Adjacency Hyperedges Matrix,
A Hypergraph Model for Constructing
Composite Objects Relationship

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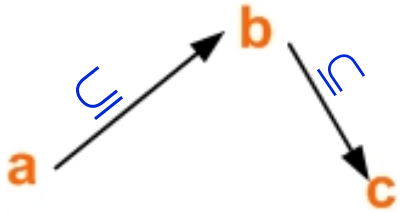
Problem definition

Objects within database potentially **have relationship**.

Can we identify and construct the relationship that called as:

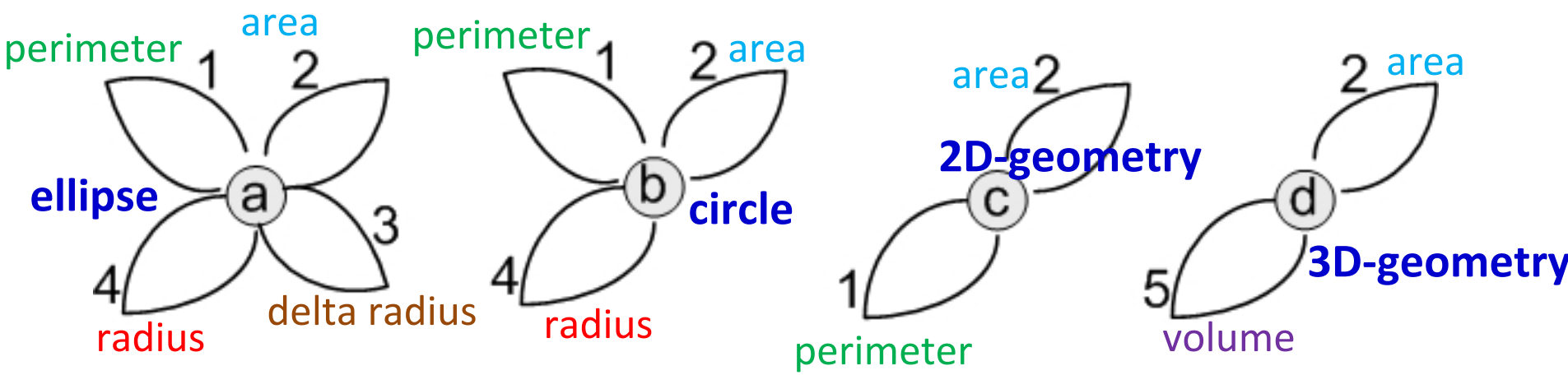


composite objects relationship: An object ***covers*** (superset, ⊆) other object(s) based-on their ***set of object features***.

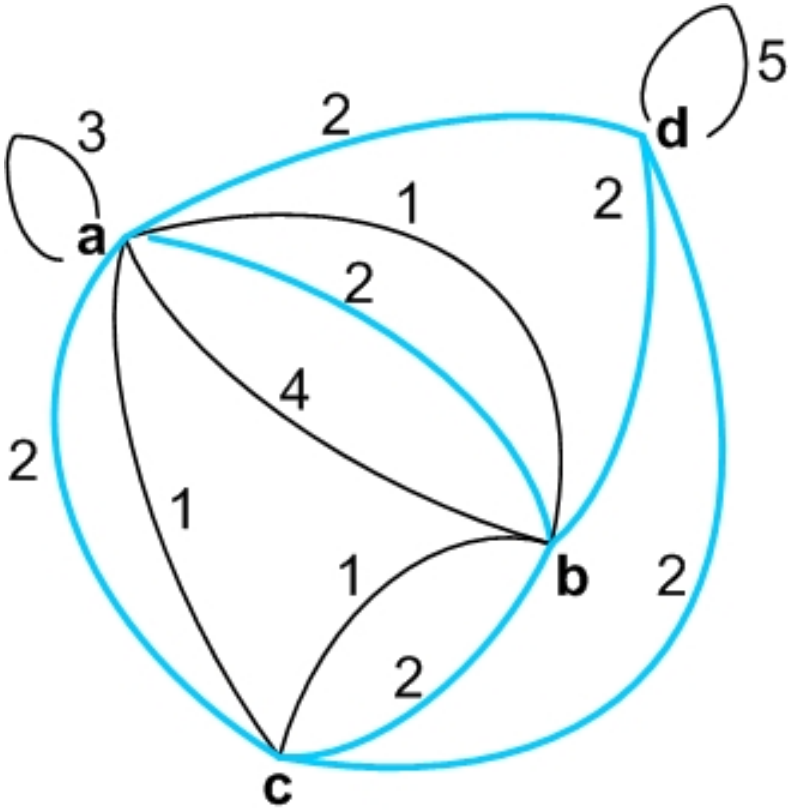
An illustration.

