$$= -R_A' + \int_0^{L/4} \frac{wLx - wlx^2}{EI} dx$$
$$= -\frac{wL^3}{24EI} + \int_0^{L/4} \left(\frac{wLx - wLx^2}{EI}\right) dx$$
$$= -\frac{wL^3}{24EI} + \frac{1}{EI}\left[\frac{wLx^2}{2} - \frac{wLx^3}{3}\right]_0^{L/4}$$
$$= -\frac{wL^3}{24EI} + \frac{1}{EI}\left[\frac{wL^3}{8} - \frac{wL^4}{12}\right]$$
$$= -\frac{wL^3}{24EI} - \frac{3wL^3}{3\times 8EI} - \frac{wL^4}{12EI}$$
$$= -\frac{4wL^3}{24EI} - \frac{wL^4}{12EI}$$
$$= -\frac{wL^3}{6EI} - \frac{wL^4}{12EI}$$
$$= -\frac{wL^3}{64EI}$$

$$\theta_c = M_c'$$
$$= -R_A' \cdot \frac{L}{4} + \int_0^{L/4} \frac{wx(L-x)}{EI}\left(\frac{L}{4} - x\right)$$
$$= -\frac{wL^3}{24EI}\frac{L}{4} + \int_0^{L/4}\left(\frac{wxL - wx^2}{EI}\right)\left(\frac{L}{4} - x\right)dx$$
$$= -\frac{wL^4}{96EI} + \int_0^{L/4}\left(\frac{wxL^2}{4EI} - \frac{wx^2L}{4EI}\right) - \int_0^{L/4}\left(\frac{wx^2L}{EI} + \frac{wx^3}{EI}\right)dx$$
$$= 8.133\times 10^{-3}\frac{wL^4}{EI} \text{ downward}$$

Ans

Ques.



$$\Sigma M_{B'} = 0$$

$$R_A' \cdot 3L = \frac{1}{2}(PL/3EI) \times 3L \times \frac{1}{3} \times 3L$$

$$\boxed{R_A' = \frac{PL^2}{6EI}}$$

~~$\theta_c = F$~~

$\theta_c = F_c'$ at c

$$\boxed{\theta_c = -0.833 \times \frac{PL^3}{EI}}$$

$$\boxed{\Delta_c = -0.667\frac{PL^3}{6EI}}$$

$\theta_c = F'$ at c

$$= \frac{PL^2}{6EI} - \frac{1}{2} \times \frac{P}{3EI} \times 3L - \frac{1}{2} \times \frac{PL}{EI} \times L$$

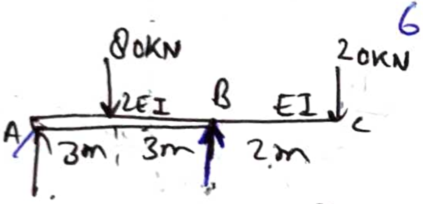
$$= -0.833 \frac{PL^3}{EI}$$ radian clockwise

$\delta_c = M_c'$

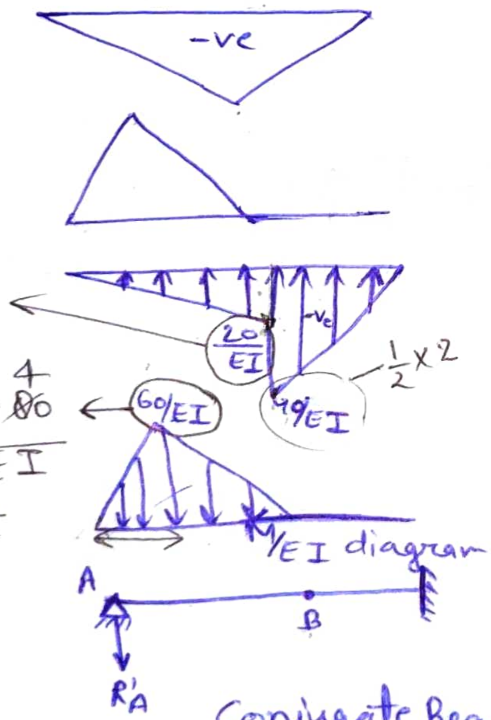
$$= R_A' \times 4L - \frac{1}{2} \times 3L \times \frac{PL}{3EI} \times 2L \rightarrow c.g. - \frac{1}{2} \times \frac{PL}{EI} \times L \times \frac{2L}{3}$$

$$= -0.667 \frac{PL^3}{6EI}$$ (downward)

Ans.



$\theta_A, \delta_c = ?$



$\frac{1}{2} \times 2 \times 20 = \frac{20}{EI}$

$\frac{1}{2} \times 2$

$\frac{1}{2} \times 3 \times 80 = \frac{}{2EI} = \frac{60}{EI}$

Conjugate Beam

$\Sigma M_B = 0$

$$R_A' \times 6 = \left[\frac{1}{2} \times \frac{20}{EI} \times 6 \times \frac{6}{3} + \frac{1}{2} \times \frac{60}{EI} \times 6 \times 3\right] \frac{1}{EI}$$

$$R_A' = \frac{110}{EI}$$

$$\theta_A = \frac{110}{EI}, \text{ clockwise}$$

∴ Deflection at C

= Moment at C in conjugate beam

$$= \frac{110}{EI} \times 8 - \frac{1}{2} \times \frac{60}{EI} \times 6 \times 5 + \frac{1}{2} \times 6 \times \frac{20}{EI} \times 4 + \frac{1}{2} \times \frac{40}{EI} \times 2 \times 1$$

$2l/3 = 2 \times \frac{6}{3} =$

$$= \frac{260}{EI}, \text{ downward.}$$

$M_A = 0$

$6R_B - 80 \times 3 - 20 \times 8 = 0$

$6R_B = 400$

$R_B = 66.7$ KN

$R_A = 33.3$ KN

$\frac{1}{2} \times 20 \times 2$

$\frac{1}{2} \times 5 \times 10$

$\frac{50}{2}$

$80 \times \frac{6}{4}$

$\frac{120}{2EI} = \frac{60}{EI}$

$20 \times 2 = 40/EI$

$= \frac{40}{2}/EI$

Energy-Methods:

$$U_e = U_i$$

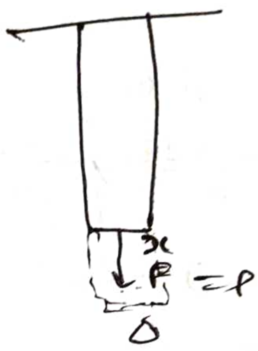
U_e = External work done by force

U_i = internal strain energy stores

$$U_e = \int_0^{x} F\,dx \qquad \left\{F = \frac{P}{\Delta}x\right\}$$

$$= \int_0^{\Delta} \frac{P}{\Delta} x\,dx$$

$$= \frac{1}{2} P \Delta$$



Work done by moment

$$U_e = \int_0^{\theta} M\,d\theta$$

$$U_e = \frac{1}{2} M \theta$$

Strain energy due to axial loading

$$\Delta = \frac{NL}{AE}$$

$$U_i = \frac{1}{2} P \cdot \Delta$$

$$= \frac{1}{2} N \cdot \delta$$

$$= \frac{1}{2} N \cdot \frac{NL}{AE}$$

$$\boxed{U_i = \frac{N^2 L}{2AE}}$$

$$\frac{d\theta}{dx} = \frac{M}{EI}$$

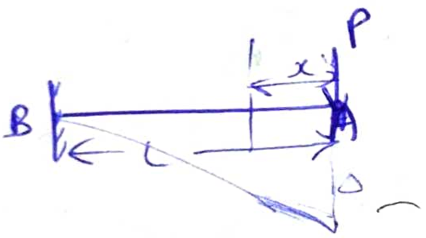
$$d\theta = \frac{M}{EI}\,dx$$

$$\theta = \int_0^L \frac{M}{EI}\,dx$$

$$U_i = \frac{1}{2} M \theta$$

$$= \int_0^L \frac{M^2\,dx}{2EI}$$

$$U_i = \frac{1}{2}\int_0^L \frac{M_x^2}{EI}\,dx$$



$$M_x = -Px \;:\; 0 \le x \le L$$

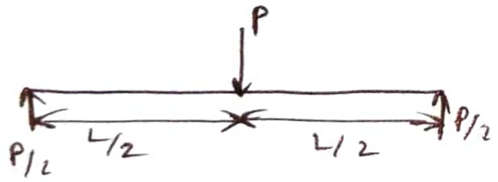
$$= \int_0^L \frac{(-Px)^2}{2EI}\,dx$$

$$= \frac{P^2L^3}{6EI}$$

$$U_e = U_i$$

$$\frac{1}{2} P \cdot \Delta = \frac{P^2L^3}{6EI}$$

$$\boxed{\Delta = \frac{PL^3}{3EI}}$$



$$M_x = +\frac{P}{2}x \quad 0 \le x \le \frac{L}{2}$$

$$= +\frac{P}{2}x \quad 0 \le x \le \frac{L}{2}$$

$$U_i = \int \frac{M_x^2\,dx}{2EI}$$

$$U_i = \int_0^{L/2} \frac{(P/2\,x)^2}{2EI}\,dx + \int_0^{L/2} \frac{(P/2\,x)^2}{2EI}\,dx$$

$$= 2\int_0^{L/2} \frac{P^2x^2\,dx}{8EI} = \int_0^{L/2} \frac{P^2x^2}{4EI}\,dx$$

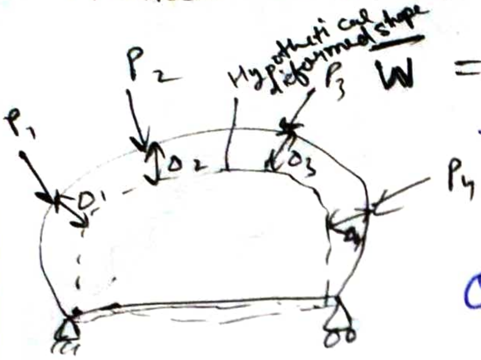
$$= \frac{P^2}{4EI}\left[\frac{x^3}{3}\right]_0^{L/2} = \frac{P^2L^3}{96EI}$$

$$U_i = U_e$$

$$\frac{1}{2} P\Delta = \frac{P^2L^3}{96EI}$$

$$\boxed{\Delta = \frac{PL^3}{48EI}}$$

Virtual Work →



W = Virtual work done

= workdone by real forces through virtual displacement (defination)

