Physarum-based network optimization

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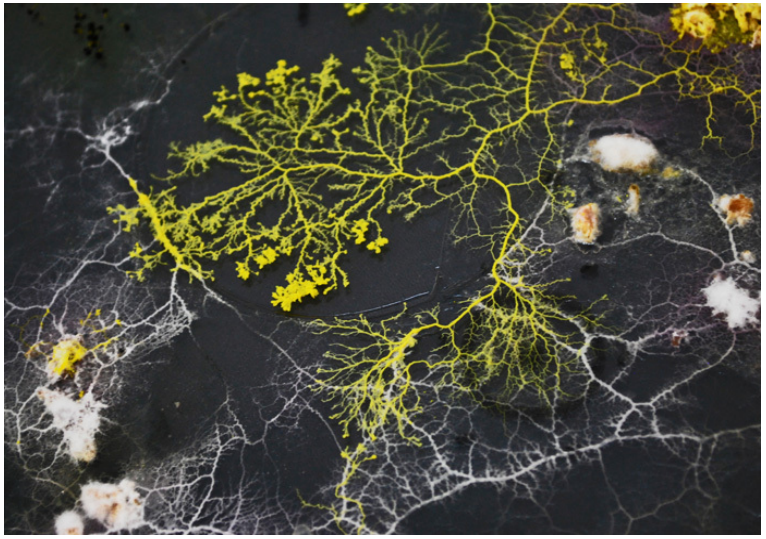
Bio-inspired algorithms

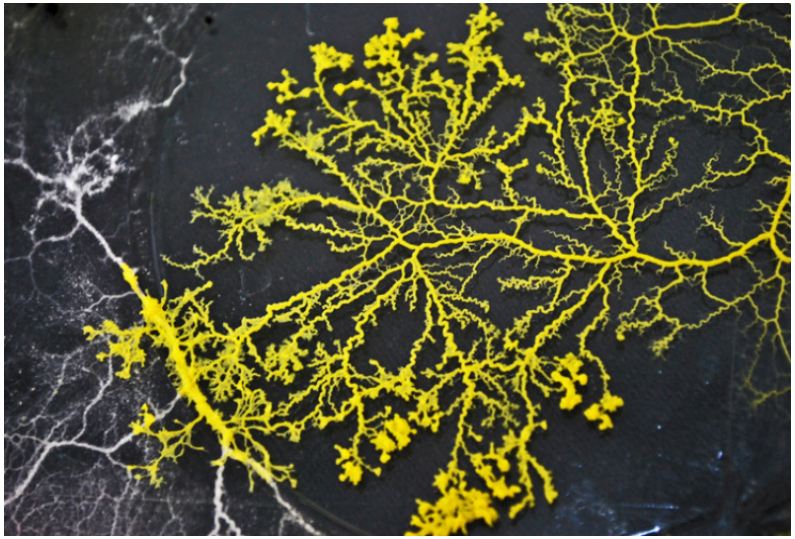
- ✓ Biological evolution ⟹ Genetic Algo
- ✓ Ant foraging ⟹ Ant Colony Algo
- ✓ Bird foraging ⟹ Particle swarm Algo
- ✓ Biological neuron ⟹ Neural network Algo

All these famous bio-inspired algorithms are proposed and applied for optimal solution in typical operational research problems or their variations because of amazing performance.

What is Physarum?

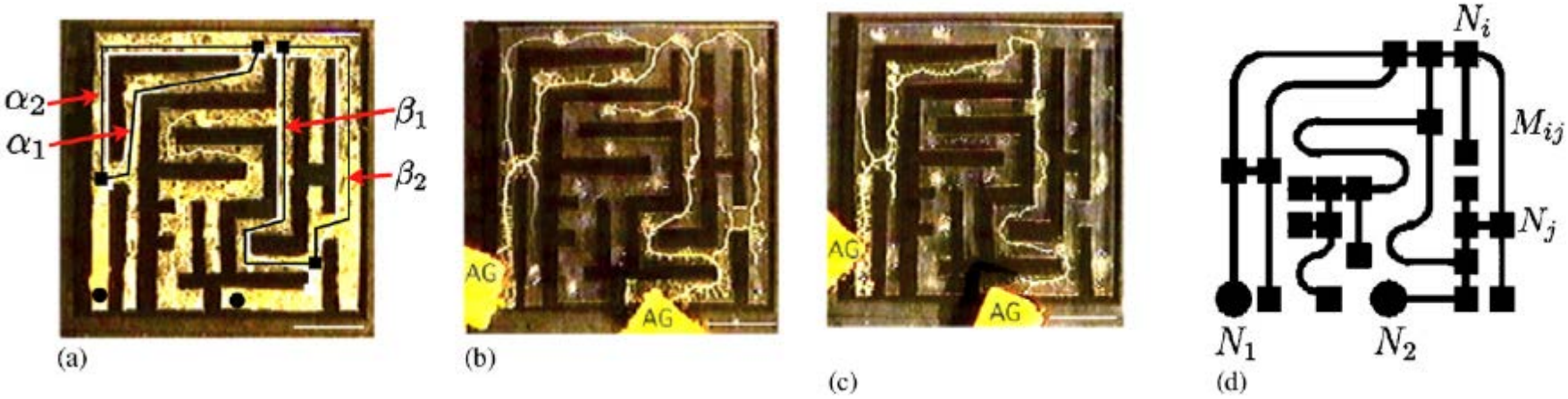
Slime mold Physarum: a large amoeboid organism with intracellular structure consisting of a network of tubes.





