Chapter 3 Hydro Logical Losses

[16 Marks]

3.1 Initial Losses (Interception and depression storage)

Interception: It is the process by which water is captured on vegetation (leaves, bark, grasses, crops etc.) during a precipitation event. Intercepted precipitation is not available for runoff or infiltration, but instead is returned to the atmosphere through evaporation. Interception losses generally occur during the first part of a precipitation event and the interception loss rate trends toward zero rather than quickly.

- → Loss of rainfall due to vegetation (from trees to grass)
- → Interception loss = rainfall- steam fall- through fall
  Interception = f (vegetation (age, density, type), season, rain, intensity antecedent conditions earlier, either in time or order)
- → Forest cover data collected by placing rain gauges under forest canopy and comparing with gauge data from open area.
- → Annual losses from 10-20% of annual precipitation.
- → Dense forest have significant losses up to 25% of rainfall
- → Importance tied to purpose of analysis:
  - Losses can be significant for annual and long term averages
  - Heavy rainfall and individual storms might not be significant
  - For abstraction from storm events standard practice is to extract from initial portion of rainfall storm (i.e., initial abstractions).

Factors affecting Interception:

1. Storm character
2. Plants character
3. Season of the year

Estimating Interception: Interception losses are described by the following equation (Horton reprinted by viessman, 1996):

$L_i = S_i + K \times E \times t$

L_i = volume of water intercepted (cm)

S_i = interception storage that will be retained on the foliage,
= f (wind, gravity, type)

K = ratio of surface area of intercepting leaving to horizontal projection of tree area,
= 100%, for light storms
= 10 to 40% for heavy storms

E = amount of water evaporating per hour during precipitation (cm/hr)

t = time, hrs

Depression storage: It refers to small low points in undulating terrain that can store precipitation that otherwise would become runoff. The precipitation stored in these depressions is then either removed through infiltration into the ground or by evaporation. Depression storage exists on previous and impervious surfaces; however, depression storage is much greater on undisturbed previous surfaces. Standard design and construction practices remove these natural depressions in order to promote drainage, which reduces depression storage.

The volume of water in depression storage at any time during a precipitation vent can be approximated as:

$$V = S_d (1 - e^{-kPe}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(linely 1982)}$$

Where, V = volume of water in depression storage,
S_d = maximum storage capacity of the depression.
P_e = rainfall excess
K = constant equal to $1/S_d$.

Depression storage assumes that all water has had a chance to infiltrate as evaporate.

3.2 Evaporation losses:

Metrological Parameters (radiation, temperature, vapor, pressure, humidity, wind)
The branch of science which deals with the study of weather is known as meteorology. The study of hydrology necessitates the collection of data on humidity, temperature, precipitation, radiation, evapo-transpiration and wind velocity for the collection of these meteorological data's, meteorological station are fixed all over the world, (at present time, about 282 meteorological stations are established in different parts of Nepal)

Radiation: Radiation measured by instrument called pyronometer.
It consists of plate divided into,

i. A central white spot
ii. A middle black spot
iii. An outer white ring

The black ring absorbs radiation white ring reflects it (ie, radiation) therefore temperature difference setup between white and black ring.

Solar Radiation: Water in the sea evaporates under solar radiation and clouds of the water vapor movers over land area. Radiated wavelengths are usually given in micrometer (μm). Radiation from the sun is short wave that from the earth is long wave. Maximum energy of shot wave radiation in the visible rage of 0.42-0.8 μm and that of long wave is 10 μm. A large part of solar radiation reaching the outer limit of the atmosphere is scattered and absorbed in the atmosphere or reflected from the cloud and the earth surface. Scattering of radiation by air molecules is most effective for the shortest wavelength. Most meteorological recording stations are equipped with radiometers to measure both incoming short wave from the sun and net radiation. The net radiation is of most important in evaporation study.

Temperature: Rate of evaporation increases with an increase in water temperature. Although there is an increase in the rate of evaporation with increase in air temperature, a high correlation does not exist between.
For the same mean monthly temperature, evaporation form a lake may be different in different months.

Vapor Pressure: The rate of evaporation is proportional to difference between the saturation vapor pressure (SVP) at the water temperature (e_w) and the actual pressure in the air (e_a)

$E_L = C(e_w - e_a)$
E_L = rate of evaporation
C = constant
e_w and e_a are in mm of mercury.

→ This equation is called **Dalton's law of evaporation.**
→ Evaporation occurs till e_w-e_a.
→ If $e_w>e_a$ condensation takes place.

Humidity: Humidity is general term used to indicate the moisture in the atmosphere. Vapor pressure and humidity are intertwined.

a. **Absolute humidity:** (vapor concentration)
- It is actual amount of water vapor present in a certain quantity of air.
- Its unit is gm/m^3.
- Absolute humidity = $\frac{\text{Actual water content}}{\text{volume of air with moisture}}$
- Saturated air at 10°C contains 9.41gm of vapor per m^3 and at 30°C contains 30.036 gm/m^3.

b. **Relative humidity (humidity):** It is the ratio of the actual vapor pressure to the saturation vapor pressure at same temperature.

Relative humidity = $\frac{ea}{es} = \frac{\text{actual vapor pressure at given temperature}}{\text{saturated vapor pressure at same temperature}}$

Dew point: It is the point at which air becomes saturated when cooled under constant pressure and wih constant water vapor content. It is thus the temperature having a saturation vapor pressure (e_s) equal to the existing or actual vapor pressure (e_s).

Specific humidity:

- Usually expressed in gm per kg.
- It is the ratio of mass of water vapor to per unit mass of moist air

$e_w = e_s = 4.584e\left(\frac{17.27t}{237.3+t}\right)$ where, t = temperature in ^oC

Humidity:

- Measured by pyrometer.
- It consists of:
  - Two glass thermometer
  - One is wet bulb thermometer (T_w)
  - Another one is dry bulb thermometer (T)
  - The difference between two ($T-T_w$) is known as wet bulb depression from measured atmosphere pressure (p_1) temperature wet bulb depression ($T-T_w$), and the relative humidity can be obtained from psychometric table or using the relation,

$$e_a = e_s - 0.0006606\, p(T-T_w)\ \left(1+\frac{Tw}{872.78}\right)$$

and $RH = \frac{ea\ x\ 100\%}{es}$

Where, RH = Relative humidity in percentage.

$e_a =$ Actual vapor pressure in millibar (mb)

e'_s = Saturated vapor pressure in millibar (mb) correcspoding to $T_{w.}$

p = Atmospheric pressure in mb

$e_s =$ Saturated vapor pressure corresponding to T.

T = Dry bulb temperature (^oC)

T_w = Wet bulb temperature (^oC)

Wind: Wind helps to remove the evaporated water vapor from the zone of evaporation, thereby creating greater scope for evaporation. The rate of evaporation increases with increase in wind velocity up to some limit (critical wind speed) and thereafter further increase in wind velocity does not have any effect on evaporation rate. This critical wind speed is function of size of water surface (i.e. large water bodies – high wind speed).

The wind speed at any height can be approximately obtained from the known wind speed at known height is given by: $\left(\frac{V}{Vo} = \frac{Z}{Zo}\right)^{0.15}$

Where,

V = wind speed at height z from ground

V_O = wind speed at anemometer level Z_o

3.3 Analytical method of evaporation estimation:

1. Water Budget Method
2. Energy Budget Method
3. Mass transfer method (Dalton's law)

1. Water Budget method:

$$P + V_{is} + V_{ig} = V_{os} + V_{og} + E_L + \Delta s + T_L$$

Where, P	=	daily precipitation
V_{is}	=	daily surface inflow into the lake
V_{ig}	=	daily ground water inflow
V_{os}	=	daily surface outflow from the lake
V_{og}	=	daily groundwater outflow
E_L	=	daily lake evaporation
Δs	=	increase in lake storage in a day
T_L	=	daily transpiration loss

All quantities are expressed in units of volume or depth.
P, V_{is}, V_{os}, Δs can only be measured
V_{ig}, V_{og}, T_L can only be estimated
If the unit of time is kept very large, estimates of evaporation will be more accurate. It is he simplest of all methods, but the least reliable.

2. Energy Budget Method:

→ It involves application of the law of conservation of energy.
→ Energy availability fir evaporation is determined by considering the incoming energy, outgoing energy and the energy stored in the water body over a known time interval.
→ Estimation of evaporation from a lake by this method has been found to give satisfactory results with errors of the order of 5% when applied to periods less than a week.

$$H_n = H_a + H_g + H_s + H_i + H_e$$

Where,
H_n = net heat energy received by the water surface (H_c (l-r)-H_b)
H_b = back (long wave) radiation from the water body. (Emission emitted radiation)
H_a = sensible heat transferred from the surface to the air.
H_g = heat flux in the ground
H_s = heat in the water body
H_e = heat energy used in evaporation = $\varrho L E_L$
(E_L = evaporation, L = latent heat of evaporation, ϱ = mass density of the fluid)
H_i = net heat conducted out of the system by water flow. (Advected heat)

→ r = albedo = $\frac{Rr}{Ri} = \frac{Reflected\ radiation}{Incoming\ radiation}$

This is the energy balance in the period of 1 day. All energy terms are in calories/sq.mm/day.

→ If time period is short H_s and H_i can be neglected as they are negligibly small.
→ All terms except H_a can either be measured or evaluated indirectly.
→ H_a is estimated using **Bowes's ratio.**

$$\beta = \frac{H_a}{\varrho L E_l} = 6.1 \times 10^{-4} \times P_a \frac{T_w - T_a}{e_w - e_a}$$

P_a = Atmospheric pressure (mm of mercury)
e_w = saturated vapor pressure of the air (mm of mercury)
e_a = actual vapor pressure of the air (mm of mercury)
T_w = temperature of the water surface. (^oC)
T_a = temperature of the air (^oC)

3. Mass Transfer Method:

In this method evaporation is considered as the turbulent transfer of vapor. Therefore, this is also known as the vapor flow approach or the aerodynamic approach, considering adiabatic atmosphere and logarithmic distribution for wind velocity.

$$E = \frac{46.08\,(e_1 - e_2)\,(V_2 - V_1)}{(T+273)\,ln\,(\frac{Z2}{Z1})} \qquad \text{equation 1}$$

Where,

E = evaporation in mm/hr
Z_1 & Z_2 = arbitrary lower and upper levels above the surface in meters.
e_1&e_2 = vapor pressure measured at Z_1 & Z_2 in mm of Hg.
V_1&V_2 = wind velocity measured at Z_1 & Z_2 in km/hr.
T = average temperature of the air in ^oC between Z_1 & Z_2.

Assuming Z_1 to lie within the thin vapor saturated film above the winter surface, e_1 equals e_s and with slight modification equation 1 may be written in the form of well-known **Dalton's Equation.**

E= (a+bv) (e_s-e_a)

Where,

e_a = vapor pressure of the air at which the wind velocity, v is obtained.
e_s = saturated vapor pressure corresponding to air temperature.

There are two cases that should be considered

i. Water temperature = air temperature

ii. Water surface temperature ≠ air temperature

Case i. occurs rarely and is empirically treated by equation

$$E_a = C\ f(v)\ (e_s - e_a)$$

Case ii. Normally and is of practical interest

$$E = C\ f(v)\ (e_s^1 - e_a)$$

Where, e_s^1 is the vapor pressure of the thin film of vapor that exist near the free surface between water and air, whose temperature is neither equal to water temperature nor to the air temperature so virtually impossible to measure.

3.4 Evaporation (Evaporation Pan)

Evaporation pan, also called evaporimeter, is shallow vessels containing water. These are placed in open to measure the loss of water by evaporation.

Lake or reservoir evaporation = Pan Coefficient x Pan Evaporation

Pan coefficient: 0.6 to 0.8

Class A evaporation pan:

It consists of a cylindrical vessel made up of galvanized 22 gauge iron sheet.

Sunken Pan (Colorado Sunken Pan)

The pan is buried into the ground such that the water level is at the ground level. The pan is painted with black tar.

$$\text{Pan Coefficient} = \frac{\text{actual evaporation from reservior}}{\text{measured evaporation from pan}}$$

Hydrology Chapter 4:

4.0 Surface Runoff [8 Hours]

4.1 Drainage basin and its quantitative characteristics

The area of land draining into a stream or a water course at a given location is known as the catchment area. It is also called drainage basin, drainage area and watershed. It is called watershed in USA and divide or ridge in UK. The areal extent of the catchment is obtained by tracing the ridge on topographic map to delineate the catchment and measuring the area by a planimeter. The basin characteristics are generally determined by its geology, geomorphology, area, slope and drainage dynamics.

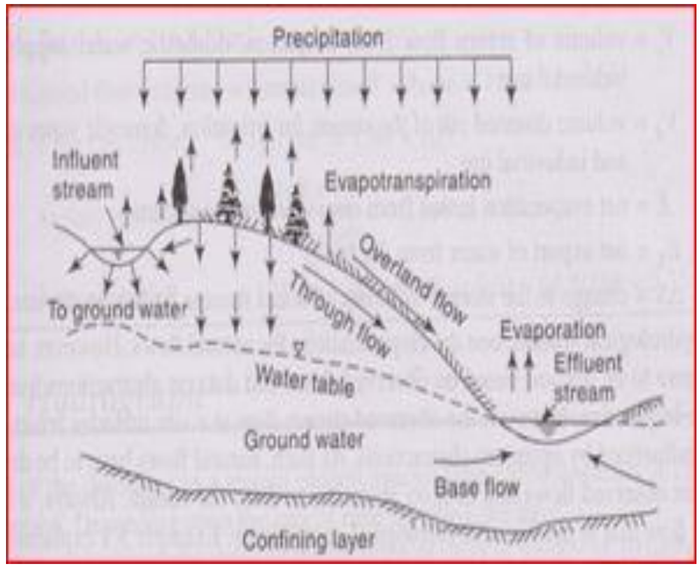
4.2 Factors affecting runoff from a catchment

1. Physiographic Factors
2. Climatic Factors

Physiographic Factors	**Climatic Factors**
1. Basin Characteristics (a) Geology (b) Shape (c) Size (d) Slope (e) Elevation (f) Drainage density = $\frac{\textbf{Total Channel Length}}{\textbf{Total drainage area}}$ (g) Land Use (h) Soil Type (i) Orientation of basin	1. Characteristics of precipitation (a) Type of precipitation (b) Intensity of Precipitation (c) Duration f rainfall magnitude (d) Rainfall distribution (e) Movement of storm
2. Infiltration Characteristics (a) Land Use and Cover (b) Soil Type and geological conditions (c) Lakes, Swamps and other storage	2. Initial Loss
3. Channel Characteristics (a) Cross Section (b) Roughness and storage capacity	3. Evaporation

4.3 Rainfall-Runoff relationship

Rainfall: The fall of moisture from atmosphere to earth in any form like rain, drizzle, snow, hail, sleet etc is called precipitation or rainfall. The term rainfall is used to denote precipitation in the form of water drops of sizes larger than 0.5mm. The maximum size of raindrop is about 6mm. Any drop larger than this size tends to break up into drops of smaller sizes during its fall from the clouds. Thus, 1cm of rainfall over a catchment area of 1 km^2 represents a volume of water equal to 10^4 m^3.



