NCERT Discrete

Pragnidhved Reddy

EE23BTECH11050

Question 11.9.3.18:
Find the sum to n terms of the sequence $8, 88, 888, 8888\ldots$

Solution:

Parameter	Value	description
$x(0)$	8	First term
$x(1)$	88	Second term
$x(n)$	$(\sum_{k=0}^{n} 8(10)^k)u(n)$	General term
$S(n)$	$S(n) = \sum_{k=0}^{n-1} x(k)$	Sum of n terms

TABLE I

INPUT PARAMETERS

From (I)

$$s(n) = x(n) \circledast u(n) \tag{1}$$

Z transform of general term

$$X(z) = \sum_{l=-\infty}^{\infty} (\sum_{k=0}^{l} 8(10)^k)u(n)z^{-l} \tag{2}$$

$$X(z) = \sum_{l=0}^{\infty} (\sum_{k=0}^{l} 8(10)^k)z^{-l} \tag{3}$$

$$\implies X(z) = \left(\frac{8z^2}{(z-10)(z-1)} \right) \quad |z| > 10 \tag{4}$$

From (1), we get

$$\implies S(z) = \left(\frac{8z^2}{(z-10)(z-1)} \right)\left(\frac{1}{z-1} \right) \tag{5}$$

$$S(z) = \left(\frac{8z^2}{(z-10)(z-1)^2} \right) \tag{6}$$

$$s(n) = \frac{1}{2\pi j} \oint_c \frac{8z^2}{(z-10)(z-1)} \tag{7}$$

$$\tag{8}$$

$$\implies s(n) = \lim_{z\to 10} \frac{1}{0!}\left(\frac{8z^{n+1}}{(z-1)^2} \right) + \lim_{z\to 1} \frac{1}{1!}\frac{d}{dz}\left(\frac{8z^{n+1}}{(z-10)} \right) \tag{9}$$

$$\implies s(n) = \left(\frac{8}{81} \right)(10^{n+1} - 9n - 10) \tag{10}$$