High competency

- Physarum drastically alters its tubular topology and flux distribution in respond to external stimulation and environment changes to maximize the efficiency of nutrient absorption and signal circulation.

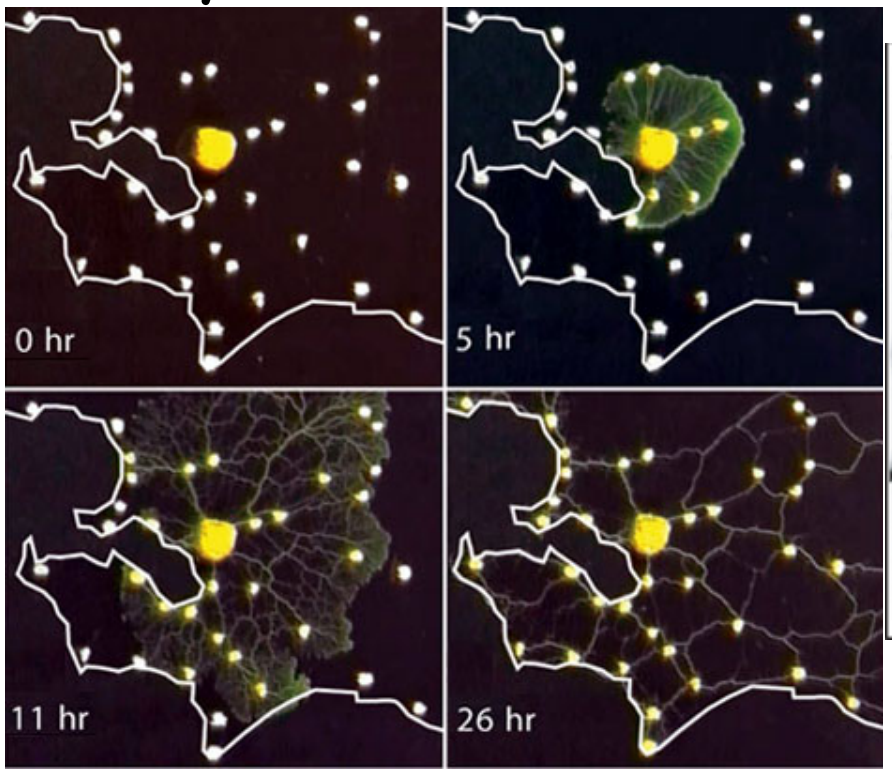


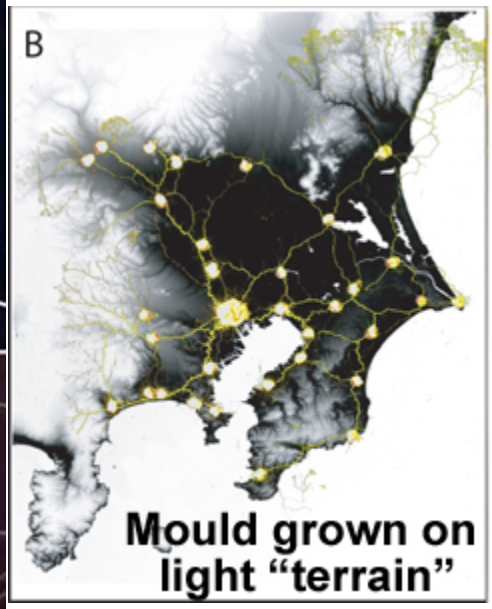
High competency

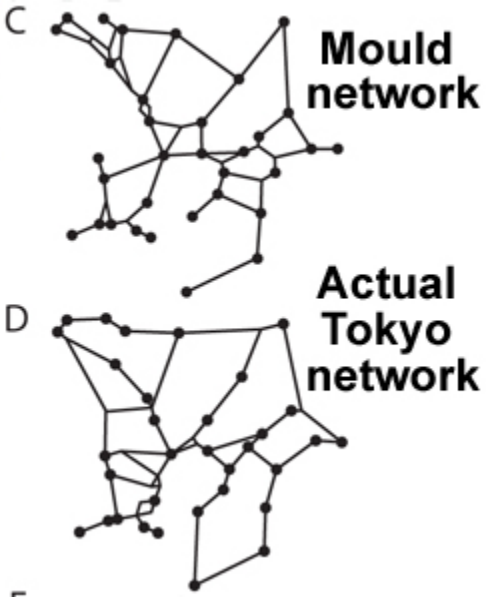
- (Tero et al, Science, 2010) In medium, feed the Physarum at the places which simulate the layout of cities around Tokyo, Physarum forms a self-organized network, whose fault tolerance, robustness and transportation efficiency are well-matched with the real rail system of Tokyo.

High competency

- Physarum simulation **vs** Real Tokyo rail system







High competency

- Subsequent research: Physarum has been showing remarkable intelligent performance in solving optimization problems, such as traveling salesman problem(Zhu et al.2013), maze solving(Tero et al.2007), 0-1 knapsack problem(Zhang et al.2013) and traffic optimization(Watanabe et al.2011).

High competency

- The essence of Physarum, I think, must be:

Network optimization

Extracted original model

$$Q_{ij} = \frac{\pi a_{ij}^4}{8\kappa} \frac{p_i - p_j}{L_{ij}} = \frac{D_{ij}}{L_{ij}} (p_i - p_j).$$