Objects within database:

ellipse, **circle**, **2D-geometry**, and **3D-geometry**.



Relationship at feature level



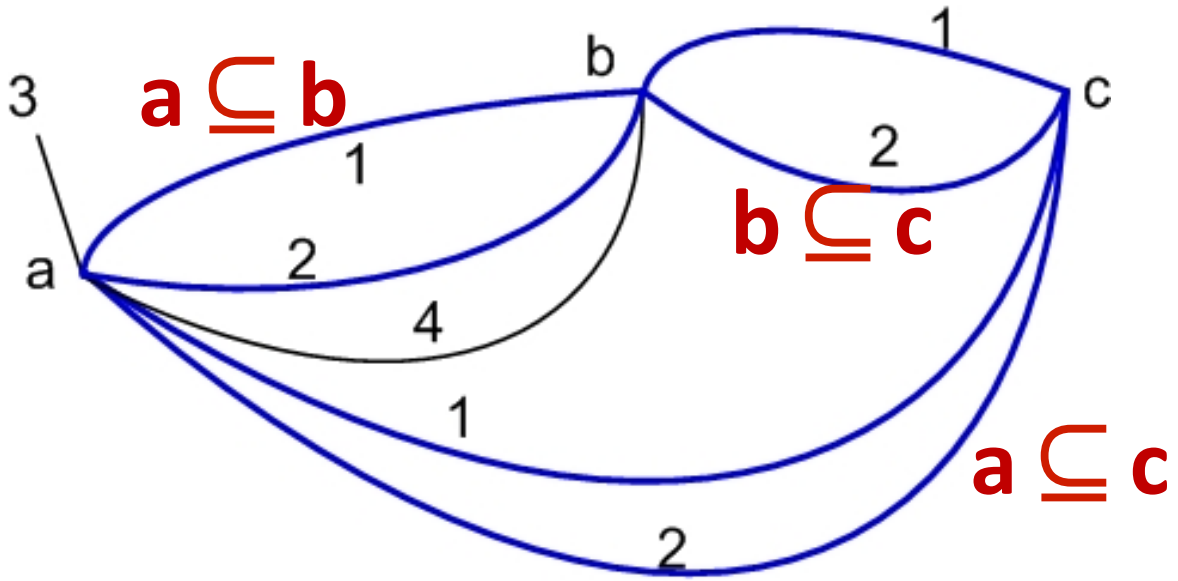
Objects a, b, c, and d, are connected at feature 2

form a **complete graph (clique)**

Objects Relationship

based-on

set of feature: superset operation ⊆



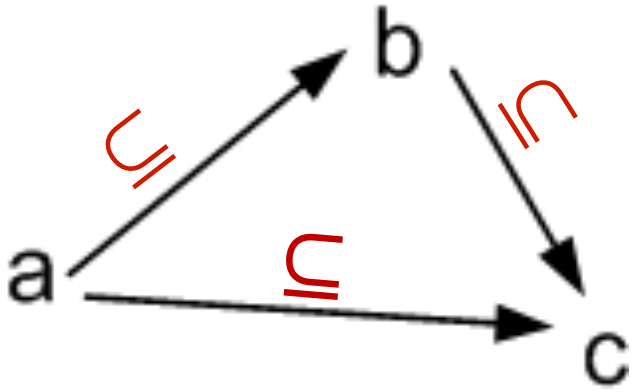
Only objects a, b, and c, form (exclude d)

composite objects relationship

based-on **superset** operation

Composite objects relationship at object level: Abstracted (higher) level

Set of feature **{1, 2}** represents **object c** (2D-geometry), **a** = **{1, 2, 3, 4}**, **b** = **{1, 2, 4}**



