NCERT Discrete

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EE23BTECH11050

Question GATE 23 ME 50:
The initial value problem $\frac{dy}{dt} + 2y = 0, y(0) = 1$ is solved numerically using the forward Euler's method with a constant and positive time step of δ.
Let y_n represent the numerical solution obtained after n steps. The condition $|y_{n+1}| \leq |y_n|$is satisfied if and only if δ does not exceed
Solution:
Numerical solution: -
By forward Euler's method formula

$$y(n+1) = y(n) + \delta f(x, y) \tag{1}$$

From question we get

$$\frac{dy}{dx} = -2y = f(x, y) \tag{2}$$

From (2) in (1)

$$y(n+1) = y(n) + \delta(-2y(n)) \tag{3}$$
$$y(n+1) = y(n)(1 - 2\delta) \tag{4}$$
$$|y(n+1)| = |y(n)||1 - 2\delta| \leq |y(n)| \tag{5}$$
$$|1 - 2\delta| \leq 1 \tag{6}$$
$$\implies 0 \leq \delta \leq 1 \tag{7}$$

From this we can say that the maximum value of δ is 1
Theoritical solution: -
By properties of Laplace transform: -

$$Y(s) = \mathcal{L}y(s) \tag{8}$$
$$\mathcal{L}y' = sY(s) - y(0) \tag{9}$$

Given equation: -

$$y' + 2y = 0 \tag{10}$$
$$\mathcal{L}(y' + 2y) = 0 \tag{11}$$

From (8) and (9)

$$sY(s) - 1 + 2Y(s) = 0 \tag{12}$$
$$\frac{1}{s+2} = Y(s) \tag{13}$$
$$y(t) = \mathcal{L}^{-1}Y(s) \tag{14}$$
$$\implies y(t) = \mathcal{L}^{-1}\left(\frac{1}{s+2}\right) \tag{15}$$
$$\mathcal{L}^{-1}\left(\frac{1}{s+k}\right) = e^{-kt}u(t) \tag{16}$$
$$\implies y(t) = e^{-2t}u(t) \tag{17}$$

The difference equation is

$$y(n+1) - y(n) = -2y(n) \tag{18}$$
$$y(n+1) = -y(n) \tag{19}$$
$$Z(y(n)) = Y(z) \tag{20}$$
$$Z(y(n+1)) = zY(z) - y(0) \tag{21}$$
$$\implies zY(z) - y(0) = -Y(z) \tag{22}$$
$$Y(z) = \frac{1}{z+1} \tag{23}$$
$$\frac{1}{z+1} \overset{Z}{\longleftrightarrow} (-1)^n u(n) \tag{24}$$

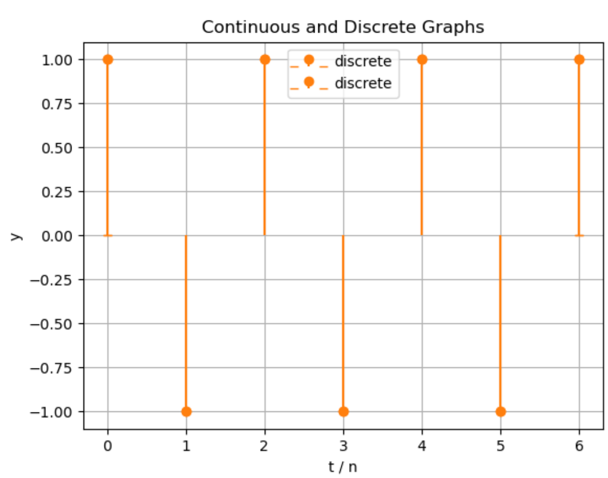

Fig. 1. simulation vs analysis