Or = work done by virtual force through real displacement

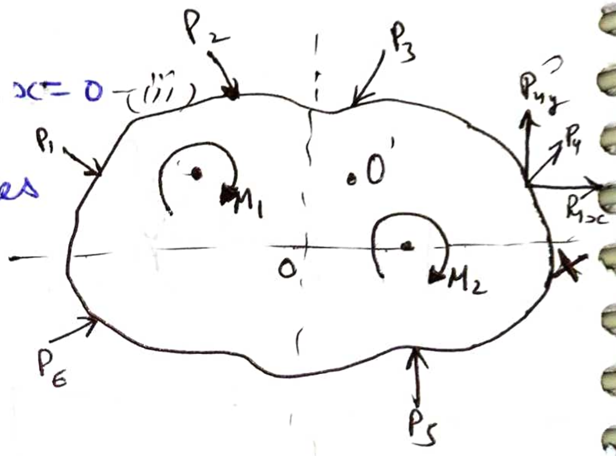
Principle of Virtual Work :

Rigid body displacement

$\Sigma P_x = 0$ --- (i)

$\Sigma P_y = 0$ --- (ii)

$\Sigma M + \Sigma P_x \cdot \delta y + \delta P_y \cdot \delta x = 0$ --- (iii)



Virtual workdone by P forces

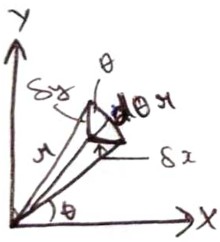
$W_e = \Sigma P_x \delta y + \Sigma P_y \delta x$

$= \delta y \, \Sigma P_x + \delta x \, \Sigma P_y$

from eqⁿ (i) & (ii) we get

$= 0 \quad \{P_x = P_y\}$

Virtual work done = 0



$\delta x = r d\theta \cdot \sin\theta$

$= y \, d\theta$

$\delta y = r d\theta \cos\theta$

$= x \cdot d\theta$

∴ VW done by real forces due to virtual diplacements

$= \Sigma M \cdot d\theta + \Sigma P_x \, \delta x + \Sigma P_y \, \delta y$

$= d\theta \Sigma M + \Sigma P_x \cdot y \, d\theta + \Sigma P_y \, x \, d\theta$

$= d\theta \left[\Sigma M + \Sigma P_x y + \Sigma P_y \cdot x \right] = 0$ (from iii)

$dw = dw_r + dw_i$

$\int dw = \int dw_r + \int dw_i$

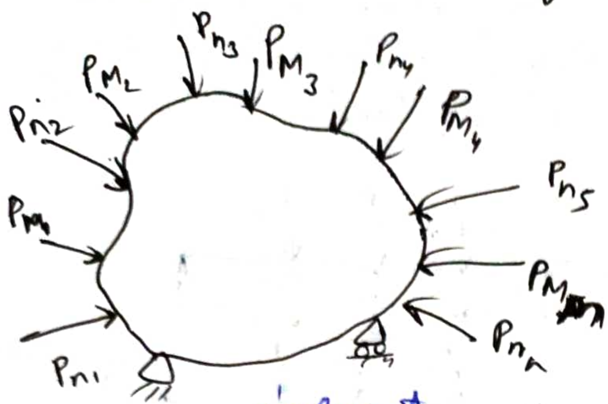
$W = W_r + W_i$

$= 0$

$$\boxed{W = W_i}$$

Betti's theorem of Principle Reciprocal work done

Maxwell's theorem of reciprocal deflection



$$W = \sum_{1}^{m} \frac{1}{2} P_{mm} \Delta_{mm} + \sum P_{mm} \Delta_{mn} + \sum \frac{1}{2} P_{nn} \Delta_{nn}$$

$$W_e = \sum \frac{1}{2} P_{nn} \cdot \Delta_{nn} + \sum P_{nn} \Delta_{nm} + \sum \frac{1}{2} P_{mm} \Delta_{mm}$$

Δ_{mm} = ~~deflection~~ displacement at m due to m system of forces

Δ_{nn} = Defl at n due to n system of forces

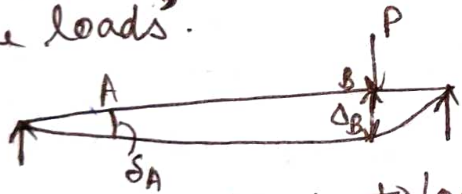
Δ_{mn} = Displacement of m due to n system of forces

$$\sum P_1 \delta_{21} = \sum P_2 \cdot \delta_{12} \text{ — Betti's}$$

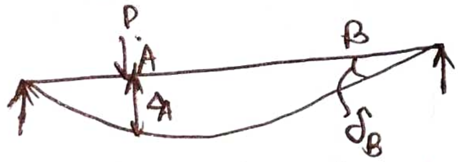
$P_1 = P_2$

$\delta_{21} = \delta_{12}$ Maxwell

"Displacement at point A due to the load at point B is same as displacement of point B due to the same load acting at point A, the displacement being measured in the directions of the loads".



Deflection at A due to load at P



Deflection at B due to load at A

$$W = \frac{1}{2} P \Delta_B \quad \text{— (i)}$$

$$W = \frac{1}{2} P \Delta_A \quad \text{— (ii)}$$

$$W_e = \frac{1}{2} P \Delta_B + P \delta_B + \frac{1}{2} P \Delta_A \quad \text{— (iii)}$$

If load P is applied first at A & then at B,

$$W = \frac{1}{2} P \Delta_A + P \delta_A + \frac{1}{2} P \Delta_B \quad \text{— (iv)}$$

from (iii) & (iv) eqn we get

$$\frac{1}{2} P \Delta_B + P \delta_B + \frac{1}{2} P \Delta_A = \frac{1}{2} P \Delta_A + P \delta_A + \frac{1}{2} P \Delta_B$$

$$\boxed{\delta_B = \delta_A} \quad \#$$

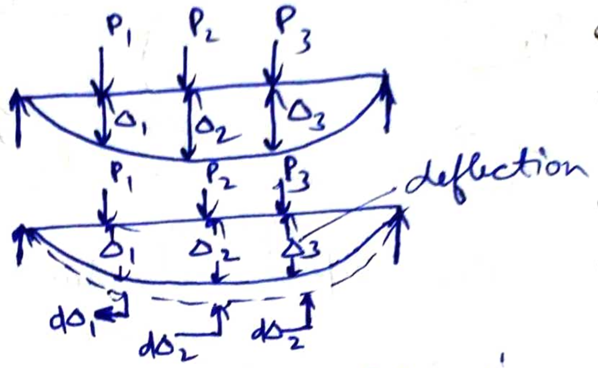
Castigliano's Theorems → In a linearly elastic structure, partial derivation of the strain energy with respect to a load is equal to the deflection of the point where the load is acting, the deflection being measured in the dir^n of the load.