Runoff: Runoff means the draining of or flowing of precipitation from a catchment area through a surface channel. For a given precipitation, the evapotranspiration, initial loss, infiltration, and detention storage requirements will have to be satisfied before the commencement of runoff. The portion of rainfall (precipitation) which reaches the stream channel by a variety of paths above and below the earth surface is called runoff.

The relationship between rainfall and the corresponding runoff is quite complex as it is influenced by host of factors relating to catchment and climate. For rough estimate, we try to correlate rainfall (P) and runoff (R) by plotting R-values against P values and drawing a best fit line between R and P and accept the result if the correlation coefficient is nearer to unity.

The equation of the straight line regression between runoff R and rainfall P is

$$R= a P + b \quad ....(4.3)$$

and the values of the coefficient a and b are given by

$$a = \frac{N(\sum PR) - (\sum P)(\sum R)}{N(\sum P^2) - (\sum P)^2}$$

$$b = \frac{\sum \boldsymbol{R} - \boldsymbol{a}(\sum \boldsymbol{P})}{\boldsymbol{N}}$$

where, N= number of observation sets R and P.

The coefficient of correlation r can be calculated as,

$$\mathbf{r} = \frac{\mathrm{N}(\sum \mathrm{PR}) - (\sum \mathrm{P})(\sum \mathrm{R})}{\sqrt{[\mathrm{N}(\sum \mathrm{P}^2) - (\sum \mathrm{P})^2]\ [\mathrm{N}(\sum \mathrm{R}^2) - (\sum \mathrm{R})^2]}}$$

The values of **r** lies between 0 and 1 as R can have only positive correlation with P.

The value of $0.6 < \mathbf{r} < 1.0$ indicates good correlation.

The term $\mathbf{r^2}$ is known as the coefficient of determination.

For large catchments, it is recommended to use exponential relationship as,

$$\mathbf{R} = \boldsymbol{\beta}\ \mathbf{P^m} \quad \text{.............................(4.5)}$$

$$\ln \boldsymbol{R} = \boldsymbol{m} \ln \boldsymbol{P} + \ln \boldsymbol{\beta}$$

where**,** β and m are constants, instead of linear relationship given by equation (4.3)

The coefficients m and β are determined by using regression analysis.

The regression equations 4.3 and 4.5 can be used to generate synthetic runoff data by using rainfall data.

4.4 Stream gauging (selection of sites, Types of gauges and measurement)

Selection of sites

(i) Site should be selected in stable bank

(ii) It should be accessible during all the time of the year

(iii) Staff gauge should be free from back water effects

(iv) Staff gauge are generally fixed to the abutment, piers or walls so that the readings can be easily taken during high as well as low floods

(v) In case of automatic stage recorder, stilling wells be provided to safeguard against debris and wave effects

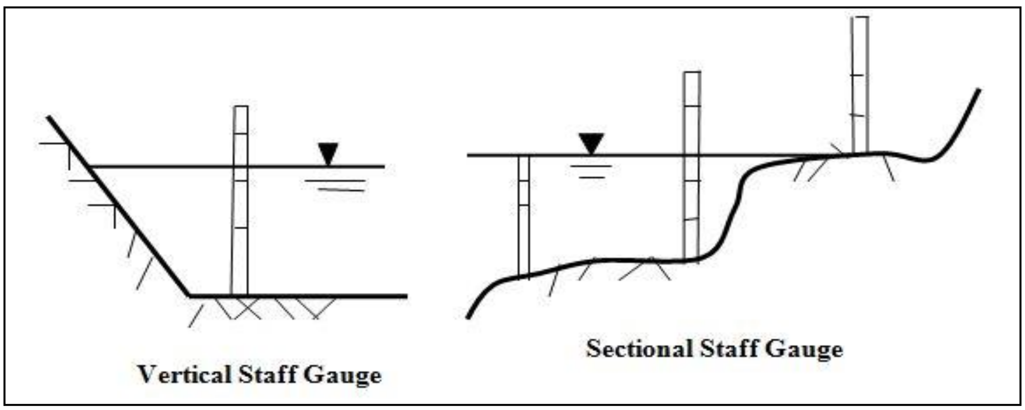
(vi) The water sage recorder has to be located above HFL to prevent it from inundation during floods

(vii) Instrument must be properly housed to protect it from weather and elements of vandalism.

Stage of river is: its water surface elevation wrt datum level (msl) or any arbitrary datum level.

River stage measurement: The discharge in stream is related to the water elevation of water surface (Stage) through a series of careful measurements. The stage of the river is observed routinely and discharge is estimated using previously determined stage-discharge relationship.

Staff Gauge: made of durable material with low coefficient of expansion rigidly fixed to a structures– abutment, pier, wall etc. vertically or inclined with accurately graduated permanent markings. Sometimes, gauges are built in sections at different locations. Care must be taken to provide overlap between various gauges and to refer the same common datum.



Wire Gauge: Used to measure water surface elevation from a bridge or similar structure. Weight is lowered by a reel to touch the water surface. A mechanical counter measures the rotation of the wheel which is proportional to the length of wire paid out. Operating range of the gauge is about 25m.

Automatic Stage Recorders:

(i) **Float gauge recorder:** In this float operating in stilling well is balanced by means of a counter weight over the pulley of a recorder. Displacement of float due to rising or lowering of water surface elevation causes an angular displacement of the pulley and hence of the input shaft of the recorder.

(ii) **Bubble gauge recorder:** Gas is made to bleed out at very small rate through an outlet placed at the bottom of river. A pressure gauge measures the gas pressure which in turn is equal to the water column above outlet. A small change in water surface elevation is felt as change in pressure from the present value at pressure gauge and this in turn is adjusted by a servo-mechanism to bring the gas to bleed at original rate under the new head. The pressure reads the new water depth which is transmitted to a recorder.

4.5 Stream flow computation by velocity area method (current meters, floats, and velocity rods**)**

Stream flow Measurement: broadly classified into

(i) Direct determination:

(a) Area-Velocity methods

(b) Dilution techniques

(c) Electromagnetic method

(d) Ultrasonic method

(ii) Indirect determination

(a) Hydraulic structures-weirs, flumes, gated structures

(b) Slope Area method

Area-Velocity method

- Measure area of cross section where stream has well defined x-section
- Site should be stable and in straight reach
- Easily accessible throughout the year
- Gauging staff should be free from backwater effects

Process:

At the selected site, section line is marked by survey markings

Depths are measured by sounding rods or sounding weights.

In large depth, high velocity, deep and mobile bed stream echo-depth recorder is used.

X-section is divided into large number of subsection by verticals. The average velocities in these sub-sections are measured by current meters or floats.

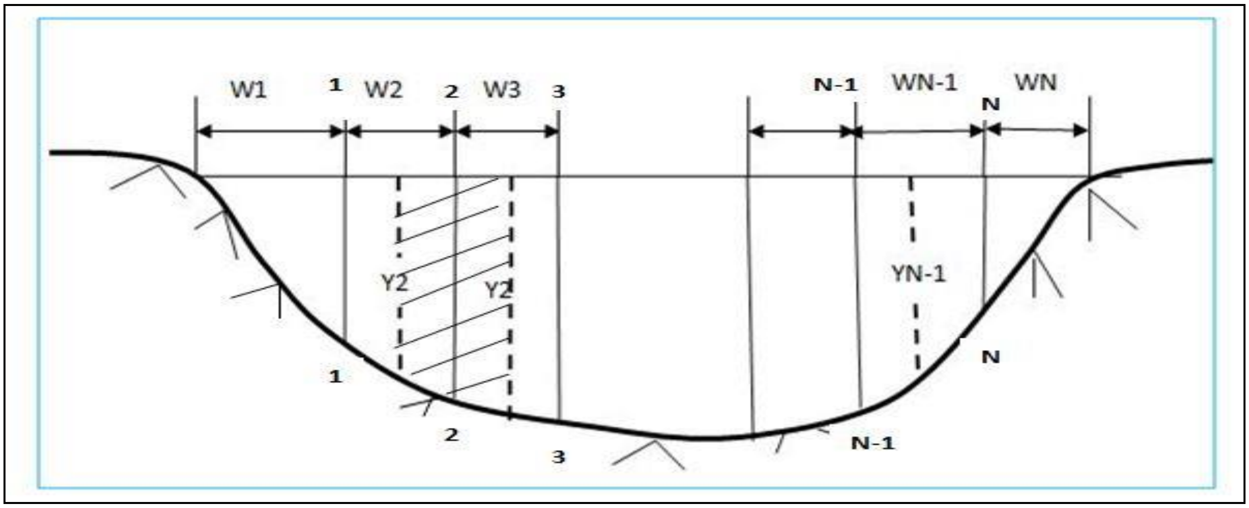
For more accuracy more numbers of sub-sections are used.

For larger segment, larger effort, time and expenditures are involved.

Guidelines to select the number of segments

- The segment width should not be greater than 1/15 to 1/20 of the width of river.
- Discharge in each segment should be less than 10% of the total discharge

- Difference in velocities in adjacent segments should not be more than 20% In natural rivers, verticals for velocity measurement need not be equally spaced.



Using the method of mid-section

$$Q = \sum_{i=1}^{N-1} \Delta Q_i$$

where, ΔQ_i = discharge in the ith segment

= (depth in the ith segment) x $\frac{1}{2}$ width to the left + $\frac{1}{2}$ width to the right) x average velocity at ith vertical

$$Q = \sum_i A_i V_i$$

For i = 2 to N-2, $\Delta Q_i = Y_i \left(\frac{W_i}{2} + \frac{W_{i+1}}{2}\right) V_i$

For 1st and last sections, the sections have triangular areas $\Delta Q_i = \overline{W}_1 Y_1$

$$\overline{W}_1 = \frac{\left(W_1 + \frac{W_2}{2}\right)^2}{2W_1}$$

and $\Delta A_N = \overline{W}_{N-1} Y_{N-1}$

$$\overline{W}_{N-1} = \frac{\left(W_N + \frac{W_{N-1}}{2}\right)^2}{2W_N}$$

Then, $\Delta Q_1 = V_1 \Delta A_1$ **and** $\Delta Q_{N-1} = V_{N-1} \Delta A_{N-1}$

where, A_i and V_i are the X-section area of the river and mean velocities of flow measured normal to the partial areas.

In shallow river, the velocity is measured at one point at 0.6d, depth d measured from the top and coefficient shall normally be applied to convert the observed velocity to the mean velocity.

In deeper rivers, velocity should be measured at 0.2 and 0,8 of the effective depth. For two or more point methods, average velocity shall be computed as,

$$V_m = 0.1\,(V_s + V_{0.2} + V_{0.4} + V_{0.6} + V_{0.8} + V_b)$$

where, V_s and V_b are velocities measured at water surface and river bed respectively and $V_{0.2}, V_{0.4}, V_{0.6}, V_{0.8}$ are velocities at 0.2, 0.4, 0.6 and 0.8 of the effective depth respectively.

Principle involved in both type of current meter:

Speed of a rotating element is directly proportional to the velocity of water. A current meter is so designed that its rotation al speed varies linearly with the stream velocity V at the location of the instrument. A typical relationship is

V= a N + b

where, a and b are the constants of the instrument and typical value of a and b are 0.65 and 0.03.

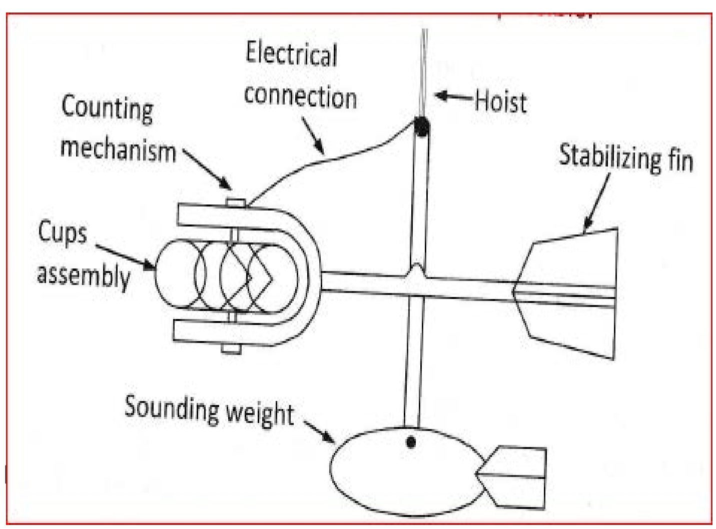
V= velocity in m/s

N= revolution per second

Each time when the wheel of the cups makes one revolution, the electrical circuit is closed and this causes a click in the headphone to be heard by the operator. The revolution per second is calculated by counting the number of such signals in a known interval of time.

Cup Type current meter.

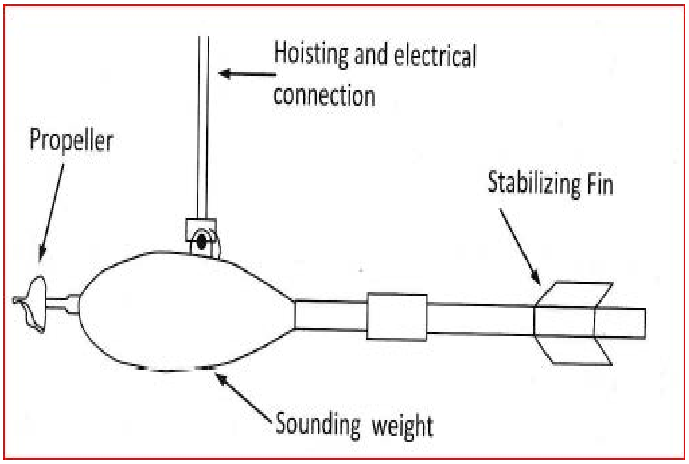
The price current meter and Gurley current meters are typical instrument under this category. The cup type current meter consists of a wheel of series of conical cups rotating about a vertical axis. It also consists of a fish weight to keep the meter cable as nearly vertical as possible.



The cup rotates in horizontal plane. The revolutions of cup assembly for a certain time is recorded and converted to stream velocity. The accuracy of this instrument is about 15%. The normal range of velocities measured by such current meter is from 0.15 to 4m/s. This type of current meter cannot be used if the vertical component of velocity is significant.

Propeller Type

It consists of a propeller mounted at the end of horizontal shaft. The revolutions of propeller for a certain time is recorded and converted to stream velocity.



Here, the rotating element is in the form of propeller facing the direction of flow which rotates about a horizontal axis. Ott or Watt type current meters are typical instruments. This instrument can register velocities in the range of 0.15 to 4m/s.

