NCERT Discrete

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Question 11.9.3.18:
Find the sum to n terms of the sequence $8, 88, 888, 8888\ldots$
Solution :

x_1	x_2	x_3	x_4
8	88	888	8888

Table 1: given inputs

By this observation we can conclude that

$$x_n = 88\ldots ntimes$$

This can also be represented as

$$x_n = 8(10)^0 + 8(10)^1 + \ldots + 8(10)^{n-1} \tag{1}$$

Now, finding the sum of the series till n terms:

$$S_n = x_1 + x_2 + x_3 + \ldots + x_n$$

On substituting (1) in the above equation we get

$$S_n = n \times 8(10)^0 + (n-1) \times 8(10)^1 \ldots + 1 \times 8(10)^{n-1} \tag{2}$$

This is an AGP. Therefore,

$$10S_n = n \times 8(10)^1 + (n-1) \times 8(10)^2 + \ldots + 1 \times 8(10)^n \tag{3}$$

Now, subtracting (2) from (3)

$$9S_n = 8(10)^1 + 8(10)^2 + \ldots + 8(10)^n - 8n$$

$$S_n = \left(\frac{8}{9}\right)\left(\left(\frac{10^n - 1}{10 - 1}\right)10 - n\right)$$

$$S_n = \left(\frac{8}{81}\right)(10^{n+1} - 9n - 10)$$