How to Elect a Leader Faster than a Tournament

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Presenter: Lily Li

CSC 2221: Introduction to Distributed Algorithms

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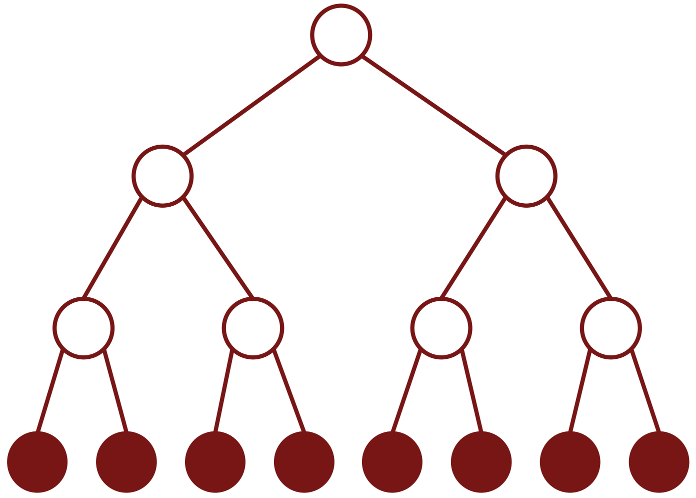
Results New and Old

Tournament Tree

Step Complexity: $O(\log n)$

Message Complexity: $O(n^2 \log n)$



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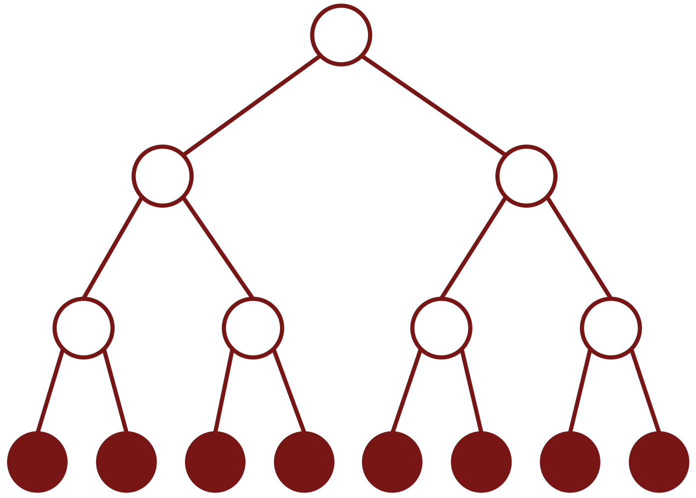

Poison Pill

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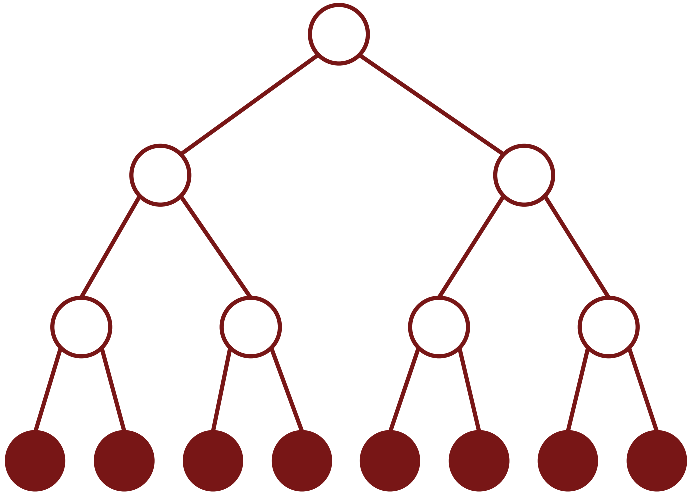