This instrument is fairly not affected by oblique flows of as much as 15^0. The accuracy of this instrument is about 1% at the threshold value.

Calibration

The relationship between the stream velocity and the revolution per second of the meter is called the calibration equation $V = a\,N + b$.

This equation is unique to each instrument. So determination of constants a and b are known as calibration of current meter. Current meters are calibrated in ponds or long channels where water is held stationary.

Float Measurement

Floats are used to measure velocity for a small stream in flood, stream with rapidly changing water surface and for preliminary analysis. $V_s = \frac{L}{t}$

where, V_s =surface velocity (m/s)

L= distance travelled (m)

t= time taken to travel by float (second)

$Q = V_{av} \times A$

where, V_{av} = average velocity (m/s) = 0.85 to0.95 times the surface velocity]

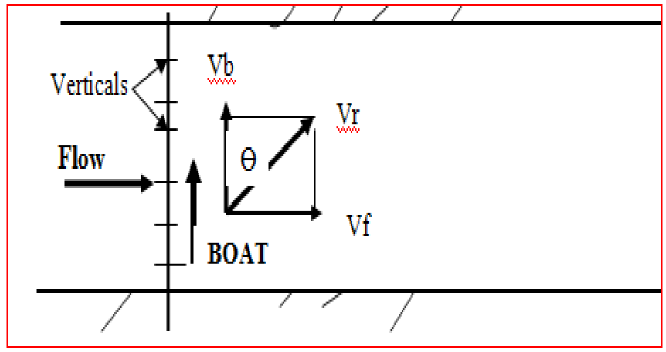
A= cross sectional area (m^2)

Types of floats:

1. Surface floats: wooden or metallic object, leaf, T-t ball
2. Subsurface float : two floats tied together by thin cord
3. Rod Float : Cylindrical rod

Moving Boat method

Suitable in deep river.



where, Vb= Velocity of boat at right angle to the stream

Vf= Flow velocity

Vr =Resultant velocity

Θ = angle made by the resultant velocity with the direction of boat

Δt = time of transit between two verticals

Convert the surface velocity to average velocity Vav= 0.85 x $V_{surface}$

Vb = Vr CosΘ and Vf= Vr SinΘ

Discharge in each segment $\Delta Q_i = \left(\frac{Y_i+Y_{i+1}}{2}\right) W_i\ V_f$

where, $W_i = v_b\ \Delta t$

Total discharge $\mathbf{Q} = \sum \Delta Q_i$

Example 1: Data pertaining to a stream gauging operation at gauging site are given below. The rating equation of the current meter is V=0.51Ns +0.03 m/s where, Ns= revolution per second. Calculate the discharge in the stream.

Distance from left water edge(m)	0	1.0	3.0	5.0	7.0	9.0	11.0	12.0
Depth (m)	0	1.1	2.0	2.5	2.0	1.7	1.0	0
Revolution of a current meter kept at 0.6 depth	0	39	58	112	90	45	30	0
Duration of Observation (s)	0	100	100	150	150	100	100	0

Solution: For the first and last sections, Average width, $\overline{W} = \frac{\left(W_1+\frac{W_2}{2}\right)^2}{2W_1} = \frac{\left(1+\frac{2}{2}\right)^2}{2x\ 1}$ = 2.0m

For rest of the section, $\overline{W} = \frac{1}{2}$ width to the left $+ \frac{1}{2}$ Width to right

$$\overline{W} = \frac{1}{2}\text{ x } 2 + \frac{1}{2}\text{ x}2 = 2.0\text{m}$$

Since, velocity is measured at 0.6depth, measured velocity is average velocity at that vertical (V).

The discharge calculation by mid-section method is shown as below:

Distance from left water edge(m)	Average width $\overline{W}$ (m)	Depth Y (m)	Ns= Rev/sec	Velocity V (m/s)	Segmental Discharge m^3/sec
0	0	0			0.00
1	2	1.1	0.39	0.2289	0.5036
3	2	2.0	0.58	0.3258	1.3032
5	2	2.5	0.747	0.4110	2.0549
7	2	2.0	0.60	0.3360	1.3440
9	2	1.7	0.45	0.2595	0.8823
11	2	1.0	0.30	0.1830	0.3660
12	0	0			0.00
				Sum	6.4539

Hence, Discharge in the stream (Q)= 6.454 m^3/s

Slope Area Method:

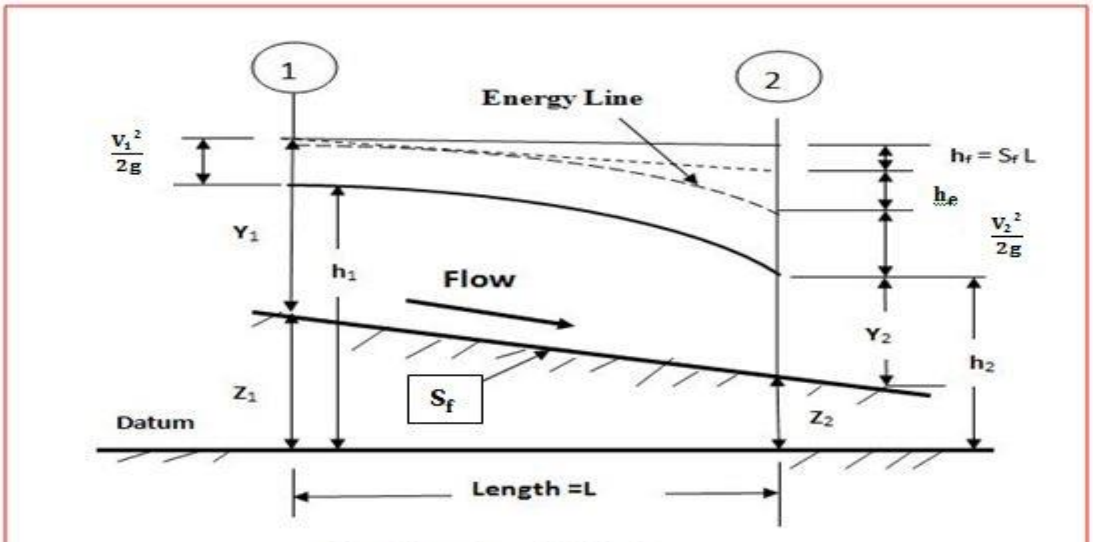


Fig: Slope Area Method

Applying Energy equation between section 1 and 2, we have

$$Z_1 + Y_1 + \frac{{V_1}^2}{2g} = Z_2 + Y_2 + \frac{{V_2}^2}{2g} + h_L$$

Where, h_L= head loss in reach.

Head loss $\mathbf{h_L}$ two parts (i) friction loss h_f + eddy loss h_e.

Denote Z+y = h, then

$$h_1 + \frac{{V_1}^2}{2g} = h_2 + \frac{{V_2}^2}{2g} + h_e + h_f$$

$$h_f = (h_1 - h_2) + \left(\frac{{V_1}^2}{2g} - \frac{{V_2}^2}{2g}\right) - h_e \text{ ....(i)}$$

If L= length of the reach, using Manning's formula for uniform flow,

$$\frac{h_f}{L} = S_f = \text{Energy Slope} = \frac{Q^2}{K^2}$$

Where, K= conveyance of the channel $= \frac{1}{n} \; A\,R^{2/3}$

In non-uniform flow average conveyance is used to estimate the average energy slope and

$$\frac{h_f}{L} = \;\bar{S}_f = \frac{Q^2}{K^2} \text{ ....(ii)}$$

Where, $K = \sqrt{K_1 K_2}\,;\; K_1 = \frac{1}{n_1}\; A_1\,{R_1}^{2/3}$ and $K_2 = \frac{1}{n_2}\; A_2\,{R_2}^{2/3}$

The eddy loss is estimated as

$$h_e = K_e \left|\frac{V_1{}^2}{2g} - \frac{V_2{}^2}{2g}\right| \text{.............................(iii)}$$

Where, Ke= eddy loss coefficient whose value varies from 0.3 to 0.8 for gradual and abrupt transition. Equation (i), (ii) and (iii) together with equation of continuity Q= $A_1V_1=A_2V_2$ and for known values of n, h_f and channel properties, discharge can be calculated by trial and error procedure using flowing sequence of calculations:

1. Assume $V_1=V_2$. This leads to $\frac{V_1{}^2}{2g} = \frac{V_2{}^2}{2g}$ and $h_f = h_1-h_2$ =F= fall in water surface between 1 and 2 section.
2. Using equation (ii) calculate the discharge Q
3. Compute $V_1= Q/A_1$ and $V_2= Q/A_2$. Calculate velocity heads and eddy loss h_e.
4. Now calculate the refined value of h_f using equation (i) and go to step 2. Repeat the calculations till two successive calculations gives values of discharge or h_f differences by negligible margin.

   This method of estimating discharge is known as S**lope-Area** method.

Stage Discharge Relationship

In this method, two steps are involved. First step is to measure the discharge and set up a relationship between stage and discharge. Once, this is done, the second stage is to reading the stage and finding the discharge. The second part is a routine operation. The current meter and other direct discharge measurements are to prepare the stage –discharge relationship for the given channel gauging section.

Example: During a flood flow the depth of water in a 10m wide rectangular channel was found to be 3m x 2.9m at two sections 200m apart. The drop in the water surface elevation was found to be 0.12m. Assuming Manning's coefficient (n) to be 0.025, estimate the fixed flood discharge through the channel.

Solution:

Let us assume sections 1 and 2 respectively.

Section 1	**Section 2**
$Y_1 = 3$m	$Y_2 = 2.9$m
$A_1 = 30$m2	$A_2 = 29$m2
$P_1 = 16$m	$P_2 = 15.8$m
$R_1 = 1.875$m	$R_2 = 1.835$m
$K_1 = \frac{1}{0.025}$ x 30 x $(1.875)^{2/3} = 1824.7$	$K_2 = \frac{1}{0.025}$ x 29 x $(1.835)^{2/3} = 1738.9$

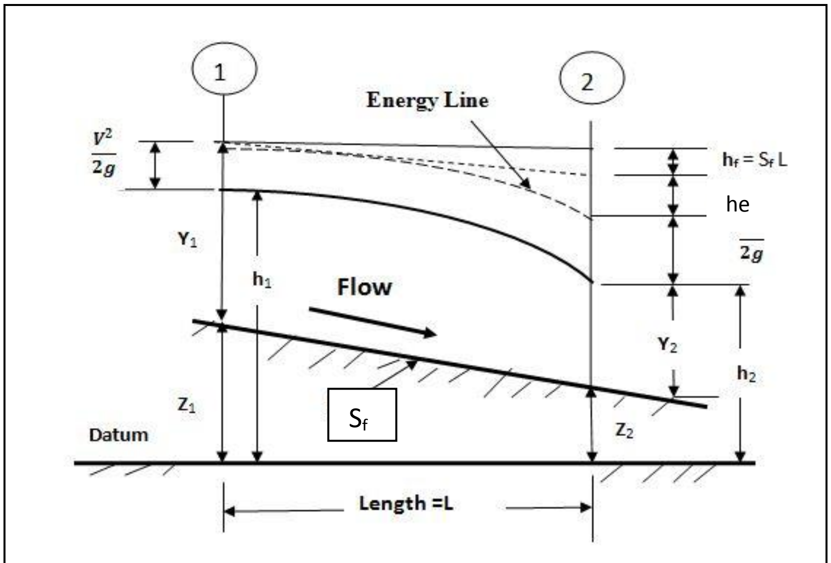
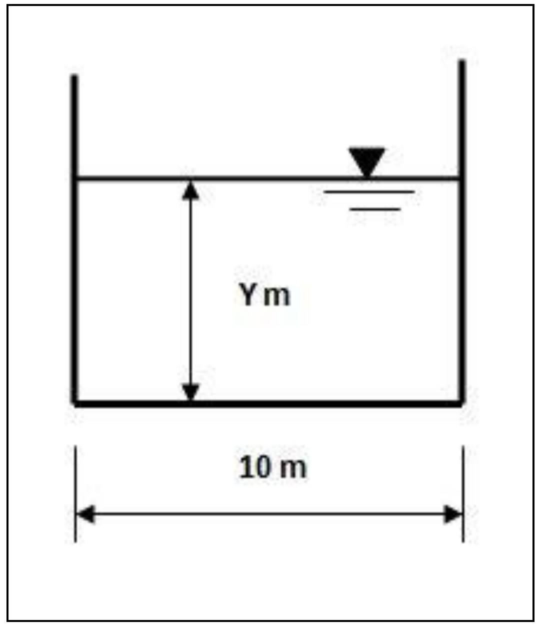


Fig: Slope Area Method



Average K for the reach = $\sqrt{K_1 \text{ x } K_2}$ = 1781.3

To start with assume fall, hf = 01.2m (given) and Eddy loss h_e=0

$$\bar{S}_f = \frac{h_f}{L} = \frac{h_f}{200}$$

$$Q = K\sqrt{\bar{S}_f} = 1781.3\sqrt{\bar{S}_f}$$

$$\frac{V_1{}^2}{2g} = \frac{\left(\frac{Q}{30}\right)^2}{2X9.81} \qquad \text{and} \qquad \frac{V_2{}^2}{2g} = \frac{\left(\frac{Q}{29}\right)^2}{2X9.81}$$

We have,

$$h_f = (h_1 - h_2) + \left(\frac{V_1{}^2}{2g} - \frac{V_2{}^2}{2g}\right) - h_e$$

$$h_f = \text{Fall} + \left(\frac{V_1{}^2}{2g} - \frac{V_2{}^2}{2g}\right) - 0 = 0.12 + \left(\frac{V_1{}^2}{2g} - \frac{V_2{}^2}{2g}\right) \quad \text{...........(i)}$$

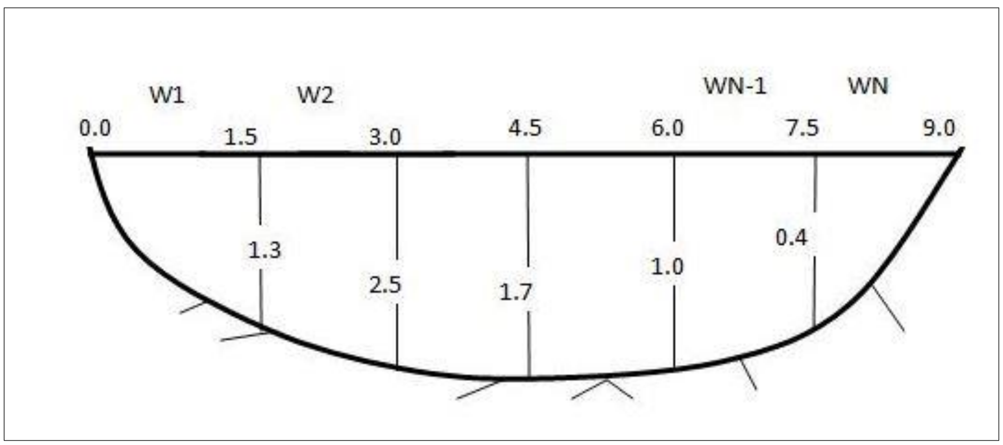
Trial	h_f	$\bar{S}_f$ (Unit of 10^{-4})	Q(m^3/s)	$\frac{V_1{}^2}{2g}$ (m)	$\frac{V_2{}^2}{2g}$ (m)	h_f equation(i) (m)
1	0.1200	6.000	43.63	0.1078	0.1154	0.1124
2	0.1124	5.622	42.24	0.1010	0.1081	0.1129
3	0.1129	5.646	42.32	0.1014	0.1081	0.1129

As per the condition, the assumed h_f is equal to computed h_f =0.1129 for the discharge 42.34m^3/s. Hence, the discharge in the channel is **42.32m^3/s**.

