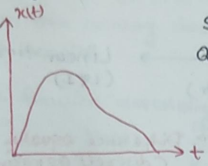
# Digital Signal Processing (DSP)

KEC- 503

4/8/22
(01)

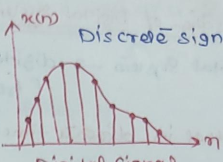
Digital signal processing is the use of digital processing, such as by computers or more specilized digital signal processors, to perform a wide variety of signal processing operation.

**Signal**




Analog

SAMPLING
OR
Quantization.
⟹




Discrete signal

Digital signal

⇒ Signal means to convey information.

**Signal Processing**

Analyse, Modify and Synthesize.

(1) Analog signal processing
(2) Digital signal processing

**Block diagram DSP**

Analog Signal I/P $x(t)$ → PRE FILTER → A/D CONVERTER → Digital Signal Processor → D/A Converter — POST FILTER → Analog O/P Signal $y(t)$

PRE FILTER ↓ low pass filter

A/D CONVERTER: Sampling & Quantization

D/A Converter ↓ Reconstruction, LPF

**Application of DSP**

- → Speech & Audio
- → Image & Video
- → Military & Space
- → Biomedical & Health care
- → Consumer electronics.
- → Signal filtering & so on

**Advantages of DSP**

- → Flexiable in operation [Design using software]
- → Accurate results.
- → Stable signal
- → Data storage - less expensive.
- → low cost.

**Disadvatages of DSP**

- → Limited speed of operation.

1. Johny R. Johnson

book - John G Prokias (Pearson)

2. S. Salivahanan → Mc Grow hill

DSP Technologies

- → Dedicated processor based DSP (application specific)
- → General purpose processor. (up or computer)

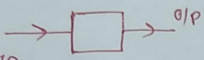
## Realization of Digital System

Digital System → digital filter (frequency selector) → Linear time invariant Sys (LTI)

We deal in discrete time system → Difference equation (discrete op making) → differential eq (continuous)

Discrete time System are of two types

FIR → finite } Impulse Response
IIR infinite



I/P
impulse
Signal

O/P

⇒ Difference equation can be implemented in 2 ways of implementation or realization of

- Hardware
- Software.

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Structural representation

- → Canonical form (No. of delay element = order of difference eqn)
- → Non canonical form
- → Recursive (FB)
- → Non-recursive

## Basic Building blocks in a DSP

① Adder

$x_1(n)$, $x_2(n)$ → (+) → $x_1(n) + x_2(n)$ or $\Sigma$

② Constant multiplier

$x(n) \xrightarrow{a} a\,x(n)$ {if $a=3 \Rightarrow 3x(n)$

③ Signal multiplier

$x_1(n)$, $x_2(n)$ → (×) → $x_1(n) \cdot x_2(n)$

# IIR Filters

⇒ The analog filter design is well developed and the techniques discussed in this chapter are all based on taking an analog filter and converting it into digital filter.

⇒ Thus, the design of IIR filter involves design of a digital filter in the analog domain and transforming the design into digital domain.

⇒ System function describing & analog filter is

$$H_a(s) = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k} \quad \text{—①}$$

where $a_k$ & $b_k$ are filter coefficient. The impulse response of these filter coefficient is related to $H_a(s)$ by Laplace transform.

$$H_a(s) = \int_{-\infty}^{\infty} h(t)\, e^{-st} dt \quad \text{—②}$$

The analog filter having the rational system fun $H(s)$ given in eq ① can also described by linear constant coefficient differential equ

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b^k \frac{x(t)}{dt^k} \quad \text{—③}$$

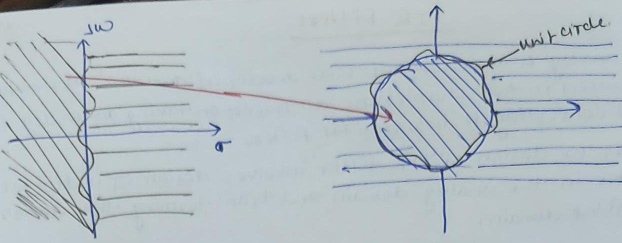
where $x(t)$ is the input signal and $y(t)$ is the ouput of the filter.

⇒ The above three characterization of an analog filter leads to three alternative methods for transforming the filter into the digital domain.

⇒ The design techniques for IIR filter are presented with the restriction that the filters be realisable and stable.

⇒ An analog filter with system fun $H(s)$ is stable if all its poles lie in the left half of the s-plane. As the result, if the conversion techniques is to be effive, the technique should posses the following properties.

1. The $j\Omega$ axis in the s-plane should map onto the unit circle in the z-plane. ~~The axis~~ This gives a direct relationship b/w the two frequency variables in the two domains.
2. The left half plane of the s-plane should map into the inside of the unit circle in the z-plane to convert a stable analog filter into a stable digital filter.




s plane          z-plane

① Derivative Approximation method (or) Backward difference method,

Principle:- This method stands on principle that the both derivative in S & Z plane are equaled to get the transformation equation.

i.e derivative in Analog Domain = Derivative in digital domain

| Design method of IIR |
|---|
| ① Derivative Approximation |
| ② Impulse Invariance |
| ③ Bilinear transformation |
| ④ Matched z-transform |

First derivative:

$$\frac{dy(t)}{dt} = \frac{y(n)-y(n-1)}{T} \quad (T\text{-sampling rate})$$

Apply L.T to LHS & z transform to RHS

$$sY(s) = \frac{Y(z)-Y(z)z^{-1}}{T}$$

$$sY(s) = Y(z)\left[\frac{1-z^{-1}}{T}\right]$$

$$\boxed{s = \frac{1-z^{-1}}{T}}$$

Second derivative :-

$$\frac{d^2y(t)}{dt^2} = \frac{\left[\frac{y(n)-y(n-1)}{T}\right] - \left[\frac{y(n-1)-y(n-2)}{T}\right]}{T}$$

$$\frac{d^2y(t)}{dt^2} = \frac{y(n)-2y(n-1)+y(n-2)}{T^2}$$

$$s^2Y(s) = \frac{Y(z)-2Y(z)z^{-1}+z^{-2}}{T^2}$$

$$s^2Y(s) = Y(z)\left[\frac{1-2z^{-1}+z^{-2}}{T^2}\right]$$

$$\boxed{s^2 = \frac{(1-z^{-1})^2}{T^2}}$$

## Realisation of Digital System

6/8/22

①

For design of digital filters, the system function $H(z)$ or the impulse response $h(n)$ must be specified. Then the digital system can be implemented or synthesized in hardware/software form by its difference equation obtained directly from $H(z)$ or $h(n)$

⇒ Each difference equation or computational algorithm can be implemented by using a digital computer or special purpose digital hardware or special programmable integrated circuit.

