Homework #6
Due by Thursday 11/20, 11:59pm

Submission instructions:

1. You should write 5 '.py' files: one for each question.
   Name your files: 'YourNetID_hw6_q1.py', 'YourNetID_hw6_q2.py', etc.
   Note: your netID follows an abc123 pattern, not N12345678.

2. In this assignment, we provided 'DoublyLinkedList.py' file (with the implementation of a doubly linked list).

   In questions where you need to use `DoublyLinkedList`, make the definition in a separate file, and use an **`import`** statement to import the `DoublyLinkedList` class.

   You should use the linked lists as a black box. **You are not allowed to put any changes in 'DoublyLinkedList.py'.** Such a change is considered a break of an abstraction barrier.

3. You should submit your homework via Gradescope. For Gradescope's autograding feature to work:
   a. Name all functions and methods exactly as they are in the assignment specifications.
   b. Make sure there are no print statements in your code. If you have tester code, please put it in a "main" function and do not call it.
   c. You don't need to submit the 'DoublyLinkedList.py' file

Question 1:

Define a `LinkedQueue` class that implements the *Queue* ADT.

Implementation Requirement: All queue operations should run in $\theta(1)$ **worst-case**.

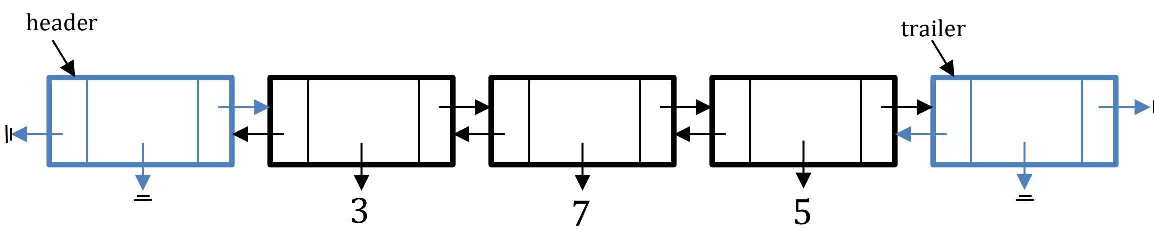
Hint: You would want to use a doubly linked list as a data member.

Question 2:

Many programming languages represent integers in a **fixed** number of bytes (a common size for an integer is 4 bytes). This, on one hand, bounds the range of integers that can be represented as an `int` data (in 4 bytes, only 2^{32} different values could be represented), but, on the other hand, it allows fast execution for basic arithmetic expressions (such as +, −, * and /) typically done in hardware.

Python and some other programming languages, do not follow that kind of representation for integers, and allows to represent arbitrary large integers as `int` variables (as a result the performance of basic arithmetic is slower).

In this question, we will suggest a data structure for positive integer numbers, that can be arbitrary large.
We will represent an integer value, as a linked list of its digits.
For example, the number 375 will be represented by a 3-length list, with 3, 7 and 5 as its elements.



Note: this is not the representation Python uses. Complete the definition of the following `Integer` class:

```
class Integer:
    def __init__(self, num_str):
        ''' Initializes an Integer object representing
        the value given in the string num_str'''

    def __add__(self, other):
        ''' Creates and returns an Integer object that
        represent the sum of self and other, also of
        type Integer'''

    def __repr__(self):
        ''' Creates and returns the string representation
         of self'''
```

For example, after implementing the `Integer` class, you should expect the following behavior:

```
>>> n1 = Integer('375')
>>> n2 = Integer('4029')
>>> n3 = n1 + n2
>>> n3
4404
```

<u>Note</u>: When adding two `Integer` objects, implement the "Elementary School" addition technique. DO NOT convert the `Integer` objects to `int`s, add these `int`s by using Python + operator, and then convert the result back to an `Integer` object. This approach misses the point of this question.

<u>Extra Credit</u>:

Support also the multiplication of two `Integer` objects (by implementing the "Elementary School" multiplication technique):

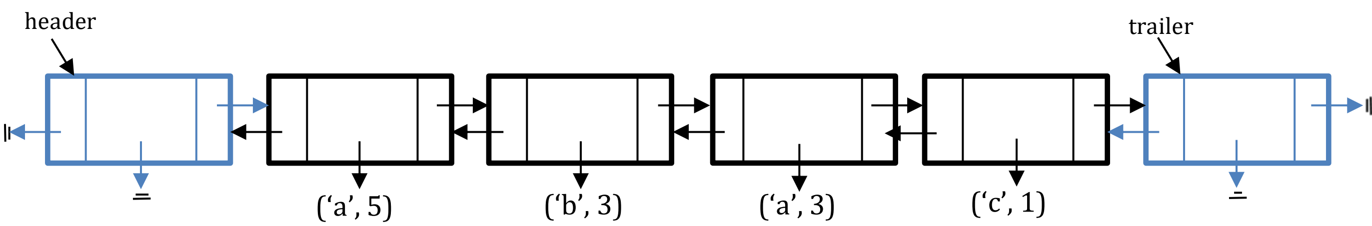
```
  def __mul__(self, other):
        ''' Creates and returns an Integer object that
        represent the multiplication of self and other,
        also of type Integer'''
```

Question 3:

In this question, we will suggest a data structure for storing strings with a lot of repetitions of successive characters.

We will represent such strings as a linked list, where each maximal sequence of the same character in consecutive positions, will be stored as a single tuple containing the character and its count.

