GATE 2021-EE-31

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GATE 21 EE 31:
The causal signal with z-transform $\frac{z^2}{(z-a)^2}$ is
Solution:
Given z transform of the signal is $\frac{z^2}{(z-a)^2}$= $\left(\frac{1}{1-az^{-1}}\right)^2$
Let the signal be $x(n)$

$$x(n) = \mathcal{Z}^{-1}\left(\frac{1}{1-az^{-1}}\right)^2 \tag{1}$$

$$\left(\frac{1}{1-az^{-1}}\right)^2 \xleftrightarrow{Z} (n+1)a^n u(n) \quad |z| > |a| \tag{2}$$

$$x(n) = (n+1)a^n u(n) \tag{3}$$

$x(n)$ should be absolutely summable. Let s(n) be the sum till n terms. Then $s(\infty)$ should be a constant

$$s(n) = \sum_{m=0}^{n-1} x(n) \tag{4}$$

$$s(n) = \sum_{m=0}^{n-1} (m+1)a^m \tag{5}$$

$$s(\infty) = \sum_{m=0}^{\infty} (m+1)a^m \tag{6}$$

$$\frac{s(\infty)}{a} = \sum_{m=0}^{\infty} (m+1)a^{m-1} \tag{7}$$

Subtract (7) from (6)

$$s(\infty)\left(1-\frac{1}{a}\right) = -\left(\frac{1}{a}\right) - \left(\sum_{m=0}^{\infty} a^m\right) \tag{8}$$

$$s(\infty)\left(\frac{a-1}{a}\right) = -\left(\frac{1}{a}\right) - \left(\frac{1}{1-a}\right) \quad |a| < 1 \tag{9}$$

$$s(\infty) = \frac{1}{(a-1)^2} \tag{10}$$

Therefore, $|a| < 1$ for x(n) to be absolutely summable.

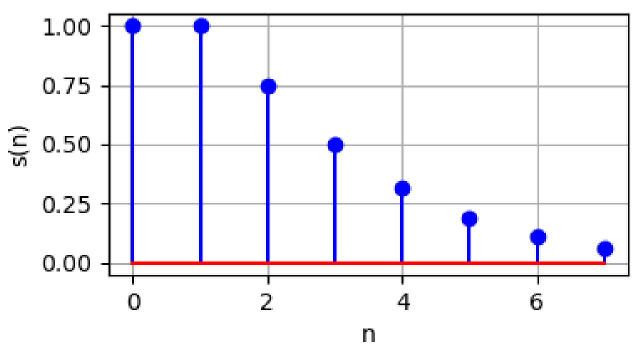

Fig. 1. graph of general term