$y(n) = 3x(n-1)$ ✓ — 1st order difference eqn

$y(n) = 3x(n-1) + 4x(n-2)$ ✓ 2nd order

### Basic Building block

The computational algorithm of an LTI digital filter can be represented as a block diagram using basic building blocks representing

⇒ The unit delay ($z^{-1}$) or storage element

⇒ The multiplier

⇒ The adder

⇒ **Delay**

unit delay

$x(n) \rightarrow \boxed{z^{-1}} \rightarrow x(n-1)$

$x(n) \rightarrow \boxed{z^{k}} \rightarrow x(n-k)$

⇒ constant multiplier

$x(n) \xrightarrow{a} a x(n)$ $\{a = 1, 2, 3 \ldots \Rightarrow 3x(n)$

⇒ **Adder**

$x_1(n),\ x_2(n) \rightarrow \oplus \rightarrow x_1(n) + x_2(n)$ or $\Sigma \rightarrow$

⇒ Signal multip

$x_1(n),\ x_2(n) \rightarrow \otimes \rightarrow x_1(n) \cdot x_2(n)$

⇒ Signal advance element

$x(n) \rightarrow \boxed{z} \rightarrow x(n+1)$

$x(n) \rightarrow \boxed{zk} \rightarrow x(n+k)$

## Discrete Time System

Consider the discrete time SISO system:

$$x[n] \longrightarrow \boxed{\begin{matrix}\text{Processor}\\ h[n]\end{matrix}} \longrightarrow y[n]$$

⇒ If the i/p signal is $x[n] = \delta[n]$ and the system has no energy at $n=0$ the o/p $y(n)$ is called impulse response of the system

$$y[n] = h[n]$$

⇒ Since the impulse response $h[n]$ provides the complete description of a discrete LTI System, we can write as

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$y(n) = \underbrace{x(n) * h(n)}_{\text{Convolution}} \quad \text{discrete time LTI System}$$

### DIGITAL SYSTEM in Block diagram form

⇒ Just by inspection, the computational algorithm can be easily written.

⇒ The hardware requirement can be easily determined

⇒ A variety of equivalent block diagram representation can be easily developed from the transfer function $H(z)$

⇒ The relationship b/w the output and the i/p can be determined

$$\boxed{H(z) = \frac{Y(z)}{X(z)}} \qquad \{H(z) \text{ transfer function of impulse response } h(n)\}$$

Transfer fun of digital system

26.08.2023

(1)

Ques (2)

## Recursive and non-recursive Realization

Causal System ⇒ causal systems are those whose o/p depend upon present value of i/p & past value of i/p but not depend on future)

$$y(n) = F[x(n), x(n-1), x(n-2) \ldots]$$

Causal System

Recursive

Non Recursive.

depend on present/ past input value of i/p.

⇒ causal system is non recursive

⇒ Recursive ⇒

⇒ Present o/p of the system depends on present input, past input & past output values.

$$y[n] = F[y(n-1) \, y(n-2) \, x(n), x(n-1), \ldots]$$

⇒ Involve feedback

⇒ Non-Recursive Realization

⇒ Present output of the system depends on present input and past input values only.

$$y(n) = F[x(n), x(n-1) \ldots]$$

⇒ Doesn't involve feedback.

### Basic structure of Digital System

IIR

Infinite impulse Response

FIR

finite impulse response

Causal IIR Systems are charaterized by the constant coefficient difference equn.

$$y(n) = -\sum_{k=1}^{N} \underbrace{a_k \, y(n-k)}_{\text{past of o/p}} + \sum_{k=0}^{M} \underbrace{b_k \, x(n-1)}_{\text{Past \& present i/p}} \quad \text{— (1)}$$

used in IIR

Or equivalently, by the real rational transfer fcn.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \quad \begin{matrix} \text{Zeros} \\ \text{Poles.} \end{matrix} \quad (2)$$

So IIR contain both zeros or pole.

Similarly FIR system are characterised by constant coefficient diff equn

$y(n) = x(n) + x(n-1) \rightarrow$ Non Recursive fcn there

Or equivalently, by the real rational fcn transfer function

$H(z) = NR$ (Numerator) it consist non recursive.

Canonic and Non-Canonic Structure

If the number of delays in the realisation block diagram is equal to the order of the difference equation or the order of the transfer fcn of a digital filter, then the realisation structure is called canonic, otherwise non canonic

IIR filter

- O/P of the system depends upon infinite number of impulse response value
- Involve f/b
- contain both poles & zeros
- can be realized using recursive realisation techniques.

FIR

- O/P of the system depends upon finite numbers of impulse response values.
- Doesn't involve f/b
- Contains only zeros.

However, here in the course we will go through only non-recursive realisation technique.

date 18.08.2022

# IIR filter Realization - 1

(1)

## Infinite Impulse Response filter (IIR)

⇒ O/P of the system depends upon infinite no of impulse response values.

⇒ Involve f/b

⇒ Contain both poles and zeros

$$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n)$$

⇒ Can be realized ~~used~~ using recursive realization technique.

Method of Realization

IIR Filter Realization →
- Direct form I & direct form II realization
- Cascade realization
- Parallel realization
- Ladder realization.

$$H(z) = \frac{Y(z)}{X(z)} \begin{matrix} \rightarrow NR \\ \rightarrow R \end{matrix}$$

### Basic realization

The o/p of a finite order linear time invariant system at time $n$ can be expressed as linear combination of the i/p and o/p

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)$$

Where $a_k$ and $b_k$ are constants with $a_0$ is not equal to zero and $M \le N$.