Example: The following data were collected during a stream-gauging operation in a river. Compute discharge.

Distance from left water edge (m)	Depth (m)	Velocity (m/s)	
		At 0.2d	At 0.8d
0.0	0.0	0.0	0.0
1.5	1.30	0.60	0.40
3.0	2.50	0.90	0.60
4.5	1.70	0.70	0.50
6.0	1.00	0.60	0.40
7.5	0.40	0.40	0.30
9.0	0.0	0.0	0.0

Solution:



Average width for the 1st and last section

$$= \frac{(W_1 + W_2)^2}{2W_1} = \frac{\left(1.5 + \frac{1.5}{2}\right)^2}{2x1.5} = 1.6875\ m$$

$$= \frac{(W_N + W_{N-1})^2}{2W_N} = \frac{\left(1.5 + \frac{1.5}{2}\right)^2}{2x1.5} = 1.6875\ m$$

Rest of the sections are = $\frac{\mathbf{1.5}}{\mathbf{2}} + \frac{1.5}{2} = 1.5m$

Distance	Average	Depth (m)	Average velocity	Segment	Segmental discharge

from left water edge (m) Col (1)	width(m) Col (2)	Col (3)	$\bar{V} = \frac{V_{0.2}+V_{0.8}}{2}$ (m/s) Col(4)	area (m^2) Col(5)=Col (2) x Col(3)	**(m^3/s)** Col6=Col(4) x Col(5)
0.0	**0**	**0**	**0**	**0**	**0**
1.5	**1.6875**	**1.3**	**0.5**	**2.193**	**1.096**
3.0	**1.5**	**2.5**	**0.75**	**3.75**	**2.812**
4.5	**1.5**	**1.7**	**0.6**	**2.55**	**1.530**
6.0	**1.5**	**1.0**	**0.5**	**1.50**	**0.75**
7.5	**1.6875**	**0.4**	**0.35**	**0.675**	**0.236**
9.0	**0**	**0**	**0**	**0**	**0**
				$\sum \boldsymbol{Sum}$	**6.424**

4.7 Development of Rating curve and its uses

Rating curve is the graphical plot between the measured discharge and the stage of the river. The stage is selected based on the good gauge site, where the rapid fluctuation of stage is not too much. The stage discharge relation is also known as rating curve. The discharge is plotted in abscissa and stage as the ordinate. Its shape is concave upward on rectilinear coordinates.

The discharge in Nepal varies in two periods-Monson and non-monsoon periods. Hence, separate rating curves could be developed.

Construction of Rating Curve:

1. Define point where rating curves are required-generally at foot of all hydraulic structures
2. Carry out survey and develop cross-section at the required point with HFL marks and magnitudes of flood peaks
3. Use Manning's Formula;

$$Q = \frac{1}{n} A R^{2/3} S^{!/2}$$

Where, Q= discharge in m3/s

A= area of cross section in m2

n= rugosity coefficient

R= hydraulic radius in m

S= slope of the stream

Equation of rating Curve:

Non-alluvial rivers exhibit permanent control. In this case single valued relationship can be used as::

$$Q = C_r (G - a)^{\beta} \ldots\ldots\ldots\ldots\ldots..(4.1)$$

Where, Q= discharge

G= gauge height

a= a constant represents gauge reading corresponding to zero discharge

C_r and β are rating curve constants. The graph can be plotted on logarithmic or arithmetic scale. Logarithmic plot is straight line and more advantageous. The best fit curve can be obtained by least square method.

$$\log Q = \beta \log(G - a) + \ log C_r$$

$$Y = \beta X + \ m$$

Value of β and m from regression given as,

$$\beta = \frac{N(\sum XY) - (\sum X)(\sum Y)}{N(\sum X^2) - (\sum X)^2}$$

$$m \ = \frac{\sum Y - \beta(\sum X)}{N}$$

Pearson product moment correlation coefficient,

$$\mathbf{r} = \frac{N(\sum XY) - (\sum X)(\sum Y)}{\sqrt{N(\sum X^2) - (\sum X)^2 [N(\sum Y^2) - (\sum Y)^2]}}$$

The variable X and Y are positively correlated and hence **r** is positive. Hence, the equation $Q = C_t (G - a)^{\beta}$ is called a ***rating equation*** of the stream and can be used to estimate the discharge Q of the stream for a given gauge reading G within the range of data used in its derivation.

Stage of zero discharge

The constant representing the stage for zero discharge in the stream is hypothetical parameter and can not be measured in the field.

Method to measure stage of zero discharge:

1. Plot Q vs G on the an arithmetic graph paper and draw a best fit curve. Extrapolate the curve by eye judgment and find “a” as the value of G corresponding to Q = 0. Using value of “a”, plot a logQ vs Log (G-a) and verify whether the data plots represent a straight line. If not select another value of “a” in the neighborhood of previously assumed value and by trial and error find acceptable value of “a”. Continue it with modification till a straight line is obtained.
2. Plot Q vs G to an arithmetic scale and fit the smooth curve. Select three points A, B, C on the curve such that their discharges are in the geometric progression.

$$\frac{Q_A}{Q_B} = \frac{Q_B}{Q_C}$$

Draw vertical lines at A and B and horizontal lines at B and C. Then two straight lines ED and BA are drawn to intersect at F as shown in fig 5.

The ordinate of F is required value of "a".

3. Plot Q vs G to arithmetic scale. Draw a smooth good fitting curve by eye judgment. Select three discharges Q1, Q2, and Q3 such that

$$\frac{Q_1}{Q_2} = \frac{Q_2}{Q_3}$$

and note down the corresponding values of stages G1, G2 and G3 such that

$$\frac{(G_1 - a)}{(G_2 - a)} = \frac{(G_2 - a)}{(G_3 - a)}$$

$$Or, \qquad a = \frac{G_1 . G_3 - {G_2}^2}{(G_1 + G_3) - 2G_2}$$

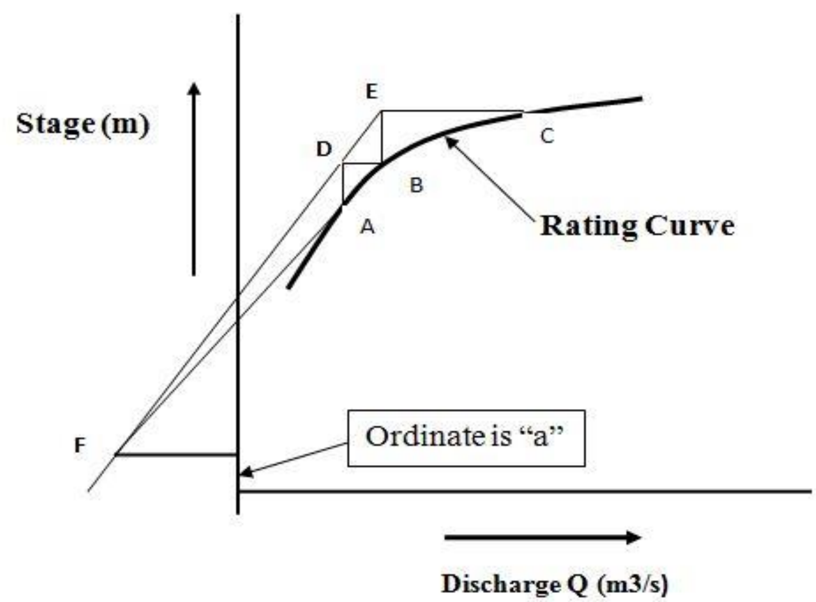


Fig : Running's Method of determining "a"

Method for extension of rating curve

1. Extension based on logarithmic plotting of rating curve or using rating equation

2. Velocity area method: Extend stag velocity and stage- area curve
3. Conveyance slope method based on Manning equation:

$$\mathbf{Q} = \frac{\mathbf{1}}{\boldsymbol{n}} \boldsymbol{A}\, \boldsymbol{R}^{2/3} \boldsymbol{S}^{1/2} = \boldsymbol{K}\sqrt{\boldsymbol{S}}$$

Where, K = conveyance of channel = $\frac{1}{n} A\, R^{2/3}$

- For different stages, Compute the value of K.
- Compute S for different stages of by using $S = \frac{Q^2}{K^2}$ from available Q-h data
- Plot and extend stage –conveyance (K) stage-slope (S) curve. For different stages, take K and S from the two curves and compute Q and $K\sqrt{S}$.

Assumptions: for higher stages the slope remains constant.

Control:

Control is combined effect of channel and flow parameters which govern the stage-discharge relationship. If the rating curve does not change with time, the control is called permanent control, In other words, the station with permanent control has single valued rating curve.

If the rating curve changes with time, it is called shifting control.

Shifting Control:

- Vegetation growth, dredging or channel encroachment: no unique rating curve
- Aggradations or degradation in alluvial channel: no unique rating curve
- Variable backwater effect: same stage indicting different discharge
- Unsteady flow effects of rapidly changing stage: for the same stage, low discharge during rising and high discharge during falling (looped rating)

  To take into account the backwater effect, secondary gauge is installed at some distance downstream of gauging site and reading of both gauges are taken. Then, the fall of water surface in the reach is computed. The relationship for the actual discharge (Q) is given by:

$$\frac{Q}{Q_O} = \left(\frac{F}{F_O}\right)^m$$

Q_O= normalized discharge at the given stage when fall = F_O when the stage in the river is same in both cases.

F= Actual Fall

m= exponent = 0.5

Correction has to be applied in case of unsteady flow due to flood wave. The actual discharge (Q) under unsteady condition is given as,

$$Q = Q_O \sqrt{1 + \frac{1}{V_w S_O}\frac{dh}{dt}}$$

Where,

Q_O= discharge under steady flow conditions

V_w= velocity of flood wave

S_O= Bed slope of river

$\frac{dh}{dt}$ = rate of change of stage

Example: 1

Three points on rating curve of a stream gauging station obtained from observed data have the following coordinates: (2m3/s, 10.65m), (4m3/s, 10.85m) and (8m3/s, 11.25m). Determine the equation of rating curve and compute the discharge in the stream corresponding to stage of 11.5m.

Solution:

Qm³/s	**G(m)**
2	**10.65**
4	**10.85**
8	**11.25**

From rating curve equation: $Q = C_r (G - a)^{\beta}$

Here, $Q_2 = \sqrt{Q_1 Q_3}$

$$a = \frac{G_1.G_3 - {G_2}^2}{(G_1+G_3) - 2G_2} = \frac{10.65\ x 11.25 - 10.85^2}{(10.65+11.25) - 210.85} = 10.45$$

Finding β, Cr algebraically,

$$\frac{Q_2}{Q_1} = \frac{C_r(G_2 - a)^{\beta}}{C_r(G_1 - a)^{\beta}}$$

$$\frac{4}{2} = \frac{C_r(10.85 - 10.45)^{\beta}}{C_r(10.65 - 10.45)^{\beta}}$$

$$\text{Or,}\quad 2 = (2)^{\beta}$$

$$\text{Or,}\quad \beta = 1$$

$$Q_1 = C_r\,(G_1 - a)^{\beta}$$

$$2 = C_r\,(10.65 - 10.45)^1$$

$$\text{Or,}\quad C_r = 10$$

The equation of rating curve is: $Q = 10(G - 10.45)^1$

For G= 11.5m, $Q = 10(11.5 - 10.45)^1 = 10.5\ m^3/s$

Example 2:

The stage discharge data of a river are given below. Derive stage-discharge (rating curve) relationship to predict the discharge for a given stage. Assume the value of zero discharge stage as 149.10m. Compute the correlation coefficient of the established relationship. Determine the discharge corresponding to a stage of 152.45m

Stage(m)	151.35	151.66	151.92	152.78	153.41	153.83	154.51	155.42	155.76
Discharge (m^3/s)	20	60	110	300	505	650	1050	1390	1600

Solution: The rating curve equation is: $Q = C_r\,(G - a)^{\beta}$

Equation in the log form is, $\log Q = \beta \log(G - a) + \log C_r$

$$\text{Or,}\quad Y = \beta X + m$$

Stage for zero flow (a) = 149.10m (given**)**

Stage(m)	Q (m3/s)	(G-a)	X=log(G-a)	Y= logQ	X^2	Y^2	XY
151.35	20	2.25	0.3522	1.301	0.124	1.693	0.458
151.66	60	2.56	0.4082	1.778	0.167	3.162	0.726
151.92	110	2.82	0.4502	2.041	0.203	4.167	0.919
152.78	300	3.68	0.5658	2.477	0.320	6.136	1.402
153.41	505	4.31	0.6345	2.703	0.403	7.308	1.715
153.83	650	4.73	0.6749	2.813	0.455	7.912	1.898
154.51	1050	5.41	0.7332	3.021	0.538	9.128	2.215
155.42	1390	6.32	0.8007	3.143	0.641	9.879	2.517
155.76	1600	6.66	0.8235	3.204	0.678	10.266	2.639
		Sum	5.4432	22.481	3.529	59.651	14.489

Here, N=9, $\sum X$=5.4432, $\sum Y$= 22.481, $\sum X^2$= 3.529, $\sum Y^2$= 59.651, $\sum XY$= 14.489,

$$\beta = \frac{N(\sum XY)-(\sum X)(\sum Y)}{N(\sum X^2)-(\sum X)^2} = \frac{9\,x\,(14.489)-(5.4432\,x\,22.481)}{9x\,3.529-(5.4432)^2} = 3.7623$$

$$m = \frac{\sum Y - \beta(\sum X)}{N} = \frac{22.481 - 3.762x5.4432}{9} = 0.2224$$

But, $\log C_r = m$

Or, $C_r = 10^m = 10^{0.2224} = 1.668$

The rating curve equation is: $Q = C_r\,(G-a)^{\beta} = 1.668(G - 149.10)^{3.7623}$

Coefficient of correlation (r) is: $= \frac{N(\sum XY)-(\sum X)(\sum Y)}{\sqrt{N(\sum X^2)-(\sum X)^2}\ \sqrt{[N(\sum Y^2)-(\sum Y)^2]}}$

$$Or, (r) = \frac{9x14.488 - 5.4432\ x\ 22.481}{\sqrt{9\ x3.529 - (5.4432)^2}\ \sqrt{[9x\ 59.651 - (22.481)^2]}} = 0.979$$

For $G = 152.45m$, $Q = 1.668(152.45 - 149.10)^{3.7623} = 157.605\ m^3/s$

4.8 Estimation of monthly flows from rainfall

Based in hydrological and meteorological data, the river basin is divided into two types:

(i) Gauged river basin (GRB)

(ii) Un-gauged river basin (UGRB)

For un-gauged river basins, the mean monthly flow and flow duration curve shall be determined by three methods:

1. **Medium Irrigation Project (MIP) Method**
2. **Water and Energy Commission Secretariat (WECS)/Department of Hydrology and Meteorology (DHM) , Method**
3. **Catchment Area Ratio (CAR) method**

1. **MIP method:** This method is used op compute the monthly discharge of the un-gauged locations (site) throughout the year. To use this method it is require to obtain one flow measured in the low flow period from November to April.
2. In this method, Nepal is divided into 7 hydrological zones. Once the catchment area and the one flow measurement in the low flow period is obtained and zone is identified, long term average monthly flow can be determined by multiplying the unit hydrograph (of the concerned region) with the measured catchment area.