$\frac{\partial U}{\partial \Delta} = \partial P$

$\frac{\partial U}{\partial P_i} = \Delta_i$

$U_e = \int P \, dx$

$\boxed{U_e = U_i}$



$U_i = f(P_1, P_2, \cdots P_n)$

P_1 load is increased by dP

U_i is inc. by dU_i

$U_i + dU_i = U_i + \frac{\partial U_i}{\partial P_i} dP_1$ --- (i)

$U_i + dU_i = U_i + dP_i \Delta_i$ --- (ii)

$U_i + \frac{\partial U}{\partial P_i} dP_i = U_i + dP_i \Delta_i$

$\boxed{\frac{\partial U_i}{\partial \Delta_i} = P_i}$

$U_i + \frac{\partial U_i}{\partial \Delta_i} d\Delta_i$

$U = \frac{1}{2}P_1\Delta_1 + \frac{1}{2}P_2\Delta_2 + \frac{1}{2}P_3\Delta_3$ --- (i)

only in dP_i $dU = \frac{1}{2} dP_1 d\Delta_1 + \frac{1}{2}P_2 d\Delta_2 + \frac{1}{2} dP_3 d\Delta_3$ --- (ii)

$U + dU = \frac{1}{2}P_1\Delta_1 + \frac{1}{2}P_2\Delta_2 + \frac{1}{2}P_3\Delta_3 + \frac{1}{2}dP_1 d\Delta_1 + P_1 d\Delta_1 + \frac{1}{2}P_2 d\Delta_2 + \frac{1}{2}P_3 d\Delta_3$ --- (iii)

If $P_1 = (P_1 + dP_1)$

$U + dU = \frac{1}{2}(P_1 + dP_1)(\Delta_1 + d\Delta_1) + \frac{1}{2}(P_2 + dP_2)(\Delta_2 + d\Delta_2) + \frac{1}{2}(P_3 + dP_3)(\Delta_3 + d\Delta_3)$ --- (iv)

final strain in both cases is same

(iii) = (iv) $\frac{1}{2}P_1 d\Delta_1 + \frac{1}{2}P_2 d\Delta_2 + \frac{1}{2}P_3 \Delta_3 = \frac{1}{2}dP_1\Delta_1$

$\frac{1}{2}(P_1 d\Delta_1 + P_2 d\Delta_2 + P_3 d\Delta_3) = \frac{1}{2}(dU - \frac{1}{2}dP_1 d\Delta_1)$

$\boxed{\frac{\partial U_i}{\partial \Delta_i} = P_i}$

$\boxed{\frac{\partial U}{\partial M_j} = \theta_j}$

P_i, M_j - loads

Δ_i, θ_j - deflections

Ans

$U_i = \int \frac{M_x^2 dx}{2EI}$

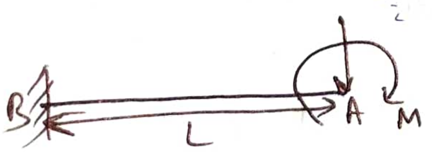
$\frac{\partial U_i}{\partial P} = \int \frac{M_x \frac{\partial M}{\partial P} dx}{EI}$

$M_x = -Px$

$\frac{\partial M_x}{\partial P} = -x$

$\Delta_A = \int_0^L \frac{Px(-x)}{EI} dx$

$= + \frac{Px^2}{3EI}$



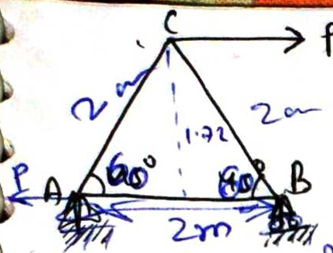
$\frac{\partial U}{\partial M} = \frac{M_x \frac{\partial M_x}{\partial M} dx}{EI}$

$M_x = -Px - M$

$\frac{\partial M_x}{\partial M} = -1$

$\theta_A = \frac{\partial U}{\partial M} = \int (Px + M)$

$= \frac{Px^2}{2EI}$

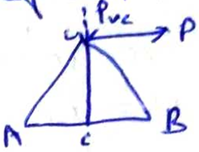


P_i = Force in Member

$$U = \frac{\Sigma P_i^2 L}{2AE}$$

$$\frac{\partial U}{\partial P} = \Sigma P_i \frac{\frac{\partial P_i}{\partial P} L}{AE}$$

{ when verticle dir^n m deflection the dummy load $\frac{\partial P_i}{\partial P_{vc}}$ }



$M_B = 0$

$R_A \times 2 + P_x (1.72) = 0$

$R_A = 0.86 P_x$

$P_{AC} = +0.98$

$P_{AB} = -0.456P$

$P_{BC} = +0.993$

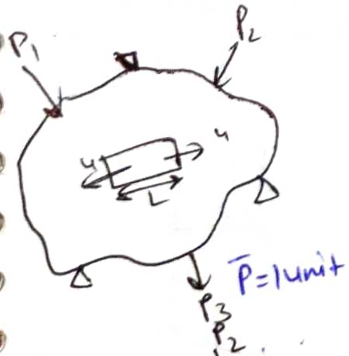
	Length	P_i	$\Delta P_i / \Delta P$	$P_i \times \frac{\partial P}{\partial P} \times L$
AB	2m	$-0.496P$	-0.496	$0.492P$
AC	2m	$0.98P$	$+0.98$	$1.9208P$
BC	2m	$0.993P$	$+0.93$	$1.9720P$
				$\Sigma\, 4.38P$

$$\Delta_{CH} = \frac{L}{AE} \times 4.38P$$

$$\Delta_{CH} = \frac{2 \times 4.38P}{AL}$$

Unit-Load Method →

$$\Sigma P\Delta = \Sigma U \cdot dL$$



P will generate $\bar{u}$

Virtual work done by unit load

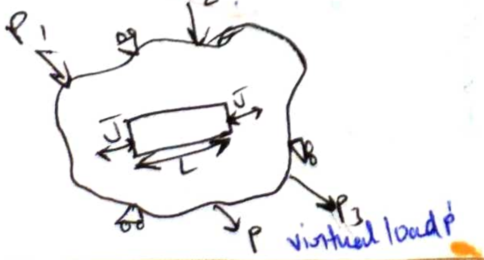
Δ_A

Internal work done by unit load

$1 \cdot \Delta_A$

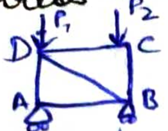
Internal work done = $\Sigma \bar{U} \cdot dL$

$$\boxed{1 \cdot \Delta_A = \Sigma\, \bar{u}\, dL}$$

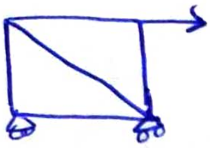


Δ_A = real displacement
$\bar{U}$ = internal force by unit load
dL = internal stress due to real load

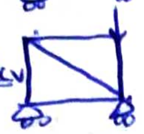
Truss:



For Δ_{CH}



For Δ_{CV}

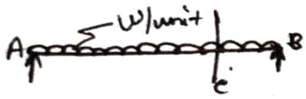


P_i = member force due to unit load = $\bar{P}_j$

dL = real displacement = $\frac{P_i L_i}{A_i E_i}$

$$1 \cdot \Delta = \sum \frac{\bar{P}_i P_i l_i}{A_i E_i}$$

Beam



$$d\theta = \frac{M}{EI} dx$$

External work done $= 1 \cdot \Delta_C$

Internal virtual work done $= \int M d\theta$

m = virtual moment (moment due to unit load)

M = moment due to real load

$$1 \cdot \Delta_C = \int M d\theta$$

$$\boxed{1 \cdot \Delta_C = \int \frac{mM\,dx}{EI}}$$

Ques

$M = -Px$ $\quad m = -1x$

$0 \le x \le L$ $\quad 0 \le x \le L$

$$\Delta_A = \int \frac{mM\,dx}{EI}$$

$$= \int_0^L \frac{Px^2\,dx}{EI}$$

$$= \frac{PL^3}{3EI}$$

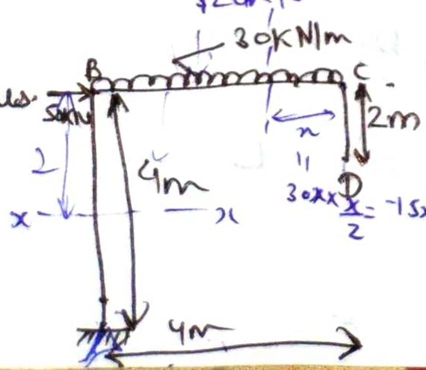
$m = -1$

$$\theta_A = \int \frac{mM\,dx}{EI}$$

$$= \int_0^L \frac{(-1)(-Px)\,dx}{EI}$$

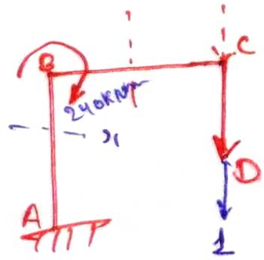
$$\theta_A = \frac{PL^2}{2EI}$$

Ques.

