NCERT Discrete

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Question 11.9.3.18:
Find the sum to n terms of the sequence $8, 88, 888, 8888\ldots$
Solution:

Parameter	Value	description
$x(0)$	8	First term
$x(1)$	88	Second term
$x(n)$	$(\sum_{k=0}^{n} 8(10)^k)u(n)$	General term
$S(n)$	$S(n) = \sum_{k=0}^{n-1} x(k)$	Sum of n terms

TABLE I

INPUT PARAMETERS

From (I)

$$s(n) = x(n) * u(n) \tag{1}$$

Z transform of general term

$$X(z) = \sum_{n=-\infty}^{\infty} (\sum_{k=0}^{n} 8(10)^k)u(n)z^{-n} \tag{2}$$

$$X(z) = \sum_{n=0}^{\infty}(\sum_{k=0}^{n} 8(10)^k)z^{-n} \tag{3}$$

$$\implies X(z) = \left(\frac{8}{(1 - 10z^{-1})(1 - z^{-1})}\right) \quad |z| > 10 \tag{4}$$

From (1), we get

$$S(z) = (X(z))(U(z)) \tag{5}$$

$$S(z) = \left(\frac{8}{(1 - 10z^{-1})(1 - z^{-1})}\right)\left(\frac{1}{1 - z^{-1}}\right) \tag{6}$$

$$S(z) = \left(\frac{8z^2}{(z - 10)(z - 1)}\right)\left(\frac{z}{z - 1}\right) \tag{7}$$

$$S(z) = \left(\frac{8z^3}{(z - 10)(z - 1)^2}\right) \tag{8}$$

$$s(n) = \frac{1}{2\pi j} \oint_c \frac{8z^3}{(z - 10)(z - 1)^2}(z^{n-1})dz \tag{9}$$

$$s(n) = \frac{1}{2\pi j} \oint_c \left(\frac{800z^n}{81(z - 10)} - \frac{8z^n}{9(x - 1)^2} - \frac{152z^n}{8(z - 1)}\right) \tag{10}$$

$$\implies s(n) = \lim_{z\to 10} \frac{1}{0!}\left(\frac{800z^n}{81}\right) - \lim_{z\to 1} \frac{1}{1!}\frac{d}{dz}\left(\frac{8z^n}{9}\right)$$
$$- \lim_{z\to 1} \frac{1}{0!}\left(\frac{152z^n}{8}\right) \tag{11}$$
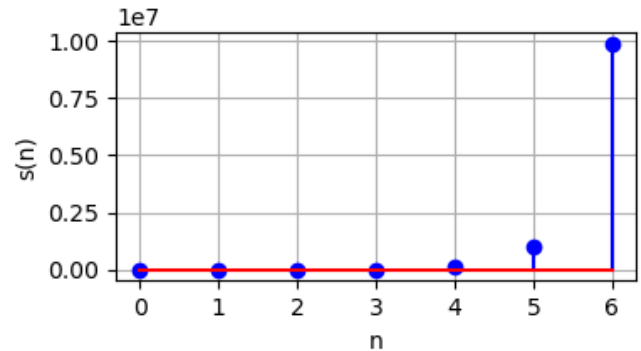
$$s(n) = \left(\frac{8}{81}\right)(10^{n+2} - 9n - 19) \tag{12}$$

Fig. 1. graph of sum of n terms