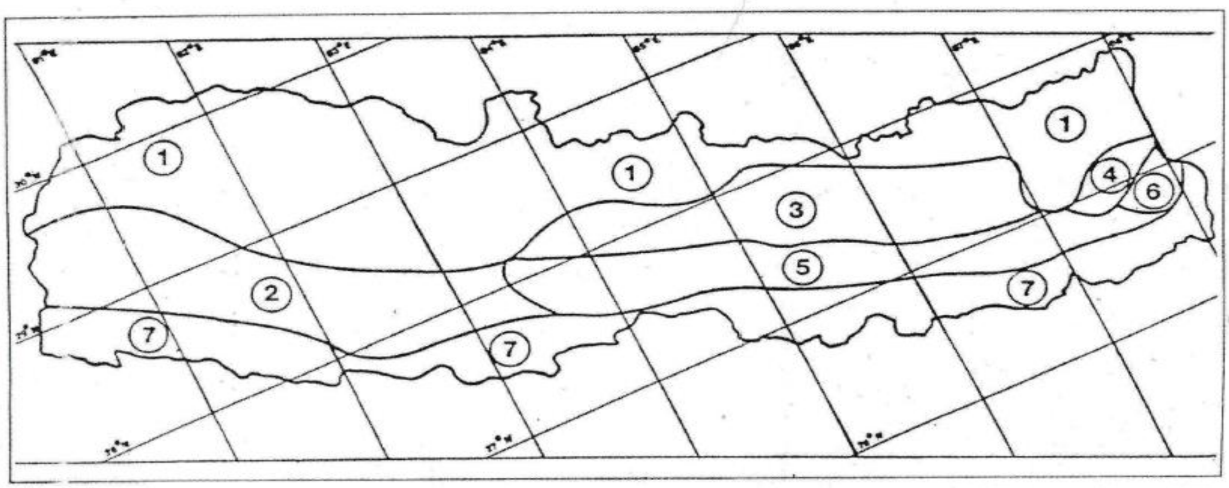
3. Hydrological zone is identified based on the location of scheme in the hydrological zoned map of Nepal.
4. MIP map is used for better result for the catchment area lesser than $100 km^2$.

If the measured date is on 15th of the particular month, the coefficient given in the table is directly used. For other date of measurement, coefficient for that date is found by interpolation.

$$\text{April Flow} = \frac{\textbf{Measured Discharge}}{\textbf{Coefficient of a particular month}}$$

Monthly Flow = **April Flow x Monthly Coefficient**

Runoff



1. Mountain catchments
2. Hills to the north of Mahabarat, river rising north of Siwaliks, inner terai.
3. Pokhara, Nuwakot, Kathmandu, Sunkosi tributaries.
4. Lower Tamur Valley
5. Rivers draining Mahabarats
6. Kankai Mai basin.
7. Rivers draining from Churia range to the terai.

Figure 5-15 : Hydrological regions of Nepal

Table : MIP non-dimensional regional hydrographs (Coefficients)

Month	Region						
	1	**2**	**3**	**4**	**5**	**6**	**7**
May	**2.60**	**1.21**	**1.88**	**2.19**	**0.91**	**2.57**	**3.50**
June	**6.00**	**7.27**	**3.13**	**3.75**	**2.73**	**6.08**	**6.00**
July	**14.50**	**18.18**	**13.54**	**6.89**	**11.21**	**24.32**	**14.00**
Aug	**25.00**	**27.27**	**25.00**	**27.27**	**13.94**	**33.78**	**35.00**
Sep	**16.50**	**20.19**	**20.83**	**20.91**	**10.00**	**27.03**	**24.00**
Oct	**8.00**	**9.09**	**10.42**	**6.89**	**6.52**	**6.08**	**12.00**
Nov	**4.10**	**3.94**	**5.00**	**5.00**	**4.55**	**3.38**	**7.50**
Dec	**3.10**	**3.03**	**3.75**	**3.44**	**3.33**	**2.57**	**5.00**
Jan	**2.40**	**2.24**	**2.71**	**2.59**	**2.42**	**2.03**	**3.30**
Feb	**1.80**	**1.70**	**1.88**	**1.88**	**1.82**	**1.62**	**2.20**
Mar	**1.30**	**1.33**	**1.38**	**1.38**	**1.36**	**1.27**	**1.40**
Apr	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**

Note: All the values for mid-month

Example: Flow measurement made on February 1^{st} on a river/stream indicated a flow of 220lps. The catchment area of the river is $60km^2$. The catchment lies in region 4. Estimate the monthly flow using MIP method.

Solution: Measured discharge = 220lps on February 1^{st}

Take coefficient of January and February for 1^{st} February.

Coefficient of February $1^{st} = \frac{2.59+1.88}{2} = 2.235$

April Flow $= \frac{220}{2.235} = 98\text{lps}$

Now multiply coefficient of each month by 98,

Month	Coefficient of Region 4	Predicted Flow (lps)
May	**2.19**	**214.62**
June	**3.75**	**367.50**
Jul	**6.89**	**675.22**
Aug	**27.27**	**2572.46**
Sep	**20.91**	**2049.18**
Oct	**6.89**	**675.22**
Nov	**5.00**	**490.00**
Dec	**3.44**	**337.12**
Jan	**2.59**	**253.82**
Feb	**1.88**	**184.24**
Mar	**1.38**	**135.24**
Apr	**1.00**	**98.00**

WECS/DHM (Hydest) Method

1. This method is developed to predict river flows of catchment area larger than $100km^2$ of un-gauged river based on hydrological theories, empirical equations and statistics.
2. In this method, the total catchment areas between 5000m to 3000m are required as input data.
3. Flow contribution per unit area (km^2) for 5000 to 3000m and from lower elevations i.e., between 3000m is assumed to be in different proportions during flood. However, for long term average monthly flows, all areas below 5000m are assumed to contribute flows equally per km^2 area.
4. The average monthly flows can be calculated by the equation:

   $$Q_{\text{mean,month}} = \text{C x (Area of Basin)}^{A1} \text{ x (Area Below 5000m} + 1)^{A2} \text{ x}$$
   $$\text{((Mean Monsoon precipiatation)}^{A3}.$$

Where,

$Q_{\text{mean,month}}$ = mean flow for a particular month in m3/s. C, A_1, A_2, and A_3 are coefficients of different months. Catchment area can be calculated from the topographical maps (maps that show contours) once the intake location is identified.

5. The input data required in the equation are total basin area, basin area below 5000m (km^2) and average monsoon precipitation (km^2) estimated from isohyetal maps.

Table: Values of Coefficients

Month	C	A_1	A_2	A_3
Jan	0.01423	0	0.9777	0
Feb	0.01219	0	0.9766	0
Mar	0.009988	0	0.9948	0
Apr	0.007974	0	1.0435	0
May	0.008434	0	1.0898	0
Jun	0.006943	0.9968	0	0.2610
Jul	0.02123	0	1.0093	0.2523
Aug	0.02548	0	0.9963	0.2620
Sep	0.01677	0	0.9894	0.2878
Oct	0.009724	0	0.9880	0.2508
Nov	0.00176	0.9605	0	0.3910
Dec	0.001485	0.9536	0	0.3607

Example equation for January:

$$Q_{\text{mean,Jan}} = 0.01423 \text{ x } (\text{Area of Basin})^{0} \text{ x } (\text{Area below } 5000\text{m} + 1)^{0.9777} \text{ x } (\text{Mean Monsoon precipiatation})^{0}$$

$$Q_{\text{mean,Jan}} = 0.01423 \text{ x } (\text{Area below } 5000\text{m} + 1)^{0.9777}$$

Example equation for July:

$$Q_{\text{mean,Jul}} = 0.02123 \text{ x } (\text{Area of Basin})^{0} \text{ x } (\text{Area below } 5000\text{m} + 1)^{1.00.93} \text{ x } (\text{Mean Monsoon precipiatation})^{0.2523}$$

$$Q_{\text{mean,Jul}} = 0.02123 \text{ x } (\text{Area below } 5000\text{m} + 1)^{1.00.93} \text{ x } (\text{Mean Monsoon precipiatation})^{0.2523}$$

Solved Example:

Calculate February and July flows for a 510km^2 catchment area whose watershed is below 5000m elevation. The average monsoon rainfall in the catchment is 1300mm. Use WECS method.

Solution: Given

Basin Area = 510 km^2

Mean monsoon rainfall = 1300mm

$$Q_{\text{mean,February}} = 0.01219 \text{ x } (\text{Area of Basin})^{0} \text{ x } (\text{Area below } 5000\text{m} + 1)^{0.9766} \text{ x } (\text{Mean Monsoon precipiatation})^{0}$$

$$= 0.01219 \text{ x } (510 + 1)^{0.9766} = 5.38 \text{ m}^3/\text{s}$$

$$Q_{\text{mean,July}} = 0.02123 \text{ x } (\text{Area below } 5000\text{m} + 1)^{1.00.93} \text{ x } (\text{Mean Monsoon precipiatation})^{0.2523}$$

$$Q_{\text{mean,Jul}} = 0.02123 \text{ x } (510 + 1)^{1.00.93} \text{ x } (1300)^{0.2523} = \mathbf{70.17\ m^3/s}$$

Catchment Area Ratio Method (CAR Method)

(i) For the two hydrologically similar catchment areas, the extension of hydrological data for proposed site under study could be done simply by multiplying the available long term data of similar catchments (HSC) with ratio of catchment areas of base (proposed site under study) and index (HSC) stations.

(ii) A more accurate result in the context of Nepal might be obtained by using Dicken's formula as:

$$Q_b = Q_i \frac{A_b}{A_i}$$

$$\text{Or,} \quad \frac{Q_b}{Q_i} = \frac{A_b}{A_i}$$

Where, Q_b= discharge of base catchment (Proposed site under study) area in m^3/s

Q_i= discharge of index catchment area in m^3/s

A_i= drainage area of index catchment in sq.km

A_b= drainage area of base catchment in sq.km

Suffix "b" stands for base station and "i" stands for index stations.

(iii) This method is useful when the hydro-meteorological data of the index station are used in the extension for the base station having similar catchment characteristics.

Discharge Measurement

1. **From Rainfall Records:** The run-off for a drainage area can be estimated from rain-fall records as

Run off = rain fall * "run-off coefficient"

The run-off coefficient takes into account the various losses and will depend upon the nature of the catchment area e.g. 0.85 for concrete pavement, 0.5-0.1 for forest area depending upon soil. This is not an accurate method of measuring run-off.

2. **Empirical Formulas:** Empirical relations to determine the stream flow relate only to a particular site and cannot be relied upon for general use. In Nepalese scenario, Medium hydropower study project (NEA 1997) has proposed a formula to estimate discharge. Input data required are mean monsoon precipitation and catchment area. Constants used are given in table below.

MHSP prediction equation constants for mean flow			
Month	C	A1	A2
January	0.03117	0.8644	0.0000
February	0.02417	0.8752	0.0000
March	0.02053	0.8902	0.0000
April	0.01783	0.9558	0.0000
May	0.01193	0.9657	0.0000
June	0.01135	0.9466	0.2402
July	0.01641	0.9216	0.3534
August	0.02592	0.9095	0.3242
September	0.02206	0.8963	0.3217
October	0.01504	0.8772	0.2848
November	0.00792	0.8804	0.2707
December	0.00538	0.8890	0.2580

Discharge for a month is calculated as:

$$\mathbf{Q_{month} = C * (catchment\ area)^{A1} \times (Mean\ monsoon\ precipitation)^{A2}}$$

Let, catchment area = 400 km^2 and Mean Monsoon Precipitation = 2400 mm, then

$$\mathbf{Q_{jan} = 0.03117*(400)^{0.8644} * (2400)^{0} = 5.53 m^3/s}$$

$$\mathbf{Q_{july} = 0.01641 * (400)^{0.9216} * (2400)^{0.3534} = 64.23\ m^3/s}$$

3. Actual Measurement

Direct measurement by stream gauging at a given site for a long period is the only precise method of evaluation of stream flow. The flow is measured by selecting a channel of fixed cross-section and measuring the water velocity at regular intervals. The velocity of flow can be measured with the help of flow method or current meter.

Hydrograph and Flow Duration Curve (FDC)

A hydrograph indicates the variation of discharge or flow with time for a specified time. The time period can be an hour, a day, a week or a month. The discharge may be in m^3/s. The common nature of hydrograph is shown below.

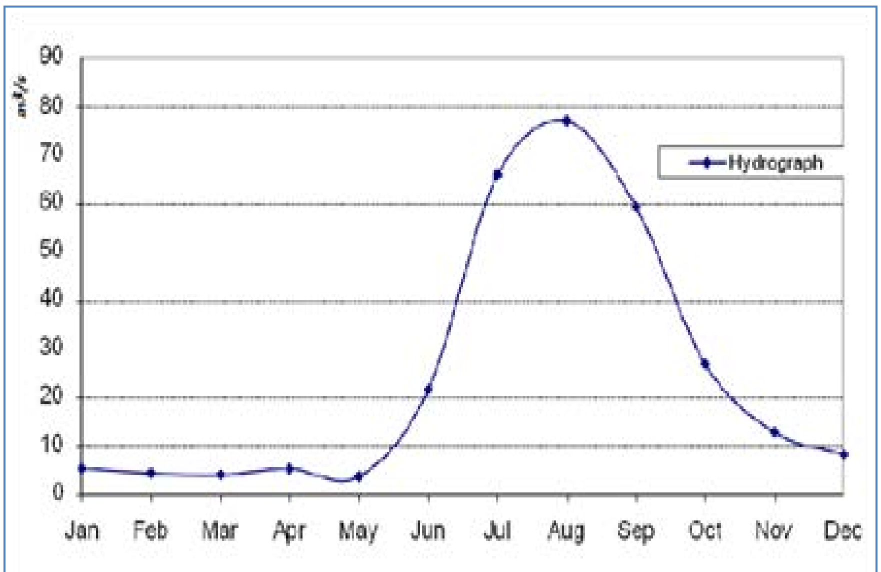


Fig. Hydrograph of a Typical Nepalese River

A flow duration curve shows the relation between flows and lengths of time during which they are available. The curve is a plot that shows the percentage of time that flow in a stream is likely to equal or exceed some specified value of interest.