Taking the z-transform of the o/p sequence $y(n)$

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty}\left[-\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k)\right] z^{-n}$$

Changing order of summation

$$Y(z) = -\sum_{k=1}^{N} a_k \left\{ \sum_{n=-\infty}^{\infty} y(n-k) z^{-n} \right\} + \sum_{k=0}^{M} b_k \left\{ \sum_{n=-\infty}^{\infty} x(n-k) z^{-n} \right\}$$

Time shifting in z-transform

$$Y(z) = -\sum_{k=1}^{N} a_k z^{-k}\, Y(z) + \sum_{k=0}^{M} b_k z^{-k}\, X(z) \quad \text{—(2)}$$

$$Y(z)\left\{1 + \sum_{k=1}^{N} a_k z^{-k}\right\} = \sum_{k=0}^{M} b_k z^{-k}\, X(z)$$

∵ Therefore

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

# For, example, the difference equation of the first order digital System may be written as

$$Y(n) = a_1 y(n-1) + x(n) + b_1 x(n-1)$$

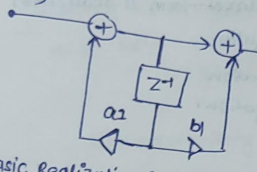
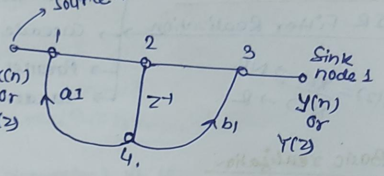



fig. Basic Realization Block diagram Representing a First order system




Signal flow graph

Direct form Realization

Standard form of the System fun is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

⇒ The block diagram representation can be drawn directly for the direct form realization. The multiplier in the feed forward paths are the numerator coefficient.

⇒ The multipliers in the f/b path are the negative of the denominator coefficient.

⇒ Since the multiplier coefficient in the structure are exactly the Coefficient of the transfer fun, they are called direct form structure.

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②

## Direct form – I Realization

The digital system structure determine directly from either eq ① or eq ② is called direct form – I

⇒ The system fun is divided into two parts connected in cascade.

⇒ The first part containing only the zeros.

⇒ followed by the part containing only the poles.

⇒ An intermediate sequence $w(n)$ is introduced.

NR

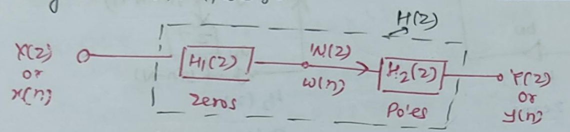
$$\boxed{w(n) = \sum_{k=0}^{M} b_k x(n-k)} \quad ①$$

$y(n) = $ ~~$\sum_{k=1} a_k y(n-k)$~~ $+ w(n)$

$$\boxed{y(n) = \sum_{k=1}^{N} a_k y(n-k) + w(n)} \quad ②$$

R

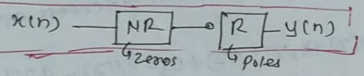
Taking Z-transform, we get



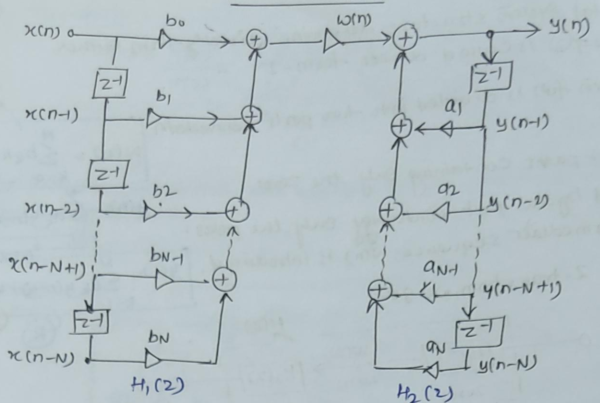

$$W(z) = X(z) \sum_{k=0}^{M} b_k z^{-k} \quad \text{and} \quad Y(z) = Y(z) \sum_{k=1}^{N} a_k z^{-k} + W(z)$$

$$Y(z) = \frac{W(z)}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

In DF-I



## Direct form - I




$H_1(z)$ zeros (Non-Recursive) $H_2(z)$ poles (Recursive)

Example 1 : Develop a direct form I realization of the difference equation,

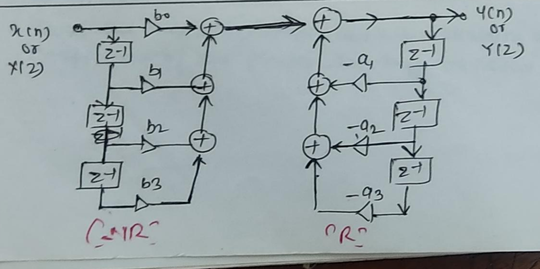
$$y(n) = b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + b_3 x(n-3) - a_1 y(n-1) - a_2 y(n-2) - a_3 y(n-3)$$

Solution ⇒ Taking z-transform of the given difference equn & simplifying we get transfer fun,

$$\Rightarrow Y(z) = b_0 X(z) + b_1 z^{-1} X(z) + b_2 z^{-2} X(z) + b_3 z^{-3} X(z) - a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z) - a_3 z^{-3} Y(z)$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)}$$

$$\Rightarrow H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$$




The corresponding direct form-I realization structure

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The direct form-I structure is non-canonic as it employs 6 delay element to realize a 3rd order transfer-fun

canonic = order of system = No. of delay element

⇒ direct form I is non-canonic

Examp Determine the direct form-I structure of the IIR filter described by the following eqn

$$y(n) = 0.5y(n-1) - 0.76y(n-2) + 0.63y(n-3) + x(n) + 0.875x(n-1)$$

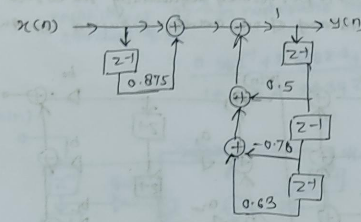
$$Y(z) = 0.5z^{-1}Y(z) + 0.76z^{-2}Y(z) + 0.63z^{-3}Y(z) + X(z) + 0.875z^{-1}X(z)$$

$$Y(z)[1 - 0.5z^{-1} + 0.76z^{-2} - 0.63z^{-3}] = X(z)[1 + 0.875z^{-1}]$$

$$H(z) = \frac{Y(z)}{X(z)} \quad \frac{1 + 0.875z^{-1}}{1 - 0.5z^{-1} + 0.76z^{-2} - 0.63z^{-3}}$$

NR

R




Direct form-II

⇒ Since we are dealing with linear system, the order of these parts can be interchanged. This property yields a second direct form realization

⇒ In direct form II, the poles of H(z) are realized first & the zeros second.

