NCERT Discrete

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Question 11.9.3.18:
Find the sum to n terms of the sequence
$8, 88, 888, 8888\ldots$
Solution:

Parameter	Value	description
$x(0)$	8	First term
$x(1)$	88	Second term
$x(n)$	$(\sum_{k=0}^{n} 8(10)^k)u(n)$	General term
$S(n)$	$S(n) = \sum_{k=0}^{n-1} x(k)$	Sum of n terms

TABLE I
INPUT PARAMETERS

From ()

$$S(n) = x(n) \circledast u(n) \tag{1}$$

Z transform of general term

$$X(z) = \sum_{l=-\infty}^{\infty} (\sum_{k=0}^{l} 8(10)^k)u(n)z^{-l} \tag{2}$$

$$X(z) = \sum_{l=0}^{\infty} (\sum_{k=0}^{l} 8(10)^k)z^{-l} \tag{3}$$

$$\implies X(z) = \frac{8}{9}\left(\frac{100z^{-1}}{1-10z^{-1}} - \frac{z^{-1}}{1-z^{-1}}\right) \quad |z| > 10 \tag{4}$$

Z-transform of unit step function is

$$\implies U(z) = \frac{1}{1-z^{-1}} \tag{5}$$

From (), we get

$$\implies S(z) = \frac{8}{9}\left(\frac{100z^{-1}}{1-10z^{-1}} - \frac{z^{-1}}{1-z^{-1}}\right)\left(\frac{1}{1-z^{-1}}\right) \tag{6}$$

Now taking inverse Z-transform of (), we get

$$\implies S(n) = \left(\frac{8}{81}\right)(10^{n+1} - 9n - 10) \tag{7}$$