CSCI 301 Homework 1

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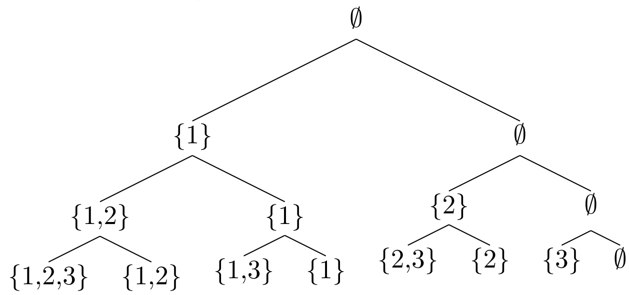
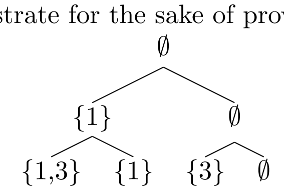
Problem 1 This problem is asking for a set containing every member of the Power Set of {1,2,3} which contains the number 2. It is possible to discover every such set by simply mapping out each subset of {1,2,3} and cherrypicking out the sets which contain two, like so:



Doing so, one can easily pick out the sets containing 2 and make a set of these elements. However, there exists an even simpler alternative: logically, the power set of {1,3} contains every set that does not include the number 2 in the power set of {1,2,3}. To illustrate for the sake of proving that statement:



So adding 2 to each of these sets gives us every subset of {1,2,3} which contains 2. So the set we are looking for is

$$\{X \in P(\{1,2,3\}) : 2 \in X\} = \{\{1,2,3\},\{1,2\},\{2,3\},\{2\}\}$$

Problem 2 The negation of a statement of the form $P \Rightarrow Q$ is $P \wedge \neg Q$. If we allow P, Q, and R represent the logical statements, "x is a rational number," and "$x \neq 0$," and "$tan(x)$ is not a rational number respectively," then it becomes clear that Problem 2 presents such a statement. Specifically, $(P \wedge Q) \Rightarrow R$. Therefore, the negation of $(P \wedge Q) \Rightarrow R$ is $(P \wedge Q) \wedge \neg R$. Translated back into English, this reads "x is a rational number and x does not equal zero, but $tan(x)$ is a rational number.

Problem 3 The set {1,2,3,4,5,6,7} contains four different odd numbers. If they are to be placed first, and must be placed in a sequence, then there are four possible layouts for such an initial placement as follows:

Odd	Odd	Odd	Odd	Even	Even	Even

Table 1: Case 1

Even	Odd	Odd	Odd	Odd	Even	Even

Table 2: Case 2

Even	Even	Odd	Odd	Odd	Odd	Even

Table 3: Case 3

Even	Even	Even	Odd	Odd	Odd	Odd

Table 4: Case 4

So there are is 1 choice (with 4 options) with, followed by 4 choices in which we must decide what numbers will actually be placed. Starting with n options for the first choice, each successive number of options is $n-1$ as we have one less option for the number we shall place each time. So the number of possible lists so far is $4 * 4^{\underline{4}}$. Extending the same logic to the placement of the remaining digits (the positive ones), we find that the remaining possible choices can be quantified by $3^{\underline{3}}$. Thus, the total number of possible lists in this scenario is $4 * 4^{\underline{4}} * 3^{\underline{3}} = 120$.

Problem 4 Note that the sets containing every 4-card hand in which all 4 cards are from different suites and in which all 4 cards are red are *mutually exclusive*. In other words, it is impossible for a 4-card hand of all red cards to *also* be a 4-card hand with one card from each suite, because two of the suites are black. This means that we can simply find all possible hands for each scenario, add them, and the resulting number is the answer. Naturally, there can be no repetition because each card in a deck is unique to that deck.

Suit1	Suit2	Suit3	Suit4

Table 5: Every card is from a different suite.

The first choice has 52 cards to choose from. However, every choice afterwards has 13 less cards because whatever suit was picked last cannot be picked again if the condition is to be satisfied. Thus, for 4 choices, the total amount of possible lists is $52 * 39 * 26 * 13 = 685,464$.

redCard1	redCard2	redCard3	redCard4

Table 6: Every card is red.

Here, the number of possible lists is simply $26^{\underline{4}}$ or 358,800, as the first choice has 26 cards, the second has 25, etc., since repetition is not allowed and half of the cards in the deck are red.

Summing up these possible lists, the total number of lists that meet the question's criteria is:

$$\prod_{i=0}^{n=3} 52 - 13x + 26^{\underline{4}} \tag{1}$$

$$= 1,044,264 \tag{2}$$