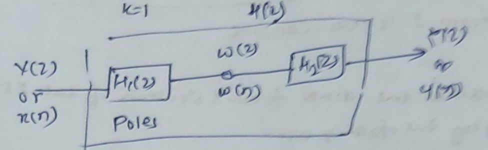
⇒ Here, the transfer-fun H(z) is broken into a product of two transfer-fun $H_1(z)$ & $H_2(z)$

poles zeros.

$$H_1(z) = \frac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}} \quad \& \quad H_2(z) = \sum_{k=0}^{M} b_k z^{-k}$$

Taking z-transform of the above eqn, we get,

$$W(z) = \frac{Y(z)}{1 - \sum_{k=1}^{M} a_k z^{-k}} \quad \text{and} \quad H(z) = W(z) \sum_{k=0}^{M} b_k z^{-k}$$

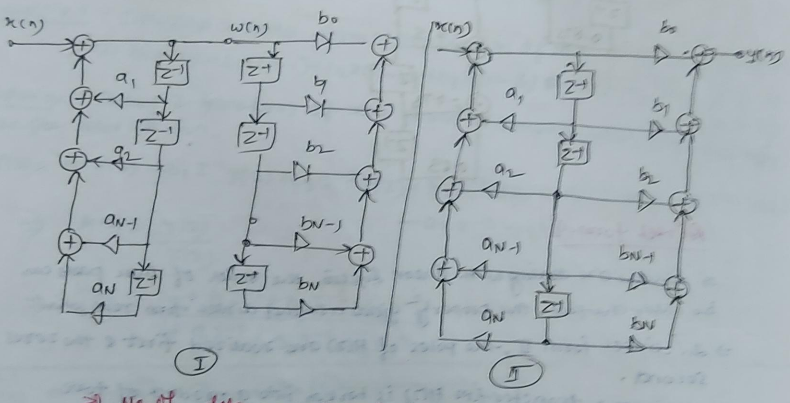



## Advantage of Direct form II

⇒ The direct form II realisation require only large M or N storage elements

⇒ when compared to direct form I realisation, the direct form II uses minimum number of storage elements & hence said to be canonic structure.

⇒ However, when the addition is performed sequentially, the direct form II need two adders instead of one adder required for the direct form I




⇒ No. of delay = order of system

⇒ Canonic structure

⇒ No. of delay is $N/2$ in direct for 2

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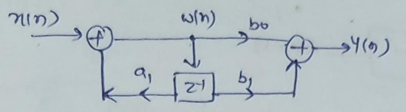
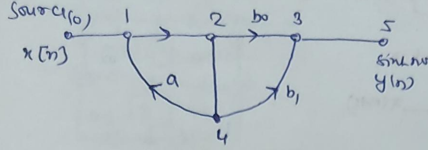
Direct form – II with signal flowgraphs



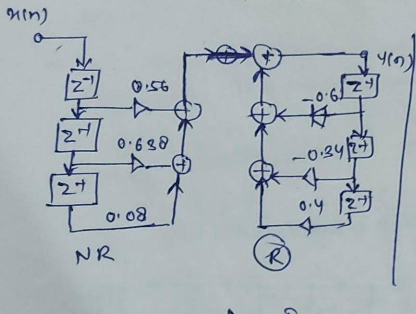


Example : Determine the direct form I and II realization for a third IIR transfer fun

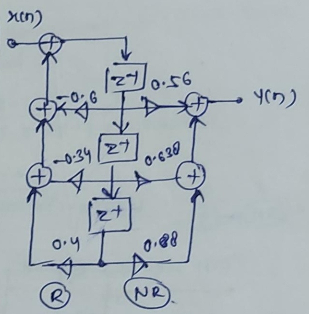
$$H(z) = \frac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2}$$

multiplying $2z^{-3}$ in numerator or denominator, we obtained standard form.

$$= \frac{0.56z^{-1} + 0.638z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{-2} + 0.4z^{-3}}$$



Direct form – I



Direct form II

(4) Delay element

$$x(n) \rightarrow \boxed{z^{-1}} \rightarrow x(n-1)$$

$$x(n) \rightarrow \boxed{z^{-k}} \rightarrow x(n-k)$$

(5) time Advanced element ⇒

$$x(n) \rightarrow \boxed{z} \rightarrow x(n+1)$$

$$x(n) \rightarrow \boxed{z^{k}} \rightarrow x(n+k)$$

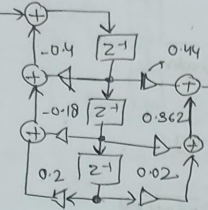
Direct form II

NR

DSP → 22/8/22

(1)

$$H(z) = \frac{0.44z^{-1} + 0.362z^{-2} + 0.02z^{-3}}{1 + 0.4z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$ (R)




## Cascade Realization

Direct form II is not the only realization. There are several other forms. Two other important forms are the cascade and the parallel form

⇒ In cascade realization, the transfer fun $H(z)$ is broken into a product of transfer function $H_1(z)$, $H_2(z)$, .. $H_k(z)$

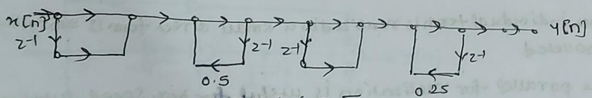
⇒ factoring the numerator & denominator polynomials of the transfer fun. $H(z)$

⇒ Each individual factor is realized ~~denominator~~ as a small Direct Form II subsystem and then cascaded

