

NCERT Discrete

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Question 11.9.3.18:

Find the sum to n terms of the sequence
8, 88, 888, 8888...

Solution:

Parameter	Value	description
$x(0)$	8	First term
$x(1)$	88	Second term
$x(n)$	$(\sum_{k=0}^n 8(10)^k)u(n)$	General term
$S(n)$	$S(n) = \sum_{k=0}^{n-1} x(k)$	Sum of n terms

TABLE I

INPUT PARAMETERS

From (I)

$$S(n) = \sum_{l=0}^{n-1} \left(\sum_{k=0}^l 8(10)^k \right) u(l) \quad (1)$$

On solving (1), we get

$$\Rightarrow S(n) = \left(\frac{8}{81} \right) (10^{n+1} - 9n - 10) \quad (2)$$

Z transform of general term

$$X(z) = \sum_{l=-\infty}^{\infty} \left(\sum_{k=0}^l 8(10)^k \right) u(n) z^{-l} \quad (3)$$

$$X(z) = \sum_{l=0}^{\infty} \left(\sum_{k=0}^l 8(10)^k \right) z^{-l} \quad (4)$$

$$\Rightarrow X(z) = \frac{8}{9} \left(\frac{100z^{-1}}{1 - 10z^{-1}} - \frac{z^{-1}}{1 - z^{-1}} \right) \quad |z| > 10 \quad (5)$$