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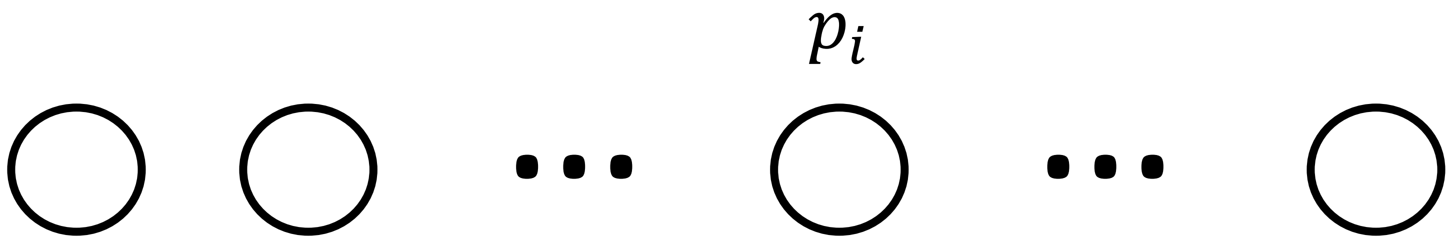
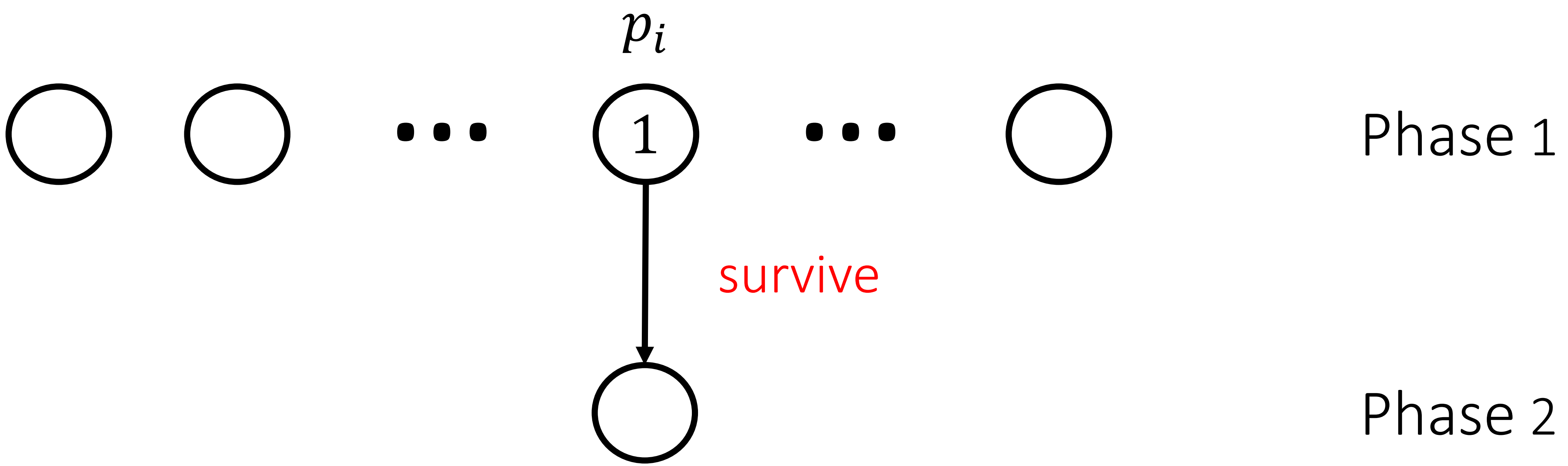
Lower bound: $\Omega(n^2)$

Algorithm Overview



Phase 1

Algorithm Overview



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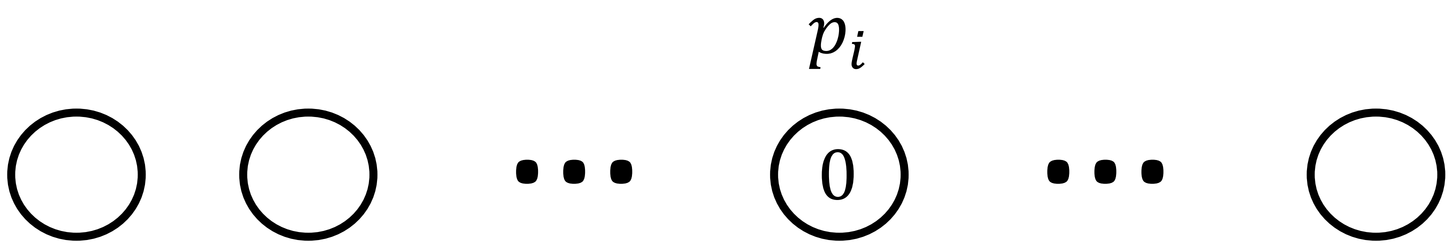