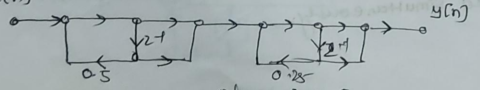
ex $H(z) = \dfrac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}} = \dfrac{(1+z^{-1})(1+z^{-1})}{(1-0.5z^{-1})(1-0.25z^{-1})}$

$\dfrac{(1+z^{-1})}{(1-0.5z^{-1})} \quad \dfrac{(1+z^{-1})}{(1-0.25z^{-1})}$

P → R, Z → NR




Direct form I




Direct form II

Example: obtained a cascade realization of the system characterized by the fun

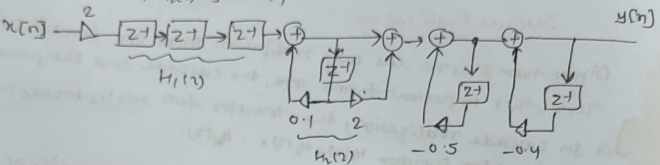
$$H(z) = \frac{2(z+2)}{z(z-0.1)(z+0.5)(z+0.4)}$$

Sol - multiplying $z^{-4}$ in numerator & denominator

$$H(z) = \frac{2z^{-3}(1+2z^{-1})}{(1-0.1z^{-1})(1+0.5z^{-1})(1+0.4z^{-1})}$$

$$= 2z^{-3}(1+2z^{-1}) \cdot \frac{1}{(1-0.1z^{-1})(1+0.5z^{-1})} \cdot \frac{1}{(1+0.4z^{-1})}$$

$2H_1(z) \cdot H_2(z) \cdot H_3(z) \cdot H_4(z)$




Example: Direct form II and cascade form realization

$$H(z) = \frac{0.44z^{-1} + 0.362z^{-2} + 0.02z^{-3}}{1 + 0.4z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$

## Parallel Realization

Parallel form is realized by first expressing the transfer fun is partial fraction form

$$H(z) = \frac{k_1}{z-p_1} + \frac{k_2}{z-p_2} + L + \frac{k_N}{z-p_N}$$

⇒ Each individual term is realized as a small direct form II subsystem & then paralleled

⇒ The parallel for realization is useful for high speed filtering application since the filter operation is performed in parallel, i.e, the processing is performed simultaneously.

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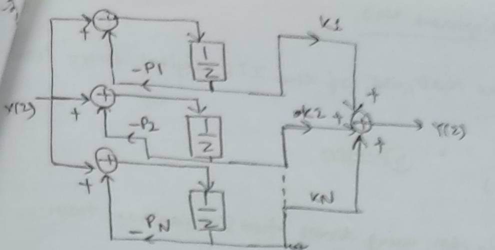



} ⇒ Parallel form realization

Example → Determine the parallel realization of the IIR digital filter transfer function.

$$H(z) = \frac{3/2\,(2z^2+5z+4)}{(2z+1)(z+2)}$$

solution ⇒ In order to find the parallel realization, the partial fraction expression of $H(z)/z$ is first determined, just as we did for inverse z-transform

$$F(z) = \frac{H(z)}{z} \quad \frac{3/2\,(2z^2+5z+4)}{z(z+\frac{1}{2})(z+2)} = \frac{A_1}{z} + \frac{A_2}{(z+\frac{1}{2})} + \frac{A_3}{(z+2)}$$

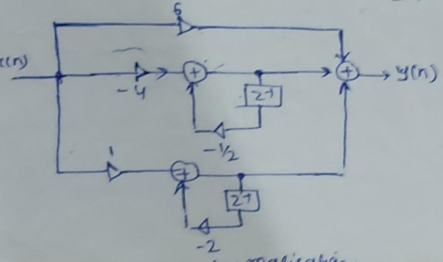
$$A_1 = zF(z)\big|_{z=0} = \frac{3/2\,(2z^2+5z+4)}{(z+\frac{1}{2})(z+2)}\Bigg|_{z=0} = 6$$

$$A_2 = (z+\tfrac{1}{2})F(z)\big|_{z=-\frac{1}{2}} = \frac{3/2\,(2z^2+5z+4)}{z(z+2)}\Bigg|_{z=-\frac{1}{2}} = -4$$

$$A_3 = (z+2)F(z)\big|_{z=-2} = \frac{3/2\,(2z^2+5z+4)}{z(z+\frac{1}{2})}\Bigg|_{z=-2} = 1$$

$$\frac{H(z)}{z} = \frac{6}{z} - \frac{4}{z+\frac{1}{2}} + \frac{1}{(z+2)}$$

$$= 6 - \frac{4z}{z+\frac{1}{2}} + \frac{z}{z+2} = 6 - \frac{4}{(1+\frac{1}{2}z^{-1})} + \frac{1}{(1+2z^{-1})}$$




Cascade for realization.

Example? obtained a cascade realiz....

## Assignment No.1

1. Determine the parallel realization of the IIR digital filter fun.

(a) $H(z) = \dfrac{3z(5z-2)}{(z+\frac{1}{2})(3z-1)}$ (b) ~~H(z)~~

2. Realized the following fun using direct form II & cascade form

$$H(z) = \frac{0.44z^{-1} + 0.36z^{-2} + 0.02z^{-3}}{1 + 0.4z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$

3. Draw the block diagram using parallel form for a LTI system whose transfer fun is

$$H(z) = \frac{1 - 3z^{-1} + 2z^{-2}}{1 + 0.3z^{-1} - 0.1z^{-2}}$$

4. Difference b/w
   (a) Recursive & Non recursive
   (b) Canonical & Non canonical form.

(5)

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(1)

## Ladder structures

Filters realized using ladder structures have desirable coefficient sensitivity properties. Small changes in the parameters have little effect on its performance.

