GATE 2021-EE-31

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EE23BTECH11050

GATE 21 EE 31:
The causal signal with z-transform $\frac{z^2}{(z-a)^2}$ is
Solution:
Given z transform of the signal is $\frac{z^2}{(z-a)^2}$= $\left(\frac{1}{1-az^{-1}}\right)^2$
Let the signal be $x(n)$

$$x(n) = \mathcal{Z}^{-1}\left(\frac{1}{1-az^{-1}}\right)^2 \tag{1}$$

$$\left(\frac{1}{1-az^{-1}}\right)^2 \overset{Z}{\longleftrightarrow} (n+1)a^n u(n) \quad |z| > |a| \tag{2}$$

$$x(n) = (n+1)a^n u(n) \tag{3}$$

Let $y(n)$ be the sum till n terms.

$$y(n) = \sum_{m=0}^{n-1} x(n) \tag{4}$$

$$y(n) = \sum_{m=0}^{n-1} (m+1)a^m \tag{5}$$

$$\frac{y(n)}{a} = \sum_{m=0}^{n-1} (m+1)a^{m-1} \tag{6}$$

Subtract (6) from (5)

$$y(n)\left(1-\frac{1}{a}\right) = n(a)^{n-1} - \left(\frac{1}{a}\right) - \left(\sum_{m=0}^{n-2} a^m\right) \tag{7}$$

$$y(n)\left(\frac{a-1}{a}\right) = n(a)^{n-1} - \left(\frac{1}{a}\right) - \left(\frac{a^{n-1}-1}{a-1}\right) \tag{8}$$

$$y(n) = \frac{na^{n+1} - (n+1)a^n + 1}{(a-1)^2} \tag{9}$$

$x(n)$ should be absolutely summable. Then $y(\infty)$ should be a constant.

$$y(\infty) = \lim_{n\to\infty} \frac{na^{n+1} - (n+1)a^n + 1}{(a-1)^2} \tag{10}$$

$$y(\infty) = \frac{1}{(1-a)^2} \quad |a| < 1 \tag{11}$$

Therefore, $|a| < 1$ for $x(n)$ to be absolutely summable.

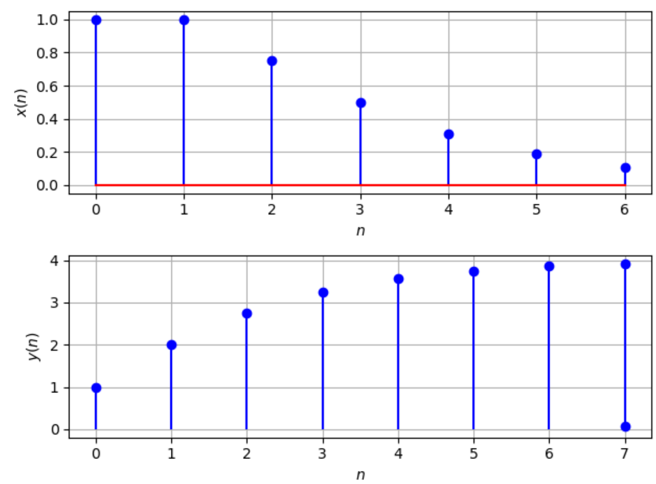

Fig. 1. graph of x(n) and y(n) for a=0.5