NCERT Discrete

Pragnidhved Reddy

EE23BTECH11050

Question 11.9.3.18:
Find the sum to n terms of the sequence $8, 88, 888, 8888\ldots$
Solution:

TABLE I
INPUT PARAMETERS

Parameter	Value	description
$x(0)$	8	First term
$x(1)$	88	Second term
$x(n)$	$(\sum_{k=0}^{n} 8(10^k)) u(n)$	General term
$S(n)$	$S(n) = \sum_{k=0}^{n-1} x(k)$	Sum of n terms

From (I)

$$s(n) = x(n) * u(n) \tag{1}$$

Z transform of general term

$$X(z) = \sum_{n=-\infty}^{\infty} (\sum_{k=0}^{n} 8(10)^k) u(n) z^{-n} \tag{2}$$

$$X(z) = 8 \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} (10)^k \right) u(n) z^{-n} \tag{3}$$

$$X(z) = 8 \left(\sum_{n=0}^{\infty} (10)^n (z^{-n}) \right) \left(\sum_{n=0}^{\infty} z^{-n} \right) \tag{4}$$

$$\implies X(z) = \left(\frac{8}{(1 - 10z^{-1})(1 - z^{-1})} \right) \quad |z| > 10 \tag{5}$$

From (1), we get

$$S(z) = (X(z))(U(z)) \tag{6}$$

$$S(z) = \left(\frac{8}{(1 - 10z^{-1})(1 - z^{-1})} \right) \left(\frac{1}{1 - z^{-1}} \right) \tag{7}$$

$$S(z) = \left(\frac{8}{(1 - 10z^{-1})(1 - z^{-1})^2} \right) \tag{8}$$

$$S(z) = \frac{-224z^{-1}}{81(1 - z^{-1})} - \frac{8z^{-2}}{9(1 - z^{-1})^2} + \frac{8000z^{-1}}{81(1 - 10z^{-1})} + 8 \tag{9}$$

$$\delta(n) \xleftrightarrow{Z} 1 \tag{10}$$

$$u(n-1) \xleftrightarrow{Z} \frac{z^{-1}}{1 - z^{-1}} \quad |z| > 1 \tag{11}$$

$$a^{n-k} u(n-k) \xleftrightarrow{Z} \frac{z^{-k}}{1 - az^{-1}} \quad |z| > a \tag{12}$$

$$(n-k) u(n-k) \xleftrightarrow{Z} \frac{z^{-k-1}}{(1 - z^{-1})^2} \quad |z| > 1 \tag{13}$$

From the above 4 equations, we get

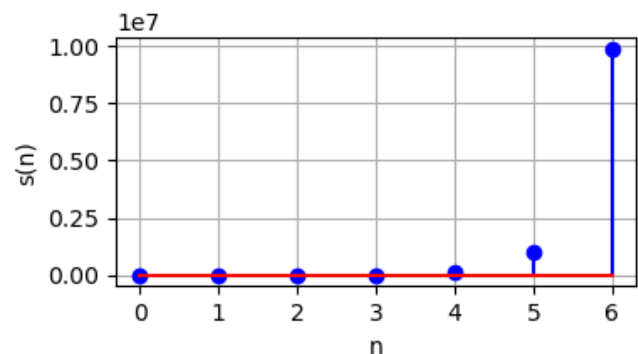$$s(n) = \frac{-224u(n-1)}{81} - \frac{8(n-1)u(n-1)}{9} + \frac{8000(10^{n-1})u(n-1)}{81} + 8\delta(n) \tag{14}$$

Fig. 1. graph of sum of n terms