→ Consider a filter represented by the transfer fun.

$$H(z) = \frac{a_N z^{-N} + a_{N-1} z^{-N+1} + \dots + a_1 z^{-1} + a_0}{b_N z^{-N} + b_{N-1} z^{-N+1} + \dots + b_1 z^{-1} + b_0} \quad \begin{cases} N(z) \\ D(z) \end{cases}$$

$$= \alpha_0 + \cfrac{1}{\beta_1 z^{-1} + \cfrac{1}{\alpha_1 + \cfrac{1}{\beta_2 z^{-1} + \dots + \cfrac{1}{\alpha_N}}}}$$

**Example** : Given the system fun

$$H(z) = (2 + 8z^{-1} + 6z^{-2}) / (1 + 8z^{-1} + 12z^{-2})$$

**Solution** : For the given system, obtain the Routh array.

| | | | |
|---|---|---|---|
| $z^{-2}$ | 6 | 8 | 2 |
| $z^{-2}$ | 12 | 8 | 1 |
| $z^{-1}$ | 4 | 3/2 | 0 |
| $z^{-1}$ | 7/2 | 1 | |
| 1 | 5/14 | 0 | |
| 1 | 1 | | |

$$\frac{12\times 8 - 6\times 8}{12} = \frac{96-48}{12} = \frac{48}{12} = 4$$

$$\frac{12\times 2 - 6\times 1}{12} = \frac{24-6}{12} = \frac{18}{12} = \frac{3}{2}$$

$$\frac{4\times 8 - 12\times \frac{3}{2}}{4} = \frac{32-18}{4} = \frac{14}{4} = 7/2$$

$$\frac{4 - 12\times 0}{4} = \frac{4}{4} = 1$$

$$\frac{\frac{7}{2}\times \frac{3}{2} - 4\times 1}{7/2} \qquad \frac{\frac{21}{4} - 4}{7/2}$$

$$\frac{21-16}{4\cdot 2}\times\frac{2}{7}$$

$$5/14$$

$$\frac{5/14 \times 1 - 0}{5/14} = \frac{5}{14}\times\frac{14}{5}$$

## Ladder structures

Filters realized using ladder structures have desirable coefficient sensitivity properties. Small changes in the parameters have little effect on its performance.

→ Consider a filter represented by the transfer fun.

$$H(z) = \frac{a_N z^{-N} + a_{N-1} z^{-N+1} + \dots + a_1 z^{-1} + a_0}{b_N z^{-N} + b_{N-1} z^{-N+1} + \dots + b_1 z^{-1} + b_0} \left\{ \begin{matrix} N(z) \\ D(z) \end{matrix} \right.$$

$$= \alpha_0 + \cfrac{1}{\beta_1 z^{-1} + \cfrac{1}{\alpha_1 + \cfrac{1}{\beta_2 z^{-1} + \dots + \cfrac{1}{\alpha_N}}}}$$

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$$\frac{21-16}{4\cdot 2}\times\frac{2}{7}$$

$$5/14$$

$$\frac{5/14\times 1 - 0}{5/14} = \frac{5}{14}\times\frac{14}{5}$$

| | | | | |
|---|---|---|---|---|
| $z^{-N}$ | $a_N$ | $a_{N-1}$ | $a_{N-2} \cdots\cdots a_1\, a_0$ | |
| $z^{-N}$ | $b_N$ | $b_{N-1}$ | $b_{N-2} \cdots\cdots b_1, b_0$ | |
| $z^{-N+1}$ | $c_{N-1}$ | $c_{N-2}$ | $c_{N-3} \cdots\cdots c_0$ | |
| $z^{-N+1}$ | $d_{N-1}$ | $d_{N-2}$ | $d_{N-3} \cdots\cdots d_0$ | |
| $z^{-N+2}$ | $e_{N-2}$ | $e_{N-3}$ | $e_{N-4} \cdots\cdots$ | |
| $z^{-N+2}$ | $f_{N-2}$ | $f_{N-3}$ | $f_{N-4} \cdots$ | |
| 1 | $g_0$ | | | |
| 1 | $h_0$ | | | |

$\alpha_0 = \frac{a_N}{b_N} = \frac{6}{12} = \frac{1}{2}$

$\beta_1 = \frac{b_N}{c_{N-1}} \quad \frac{12}{4} = 3$

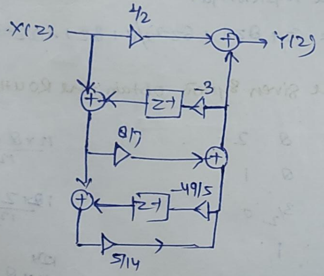
$\alpha_1 = \frac{c_{N-1}}{d_{N-1}} = \frac{4}{7/2} = \frac{8}{7}$

$\beta_2 = \frac{d_{N-1}}{e_{N-1}} \quad \frac{7/2}{5/14} = \frac{7}{2} \times \frac{14}{5} = \frac{49}{5}$

$\alpha_2 = \frac{e_{N-1}}{f_{N-1}} = \frac{5/14}{1} = 5/14$

$$H(z) = \frac{1}{2} + \cfrac{1}{3z^{-1} + \cfrac{1}{\frac{8}{7} + \cfrac{1}{(49/5)z^{-1} + \cfrac{1}{(5/14)}}}}$$

Realization




Example 2

$$H(z) = \frac{z^{-2} + 2z^{-1} + 1}{3z^{-2} + 3z^{-1} + 1}$$

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## Basic Structure of FIR System

The system fun of the FIR system whose realization take the form of non-recursive computational algorithm may be written as.

$$H(z) = \sum_{n=0}^{M-1} h(n) z^{-n}$$

FIR filters are often preferred in many application, since they provide an exact linear phase over the whole frequency range and they are always BIBO stable independent of the filter coefficient.

### Direct Form Realization of FIR System

$$y(n) = \sum_{k=0}^{M-1} h(k) x(n-k)$$

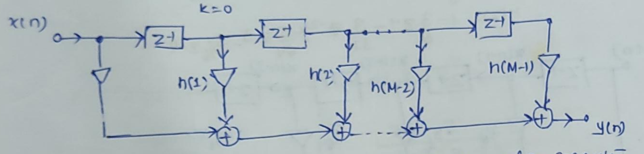
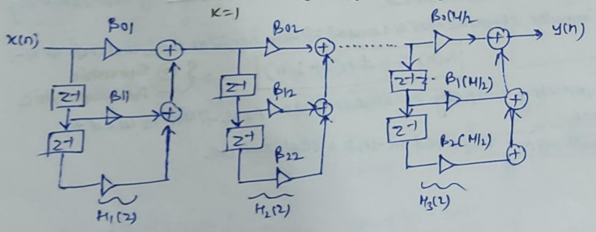