Poiseuille flow

$$\sum_i Q_{i1} + I_0 = 0, \quad \sum_i Q_{i2} - I_0 = 0,$$

$$\sum_i Q_{ij} = 0 \quad (j \neq 1, 2).$$

Conservation law

$$\sum_i \frac{D_{ij}}{L_{ij}} (p_i - p_j) = \begin{cases} -1 & \text{for } j = 1, \\ +1 & \text{for } j = 2, \\ 0 & \text{otherwise.} \end{cases}$$

Poisson equation

$$\frac{\mathrm{d}}{\mathrm{d}t} D_{ij} = f(|Q_{ij}|) - rD_{ij},$$

Conductivity evolution

Our own advanced model

- Fit for directed graphs and multi-source multi-sink demands rather than undirected graphs and single-source single-sink conditions, which Tero proposes.

$$\sum_i \left(\frac{D_{ij}}{L_{ij}} + \frac{D_{ji}}{L_{ji}}\right)(p_i - p_j) = \begin{cases} -I_{S_j} & for\ each \quad S_j \in S^{(\zeta)}, \\ +O_{D_j} & for\ each \quad D_j \in D^{(\zeta)}, \\ 0 & otherwise. \end{cases}$$

$$Q_{ij} = \begin{cases} \frac{D_{ij}(p_i - p_j)}{L_{ij}} & for\ \ p_i \geq p_j, \\ 0 & for\ \ p_i < p_j, \end{cases} \qquad \frac{dD_{ij}}{dt} = \begin{cases} f(|Q_{ij}|) - \psi D_{ij} & for\ \ Q_{ij} \geq 0, \\ -\psi D_{ij} & for\ \ Q_{ij} < 0. \end{cases}$$

Our Work

Solving

A: **Shortest Path Tour Problem（SPTP）**

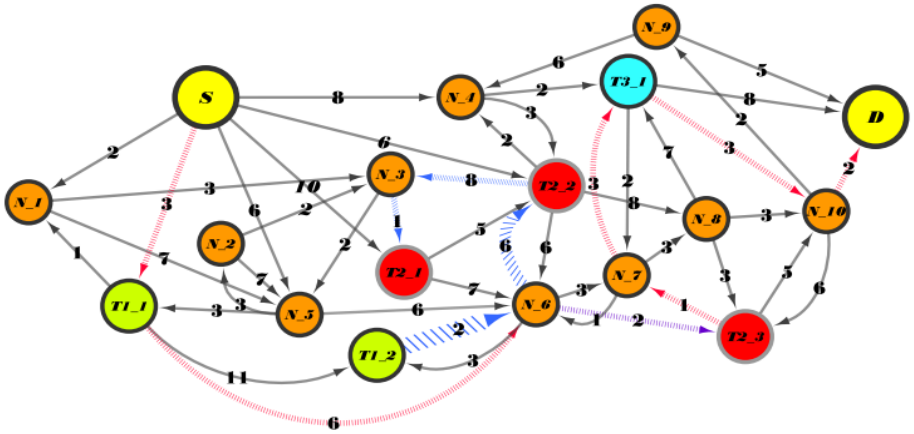
B: **Linear Transportation Problem （LTP）**