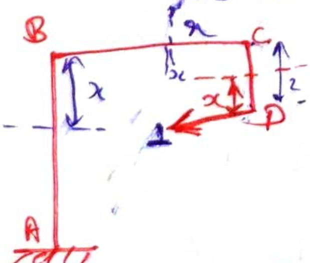


$EI = 12 \times 10^{13}\ N\text{-}mm^2$

Horizontal & Vertical displacement at D



Verticle



Horizontal

Portion	AB	BC	CD
Origin	B	C	D
Limit	0—4	0—4	0—2
M	$-240-50x$	$-15x^2$	0
M_{DH}	moment = AB = 240 $-(2-x)$	-2	$+1\cdot x$
m_{DV}	-4	$-1\cdot x$	0

$\left. \frac{1}{(2-x)} \right]^2$

$U = \int \frac{M\,m\,dx}{EI}$

$$\Delta_{DV} = \int_0^2 0 + \int_0^4 \frac{-15x^2(-x)dx}{EI} + \int_0^4 \frac{-(240+50x)x\,dx}{EI}$$

$~~240 = 30 \times 2~~$

$30 \times 4 \times 2 = 240$

$$= \int_0^4 \frac{15x^3\,dx}{EI} + \int_0^4 \frac{(960+200x)dx}{EI}$$

$$= \frac{15 \times 256}{4EI} + \frac{960 \times 4 + \frac{200 \times 16}{2}}{EI}$$

$$= \frac{6400}{EI}$$

$$\Delta_{DV} = \frac{6400}{12 \times 10^4} = 0.533\,m = 53.33\,mm.$$

$~~= 533.33 \times 10^{-13}~~$

$~~= 53.~~$

$$\Delta_H = \int_0^2 0 + \int_0^4 \frac{-15x^2 \times -2\,dx}{EI} + \int_0^4 \frac{(-240-50x)(-2)dx}{EI}$$

$$= \int_0^4 \frac{30x^2\,dx}{EI} + \int_0^4 \frac{480+100x\,dx}{EI}$$

$$= \frac{640}{EI} + \frac{2720}{EI}$$

$$= 373.33 / 12 \times 10^4$$

$(30 \times 4 \times 2$

$$= 0.0031\,m$$

$$= 3.1\,mm$$

$= 0.020$

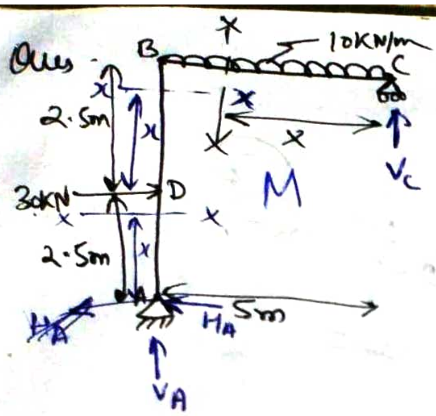
$= 0.03\,m$

$= 3.0\,m$

120×2

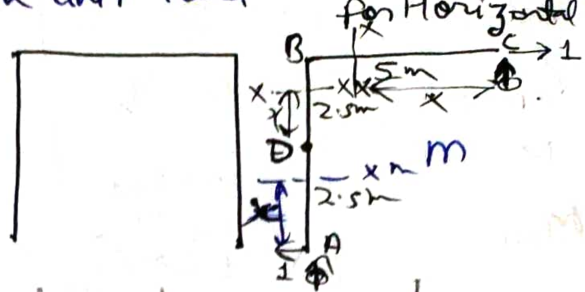
240

Ans



Remove all loads & redraw with unit load

for Horizontal



find support rxn

$\Sigma M_A = 0$

$-V_C \times 5 + 10 \times 5 \times 2.5 + 30 \times 2.5 = 0$

$-V_C \times 5 = -200$

$\boxed{V_C = 40\,KN}$

$\Sigma F_y = 0$ ↑+ ↓-

$40 + V_A - 50$

$\boxed{V_A = 10KN}$

$\Sigma F_x = 0$ →+ ←-

$-H_A + 30 = 0$

$\boxed{H_A = 30KN}$

~~Remove~~

Region	origin	limit	M	m	EI
AD	A	0-2.5	$30 \times x$	$1 \times x$	EI
DB	D	0-2.5	$30(2.5+x) - 30x$	$1(2.5+x)$	EI
BC	C	0-5	$40x - \frac{10x^2}{2}$	1×0	EI

$$\Delta_{Hc} = \frac{1}{EI}\left[\left(\int_0^{2.5} 30x \times x\,dx\right) + \int_0^{2.5}\left(30(2.5+x) - 30x\right) \times (2.5+x)\,dx + \int_0^5 (40x - 5x^2)\,dx \times 0\right]$$

$$\Delta_{HC} = \frac{1}{EI}\left[\frac{30 \times (2.5)^3}{3} + \left(75 \times 2.5 + (2.5)^2 \times 15 - 15 \times (2.5)^2 \times 2.5 \times 2.5 + \frac{(2.5)^2}{2}\right) + \left(20 \times (2.5)^2 \times 0 - \frac{5 \times (2.5)^3}{3} \times 0\right)\right] \times 0$$

$EI = 8 \times 10^{-11}\,Nm^2$

$I = 400 \times 10^{-6}\,m^4$

$E = 200\frac{KN}{mm^2} = \frac{200 \times 1000\,N}{10^{-6}\,m^2}$

$= 8 \times 10^{-5}\,m^2$

$$= \frac{1}{EI}\left[156.25 + 312.5 - 26.09\right]$$

1757.01

$= \frac{442.66}{EI} + \frac{1}{EI} \times 1919.06$

$= \frac{442.66}{}$

$= 5.$

$\frac{1919.06}{8 \times 10^{-11}}$

239.9×10^{5}

Clock = +

Anti = -

↑ = +

↓ = -

← -

→ +

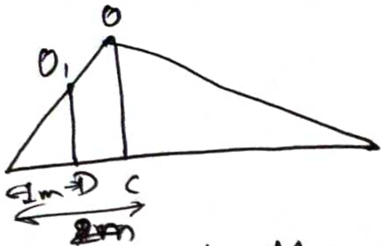
23/09/22

Unit - 4

Influence line diagram & rolling load

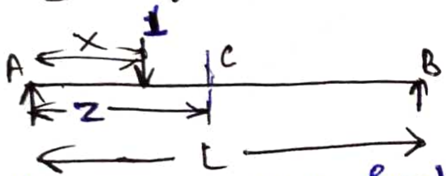
(a) Dead Load → No change in position

(b) Live Load → change in position during the life of the beams called live loads.



ILD for M_c

Ques.



ILD for R_A →

$R_A \times L = 1(L-x)$ $\{M_B = 0\}$

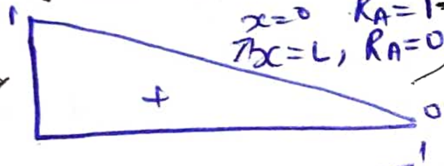
Sabunit load ko A pr rakhe

$$\boxed{R_A = \frac{L-x}{L}}$$

$x=0$, $R_A = 1 =$ At R_A

$x = L$, $R_A = 0 =$ At R_B

IDL for R_A →

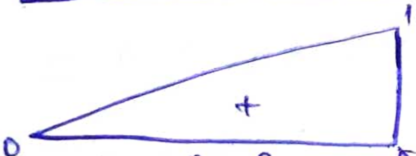


ILD for R_B

$R_B \times L = 1 \cdot x$

$$\boxed{R_B = \frac{x}{L}}$$

IDL for R_B →



At A ⇒ $x = 0$, $R_B = 0$

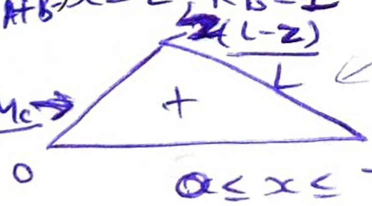
At B ⇒ $x = L$, $R_B = 1$

for ILD M_c

$M_c = R_B(L-z)$

$= \frac{x}{L}(L-z)$

ILD for M_c →



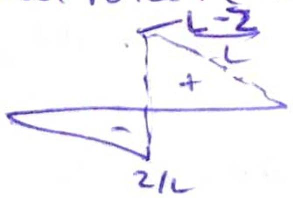
$0 \le x \le z$

$x = 0$, $M_c = 0$

$x = z$, $M_c = \frac{z(L-z)}{L}$

$x \ge z$

ILD for Shear force F_c →



$F_c = -R_B$

$x = -\frac{x}{L}$

$0 \le x \le z$

$x = 0$, $F_c = 0$

$x = z$, $F_c = -\frac{z}{L}$

$x > z$

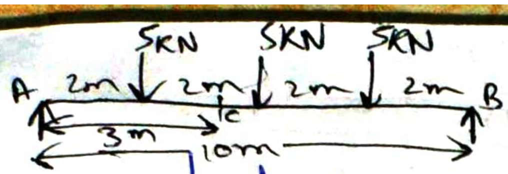
$F_c = R_B = \frac{L-x}{L}$

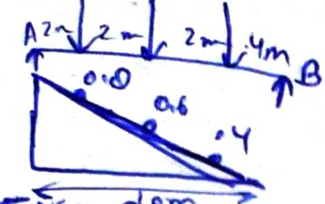
$SF_c = R_A - 1$

$= \left(\frac{l-x}{l}\right) - 1 = -\frac{x}{l}$

$SF_c = \left(\frac{l-z}{l}\right)$

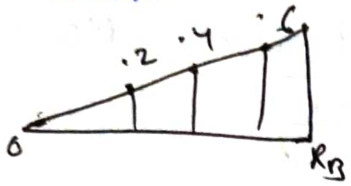
ILD are drawn for various stress resultant like rxn, shear force, Bending moment, torsion at specified points. ILD for a stress resultant is the one in which ordinate represent the value of the stress resultant for the position of unit load at the corresponding abscissa.