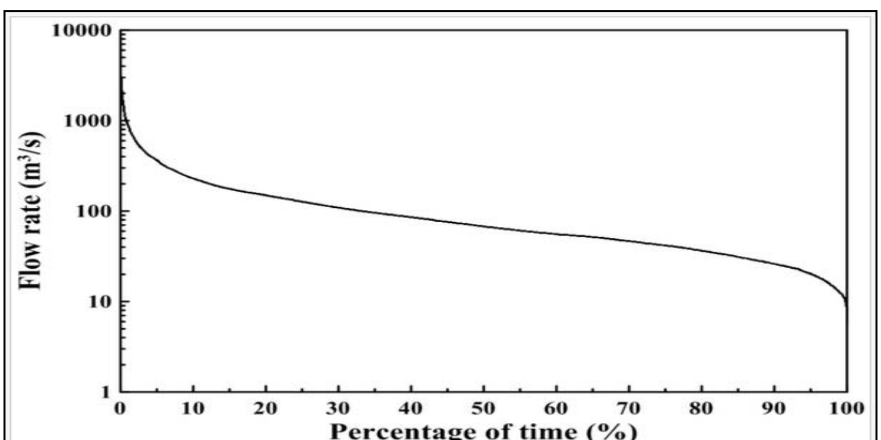


Fig. Flow Duration Curve of a River

Flow duration curve can be plotted from a hydrograph. The empirical formula for FDC developed by NEA is given below.

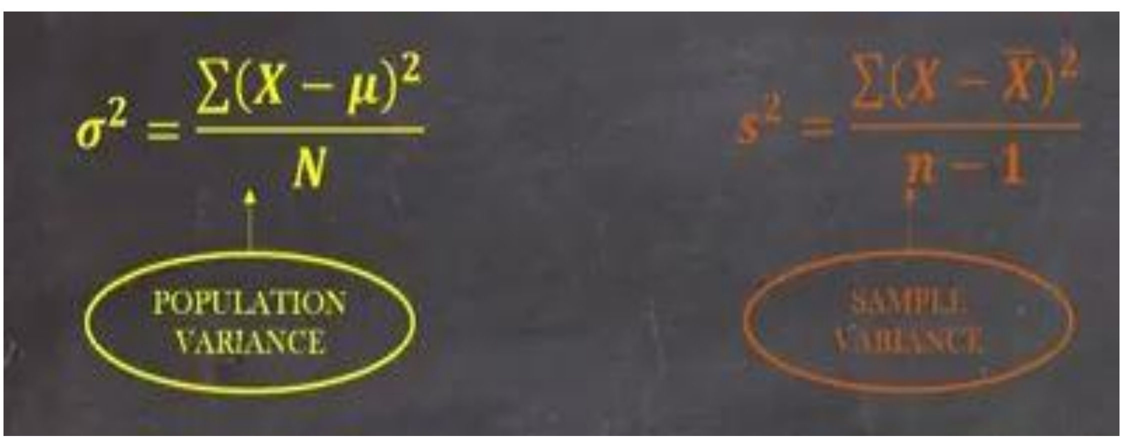
$Q_{required\ \%}$ = C x (Catchment area)^b x (mean monsoon precipitation)^a

For example, for discharge of minimum of 45% of the time of above example

$Q_{45\%} = 0.008915 \times 400^{0.9239} \times 2600^{0.2018} = 11.05\ m^3/sec.$

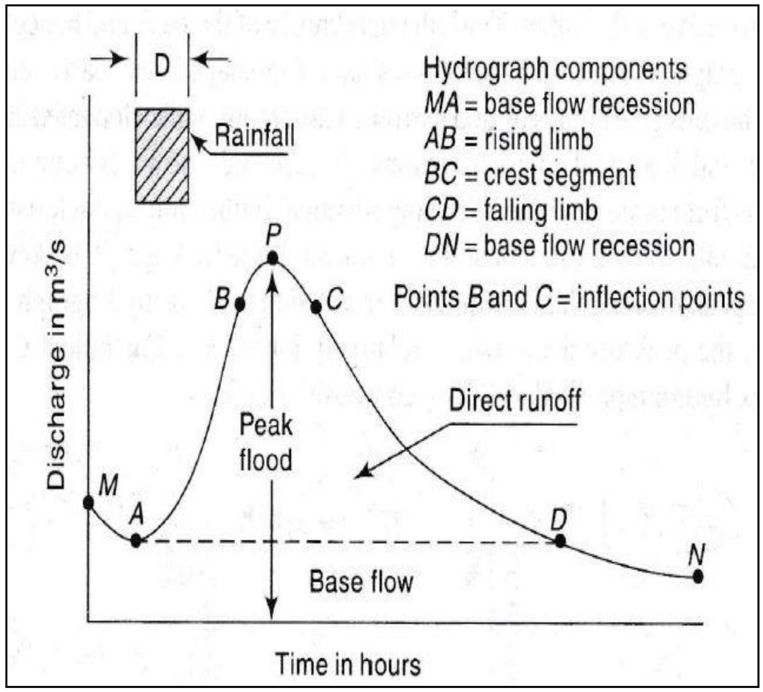
For values of coefficient **a, b** and **c**, the table given below is referred.

Daily Flow Duration Curve's coefficients developed by NEA			
Dependent variable	**b**	**a**	**c**
Maximum flow(Q_o)	**0.812**	**0.5337**	**0.061411**
25% exceedance	**0.9279**	**0.2986**	**0.012434**
45% exceedance	**0.9239**	**0.2018**	**0.008915**
65% exceedance	**0.9044**	**0**	**0.024831**
85% exceedance	**0.9256**	**0**	**0.014491**
95% exceedance	**0.9531**	**0**	**0.008645**
Minimum flow	**1.1689**	**0**	**0.000738**



Chapter 5: Hydrograph Analysis

5.1 Components of a Hydrograph:



The graphical presentation of the instantaneous rate of discharge (Q) of a storm with respect to time (T) is known as a hydrograph. The hydrograph is the response of the catchment to a rainfall input. It is the result of physiographic and hydro-meteorological effects of the watershed.

The hydrograph may have single peak or multiple peaks depending upon the nature of the storm and characteristics of the catchment. If the subsequent storm burst does not occur before the direct run off of the present storm ceases, single peaked hydrograph results. Such a storm is known as an isolated storm. If the next rainfall occurs before the direct run off of the previous rainfall ceases, there will be multiple peaked hydrograph. Such a storm is called as complex storm.

A single peaked hydrograph produced by storm consists of a rising limb, a crest segment and recession limb. The volume of water due to stream flow during any period can be obtained from the hydrograph by integrating between time t_1 and t_2. Thus volume of water = $\int_{t1}^{t2} Q\,dt$

The volume of stream flow during any period is equal to the area of the hydrograph between two successive time intervals.

Components of hydrograph:

(a) Rising Limb: It is the ascending portion of the hydrograph. It is influenced by storm and basin characteristics. The rising limb rises slowly in the early stage of flood but more rapidly towards the end portion. This is because in the initial stage the losses are high. The flow begins to build up the channel as the storm duration increases. It gradually reaches the peak when maximum area contributes.

(b) Peak or Crest Segment:
It is the part which contains peak flow-interest to hydrologists. Peak of hydrograph occurs when all portions of basins contribute at the outlet simultaneously at the maximum rate. Depending upon the rainfall-basin characteristics, the peak may be sharp, flat or may have several well defined peaks.

(c) Recession Limb:

Recession limb represents withdrawal of water from the storage built up in the basin during the earlier phase of the hydrograph. It extends from the point of inflection at the end of the crest to the beginning of natural groundwater flow. The recession limb is affected by basin characteristics only and independent of the storm.

Equation for recession curve:

$$Q_t = Q_o K_r^t$$

$Qo = \text{initial discharge}$

$Qt = \text{dischrge at a time interval of } t \text{ days}$

$Kr = \text{recession constant}$

Alternate form is:

$$Q_t = Q_o e^{-at}$$

Where

$a = -\ln K_r$

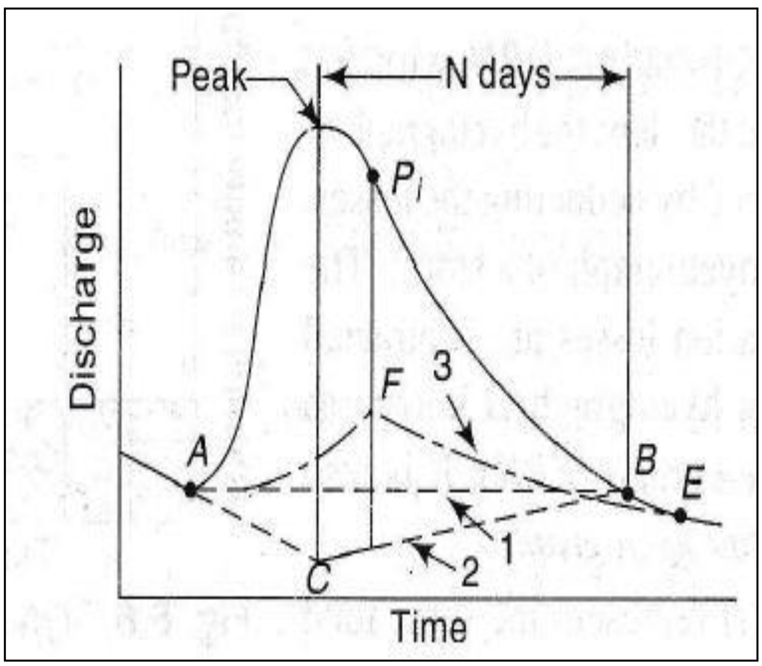
Time to peak: time laps between starting of the rising limb to the peak.

Time lag: The time interval between the center of the hyetograph and the peak discharge is called the basin lag. The basin lag depends upon the catchment and storm characteristics.

Time of concentration: Time taken by a drop of water to travel from the remotest point to the outlet.

Time base of hydrograph: Time between starting of runoff hydrograph to the end of direct runoff due to storm.

5.2 Separation of a base Flow



It is necessary to separate the observed hydrograph into its components parts namely surface runoff, interflow and ground water flow. However, it is standard practice to separate the hydrograph into two parts only i.e., direct runoff and base flow. The base flow is deducted from total storm hydrograph to obtain the surface flow hydrograph.

Method to separate Base Flow:

The separation of base flow is achieved by joining with a straight line the beginning of the surface run off to a point on the recession limb, which represents the end of direct run off.

The various graphical methods for base flow separation are depicted in the figure above.

Method 1: Straight line method:

Join the beginning of surface runoff to a point on the recession limb representing the end of direct run off.

A- beginning of direct run off

B- End point of direct runoff. This point is difficult to locate exactly. The end point is located by expert judgment or empirical equation.

Empirical equation to find end of direct run off: $\mathbf{N=0.83\ A^{0.2}}$

N= time interval from the peak to the end of direct run off (in days)

A= Basin or drainage area (in km^2)

Point A and B are joined and two flows are separated.

Method 2:

Extend the base flow curve prior to the commencement of surface run off till it intersects the ordinate drawn at the peak point. Join this point to the end point of direct run off.

Method 3:

Extend the base flow recession curve backwards after the depletion of flood water till it intersects the ordinate at the point of inflection. Join this point to the beginning of the surface run off by smooth curve.

Direct Run off Hydrograph (DRH): The surface run off hydrograph obtained after separating the base flow is called the direct run off hydrograph (DRH).

Rainfall Excess: If the initial loss and the infiltration loss are subtracted from the total rainfall, the remaining portion of rainfall is called the rainfall excess.

Rainfall Excess = Total rainfall – (initial loss + infiltration loss)

Effective Rainfall:

This is the portion of the rainfall which causes direct run off. As the direct run off includes the surface run off and interflow, the effective rainfall is slightly greater than the rainfall excess.

The effective rainfall can be obtained from the hydrograph. The volume of the direct run off obtained from the area of the hydrograph after separating the base flow. The effective rainfall is then obtained from the volume of the direct run off divided by the area of catchment.

$$\text{Effective Rainfall} = \frac{\text{Direct Run off Volume}}{\text{Area of Catchment}}$$

5.3 Unit Hydrographs, their uses and limitations

Unit hydrograph is defined as the direct runoff hydrograph (DRH) resulting from one-unit depth of rainfall excess generated uniformly over the basin at a constant rate for an effective duration (D). The term unit refers to a unit depth of rainfall excess which is 1cm in SI unit and 1inch in FPS unit.

Duration of unit hydrograph (D-hour UH) indicates the duration of rainfall excess.

The unit hydrograph gives the response of the catchment due to 1cm **of effective rainfal**l which occurs due to a storm of unit duration. It relates the direct run off hydrograph and the unit effective rainfall. Thus the unit hydrograph reflects the combined effect of all physical characteristics of catchment. This is to be noted that the average intensity of effective rainfall is equal to $\frac{1}{D}$ cm/hr for the unit hydrograph where D is duration

Features of unit hydrograph:

1. rainfall excess (re) = 1cm, runoff depth (rd) = 1cm
2. Continuity: Total depth of rainfall excess = Total depth of direct run off
3. Run off volume (Vd) = Basin area (A) x rd = A x 1 cm
4. Rainfall intensity = 1/D in cm/hour
5. Lumped response: Catchment as a single unit
6. Initial loss absorbed by the basin, no effect of antecedent storm condition

Uses of Unit Hydrograph:

- **i.** used to develop the flood hydrograph for extreme rainfall magnitudes for **design of hydraulic structures**
- **ii.** can be used for the watershed simulation models
- **iii.** used for flood forecasting and warning system
- **iv.** used to extend the flood flow records based on rainfall records
- **v.** Once the UH is prepared for duration D hr of a basin, the storm hydrograph of any other intensities (different intensities) but of same duration of that basin can be developed.