SPTP

- A new operational research problem (Festa et al , EJOR , 2013) : find the shortest path between a given pair of nodes in directed weighted graph. Such path extends with the constraint of successively and sequentially passing through all given disjoint node subsets.
- The solution of SPTP directs multilevel process or transformation of information and physical goods with kinds of path costs. In production process chain, raw materials are transported from places of origin, successfully processed under the constrain of procedure order by processing plants, which are responsible for different process degree, and ultimately become finished goods which will be sent to commercial agents.

SPTP

- Our model takes advantage of polynomial time reduction result, a modified direct graph Gw, based on which, a number of sink-constraint shortest path trees (SCSPTs) of Physarum are simultaneously constructed through parallel processing module and then the application of directed Physarum algorithm in Gw outputs the optimal result. The complexity of this model is O(n3).

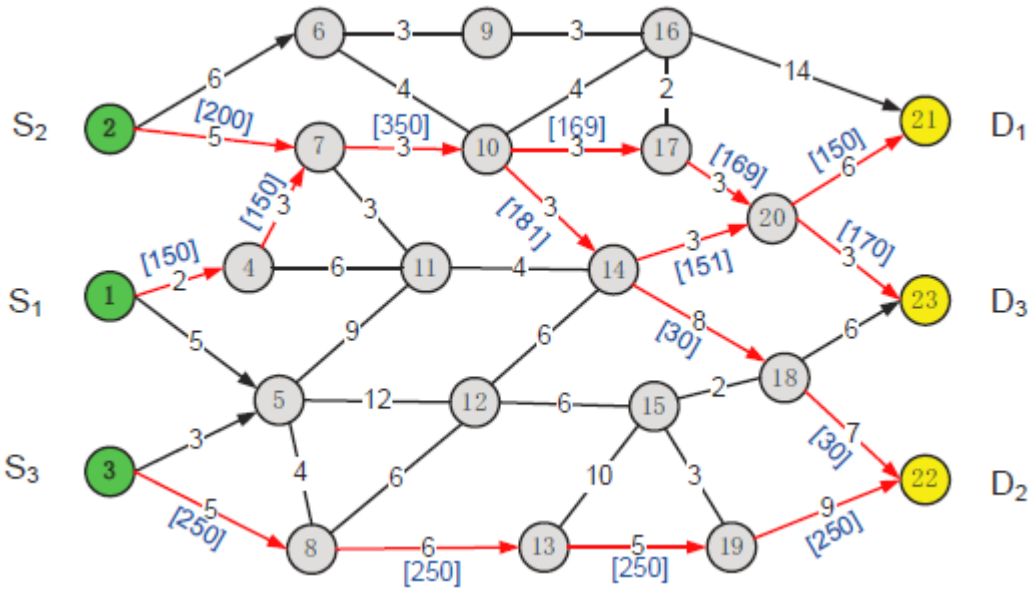


LTP

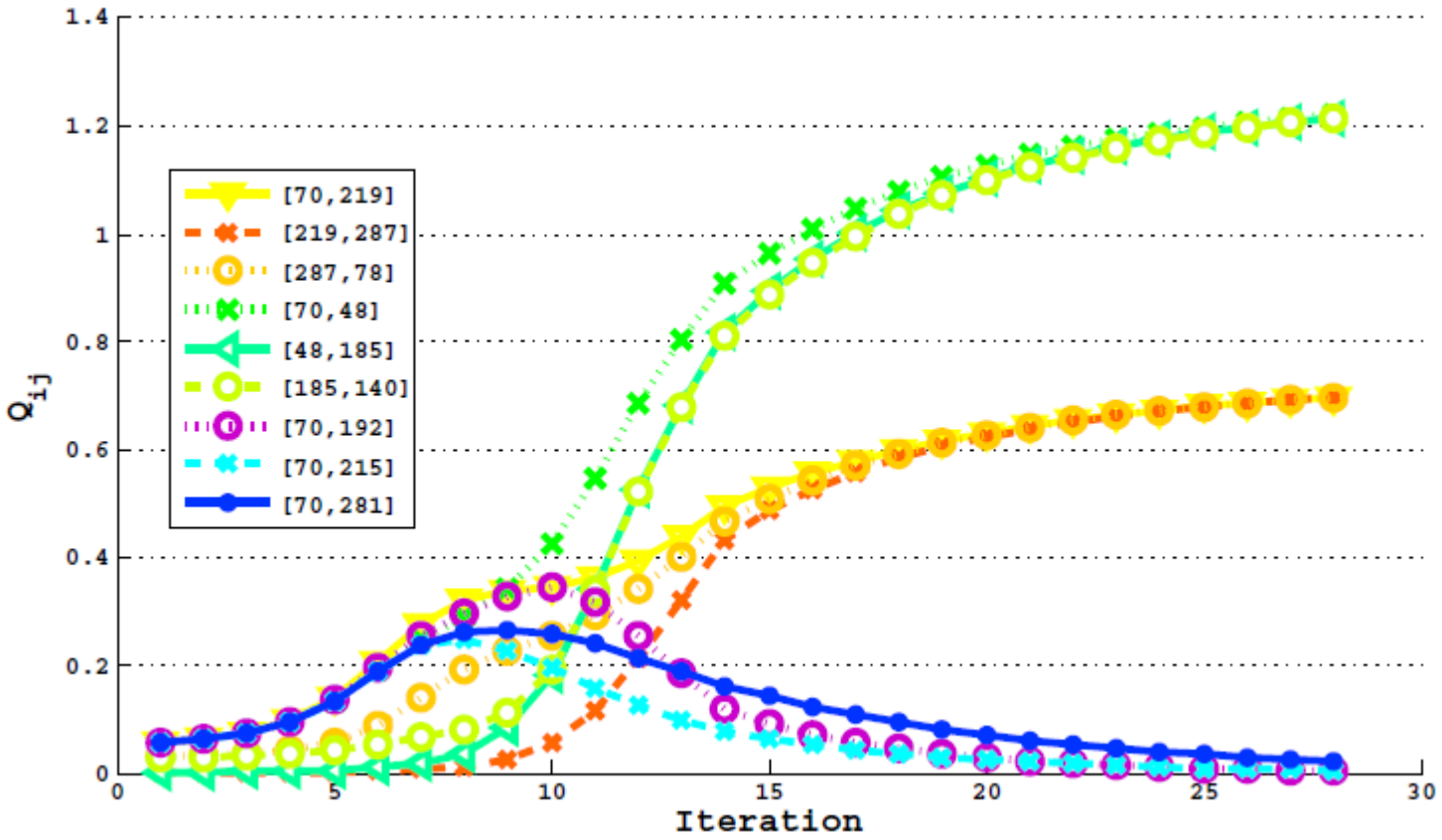
- In transportation problem, if the cost on a route is directly proportional to the amount of the shipment, the problem is LTP. The problem seeks for the least total cost of global transportation from multiple supply sources to multiple demand destination, which is the status of the most efficient resource distribution.
- By applying multi-source multi-sink Physarum algorithm, the optimal transportation strategy with minimum total cost can be gained. The complexity of this model is O(n3).

LTP

- Illustrate example



Convergence process



New Physarum model: a bio-inspired algorithm for shortest path tour problem

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Abstract