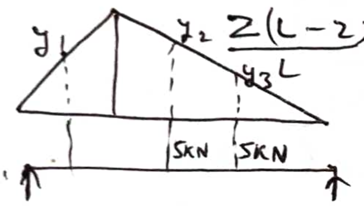
$R_A = 5 \times 0.8 + 5 \times 0.6 + 5 \times .4$

$= 9KN$



$R_B = 5 \times 0.2 + 5 \times 0.4 + 5 \times 0.1$

$= 6KN$



$\frac{Z(L-Z)}{L} = \frac{3(10-3)}{10} = 2.1$

$y_1 = \frac{2.1}{3} \times 2 = 1.4m$

$y_2 = \frac{2.1}{7} \times 6 = 1.8m$

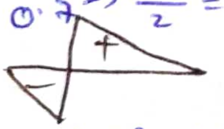
$y_3 = \frac{2.1}{7} \times 4 = 1.2m$

$M_C = 5y_1 + 5y_2 + y_3$

$= 5 \times 1.4 + 5 \times 1.8 + 5 \times 1.2$

$= 7 + 9 + 6$

$= 22 KN\text{-}m$



$0.7 \to \frac{L-2}{2} = \frac{10-3}{10}$

$2/3 = \frac{3}{10} = 0.3$

$y_1 = \frac{0.3}{3} \times 2 = 0.2m$

$y_2 = 0.6$

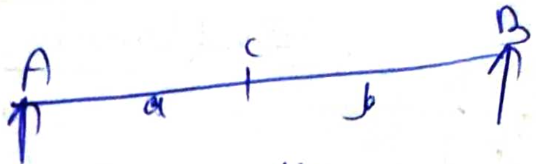
$y_3 = 0.4$

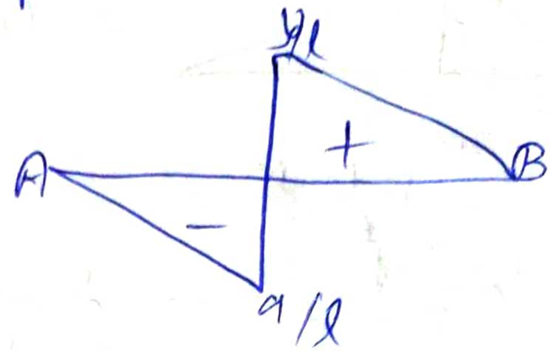
$F_C = 5y_1 + 5 \times 0.6 + 5 \times 0.4$

$= -5 \times 0.2 + 5 \times 0.6 + 5 \times 0.4$

$= -1 + 3 + 2$

$= 4$





4. ILD for M_D

When unit load at AD

$M_D = R_A(Z) - 1(Z - x)$

$= \left(\frac{L-x}{L}\right)Z - (Z - x)$

$0 < x < L$

at $x = 0$, $M_D = \left(\frac{L-0}{L}\right)Z - (Z-0) = 0$

$x = 2$, $M_D = \left(\frac{L-Z}{L}\right)Z$

when unit loat is in DB

$M_D = R_A \times Z$

$= \left(\frac{L-x}{L}\right)Z \quad \because Z < x < L$

at $x = 2$, $M_D = \left(\frac{L-Z}{L}\right)Z$

$x = L$, $M_D = 0$

When UL is in B, C

$M_D = R_A Z = -\left(\frac{(a-x)}{L}\right)Z$

$\because 0 < x < a$

at $x = 0$, $M_D = -\frac{a}{L} Z$

at $x = a$, $M_D = 00$

5. ILD for SF_E

when unit load is in AB

$SF_E = 0$

when UL is in BE

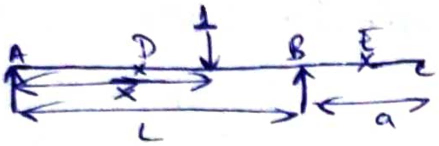
$SF_E = 0$

when UL is in EC

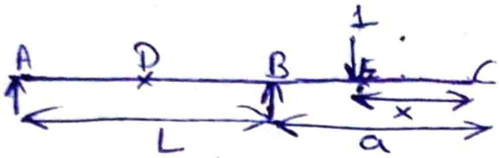
$SF_E = 1$

Ans.

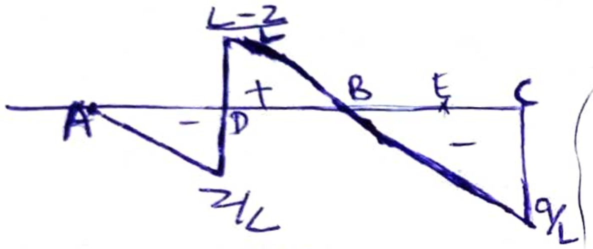
1.



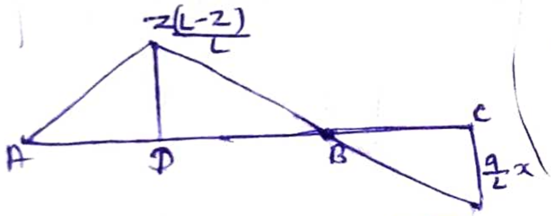
2.



3.



4.



5.

6.

1. ILD for Rxn at A

when unit load is in AB

(i) $M_B = 0$, $R_A \times L = L - x$

$R_A = \frac{L-x}{L}$, linear variation

$0 < x < L$, when $x = 0$, $R_A = \frac{L-0}{L} = 1$

$x = L \Rightarrow R_A = 0$

2. (i) ILD for R_B when unit load AB

(i) $M_A = 0$, $R_B \times L = 1 \times x$ (linear variation)

$$\boxed{R_B = \frac{x}{L}}$$

$0 < x < L$, at $x = 0$, $R_B = 0$

$x = L$, $R_B = 1$

(ii) when unit load at in BC

$M_A = 0 \Rightarrow R_B L = 1 \times [L + (a - x)]$

$R_B = \frac{L + (a-x)}{L}$

$0 < x < a$

at $x = 0$, $R_B = \frac{L+a}{L}$

at $x = a$, $R_B = 1$

(ii) when unit load is in BC

$M_B = 0$, $R_A \times L = -1(a - x)$

$R_A = \frac{x-a}{L}$

$0 < x < a$, at $x = 0$, $R_A = -\frac{a}{L}$

at $x = a$, $R_A = 0$

6. ILD for M_E

when 1 load is in AB & BE

$M_E = 0$

when 1 load is EC

$M_E = -1(z_1 - x)$

$0 < x < z_1$

at $x = 0$, $M_E = -z_1$

at $x = z$, $M_E = 0$

3. ILD for SF_D

when Load is in portion AD

$SF_D = R_A = R_A - 1$ $\quad \because R_A = \frac{L-x}{L}$

$= \frac{L-x}{L} - 1$ $\quad 0 < x < z$

at $x = 0$, $SF_D = \frac{L-0}{L} - 1 = 0$

$x = z$, $SF_D = \frac{L-z}{L} - 1 = -\frac{z}{L}$

when unit load is in DB

$SF_D = R_A = \frac{L-x}{L}$, Linear variation

$\because z < x < L$

at $x = z$, $SF_D = \frac{L-z}{L}$

at $x < L$, $SF_D = 0$

when load at BC

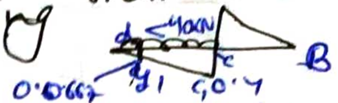
$SF_D = R_B = -\frac{x}{L}$ $\quad \because 0 < x < a$

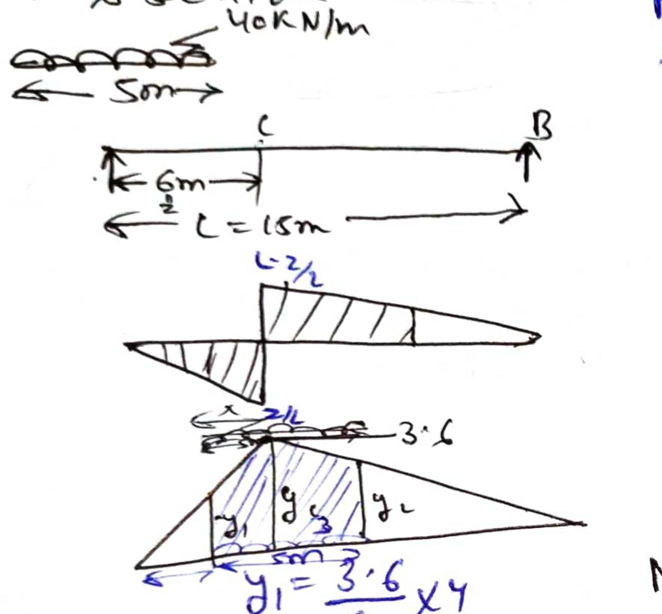
at $x = 0$, $SF_D = 0$

at $x = a$, $SF_D = -\frac{a}{L}$

Ans-

Examp
A simply supported beam has a span of 15m UDL of 40KN/m & 5m long crosses the girder from left to right. Draw the ILD for shear force and B.M at section 6m from left end. Use the diagrams to Calculate the max. shear force & Bending moment at this section





$y_1 = \frac{3.6}{6} \times 4$

$= 2.4$

$\boxed{y_2 = y_1}$

$\frac{x}{5} = \frac{z}{L} = \frac{6}{15} = 0.4$

$x = 0.4 \times 5$

$= 2.0$

$M_c = w \times \left[\frac{1}{2}(y_1+y_2) \times 2 + \frac{1}{2}(y_2+y_3) \times 3\right]$