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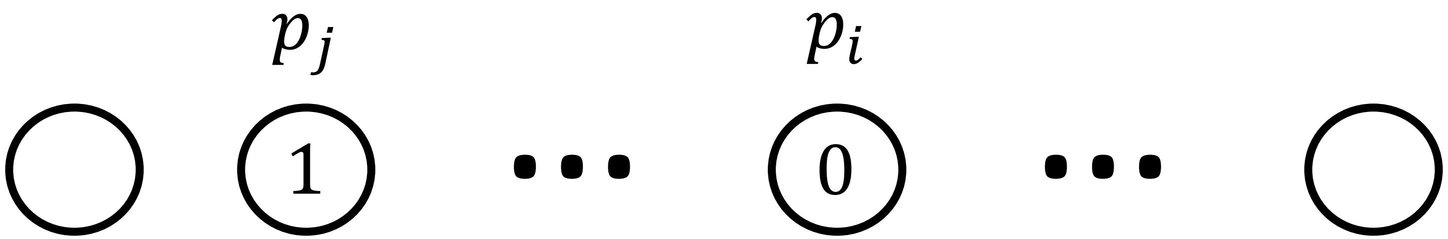

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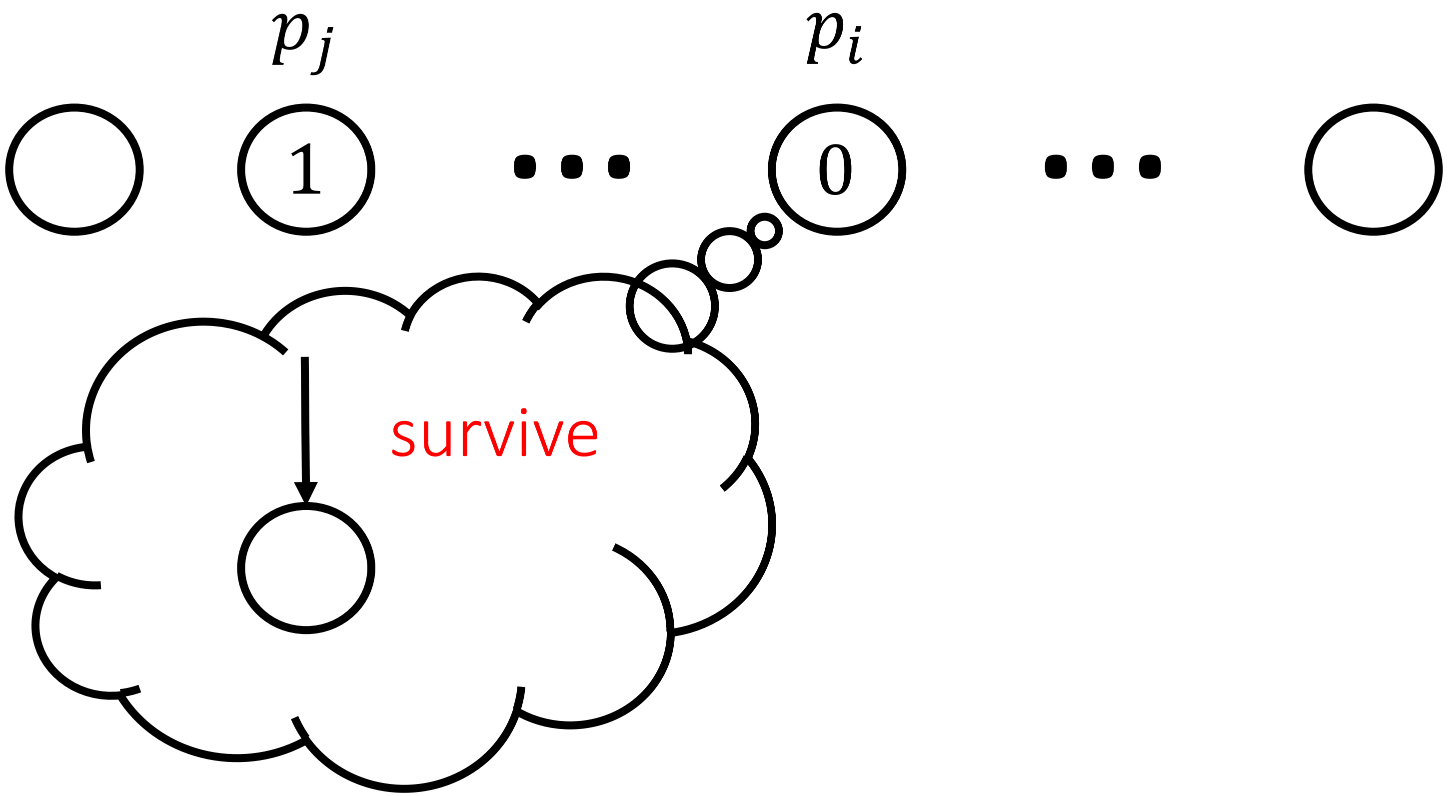
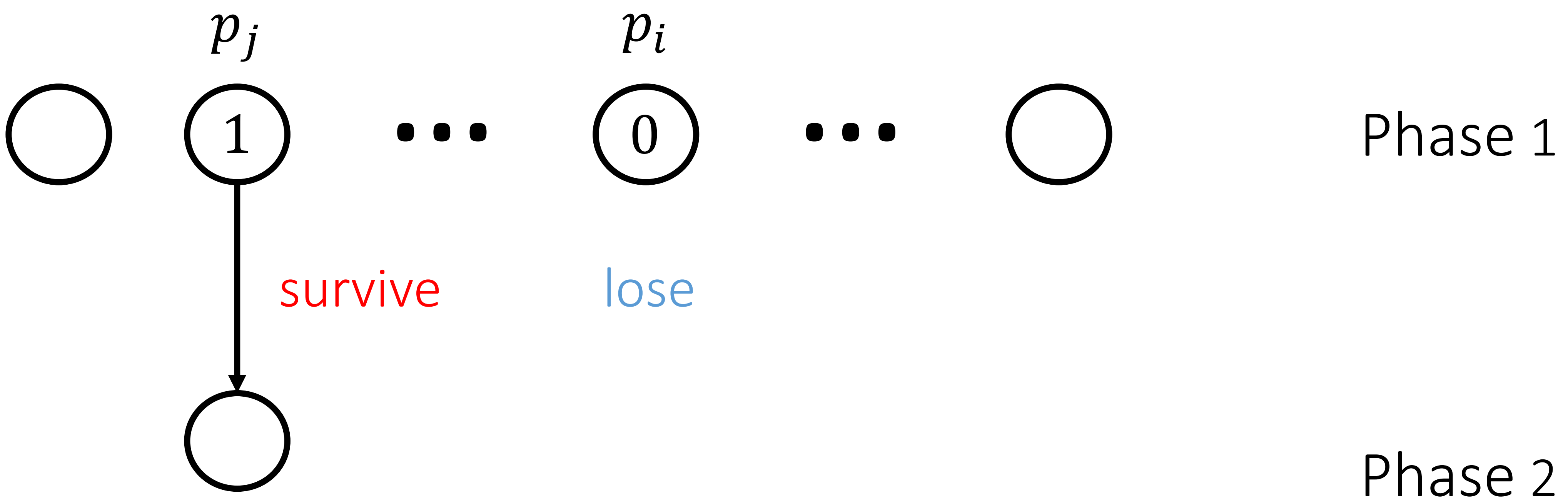