A bio-inspired model is proposed to solve shortest path tour problem(SPTP) in this paper, whose goal is to find the shortest path between a given pair of nodes in directed graph with nonnegative arc costs. The path extends with the constraint of successively and sequentially passing through all given disjoint node subsets. The characteristics and properties of the slime mould *Physarum Polycephalum*, a large single-celled amoeboid organism, determine its high competency level for solving SPTP. The proposed model takes advantage of polynomial time reduction result, a modified direct graph, based on which, a number of sink-constraint shortest path trees (SCSPTs) of Physarum are simultaneously constructed through parallel processing module and then the application of directed *Physarum* algorithm in the modified

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The optimal path tour problem

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Abstract

Solving variations of classical operational research problems, especially transportation problem (TP) and supply chain network (SCN) design, which are concerning optimally allocating scarce resources to new demanding requirements, has been becoming central in research community. In this paper, we define and study the optimal path tour problem ($\mathcal{OPTP}$), whose goal is to select a set of paths and to decide their corresponding loads from a given source node subset to a given sink node subset in a directed weighted graph. The constraint of such paths is that they have to sequentially and successively pass through all given disjoint node subsets. The optimal transportation strategy guarantees the least total cost of the transportation under certain supply-demand conditions. Two kinds of $\mathcal{OPTP}$ are modeled respectively

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Our future research

- Modify Physarum algorithm to make it fit for optimization problem in uncertain or dynamic environment, which asks for the optimal strategy immediately.
- Simulate competition of two networks with Physarum.

Thanks ~.~