$= 40 \times \frac{1}{2}(2.4+2.4) \times 2 + \frac{1}{2}(2.4+2.0) \times 3]$

$= 600 KN\text{-}m$

Max S.F

max (+ve) S F = Head of UDL touches the section

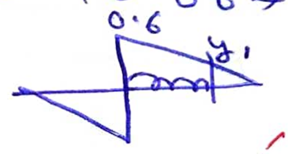
$= w \times$ Area of cc_1y_1

$= w \times \frac{1}{2}(y_1+c_1) \times 5$

$= 40 \times \frac{1}{2}(0.0667+0.4) \times 5$

Negative F_c = 46.667

Max +ve F =

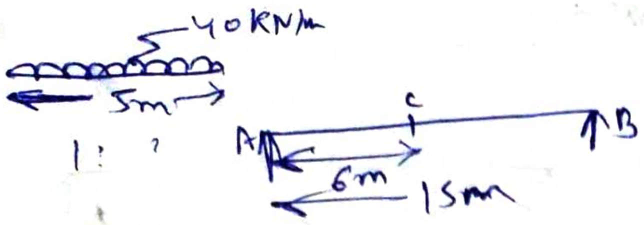


$y_1 = \frac{0.6}{9} \times 4 = \frac{4}{9} \times 0.6$

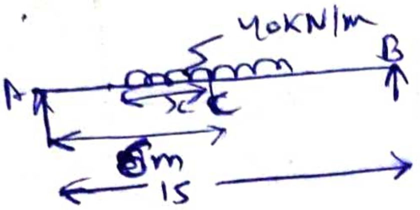
$= 0.26$

$= 40 \times \left(\frac{0.6+0.267}{2}\right) \times 7$

$= 86.67 KW$



For max^n bending moment at the section it takes place the section x divides the load in the same ratio as it divides the beam

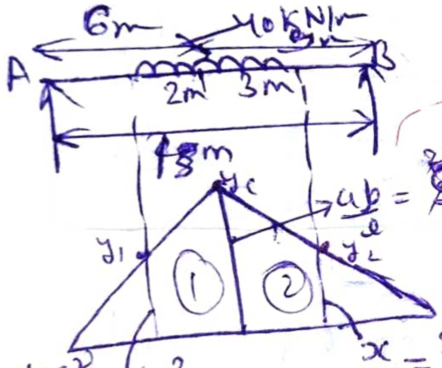


Only for UDL Trapezium used

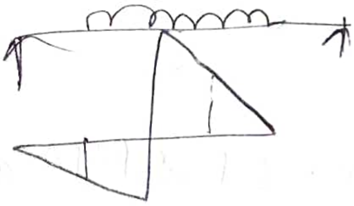
$$\frac{x}{6} = \frac{\cancel{5}}{\cancel{15}\,3}$$

$$\boxed{x = 2m}$$

$y_c = \frac{ab}{l}$



$y_c = \frac{ab}{l} = \frac{\cancel{6} \times \cancel{9}}{\cancel{15}\,3} \quad \frac{9 \times \cancel{6}\,2}{\cancel{15}\,5} = \frac{18}{5} = 3.6m$



$\frac{x}{l} = \frac{\cancel{4}\,(6-2)}{\cancel{6}} \times \frac{\cancel{18}\,3}{5} = \frac{12}{5}$

$= 2.4m$

$\frac{x}{l} = \frac{\cancel{26}\,(9-3)}{\cancel{20}} \times \frac{\cancel{18}\,6}{5} = \frac{12}{5}$

$= 2.4m$

— Same Absolute max shear

$\frac{1}{2}$(sum of parallel sides) × height

$M_c = \frac{1}{\cancel{2}}(2.4 + 3.6) \times \cancel{2} \times 40 + \frac{1}{2}(2.4 + 3.6) \times 3 \times 40$

given distance

$= 240 + 360$

$= 600KN\,m$

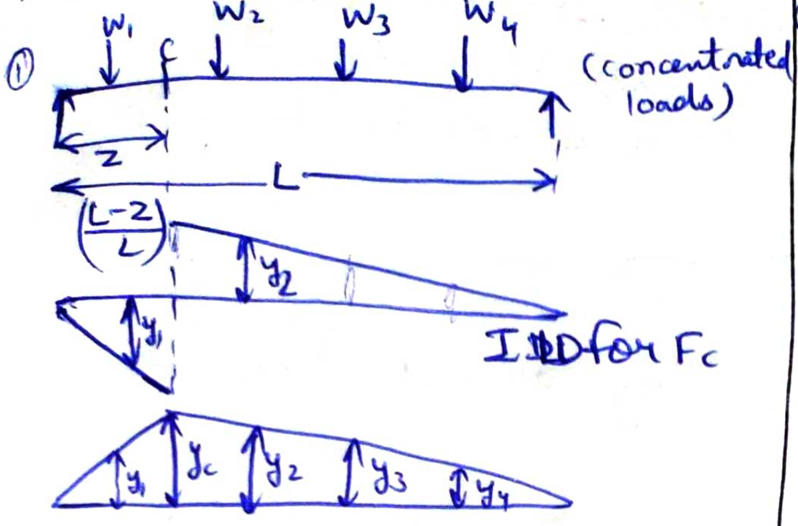
$y_1 \Rightarrow x = \frac{l}{L} \times 6 \rightarrow A$ to y_c

$y_2 = x = \frac{l}{L} \times 9 \rightarrow y_c$ to B

* Use of Influence line Diagrams :-

(a) To determine S·F & B·M at a Section

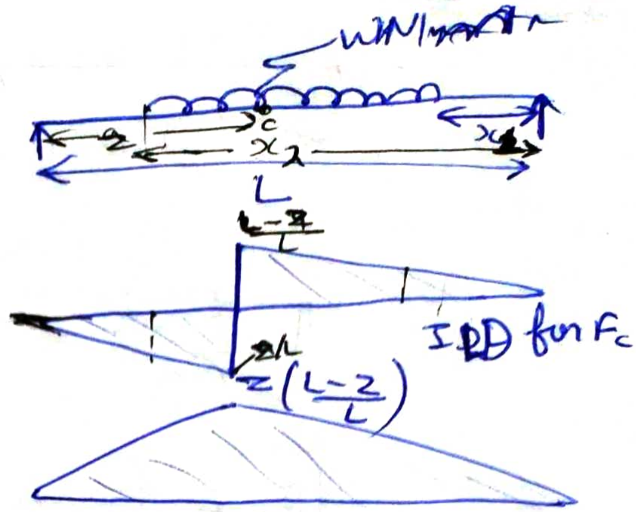
① (concentrated loads)



$\boxed{F_c = \pm W_1 y_1 \pm W_2 y_2 \pm W y_3 \pm W y_4}$

$\boxed{F_c = -W_1 y_1 + W y_2 + W y_3 + W y_4}$

②

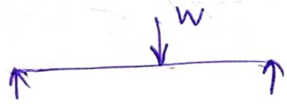


$df_c = \omega \cdot dx \cdot y$

$F_c = \int \omega \, y \, dx$

$\boxed{F_c = \omega \int_{x_1}^{x_2} \text{area}}$

Max · S·F & B·M at a Section :— for moving loads



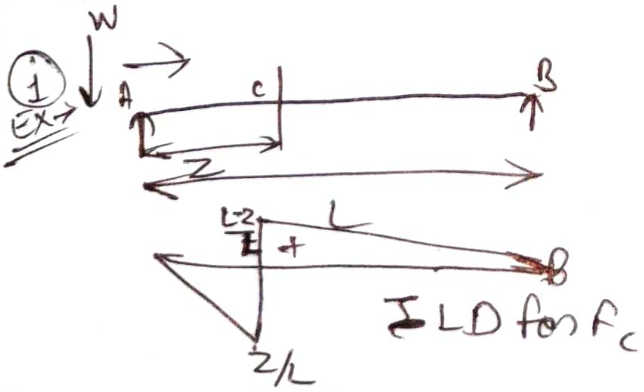
① Single Load (point)

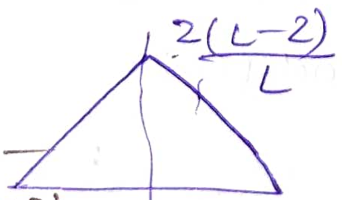
② UDL larger than ~~Length of~~ the span

③ UDL ~~lengt Stress~~ shorter than span

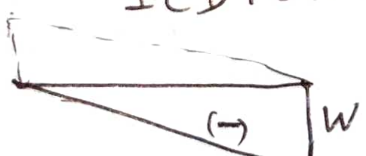
④ A train of point loads

Ex→ ①





Max · M : $= + \frac{Wz(L-z)}{L}$

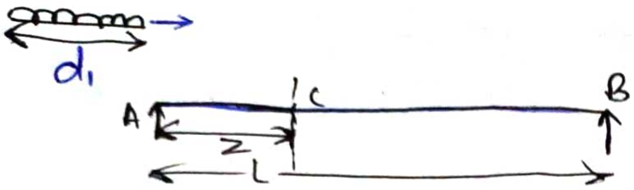


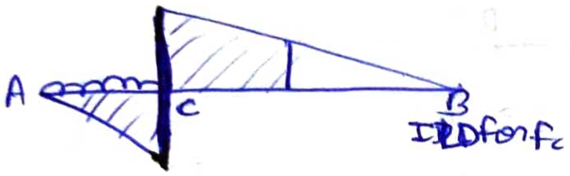
Absolute section max shear force

Max − ve S.F
= Load in on section (Left of section)
$= -W \frac{z}{L}$

Max + ve SF
= Load at section (right of section)