Phase 1

Phase 2

Algorithm Overview



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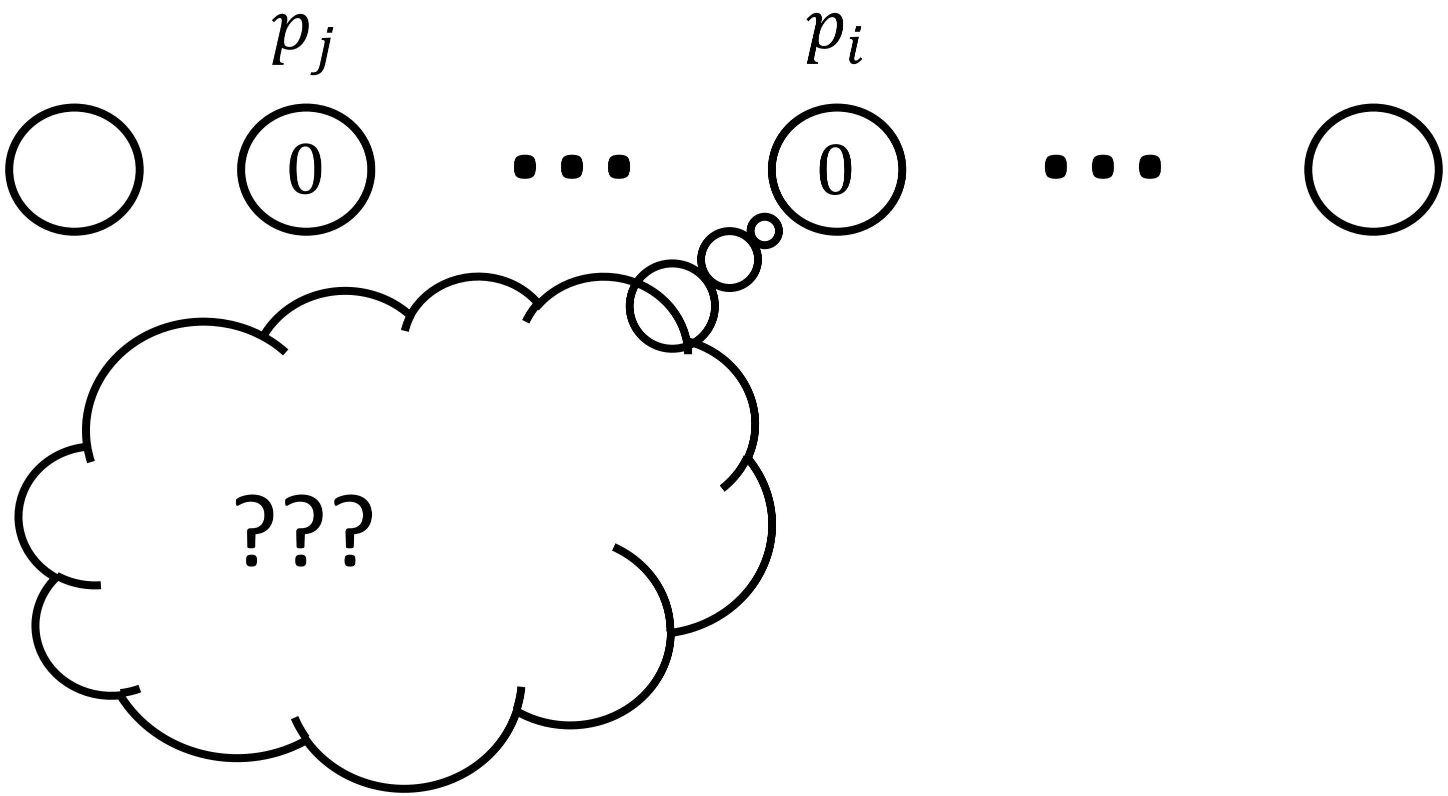