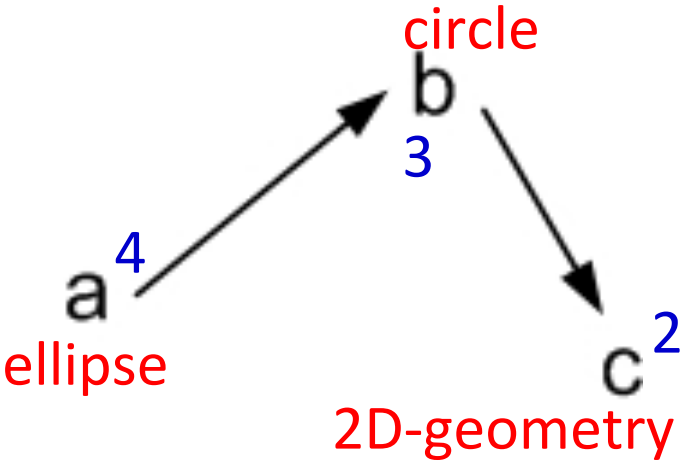
Composite object relationship is

poset (**partially order set**)

Transitive path, **edge** (a,c) can be **reduced (edge-induced)**

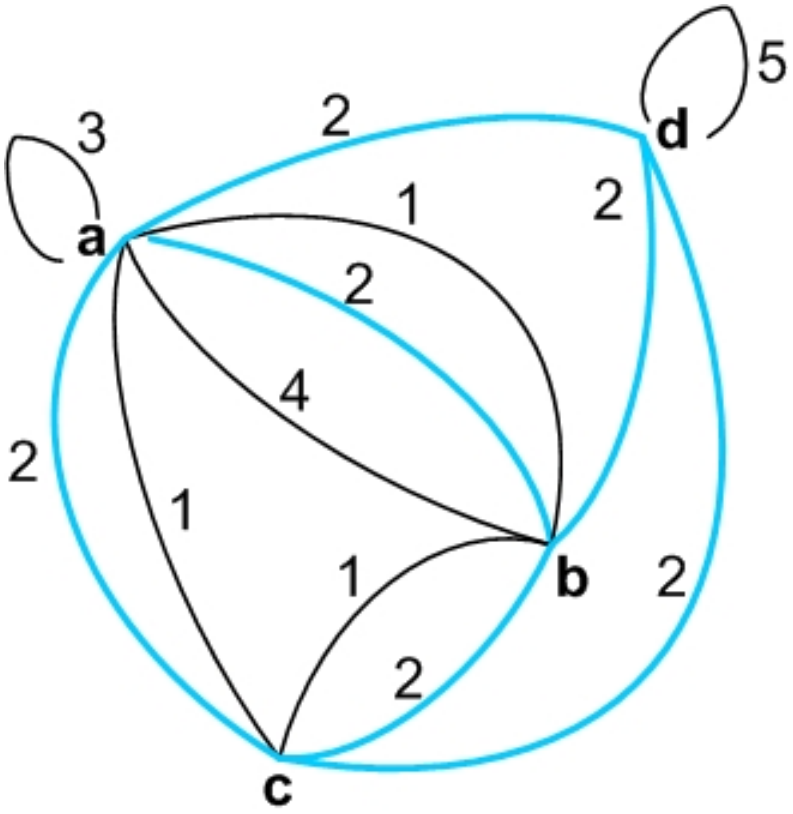
Optimum cost traversing

Each object has **longest path connection**, as **Hasse diagram**



Optimum cost traversing for each longest path based-on vertex degree and still preserve poset relation

Why we need alternative graph representation?