For example, the string “aaaaabbbaaac” will be represented as the following list:



Complete the definition of the following `CompactString` class:

```
class CompactString:
    def __init__(self, orig_str):
        ''' Initializes a CompactString object
        representing the string given in orig_str'''

    def __add__(self, other):
        ''' Creates and returns a CompactString object that
        represent the concatenation of self and other,
        also of type CompactString'''

    def __lt__(self, other):
        ''' returns True if”f self is lexicographically
        less than other, also of type CompactString'''

    def __le__(self, other):
        ''' returns True if”f self is lexicographically
        less than or equal to other, also of type
        CompactString'''

    def __gt__(self, other):
        ''' returns True if”f self is lexicographically
        greater than other, also of type CompactString'''

    def __ge__(self, other):
        ''' returns True if”f self is lexicographically
        greater than or equal to other, also of type
        CompactString'''

    def __repr__(self):
        ''' Creates and returns the string representation
        (of type str) of self'''
```

For example, after implementing the `CompactString` class, you should expect the following behavior:

```
>>> s1 = CompactString('aaaaabbbaaac')
>>> s2 = CompactString('aaaaaaacccaaaa')
>>> s3 = s2 + s1 #in s3's linked list there will be 6 'real' nodes
>>> s1 < s2
False
```

Note: Here too, when adding and comparing two `CompactString` objects, DO NOT convert the `CompactString` objects to `str`s, do the operation on `str`s (by using Python +, <, >, <=, >= operators), and then convert the result back to a `CompactString` object. This approach misses the point of this question.

Question 4:

In this question, we will demonstrate the difference between shallow and deep copy. For that, we will work with *nested doubly linked lists of integers*. That is, each element is an integer or a `DoublyLinkedList`, which in turn can contain integers or `DoublyLinkedList`s, and so on.

a. Implement the following function:

```
def copy_linked_list(lnk_lst)
```

The function is given a nested doubly linked lists of integers `lnk_lst`, and returns a **shallow copy** of `lnk_lst`. That is, a new linked list where its elements reference the same items in `lnk_lst`.

For example, after implementing `copy_linked_list`, you should expect the following behavior:

```
>>> lnk_lst1 = DoublyLinkedList()
>>> elem1 = DoublyLinkedList()
>>> elem1.add_last(1)
>>> elem1.add_last(2)
>>> lnk_lst1.add_last(elem1)
>>> elem2 = 3
>>> lnk_lst1.add_last(elem2)

>>> lnk_lst2 = copy_linked_list(lnk_lst1)

>>> e1 = lnk_lst1.header.next
>>> e1_1 = e1.data.header.next
>>> e1_1.data = 10

>>> e2 = lnk_lst2.header.next
>>> e2_1 = e2.data.header.next
>>> print(e2_1.data)
10
```

b. Now, implement:
`def` `deep_copy_linked_list(lnk_lst)`
The function is given a nested doubly linked lists of integers `lnk_lst`, and returns a **deep copy** of `lnk_lst`.

For example, after implementing `deep_copy_linked_list`, you should expect the following behavior:

```
>>> lnk_lst1 = DoublyLinkedList()
>>> elem1 = DoublyLinkedList()
>>> elem1.add_last(1)
>>> elem1.add_last(2)
>>> lnk_lst1.add_last(elem1)
>>> elem2 = 3
>>> lnk_lst1.add_last(elem2)

>>> lnk_lst2 = deep_copy_linked_list(lnk_lst1)

>>> e1 = lnk_lst1.header.next
>>> e1_1 = e1.data.header.next
>>> e1_1.data = 10

>>> e2 = lnk_lst2.header.next
>>> e2_1 = e2.data.header.next
>>> print(e2_1.data)
1
```

Note: `lnk_lst` could have **multiple levels** of nesting.

Question 5:

In this question, we will implement a function that merges two sorted linked lists:

```
def merge_linked_lists(srt_lnk_lst1, srt_lnk_lst2)
```

This function is given two doubly linked lists of integers `srt_lnk_lst1` and `srt_lnk_lst2`. The elements in `srt_lnk_lst1` and `srt_lnk_lst2` are sorted. That is, they are ordered in the lists, in an ascending order.

When the function is called, it will **create and return a new** doubly linked list, that contains all the elements that appear in the input lists in a sorted order.

For example:
if `srt_lnk_lst1`= [1 <--> 3 <--> 5 <--> 6 <--> 8],
and `srt_lnk_lst2`=[2 <--> 3 <--> 5 <--> 10 <--> 15 <--> 18],
calling: `merge_linked_lists(srt_lnk_lst1, srt_lnk_lst2)`, should create and return a doubly linked list that contains:
[1 <--> 2 <--> 3 <--> 3 <--> 5 <--> 5 <--> 6 <--> 8 <--> 10 <--> 15 <--> 18].

The `merge_linked_lists` function is not recursive, but it defines and calls `merge_sublists` - a nested helper **recursive** function.

Complete the implementation given below for the `merge_linked_lists` function:

```
def merge_linked_lists(srt_lnk_lst1, srt_lnk_lst2):
    def merge_sublists( ______________________________ ):
        ____________________________________________________
        ____________________________________________________
        ____________________________________________________
        ____________________________________________________
        ____________________________________________________


    return merge_sublists( _________________________________ )
```

Notes:

1. You need to decide on the signature of `merge_sublists`.
2. `merge_sublists` has to be **recursive.**
3. An efficient implementation of `merge_sublists` would allow `merge_linked_lists` to run in **linear time.** That is, if n_1 and n_2 are the sizes of the input lists, the runtime would be $\theta(n_1 + n_2)$.