Limitations of Unit Hydrograph:

- (i) Cannot be applied for very large catchment area more than 5000km^2 as rainfall is not uniformly distributed in such case and cannot be used for basin lesser than 2km^2.
- (ii) Not suitable for long basin
- (iii) Applicable for short duration
- (iv) Precipitation must be from rainfall only. Snow melt run off cannot be satisfactorily represented. UH not applicable for the basin having large storage.
- (v) Principle of linearity of response is not strictly correct. Unit hydrograph derived from very light rainfalls have generally lower peaks than those derived from very heavy rainfalls.
- (vi) UH not applicable for basins having high variations of rainfall intensities
- (vii) UH theory not very accurate. The accuracy obtained is $\pm 10\%$
- (viii) The base period of direct run off is not exactly the same for all storms of same duration but of different intensities.

5.4 Derivation of Unit Hydrographs from isolated (single) storm

In this case, stream flow data and basin area are given. Single storm means all of the rainfall excess occurs at a reasonably uniform rate over a fairly short time period.

Steps followed:

1. Select a single peak isolated hydrograph. (Let Q_1, Q_2, Q_3, Q_4….. be the ordinates of the isolated hydrograph)
2. Take the rainfall which produces this single isolated peak
3. Separate the base flow from the total run off.
4. From the ordinates of the total run off hydrograph at the regular time interval, deduct the corresponding ordinates of base flow to obtain the ordinates of the DRH. Let Q_1, Q_2, Q_3, Q_4….. be the ordinates of the DRH which can be obtained as $O_1 = Q_1 - Q_b$, $O_2 = Q_2 - Q_b$, $O_3 = Q_3 - Q_b$ and so on.
5. Compute the volume of direct run off.

   Volume of DRH (V_d) = $\sum \mathbf{O_t}\, \Delta \mathbf{t}$
6. Divide the volume of DRH by the area of the drainage basin to obtain the runoff depth (r_d), $\mathbf{r_d} = \frac{\mathbf{V_d}}{\mathbf{A}}$

   Depth of Direct Runoff $(r_d) = \frac{\text{Volume of Direct Runoff}}{\text{Area of Catchment}}$
7. Divide each of the ordinates of DRH by the runoff depth (r_d) to obtain the ordinate of the unit hydrograph.

   $u_1 = \frac{O_1}{r_d}, u_2 = \frac{O_2}{r_d}, \quad u_3 = \frac{O_3}{r_d}, \qquad u_4 = \frac{O_4}{r_d} \ldots \ldots$

   Ordinate of Unit hydrograph (u) = $\frac{\text{Ordinate of DRH (O)}}{\text{Depth of Direct Runoff}(r_d)}$
8. Check whether total depth of runoff is equal to total rainfall excess.
9. Plot the ordinates of unit hydrograph against time since the beginning of direct runoff.

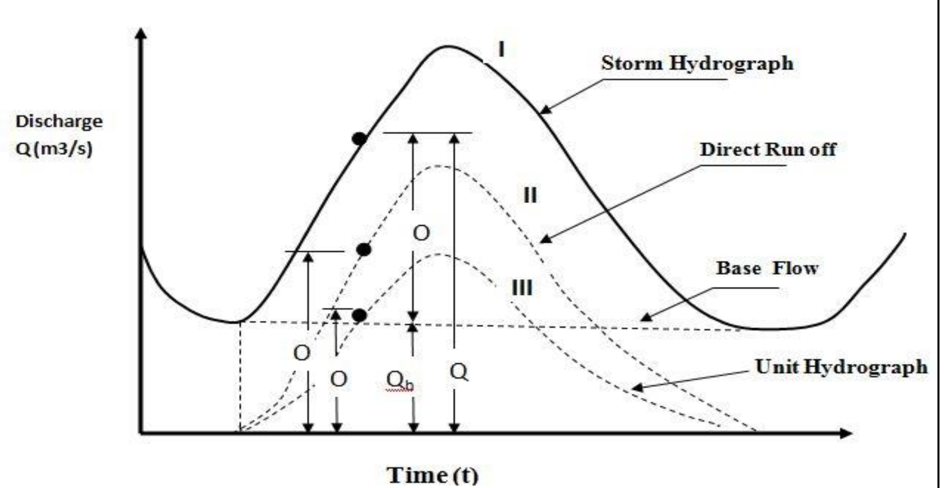


Fig: Derivation of UH from Single storm

Example 1:

Given below are the observed flows from a storm of 4hr duration on a stream with a catchment area of 613km^2. Derive 4hr unit hydrograph. Make suitable assumptions regarding base flow.

Time(hr)	0	4	8	12	16	20	24	28	32	36	40	44	48
Observed flow(m3/s)	10	110	225	180	130	100	70	60	50	35	25	15	10

Solution:

Catchment area (A) = 613km^2

Assume base flow (Q_b) = 10 m^3/s

Direct runoff (O) = $Q - Q_b$

Volume of runoff (V) = $\sum O\ \Delta t$

Runoff depth $\mathbf{r_d} = \frac{\mathbf{V}}{\mathbf{A}}$

Divide O by r_d to get UH ordinate. Δt is same for each runoff ordinate.

$\Delta t = 4hr$ = 4 x 3600 second

(V) $= \sum O\ \Delta t = \Delta t \sum O$ = 4 x 3600 x 890

$$\mathbf{r_d} = \frac{\mathbf{V}}{\mathbf{A}} = \frac{4 \times 3600 \times 890}{613 \times 10^6} = 0.02m = 2cm$$

Computation of UH

Time (hr)	0	4	8	12	16	20	24	28	32	36	40	44	48
Q (m3/s)	10	110	225	180	130	100	70	60	50	35	25	15	10
Q_b (m^3/s)	10	10	10	10	10	10	10	10	10	10	10	10	10
$O=Q-Q_b$ (m^3/s)	0	100	215	170	120	90	60	50	40	25	15	5	0
UH (m^3/s)	0	50	107.5	85	60	45	30	25	20	12.5	7.5	2.5	0

Example 2:

The peak of a flood hydrograph due to a storm is 470m^3/s. The mean depth of rainfall is 8cm. Assume an average infiltration loss of 0.25cm/hr and a constant base flow of 15m3/s, estimate the peak discharge of a 6hr-unit hydrograph for this catchment.

Solution:

Peak discharge (Q) = 470 m^3/s

Base flow (Q_b) = 15m^3/s

Peak of DRH (O) = Peak of storm discharge (Q)-base flow (Q_b)

= 470-15 = 455 m^3/s

Rainfall =8cm

Loss in 6hour (due to infiltration) =0.25 x6 =1.5cm

Effective Rainfall (Re) = 8-1.5= 6.5cm

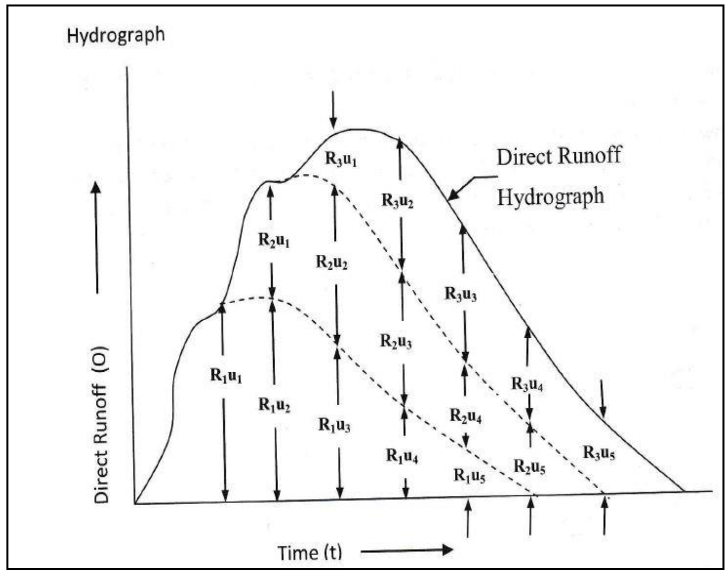
Peak of 6hour UH $= \frac{O}{R_e} = \frac{455.0}{6.5} = 70\ m^3/s$

5.4 Derivation of Unit Hydrograph from Complex storm

Complex hydrograph consists of several peaks. Principle of superposition is used in its derivation.

Multiple storms: These are relatively long and of varying intensities of rainfall. The storms are divided into number of equal periods and fairly constant rate of rainfall for each period.

Duration of UH = Duration of period of each storm



Procedure:

1. Assume a common unit hydrograph to exist.
2. Let u_1, u_2, u_3…….Un be the ordinate of unit hydrograph.
3. From storm hydrograph, deduct base flow (Qb) to calculate the direct runoff hydrograph (DRH). Let O_1, O_2, O_3….. On be the ordinate of DRH. The ordinate of DRH can be obtained from $O_1=Q_1$-Qb, $O_2=Q_2-Q_b$, $O_3= Q_3-Q_b$, $O_4=Q_4-Q_b$……..
4. Now, if rainfall excess made up of three consecutive durations of D-h and rainfall excess values of R_1, R_2, R_3 and the ordinates of the composite DRH be drawn at a time interval of D hour at various time intervals 1D, 2D, 3D, 4D…… from the start of the DRH.

Then,

$O_1= R_1U_1$

$O_2= R_1U_2+R_2U_1$

$O_3=R_1U_3+R_2U_2+R_3U_1$

$O_4=R_1U_4 +R_2U_3+R_3U_2+R_4U_1$

……………………………………..
……………………………………..
$$O_n = \sum_{m=1}^{K} R_m U_{n-m+1}$$
Where,
K= n if $n \leq m$

K= m if $n > m$

n= Number of runoff ordinates
m=Number of pulses of rainfall excess
O_n= DRH Ordinate
R_m= Excess Rainfall

U_{n-m+1}= UH Ordinate

This is because the rain occurring after the time step n, cannot contribute to runoff at time step n. For instance, if n=9 and m=4, the DRH will have 9- 4+1 =6 Ordinates.
For complex multi peaked hydrograph, solution of above equation can be obtained by least square regression.

Solved Example:

In a storm, the rainfall excess of 0.5cm, 0.7cm, 0.0cm, and 0.8cm occurred in four successive hours. The storm hydrograph due to this storm has the hourly ordinates (Q) as given below: 0.5, 44.5, 110.5, 85.5, 102.8, 94.0, 38.4, 18.6, 10.9, 5.3, 2.9, 0.5 (cumecs). If there is a constant base flow of 0.5cumecs, find the hourly ordinates of unit hydrograph.

Solution:

n=number of runoff ordinates =10

m=number of periods (pulses) of rainfall excess= 4

Number of UH ordinates = n-m+1= 10-4+1=7

U_{n-m+1}=UH Ordinate

Let the hourly ordinate of unit hydrograph be U1, U2, U3, U4…..

The hourly ordinates (O) of the direct hydrograph (DRH) are obtained by subtracting base flow in the ordinate of storm hydrograph.

The ordinates of DRH are: 0.0. 44.0, 110.0, 85.0, 102.3, 93.5, 37.9, 18.1, 10.4, 4.8, 2.4, 0.0 (cumecs).

The depths of effective rainfall are:

$R_{e1} = 0.5cm, R_{e2} = 0.7cm, R_{e3} = 0.0cm, \ R_{e4} = 0.8cm,$

Ordinate of UH	Ordinate of DRH from R_{e1}	Ordinate of DRH from R_{e2}	Ordinate of DRH from R_{e3}	Ordinate of DRH from R_{e4}	Ordinate of DRH (given)	$O_1 + O_2 + O_3$	Ordinate of UH (cumecs)
O	0	-	-	-	0	0	0
U_1	$0.5U_1$	0	-	-	44	$0.5U_1$	$U_1 = 88$
U_2	$0.5U_2$	$0.7U_1$	0	-	110	$0.5U_2 + 0.7U_1$	$U_2 = 96.8$
U_3	$0.5U_3$	$0.7U_2$	0	0	85	$0.5U_3 + 0.7U_2$	$U_3 = 34.5$
U_4	$0.5U_4$	$0.7U_3$	0	$0.8U_1$	102.3	$0.5U_4 + 0.7U_3 + 0.8U_1$	$U_4 = 15.5$
U_5	$0.5U_5$	$0.7U_4$	0	$0.8U_2$	93.5	$0.5U_5 + 0.7U_4 + 0.8U_2$	$U_5 = 10.4$
U_6	$0.5U_6$	$0.7U_5$	0	$0.8U_3$	37.9	$0.5U_6 + 0.7U_5 + 0.8U_3$	$U_6 = 6.0$
U_7	$0.5U_7$	$0.7U_6$	0	$0.8U_4$	18.1	$0.5U_7 + 0.7U_6 + 0.8U_4$	$U_7 = 3.0$
O	0	$0.7U_7$	0	$0.8U_5$	10.4	$0.7U_7 + 0.8U_5$	0
		0	0	$0.8U_6$	4.8	$0.8U_6$	
			0	$0.8U_7$	2.4	$0.8U_7$	
				0	0		

Check for other Computation:

$0.5U_8 + 0.7U_7 + 0.8U_5 = 10.4$

0r, $0.5U_8 + 0.7 \text{ x}3 + 0.8 \text{ x}10.4 = 10.4$

Or, $0.5U_8 = 10.4 - 2.1 - 8.32 = 0$

Or, $U_8 = 0$

5.5 Derivation of Unit Hydrograph of Different durations

Ideally unit hydrograph are developed from simple isolated storms and if the durations of various storms do not differ vary much within a band of $\pm 20\%D$, they would be grouped under one average duration of D-h. If no adequate data is available, conversion method may be used. The approach depends if the new duration is shorter or longer than the duration of available unit hydrographs.

Two methods are available for this purpose.

1. Method of superposition
2. The S-curve

1. **Method of superposition**:

If a D-h unit hydrograph is available and it is desired to develop a unit hydrograph of nD-h, where n is an integer, it is easily accomplished by method of superposing n unit hydrographs with each graph separated from the previous one by D-h. Let there are three 2-h unit hydrographs A, B, C. Curve of hydrograph B begins after 2 after A and C begins 20h after B. The combination of these three curves will give a DRH of 3cm due to a rainfall excess of 6-h duration. Thus a 6-h hydrograph can be obtained by dividing the sum of ordinates of three 2hour unit hydrograph lagged from each other by 2 hour.

Example:

From the observation of a storm the ordinates of a two-hour hydrograph are obtained as given below. From this data draw a 6hour unit hydrograph.

Time(hr)	2	4	6	8	10	12	14	16	18	20	22	24
Flow(m^3/s)	0	7	20	40	29	17	11.5	7	5.5	3.0	2.0	0

Solution:

The ordinates of the 6hour unit hydrograph are obtained by adding the ordinates of the 2 hours 3 hydrographs lagged by 2 hours and dividing the sum by 3 as shown below:

Time(hr)	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28
Flow (m^3/s)	0	7	20	40	29	17	11.5	7	5.5	3.5	3.0	2.0	0		
		0	7	20	40	29	17	11.5	7	5.5	3.5	3.0	2.0	0	
			0	7	20	40	29	17	11.5	7	5.5	3.5	3.0	2.0	0
Total Flow(m3/s)	0	7	27	67	89	86	57.5	35.5	24	16	12	8.5	5	2	0

The ordinates of 6hr unit hydrograph are:

0	2.3	9	22.3	29.7	28.7	19.2	12	8	5.3	4	2.8	1.7	0.67	0

These ordinates plotted in graph give 6hour unit hydrograph.

