

Lecture 2: Poly-time Hierarchy - 23 26 May

*Lecturer: Valentine Kabanets**Scribe: Lily Li*

2.1 Introduction

Polynomial hierarchy PH is the generalization of the classes P, NP, and coNP. There are an infinite number of subclasses in PH which are conjectured to be distinct (stronger version of $P \neq NP$). Three definitions of PH are as follows:

1. defined as the set of languages defined via polynomial-time predicates combined with a constant number of alternating for all and exists quantifiers, generalizing the definitions of NP and coNP.
2. defined in terms **alternating TM** which generalize NTM.
3. defined using **oracle TM**.

2.2 Polynomial Hierarchy

Definition 2.1 For $i \geq 1$, a language L is in sup_2^P if there exists a polynomial-time TM M and a polynomial q such that

$$x \in L \iff \exists u_1 \in \{0, 1\}^{q(|x|)} \forall u_2 \in \{0, 1\}^{q(|x|)} \dots Q_i u_i \in \{0, 1\}^{q(|x|)} M(x, u_1, \dots, u_i) = 1$$

where Q_i is a \forall or a \exists depending if i is even or odd. **Polynomial hierarchy (PH)** is $\text{PH} = \cup_i \sum_2^P$.

2.3 Alternating TM

2.4 Oracle Machines