② UDL longer than the span $d_1 > L$





IDD for F_c

$\underbrace{\frac{1}{2} \cdot w \times \frac{z}{L} \times L}_{\text{for -ve part}}, \; \underbrace{w \cdot \frac{1}{2} \frac{(L-z)^2}{L}}_{\text{+ve part}}$

ILD for M_c

Max S.F at $B = w \times \frac{1}{2} \times 1 \times L$

$w \times \frac{1}{2} \frac{z \times (L-z)}{L} \times L$

$M_c = \frac{wz(L-z)}{2}$

Absolute

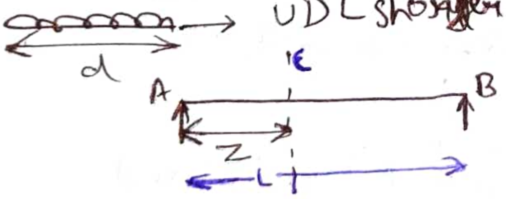
$w \times$ Area of ILD for F_c ...

$F_c = w \times \frac{1}{2} (L-z) \frac{(L-z)}{L}$

$= \frac{w(L-z)^2}{2L}$

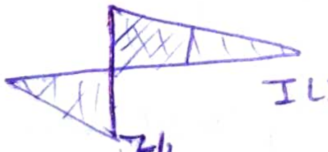
For max M_c, $\frac{dM_c}{dz} = 0$

③ UDL shorter than the span $d_1 < L$



max -ve S.F = ~~load~~ of load touches section

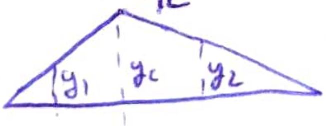
$= w \times \frac{1}{2} (y_1 + y_2) \times d$

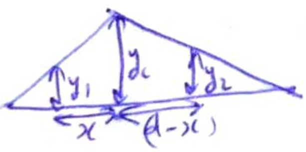


ILD for F_c

When tail of load touches the section

$= w \times \frac{1}{2} \times (y_c + y_2) \times d$





$M_c = w \times \frac{(y_1 + y_c)}{2} x + w \frac{(y_2 + y_c)}{2} (d-x)$

$\frac{dM_c}{dx} = 0$

$y_1 + y_c - (y_2 + y_c) = 0$

$\boxed{y_1 = y_2}$

for $y_c = \frac{z(L-z)}{L}$

$\frac{dy_c}{dz} = 0 = L - 2z$

$\boxed{z = \frac{L}{2}}$

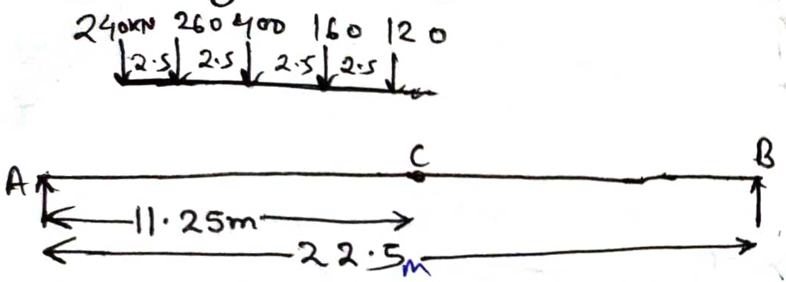
$\frac{y_c}{z}(z-x) = \frac{y_c}{(L-z)} [(L-z) - (d-x)]$

$(z-x)(L-z) = z[(L-z) - (d-x)]$

$xL = zd$ or $\boxed{\frac{x}{d} = \frac{z}{L}}$

~~Ques~~. 1

(4) Train of Concentrated loads
or Moving series of Point loads →



The ILD for shear force at centre of span is as shown in fig
for max. negative shear force the leading load 120KN should

The max ILD ordinate for shear force at C

$$= \frac{11.25}{22.5} = 0.5$$

~~∴ Max. negative S.F at mid span → 365.5KN~~

~~$= 240 \times 0.5 + \left[160 \times \frac{8.75}{11.25} + 400 \times \frac{6.25}{11.25} + 260 \times \frac{3.75}{11.75} + 240 \times \frac{1.25}{11.75}\right] 0.5$~~

~~$= 60 + [124.4 + 222.2 + 06.7 + 26.7] 0.5$~~

~~$= 290 KN$~~

for max shear force at C, the leading load should be at C

Max shear force at C

$$= 240 \times 0.5 + \left[260 \times \frac{8.75}{11.25} + 400 \times \frac{6.25}{11.25} + 160 \times \frac{3.75}{11.25} + 120 \times \frac{1.25}{11.25}\right] 0.5$$

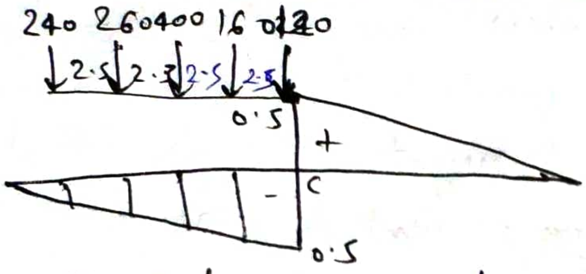
$$= 120 + [202.2 + 222.2 + 53.3 + 13.3] \times 0.5$$

$$= 365.5 KN$$

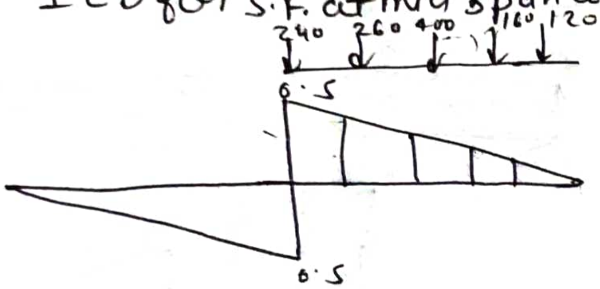
~~When~~ leading 240 kN load is one

$$+ve\ S.F\ at\ C = \left[-240 \times \frac{8.75}{11.25} + 260 \times 1 + 400 \times \frac{6.25}{11.25} + 160 \times \frac{3.75}{11.25} + 120 \times \frac{1.25}{11.25}\right] 0.5$$

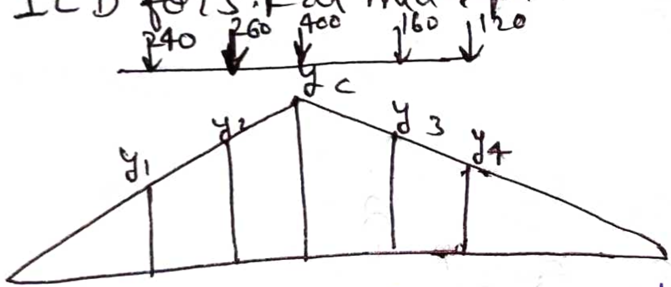
$$= 181.067 KN$$



ILD for S.F. at mid span with loading for max. -ve S.F



ILD for S.F at mid span with loading for max +ve S.F



Wrong

Hence max. shear force at mid-span C = 365.5 KN

Let C.G of loads from leading wheel load be at a distance x

Then,
$$x = \frac{160 \times 2.5 + 400 \times 5 + 260 \times 7.5 + 240 \times 10}{120 + 160 + 400 + 260 + 240}$$

$$= 5.7203\,m$$

This is nearer to 400 kN load. Hence max. moment is likely to occur under this load its distance from CG

$$= 5 - 5.720$$

$$= -0.7203\,m$$

∴ its position from end A

$$= 11.25 - \left(-\frac{0.7203}{2}\right)$$

$$= 11.610\,m$$

ILD ordinate under this load is

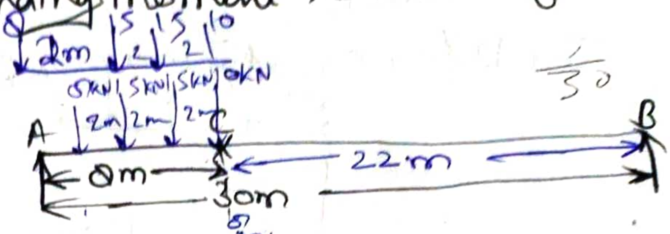
$$y_c = \frac{z(L-z)}{L} = \frac{11.610(22.5 - 11.25)}{22.5} = 5.005\,m$$