In 2-graph, each edge only connects two vertices

We need **edge** can **connect more than two vertices:**

Hypergraph

Laplacian Hypergraph:
Adjacency Hyperedges Matrix, A_e

S = {a, b, c, d} = {{1, 2, 3, 4}, {1, 2, 4}, {1, 2}, {2, 5}}

F = {1, 2, 3, 4, 5} = {
{a, b, c}, {a, b, c, d}, {a}, {a, b},
{d}}

Hypergraph H = (S, F)

	1	2	3	4	5
a	1	1	1	1	
b	1	1		1	
c	1	1			
d		1			1

d(c) = 2

δ(1) = 3

Laplacian hypergraph of H

	1	2	3	4	5
1	**3**	3	1	2	
2	3	**4**	1	2	1
3	1	1	**1**	1	
4	2	2	1	**2**	
5		1			**1**

A_e

D_e

$|1 \cap 4| = 2$

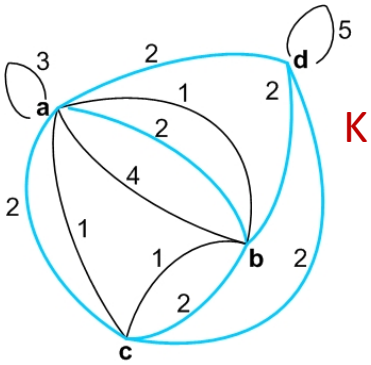
Objects Connection: Partially similar, P_K

$K \subseteq S$ from $H = (S, F)$

$$n(K) > |F_K|$$

$$\exists e_i \epsilon F_K, \delta(e_i) > 1$$

Stepping Solution of Our Model



$K \subseteq S = \{a, b, c, d\}$

Incident matrix (object and feature)

⇓

$H = (S, F)$

⇓

$H(K) = (S, F(K)), K \subseteq S$

⇓

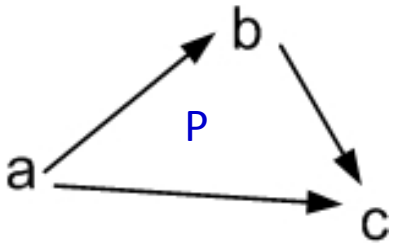
$H_K = (K, F_K)$

⇓

K is partially similar

Hypergraph construction

Multi-objects connection



P ⊆ K is composite objects form as poset (P, ⊆)

$$\pi_v(u) = \begin{cases} v = u, & identical\ and\ d(v) = d(u) \\ v \supset u, & antisymmetri\ and\ d(v) > d(u) \end{cases}$$

Composite object construction

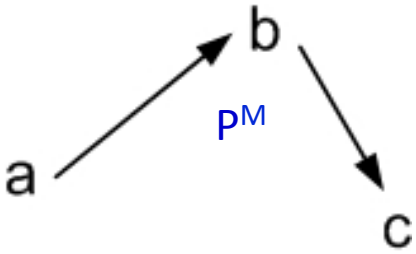
⇓

Q ⊆ P is **identical composite objects** (if any), as P^Q

⇓

P^M is **longest path** of P^Q as Hasse diagram

Reducing of identical objects and transitive path



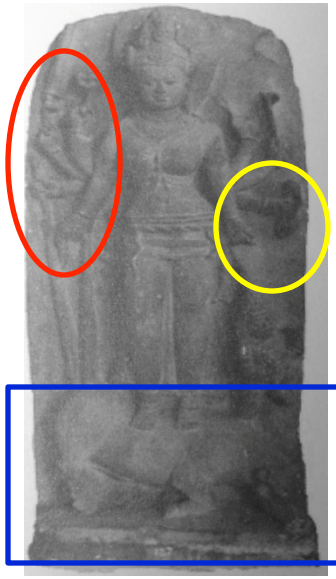
⇓

K_{P^M} **number of forest graphs** (posets and isolated objects)

