NCERT Discrete

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EE23BTECH11050

Question 11.9.3.18:
Find the sum to n terms of the sequence
$8, 88, 888, 8888 \ldots$
Solution:

Parameter	Value	description
$x(0)$	8	First term
$x(1)$	88	Second term
$x(n)$	$(\sum_{k=0}^{n} 8(10)^k)u(n)$	General term
$S(n)$	$S(n) = \sum_{k=0}^{n-1} x(k)$	Sum of n terms

TABLE I

INPUT PARAMETERS

From (I)

$$s(n) = x(n) * u(n) \tag{1}$$

Z transform of general term

$$X(z) = \sum_{n=-\infty}^{\infty} (\sum_{k=0}^{n} 8(10)^k)u(n)z^{-n} \tag{2}$$

$$X(z) = 8\sum_{n=0}^{\infty} \left(\sum_{k=0}^{n}(10)^k\right) u(n)z^{-n} \tag{3}$$

$$X(z) = 8 \left(\sum_{n=0}^{\infty}(10)^n(z^{-n})\right) \left(\sum_{n=0}^{\infty} z^{-n}\right) \tag{4}$$

$$\implies X(z) = \left(\frac{8}{(1 - 10z^{-1})(1 - z^{-1})}\right) \quad |z| > 10 \tag{5}$$

From (**??**), we get

$$S(z) = (X(z))(U(z)) \tag{6}$$

$$S(z) = \left(\frac{8}{(1 - 10z^{-1})(1 - z^{-1})}\right) \left(\frac{1}{1 - z^{-1}}\right) \tag{7}$$

$$S(z) = \left(\frac{8}{(1 - 10z^{-1})(1 - z^{-1})^2}\right) \tag{8}$$

$$s(n) = \frac{1}{2\pi j} \oint_c \frac{8}{(1 - 10z^{-1})(1 - z^{-1})^2}(z^{n-1})dz \tag{9}$$

$$s(n) = \frac{1}{2\pi j} \oint_c \left(\frac{800z^{n-1}}{81(1 - 10z^{-1})}\right) dz$$
$$- \oint \left(\frac{8z^{n-2}}{9(1 - z^{-1})^2}\right) dz - \oint \left(\frac{152z^{n-1}}{8(1 - z^{-1})}\right) dz \tag{10}$$

$$\implies s(n) = \lim_{z\to 10} \frac{1}{0!} \left(\frac{800z^n}{81}\right) - \lim_{z\to 1} \frac{1}{1!} \frac{d}{dz} \left(\frac{8z^n}{9}\right)$$
$$- \lim_{z\to 1} \frac{1}{0!} \left(\frac{152z^n}{8}\right) \tag{11}$$
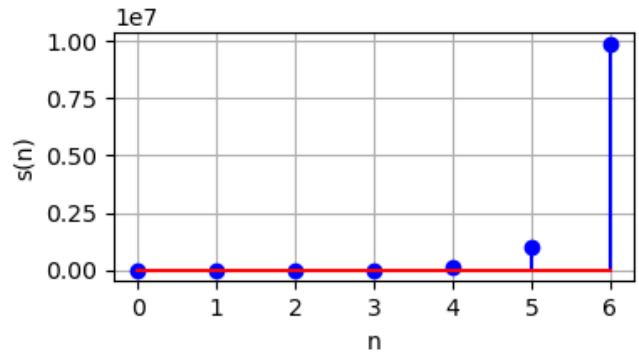
$$s(n) = \left(\frac{8}{81}\right) (10^{n+2} - 9n - 19) \tag{12}$$

Fig. 1. graph of sum of n terms