Section 13: Modules and Vector Spaces

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13.1 Definitions

Let R be a ring (i.e. commutative ring with unity).

Definition 13.1 *An **R-module** is a set M together with an addition operation on M and a function $\mu : R \times M \leftarrow M$, such that M under addition forms an abelian group and for all $c, d \in R$ and $\alpha, \beta \in M$:*

Scalar Mult: $\mu(c, \mu(d, \alpha)) = \mu(cd, \alpha)$;

Set Distrib: $\mu(c + d, \alpha) = \mu(c, \alpha) + \mu(d, \alpha)$;

Scalar Distrib: $\mu(c, \alpha + \beta) = \mu(c, \alpha) + \mu(c, \beta)$;

Scalar Id: $\mu(1_R, \alpha) = \alpha$.

You should think of these as more general vector spaces (R need not be a field here).