

NCERT Discrete

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Question 11.9.3.18:

Find the sum to n terms of the sequence
8, 88, 888, 8888...

Solution:

Parameter	Value	description
$x(0)$	8	First term
$x(1)$	88	Second term
$x(n)$	$(\sum_{k=0}^n 8(10)^k)u(n)$	General term
$S(n)$	$S(n) = \sum_{k=0}^{n-1} x(k)$	Sum of n terms

TABLE I
INPUT PARAMETERS

From (I)

$$s(n) = x(n) * u(n) \quad (1)$$

Z transform of general term

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\sum_{k=0}^n 8(10)^k \right) u(n) z^{-n} \quad (2)$$

$$X(z) = \sum_{n=0}^{\infty} \left(\sum_{k=0}^n 8(10)^k \right) z^{-n} \quad (3)$$

$$\Rightarrow X(z) = \left(\frac{8z^2}{(z-10)(z-1)} \right) \quad |z| > 10 \quad (4)$$

From (1), we get

$$\Rightarrow S(z) = \left(\frac{8z^2}{(z-10)(z-1)} \right) \left(\frac{1}{z-1} \right) \quad (5)$$

$$S(z) = \left(\frac{8z^2}{(z-10)(z-1)^2} \right) \quad (6)$$

$$s(n) = \frac{1}{2\pi j} \oint_c \frac{8z^2}{(z-10)(z-1)^2} (z^{n-1}) dz \quad (7)$$

$$s(n) = \frac{1}{2\pi j} \oint_c \left(\frac{800z^{n-1}}{81(z-10)} - \frac{8z^{n-1}}{9(z-1)^2} - \frac{152z^{n-1}}{8(z-1)} \right) dz \quad (8)$$

$$\Rightarrow s(n) = \lim_{z \rightarrow 10} \frac{1}{0!} \left(\frac{800z^{n-1}}{81} \right) - \lim_{z \rightarrow 1} \frac{1}{1!} \frac{d}{dz} \left(\frac{8z^{n-1}}{9} \right) - \lim_{z \rightarrow 1} \frac{1}{0!} \left(\frac{152z^{n-1}}{8} \right) \quad (9)$$

$$s(n) = \left(\frac{8}{81} \right) (10^{n+1} - 9n - 10) \quad (10)$$

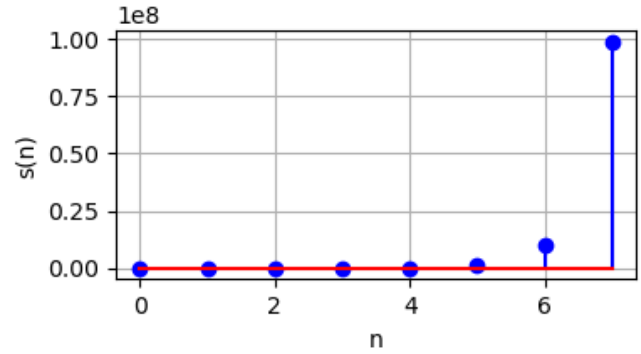


Fig. 1. graph of sum of n terms