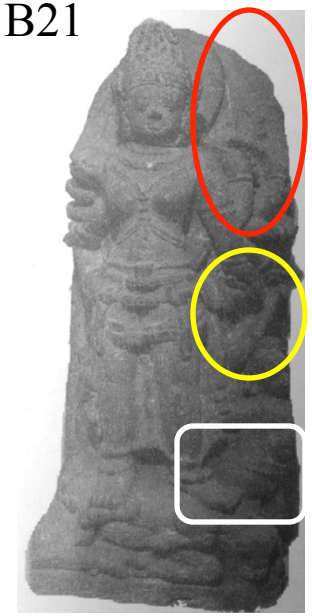
Forest graph configuration

1. {a, b, c}
2. {d}

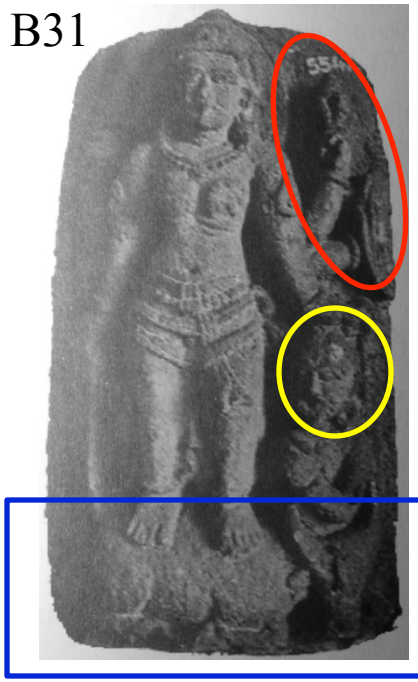
B01



B21



B31



B26

B61

B60

Non-uniform object features, there is a feature not exist on an object in reality (not missing).

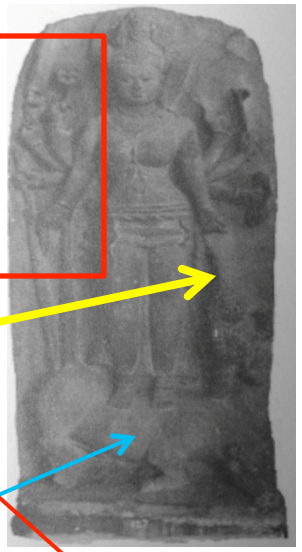
An **object feature** represented as **pair of <feature, feature-value>**

Object feature representation

Share common feature
"Number of hands"

10

8



"Demon figure"

"Lion figure"

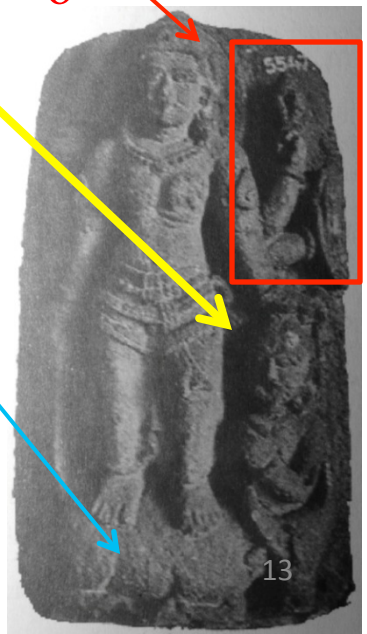
4

"Durga stands on buffalo"