∴ absolute max. BM = $240y_1 + 260y_2 + 400y_3 + 160y_4 + 120y_5$

$$= 240 \times \frac{9.11}{11.610} + 260 \times \frac{6.61}{11.61} + 400 \times 1 + 160 \times \frac{4.}{11.61} + \frac{120 \times 1.61}{11.61} \times 5.005$$

$$= 5222.0\ kNm$$

Ques. four point loads, 8, 15, 15 & 10KN have centre to centre spacing of 2m between consecutive loads and they transverse a girder of 30m span from left to right with 10KN load leading. Calculate the max. bending moment & shear force at 8m from the left support



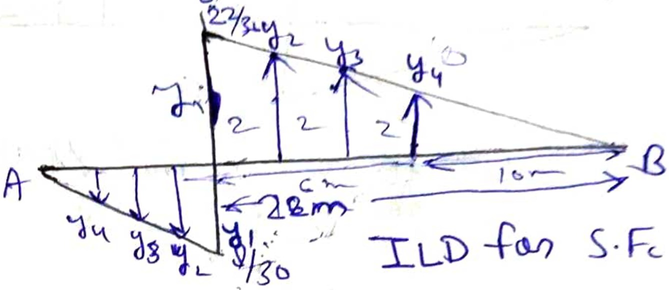
Solⁿ:

max −ve S.F$_c$ = 10KN at C

$F_c = 10y_1 + 15y_2 + 15y_3 + 8y_4$

$= 10\times\frac{8}{30} + 15\times\frac{6}{30} + 15\times\frac{4}{30} + \frac{8\times2}{30}$

$= 8.2\,KN$

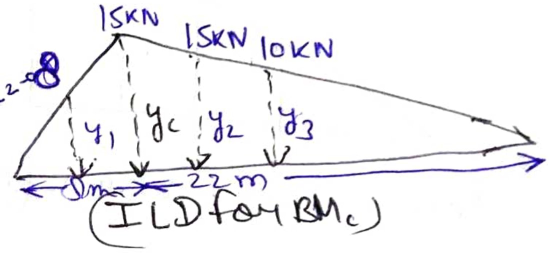


ILD for S.F$_c$

max +ve S.F$_c$ = $10y_1 + 15y_2 + 15y_3 + 8y_4$

$= 10\times\frac{16}{30} \times 15\times\frac{18}{30} + 8\times\frac{22}{30}$

$= 30.2\,KN$



(ILD for BM$_c$)

Check for another position W_3 = 15KN load is on the section

$F_c = 10\times\frac{18}{30} + 15\times\frac{20}{30} + 15\times\frac{22}{30} - 8\times\frac{6}{30}$

$= 25.4\,KN$

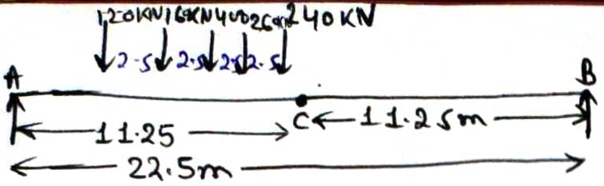
Load crossing	(Avg. load) AC W_{1av}	BC W_{2av}	Remark
10KN	$\frac{38}{8}$ >	$\frac{10}{22}$	$W_{1av} > W_{2av}$
15KN	$\frac{23}{8}$ >	$\frac{25}{22}$	$W_{1av} > W_{2av}$
15KN	$\frac{8}{8}$ <	$\frac{40}{22}$	$W_{1av} < W_{2av}$

$y_c = \frac{8\times22}{30} = 5.867$

$M_c = 15y_c + 8y_1 + 15y_2 + 10y_3$

$= 15y_c + 8\times\frac{6}{8}y_c + 15\times\frac{8}{22}y_c + 10\times\frac{18}{22}y_c$

$= 251.27\,Nm$



The max ILD ordinate for shear force at AC

$= \frac{11.25}{22.5} = 0.5$

$\therefore$ Max. -ve' S.F at mid span

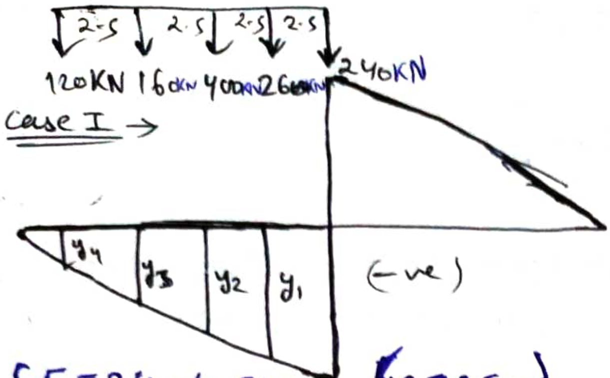
$= 240 \times 0.5 + \left[260 \times \frac{10.75}{11.25} + 400 \times \frac{8.25}{11.25} + 160 \times \frac{5.75}{11.25} + 120 \times \frac{3.25}{11.25}\right] \times 0.5$

$= 120 + [248.4 + 293.3 + 81.78 + 34.7] \times 0.5$

$= 449.1\,KN$

$= 365.5\,KN$

Case I →



$S.F = 240 \times 0.5 + 260 \times \left(\frac{11.25 - 2.5}{11.25} \times 0.5\right) + 400 \times 0.278 + 160 \times 0.168 + 120 \times 0.056$

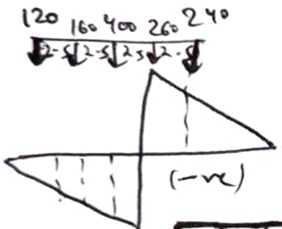
$= 365.94\,KN$

$= 240 \times 0.5 +$

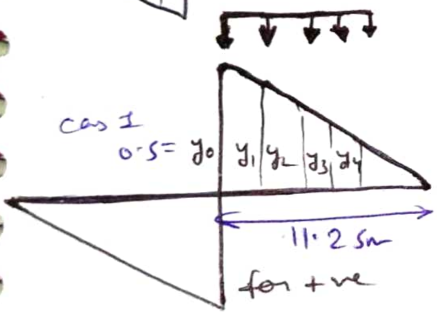
Case II →

$S.F = -240 \times 0.389 + 260 \times 0.5 + 400 \times 0.389 + 160 \times 0.270 + 120 \times 0.168$

$= 256.88\,KN$



hence, the Max Negative S.F is 365.98 obtained by Case 1 loading conditions.



$y_1 = \frac{0.5}{11.25} \times 8.75 = 0.389m$

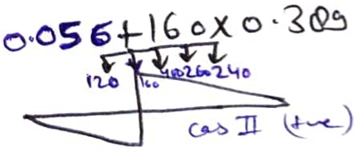
$y_2 = \frac{0.5}{11.25} \times 6.25 = 0.278m$

$y_3 = 0.168\,m$

$y_4 = 0.045 \times 1.25 = 0.056m$

Case I: $S.F_1 = 120 \times 0.5 + 400 \times 0.278 + 260 \times 0.168 + 240 \times 0.056 + 160 \times 0.389$

$=$

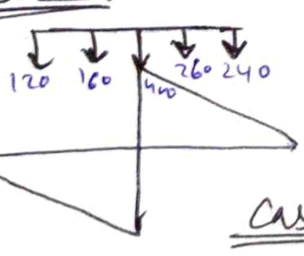


Case II:

$S.F_2 = -120 \times 0.389 + 160 \times 0.5 + 400 \times 0.389 + 260 \times 0.270 + 240 \times 0.168$

$= 301.52\,KN$

Case III: +ve'



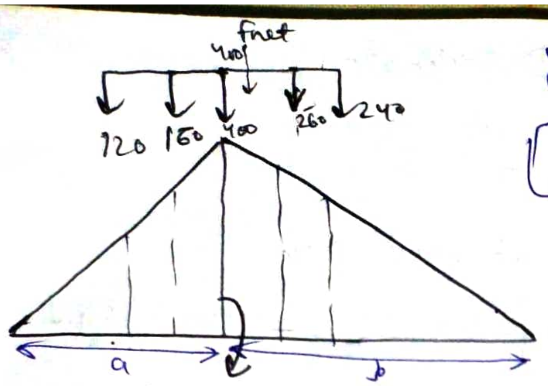
$S.F = -120 \times 0.270 - 160 \times 0.389 + 0.5 \times 400 + 260 \times 0.389 + 240 \times 0.278$

$= 271\,kN$

Case IV: No need

$\therefore$ The S.F in case 3 is decreased & it will further decrease.

So Max. Shear force for the given loading conditions is 301.52KN i.e., Case II



$$y_o = \frac{11.25 \times 11.25}{22.5} \quad \left\{\frac{a\,b}{l}\right\}$$

$y_o = 5.625\,m$ {becoz of more than 5 distance so 400kN is max.}

$$y_1 = \frac{5.625}{11.25} \times 8.75$$

$$\boxed{y_1 = 4.375\,m}$$

$$y_2 = 0.5 \times 6.25$$
$$= 3.125$$

$$\boxed{y_2 = 3.125\,m}$$

$$BM_{(absolute)} = 400 \times 5.625 + 260 \times 4.375 + 240 \times 3.125 + 160 \times 4.375 + 120 \times 3.125$$

$$BM_{(absolute)} = 5211.3 \text{ KN-m}$$