Fig. Direct Form Realization Structure of an FIR system

### Cascade form Realization

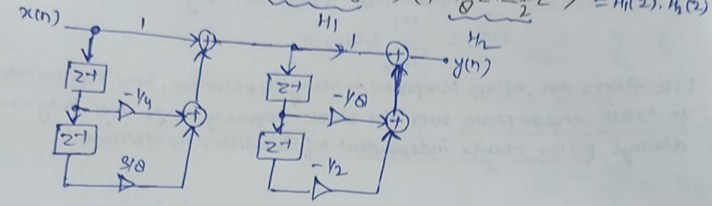
As an alternative to the Direct-form, we factor the FIR transfer function H(z) as a product of second order factor given by

$$H(z) = \prod_{k=1}^{M} (B_{0k} + B_{1k} z^{-1} + B_{2k} z^{-2})$$



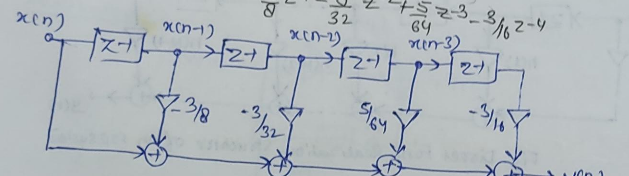
**Example** Obtain direct form and cascade form realisation for the transfer fun of an FIR System given by

Cascade form: $H(z) = \underbrace{(1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2})}_{H_1}\underbrace{(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2})}_{H_2} = H_1(z) \cdot H_2(z)$




Direct form expanding the transfer fun H(z) we get

$$H(z) = 1 - \frac{3}{8}z^{-1} - \frac{3}{32}z^{-2} + \frac{5}{64}z^{-3} - \frac{3}{16}z^{-4}$$




⇒ Realization of Linear Phase FIR System

⇒ If the impulse response is symmetric about its origin, linear phase results.

⇒ If all zero filter has a linear phase response, a special non-recursive structure that reduces the number of multiplication by approximately one half can be implemented.

→ The impulse response for a causal filter begin at zero and ends at M-1

$$\boxed{h(n) = \pm h(M-1-n)} \rightarrow \begin{cases} + & \text{Symmetric} \\ - & \text{Anti-symmetric} \end{cases}$$

→ The symmetry property of linear phase FIR filter is used to reduce the multipliers required in these realization.

## Parallel form Realization

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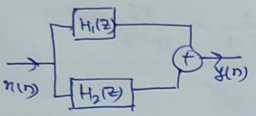
Every input signal is processed by different subsystem

$$H(z) = H_1(z) + H_2(z) + \dots$$

or

$$= C + H_1(z) + H_2(z) + \dots$$

⇒ Solved by Partial fractional Residual method (Direct form-II)




Q. $H(z) = \dfrac{1 - z^{-1}}{1 - 0.2z^{-1} + 0.15z^{-2}}$

multiply & divide by height power of z

$$= \frac{z^2(1 - z^{-1})}{z^2(1 - 0.2z^{-1} + 0.15z^{-2})} = \frac{z^2 - z}{z^2 - 0.2z - 0.15}$$

$$H(z) = \frac{z(z-1)}{z^2 - 0.2z - 0.15} \Rightarrow \frac{H(z)}{z} = \frac{z-1}{z^2 - 0.2z - 0.15}$$

$$\boxed{\frac{H(z)}{z} = \frac{z-1}{(z-0.5)(z+0.3)}}$$

$$\frac{1}{(z-P_1)(z-P_2)}$$
⇓
By Partial

$$\frac{H(z)}{z} = \frac{A_1}{(z-0.5)} + \frac{A_2}{(z+0.3)}$$

$$A_1 = (z-P_1)\frac{H(z)}{z}\Big|_{z=P_1} = \frac{\cancel{(z-0.5)}(z-1)}{\cancel{(z-0.5)}(z+0.3)}\Big|_{z=0.5}$$

$$A_1 = \frac{(z-1)}{(z+0.3)}\Big|_{z=0.5} = \frac{-0.5}{0.8} = -0.625$$

$$A_2 = (z-P_2)\frac{H(z)}{z}\Big|_{z=P_2} = \frac{\cancel{(z+0.3)}(z-1)}{(z-0.5)\cancel{(z+0.3)}}\Big|_{z=-0.3}$$

$$A_2 = \frac{-1.3}{-0.8} = 1.625$$

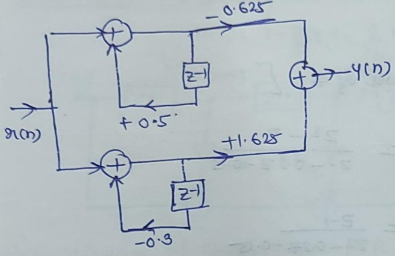
Now $\frac{H(z)}{z} = \underbrace{\frac{-0.625}{z-0.5}}_{H_1(z)} + \underbrace{\frac{1.625}{z+0.3}}_{H_2(z)}$

Now realize into Direct form-II

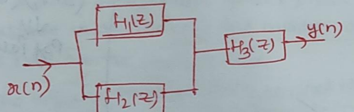
$$H(z) = \frac{-0.625z}{z-0.5} + \frac{0.625z}{z+0.3}$$

$$H(z) = \frac{-0.625\cancel{z}}{\cancel{z}(1-0.5z^{-1})} + \frac{1.625\cancel{z}}{\cancel{z}(1-0.3z^{-1})}$$

$$= \frac{-0.625}{(1-0.5z^{-1})} + \frac{1.625}{(1-0.3z^{-1})}$$




Q. A system has a transfer fun as shown




$H_1(z)$ & $H_2(z)$ has poles at $+j$ and $-j$

$H_1(z)$ has 2 zeros at origin

$H_2(z)$ has 1 zeros at origin

$H_3(z) = 1 + 1.5z^{-1} - 1.5z^{-2} - z^{-3}$

Realize the circuit?

$$H(z) = \frac{(z-z_1)(z-z_2)\ldots}{(z-p_1)(z-p_2)\ldots}$$

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$$H_1(z) = \frac{z^2}{(z-1)(z+1)}$$

