

NCERT Discrete

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EE23BTECH11050

Question 11.9.3.18:

Find the sum to n terms of the sequence
8, 88, 888, 8888...

Solution:

Parameter	Value	description
$x(0)$	8	First term
$x(1)$	88	Second term
$x(n)$	$(\sum_{k=0}^n 8(10^k) u(n))$	General term
$S(n)$	$S(n) = \sum_{k=0}^{n-1} x(k)$	Sum of n terms

TABLE I

INPUT PARAMETERS

From (I)

$$s(n) = x(n) * u(n) \quad (1)$$

Z transform of general term

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\sum_{k=0}^n 8(10^k) u(n) \right) z^{-n} \quad (2)$$

$$X(z) = 8 \sum_{n=0}^{\infty} \left(\sum_{k=0}^n (10^k) \right) u(n) z^{-n} \quad (3)$$

$$X(z) = 8 \left(\sum_{n=0}^{\infty} (10)^n (z^{-n}) \right) \left(\sum_{n=0}^{\infty} z^{-n} \right) \quad (4)$$

$$\Rightarrow X(z) = \left(\frac{8}{(1 - 10z^{-1})(1 - z^{-1})} \right) \quad |z| > 10 \quad (5)$$

From (1), we get

$$S(z) = (X(z))(U(z)) \quad (6)$$

$$S(z) = \left(\frac{8}{(1 - 10z^{-1})(1 - z^{-1})} \right) \left(\frac{1}{1 - z^{-1}} \right) \quad (7)$$

$$S(z) = \left(\frac{8}{(1 - 10z^{-1})(1 - z^{-1})^2} \right) \quad (8)$$

$$S(z) = \frac{-224}{81(z-1)} - \frac{8}{9(z-1)^2} + \frac{8000}{81(z-10)} + 8 \quad (9)$$

$$S(z) = \frac{-224z(z)^{-1}}{81(z-1)} - \frac{8z(z)^{-1}}{9(z-1)^2} + \frac{8000z(z)^{-1}}{81(z-10)} + 8 \quad (10)$$

$$\delta(n) \xleftrightarrow{Z} 1 \quad (11)$$

$$u(n-1) \xleftrightarrow{Z} \frac{z^{-1}}{1 - z^{-1}} \quad |z| > 1 \quad (12)$$

$$a^{n-k} u(n-k) \xleftrightarrow{Z} \frac{z^{-k}}{1 - az^{-1}} \quad |z| > a \quad (13)$$

$$(n-k) u(n-k) \xleftrightarrow{Z} \frac{z^{-k-1}}{(1 - z^{-1})^2} \quad |z| > 1 \quad (14)$$

From the above 4 equations, we get

$$s(n) = \frac{-224u(n-1)}{81} - \frac{8(n-1)u(n-1)}{9} + \frac{8000(10^{n-1})u(n-1)}{81} + 8\delta(n) \quad (15)$$

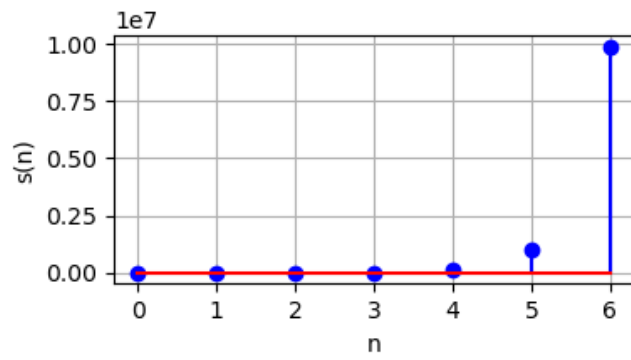


Fig. 1. graph of sum of n terms