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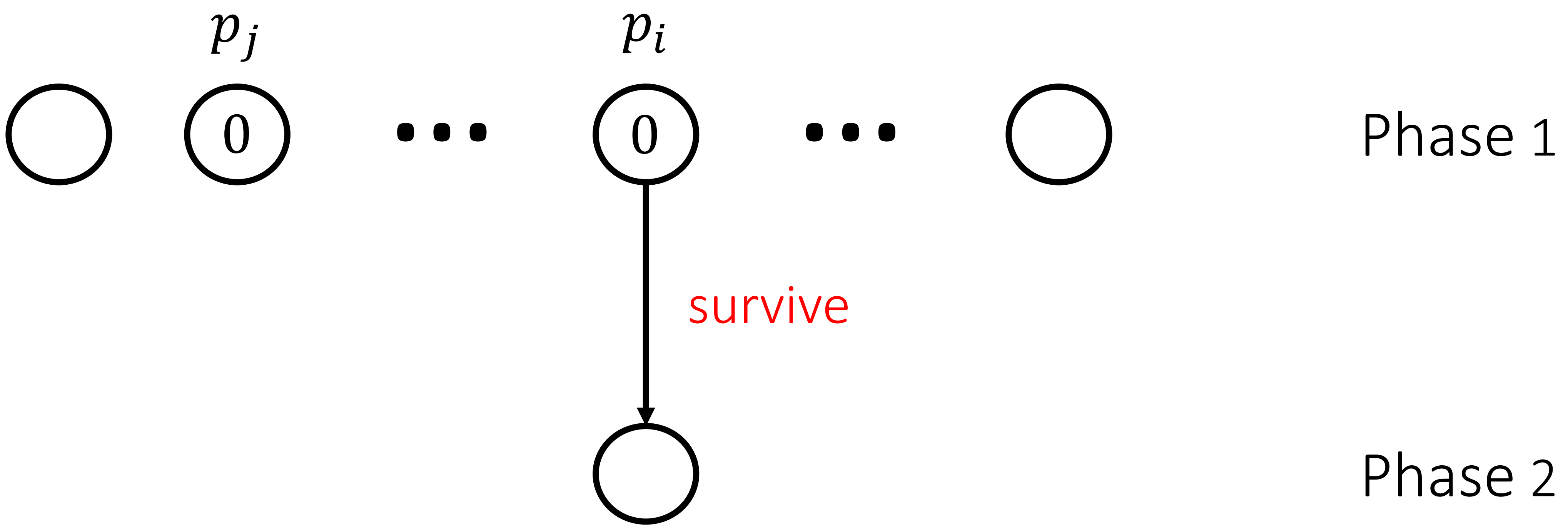

Local Variables and Communication Primitives

Local variables of process p_j:

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High-Level Protocol **`COMMUNICATE<`**`p`**`>`** (where `p` is a procedure which broadcasts a message and wait for a response):

Executes `p` and waits for $\left\lceil \frac{n}{2} \right\rceil + 1$ processes to respond before proceeding.

`p` can be `broadcast(v)` or `collect( )` where

1. If p_j receives `broadcast(v)` from p_i, store $S_j[i] \leftarrow v$ and send p_i $\langle ack \rangle$.
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`random(r):` outputs 1 with probability `r` and 0 with probability `1-r`.

(Simple) Poison Pill

```
Algorithm 1. Code for process p_i during one phase.
Initialize: S_i = [w,...,w], V_i = EMPTY
    S_i[i] = c
    COMMUNICATE<broadcast(S_i[i])>
    S_i[i] = random(1/sqrt(n))
    COMMUNICATE<broadcast(S_i[i])>
    V_i = COMMUNICATE<collect()>
    if(S_i[i] = 0 and exists k: p_k is active)
        return 0
    return 1
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Lemma 1. If a process receives 1 from `random` at time t, then all processes which received 0 from `random` at any time $\geq t$ will return 0.

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This randomized algorithm can be used to solve the tight renaming problem in $\mathrm{O}(\log^2 \mathrm{n})$ steps using $\mathrm{O}(\mathrm{n}^2)$ messages.