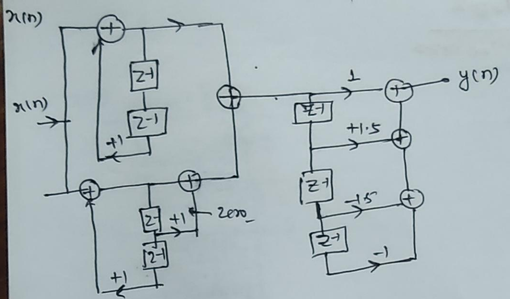
$$H_2(z) = \frac{z}{(z+1)(z-1)}$$

$$H_3(z) = 1+0.5z^{-1}-1.5z^{-2}-z^{-3}$$

$$\boxed{H_1(z) = \frac{z^2}{(z^2-1)} = \frac{1}{1-z^{-2}}}$$

$$H_2(z) = \frac{z}{z^2-1}$$

$$\boxed{H_2(z) = \frac{z^{-1}}{1-z^{-2}}}$$




Q. A system has an impulse response
$h(n) = (0.5)^n u(n) + n(0.2)^n u(n)$
Realize the system using parallel form

multiply by exponential sequence property
$a^n \cdot x(n) \xleftrightarrow{Z.T} X(z/a)$

$Z.T[a^n u(n)]$
$= \frac{z}{z-a}$

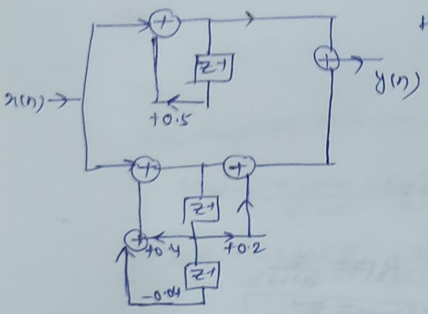
$n a^n u(n) \to \frac{az}{(z-a)^2}$

using z transform
$h(n) \longrightarrow H(z)$

$$H(z) = \underbrace{\frac{z}{(z-0.5)}}_{H_1(z)} + \underbrace{\frac{0.2z}{(z-0.2)^2}}_{H_2(z)}$$

$$= \frac{1}{(1-0.5z^{-1})} + \frac{0.2z}{z^2+0.04-0.4z}$$

$$= \frac{1}{1-0.5z^{-1}} + \frac{0.2z^{-1}}{1-0.4z^{-1}+0.04z^{-2}}$$




$$H(z) = \frac{1}{(1-0.5z^{-1})} + \frac{0.2z^{-1}}{1-0.4z^{-1}+0.04z^{-2}}$$

Q- $H(z) = \dfrac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}$

26.08.2023

1

$$H(z) = \frac{z^{-2} + 2z^{-1} + 1}{3z^{-2} + 3z^{-1} + 1}$$

| | | | |
|---|---|---|---|
| $z^{-2}$ | 1 | 2 | 1 |
| $z^{-2}$ | 3 | 3 | 1 |
| $z^{-1}$ | 1 | $\frac{2}{3}$ | 0 |
| $z^{-1}$ | 1 | 1 | |
| $1 = z^0$ | $-\frac{1}{3}$ | 0 | |
| $1 = z^0$ | 1 | | |

Solution

$\frac{3\times2 - 3\times1}{3} = 1 \qquad \frac{3\times1 - 1\times1}{3} = \frac{2}{3}$

$\frac{1\times3 - 3\times\frac{2}{3}}{1} = 1 \qquad \frac{1-0}{1} = 1$

$\frac{\frac{2}{3} - 1}{1} = \frac{2-3}{3} = \frac{-1}{3}$

$\frac{-\frac{1}{3}\times1 - 0}{1/3} = 1$

$\alpha_0 = \frac{1}{3} \qquad \beta_2 = -3$

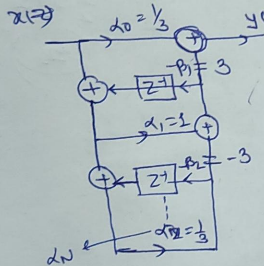
$\beta_1 = 3 \qquad \alpha_2 = -1/3$

$\alpha_1 = 1$

$\alpha_0 = \frac{a_N}{b_N} \qquad \beta_1 = \frac{b_N}{c_{N-1}}$

$\alpha_1 = \frac{c_{N-1}}{d_{N-1}} \qquad \beta_2 = \frac{d_{N-1}}{e_{N-1}}$

$\alpha_2 = \frac{e_{N-1}}{f_{N-1}}$



By continued fraction method

$$H(z) = \frac{2z^{-2}+2z^{-1}+1}{z^{-2}+4z^{-1}+2}$$

$z^{-2}+4z^{-1}+2 \overline{)\, 2z^{-2}+2z^{-1}+1 \,(} \; 2 \; - \alpha_0$

$\quad \underline{2z^{-2} \pm 8z^{-1} \pm 4}$

$\quad -6z^{-1}-3 \,)\, z^{-2}+4z^{-1}+2 \,( \; -1/6\, z^{-1} \; - \beta_1$

$\qquad \underline{z^{-2} + \frac{1}{2}z^{-1}}$

$\qquad \frac{7}{2}z^{-1}+2 \,)\, -6z^{-1}-3 \,( \; -12/7 \; - \alpha_1$

$\qquad\quad \underline{-6z^{-1} - \frac{24}{7}}$

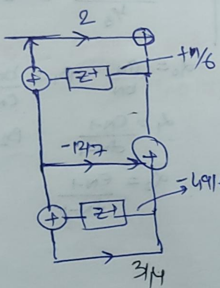
$\qquad\quad \frac{3}{7} \,)\, \frac{7}{2}z^{-1}+2 \,( \; 49/6\, z^{-1} \; \beta_2$

$\qquad\qquad \underline{\frac{7}{2}z^{-1}}$

$\qquad\qquad 2 \,)\, \frac{3}{7} \,( \; 3/14 \; \alpha_2$

$\qquad\qquad\quad \underline{3/7}$

$\qquad\qquad\quad 0$




H.W $\quad H(z) = \dfrac{2+8z^{-1}+6z^{-2}}{1+8z^{-1}+12z^{-2}}$