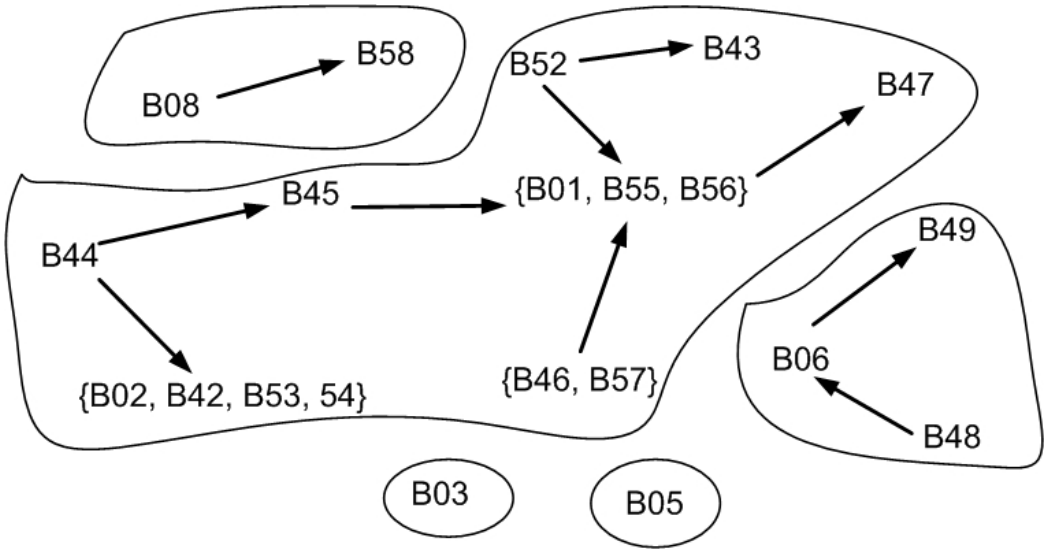
6

An **Object Feature** represented as **pair of <feature, feature-value>**

Each feature and feature-value are ***encoded*** *as* ***categorical value***

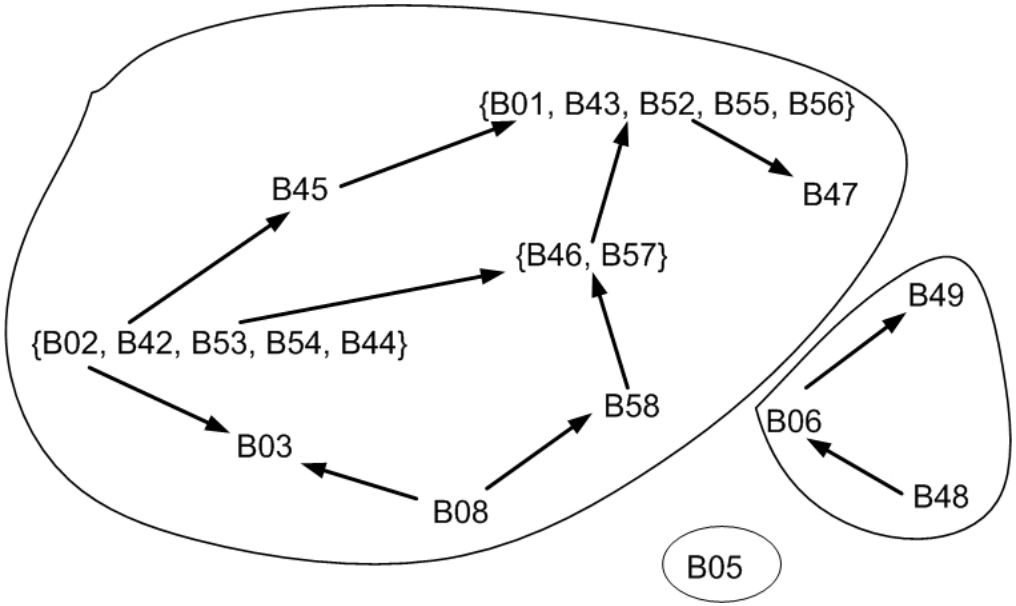


Durga sculptures: original and synthetic objects



Based-on pair of <feature, feature-value>

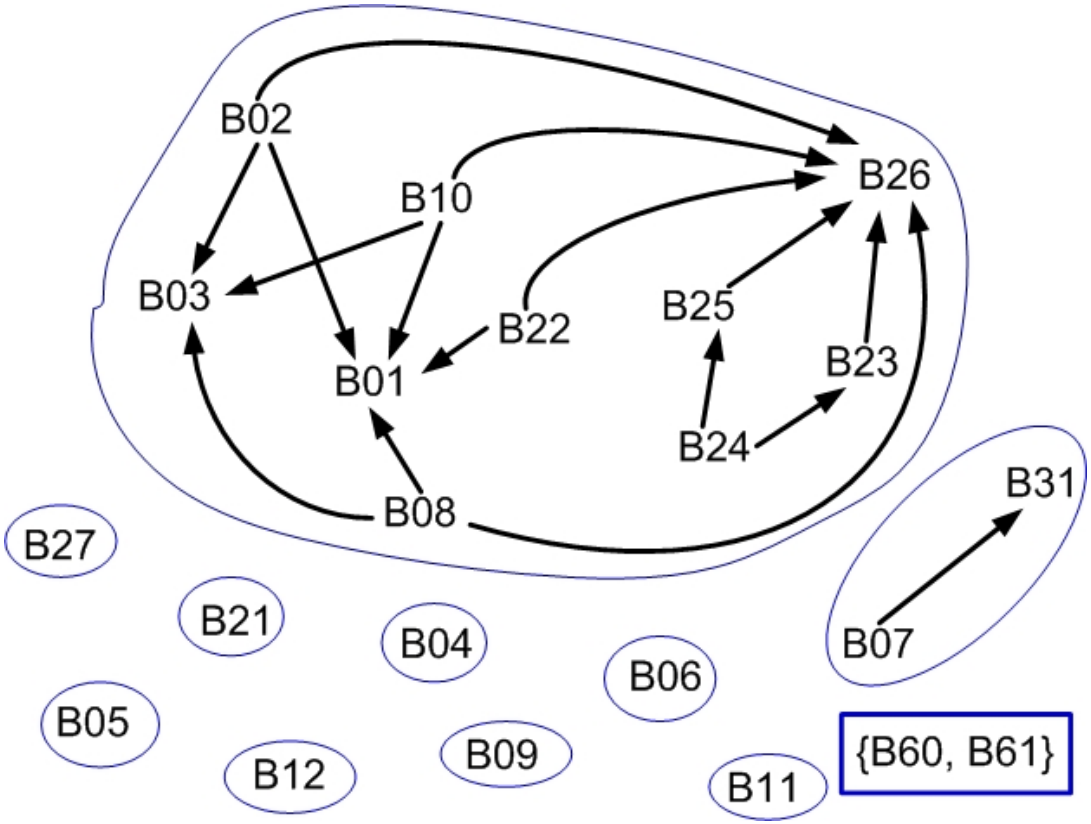
Notes, objects within set "{}" are identical objects based on set of pair <feature, feature-value> or based-on set of feature exclude feature-value.



Based-on feature exclude feature-value

Original objects of Durga and Ganesha sculptures

There is no composite objects relationship based-on pair of <feature, feature-value>, all objects are **isolated objects**



Based-on feature exclude feature-value

Three composite objects composition relationships (posets)

- Eight isolated objects
- **One of posets** is identical objects {B60, B61}, it is **Ganesha cluster**

Conclusions

The proposed model effective **able recognize and construct composite objects relationship** based-on pair of <feature, feature-value>, or based-on feature exclude feature-value.

Each object has longest path connection to other objects.

This model can present maximal group of clusters as **poset**, **isolated objects**, or combination of them together.

Future Works

- Hyperedge represents **encoded feature and feature-value**, the model provides flexibility, and **open possibility** applied on different application domains.

- Explore objects relationship on **objects** as temporal and spatial hypergraph, such as in chemioinformatics.

Future Works

- **Object-oriented modeling** in designing class relationship.

- Constructing composite object relationship by **creating new generated objects as abstract objects** from objects that originally identified only as *partially similar*.

- Use for **indexing in graph database**.

Future Works

Seeking **mathematical formulation** can indicate objects in partially similar potentially form composite object relationship before invoking composite objects construction algorithm.

Parallel computation applies on this model.

Thank You