The S-Curve

The S-curve is also known as S-hydrograph. A hydrograph produced by continuous effective rainfall at a constant rate for an infinite period. It is a curve obtained by summation of an infinite series of D-h unit hydrographs spaced D-h apart.

The average intensity of excess rainfall producing the S-curve is $\frac{1}{D}\left(\frac{cm}{H}\right)$ and the equilibrium discharge, $Q_S = \left(\frac{A}{D} x\ 10^4\right)\frac{m3}{h}$

Where, A= area of the catchment in km^2 and D= duration in hours of rainfall excess of the unit hydrograph used deriving the S-curve. Alternately,

$$Q_S = \left(\frac{A}{D} x \frac{10^4}{3600}\right)\frac{m^3}{s} = 2.778\ \frac{A}{D}\ \frac{m^3}{s}$$

$Q_S =$ maximum rate at which ER of $\frac{1}{D}\frac{cm}{h}$ can drain out of catchment area A.

Example: Given the ordinates of a 4-h unit hydrograph as below. Derive the ordinates of 12-h unit hydrograph for the same catchment using S-curve method.

T(hour)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinate of 4h UH (m3/s)	0	20	80	130	150	130	90	52	27	15	5	0

Computation of S-Curve method:

Time (hour) (Col 1)	Ordinate of 4-h UH(m^3/s) (Col 2)	S-Curve addition (m^3/s) (Col 3)	S-curve ordinate (m^3/s) (Col 2+Col 3) Col 4	S-curve Lagged by 12-h (m^3/s) Col 5	Col 4-Col 5 Col 6	$\frac{\text{Col 6}}{12/4}$= 12-h UH ordinate($m^3$/s) Col 7
0	0	-	0	-	0	0
4	20	0	20	-	20	6.7
8	80	20	100	-	100	33.3
12	130	100	230	0	230	76.7
16	150	230	380	20	360	120
20	130	380	510	100	410	136.7
24	90	510	600	230	370	123.3
28	52	600	652	380	272	90.7

32	27	652	679	510	169	56.3
36	15	679	694	600	94	31.3
40	5	694	699	652	47	15.7
44	0	699	699	679	20	6.7
48		699	699	694	5	1.7
52			699	699	0	0

Example 2

The ordinate of 4hour UH of a basin of area 25km^2 are given below:

T(hour)	0	4	8	12	16	20	24	28	32	36	40	44	48	52
UH(m3/s	0	30	55	90	130	170	180	160	110	60	35	20	8	0

Calculate the following:

(a) 4-hr DRH for a rainfall of 3.25cm with Ø-index of 0.25cm

(b) A 12-hr UH by using the method of superposition

(c) A 12-hr UH by using the S-Curve

Solution:

(a) Rainfall= 3,25cm**,** Ø-index = 0.25cm

Rainfall Excess (R_e) = 3.25-0.25 = 3.0 cm

We, Know, DRH = UH x R_e

Computation of DRH

T(hour)	0	4	8	12	16	20	24	28	32	36	40	44	48	52
UH(m3/s	0	30	55	90	130	170	180	160	110	60	35	20	8	0
DRH(m3/s	0	90	165	270	390	510	540	480	330	180	105	60	24	0

(b) Required duration of UH (D') = 12hr

(c) Given duration (D) = 4hr

(d) n= $\frac{D'}{D} = \frac{12}{3} = 3 \text{ (integer)}.$ So, method of superposition will be simpler.

UHa= UH lagged by 4hr, UHb = UH lagged by 8 hr

UH1= UH + UHa + UHb

12 hr- UH = $\frac{UH_1}{\frac{D'}{D}} = \frac{UH_1}{3}$

Computation of 12hr UH using method of superposition

T(hr)	UH	UH_a	UH_b	UH1 ($UH + UH_a + UH_b$)	12hr UH(m3/s) (UH1/3)
0	0	-	-	0	0
4	30	0	-	30	10
8	55	30	0	85	28.3
12	90	55	30	175	58.3
16	130	90	55	275	91.7
20	170	130	90	390	130
24	180	170	130	480	160
28	160	180	170	510	170
32	110	160	180	450	150
36	60	110	160	330	110
40	35	60	110	205	68.3
44	20	35	60	115	38.3
48	8	20	35	63	21
52	0	8	20	28	9.3
-	-	0	8	8	2.7
-	-	-	0	0	0

c) S-curve addition= Ordinate of S-curve at (t-D)

Ordinate of S-Curve ((S_1) = Ordinate of UH+S-curve addition

S_2=S_1 lagged by 12 hr

12 hr-UH = $\frac{(S_1 - S_2)}{\left(\frac{D'}{D}\right)} = \frac{(S_1 - S_2)}{3}$

Computation of 12hr UH using S-curve method

T(hr)	UH	S-curve addition	S-curve (S_1)	S_2	$\frac{(S_1 - S_2)}{3}$	12hr UH (m^3/s)
0	0	-	0	-	0	0
4	30	0	30	-	30	10
8	55	30	85	-	85	28.3
12	90	85	175	0	175	58.3
16	130	175	305	30	275	91.7
20	170	305	475	85	390	130
24	180	475	655	175	480	160
28	160	655	815	305	510	170
32	110	815	925	475	450	150
36	60	925	985	655	330	110
40	35	985	1020	815	205	68.3
44	20	1020	1040	925	115	38.3
48	8	1040	1048	985	63	21
52	0	1048	1048	1020	28	9.3
56	-	-	1048	1040	8	2.7
60	-	-	1048	1048	0	0
64	-	-	1048	1048	0	0

Module 3

Lecture 5: Derivation of S-curve and discrete convolution equations

S – Curve method

- It is the hydrograph of direct surface discharge that would result from a continuous succession of unit storms producing 1cm(in.)in t_r –hr
- If the time base of the unit hydrograph is T_b hr, it reaches constant outflow (Q_e) at T hr, since 1 cm of net rain on the catchment is being supplied and removed every t_r hour and only T/t_r unit graphs are necessary to produce an S-curve and develop constant outflow given by,

$$\mathbf{Qe = (2.78 \cdot A) / t_r}$$

where

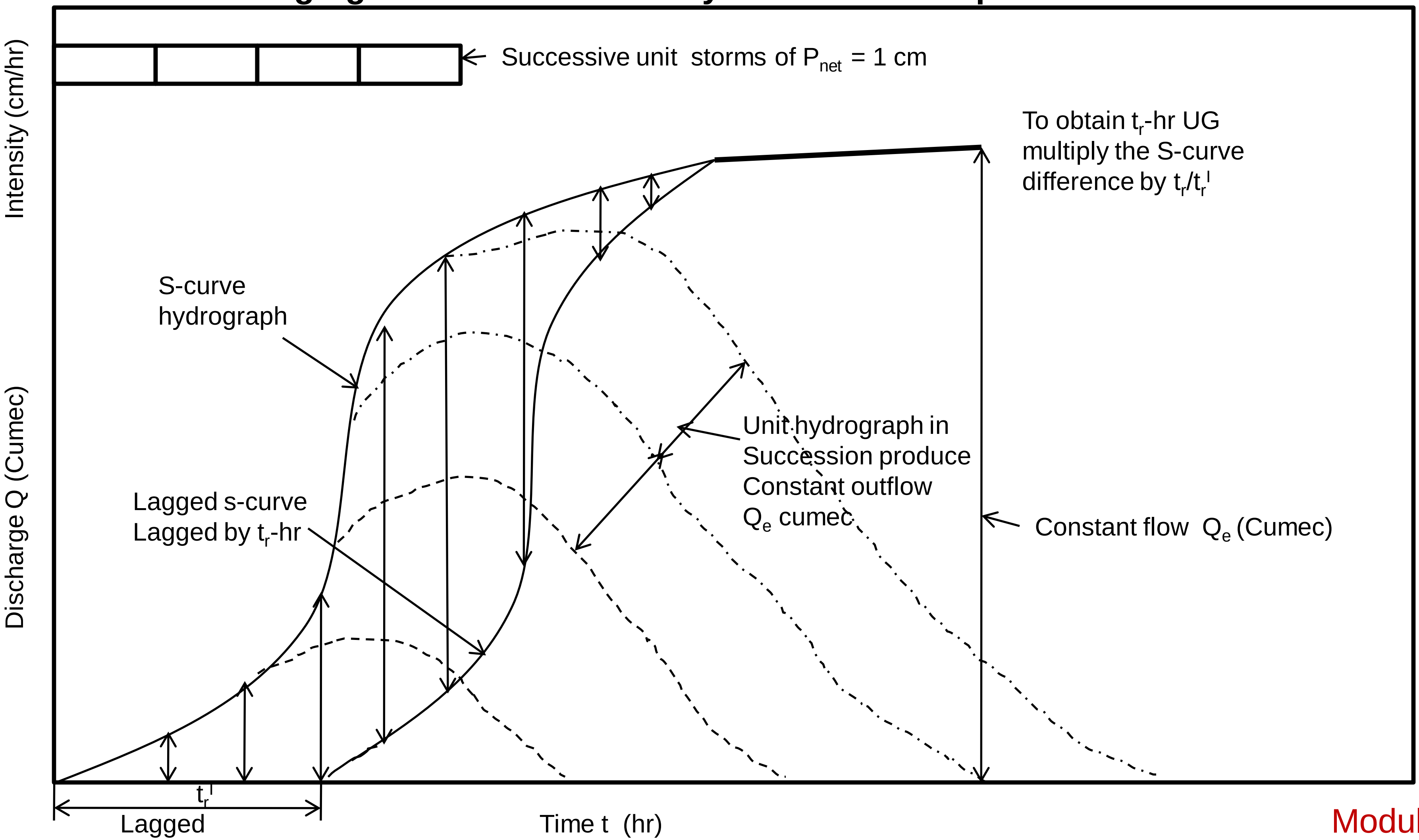
Qe = constant outflow (cumec)

t_r = duration of the unit graph (hr)

A = area of the basin (km^2 or acres)

S – Curve method Contd…

Changing the duration of UG by S-curve technique



Example Problem

- Convert the following 2-hr UH to a 3-hr UH using the S-curve method

Time (hr)	2-hr UH ordinate (cfs)
0	0
1	75
2	250
3	300
4	275
5	200
6	100
7	75
8	50
9	25
10	0

Solution

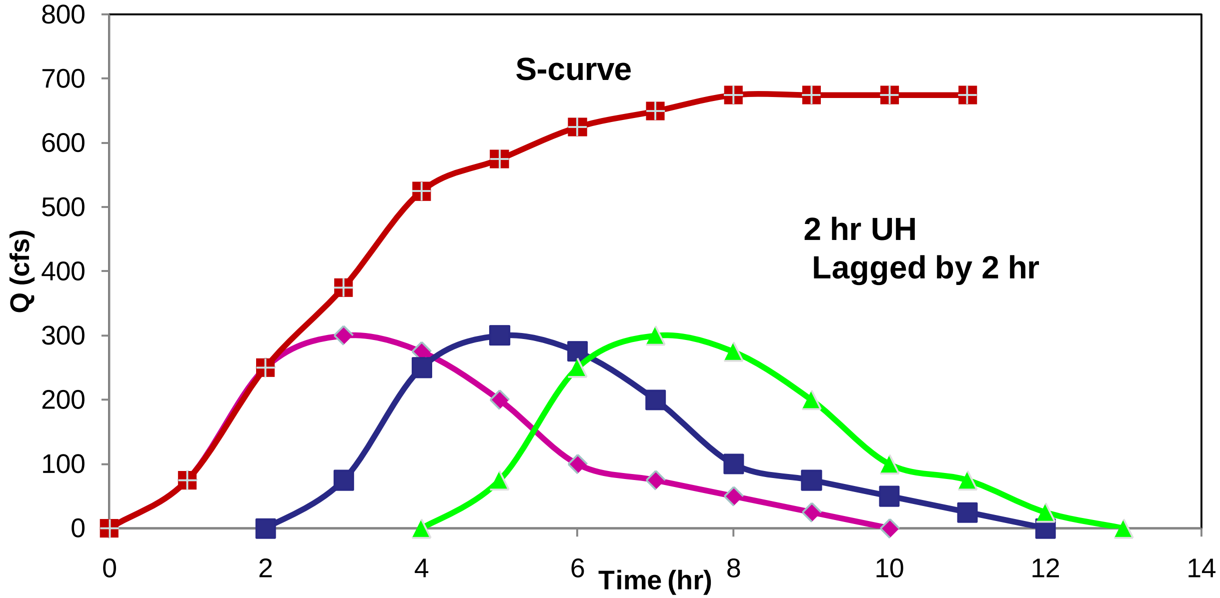
Make a spreadsheet with the 2-hr UH ordinates, then copy them in the next column lagged by D=2 hours. Keep adding columns until the row sums are fairly constant. The sums are the ordinates of your S-curve

Example Problem Contd…

Time (hr)	2-hr UH	2-HR lagged UH's					Sum
0	0						0
1	75						75
2	250	0					250
3	300	75					375
4	275	250	0				525
5	200	300	75				575
6	100	275	250	0			625
7	75	200	300	75			650
8	50	100	275	250	0		675
9	25	75	200	300	75		675
10	0	50	100	275	250	0	675
11		25	75	200	300	75	675

Example Problem Contd…

Draw your S-curve, as shown in figure below



Make a spreadsheet with the 2-hr UH ordinates, then copy them in the next column lagged by D=2 hours. Keep adding columns until the row sums are fairly constant. The sums are the ordinates of your S-curve.

Example Problem Contd…

Time (hr)	S-curve ordinate	S-curve lagged 3hr	Difference	3-HR UH ordinate
0	0		0	0
1	75		75	50
2	250		250	166.7
3	375	0	375	250
4	525	75	450	300
5	575	250	352	216
6	625	375	250	166.7
7	650	525	125	83.3
8	675	575	100	66.7
9	675	625	50	33.3
10	675	650	25	16.7
11	675	675	0	0

Example Problem

Find the one hour unit hydrograph using the excess rainfall hyetograph and direct runoff hydrograph given in the table

Time (1hr)	Excess Rainfall (in)	Direct Runoff (cfs)
1	1.06	428
2	1.93	1923
3	1.81	5297
4		9131
5		10625
6		7834
7		3921
8		1846
9		1402
10		830
11		313

Discrete Convolution Equation

Suppose that there are M pulses of excess rainfall.

If N pulses of direct runoff are considered, then N equations can be written Q_n in terms of N-M+1unknown values of unit hydrograph ordinates, where n= 1, 2, …,N.

$$Q_n = \sum_{m=1}^{m^*} P_m U_{n-m+1} \qquad m^* = \min(n,M)$$

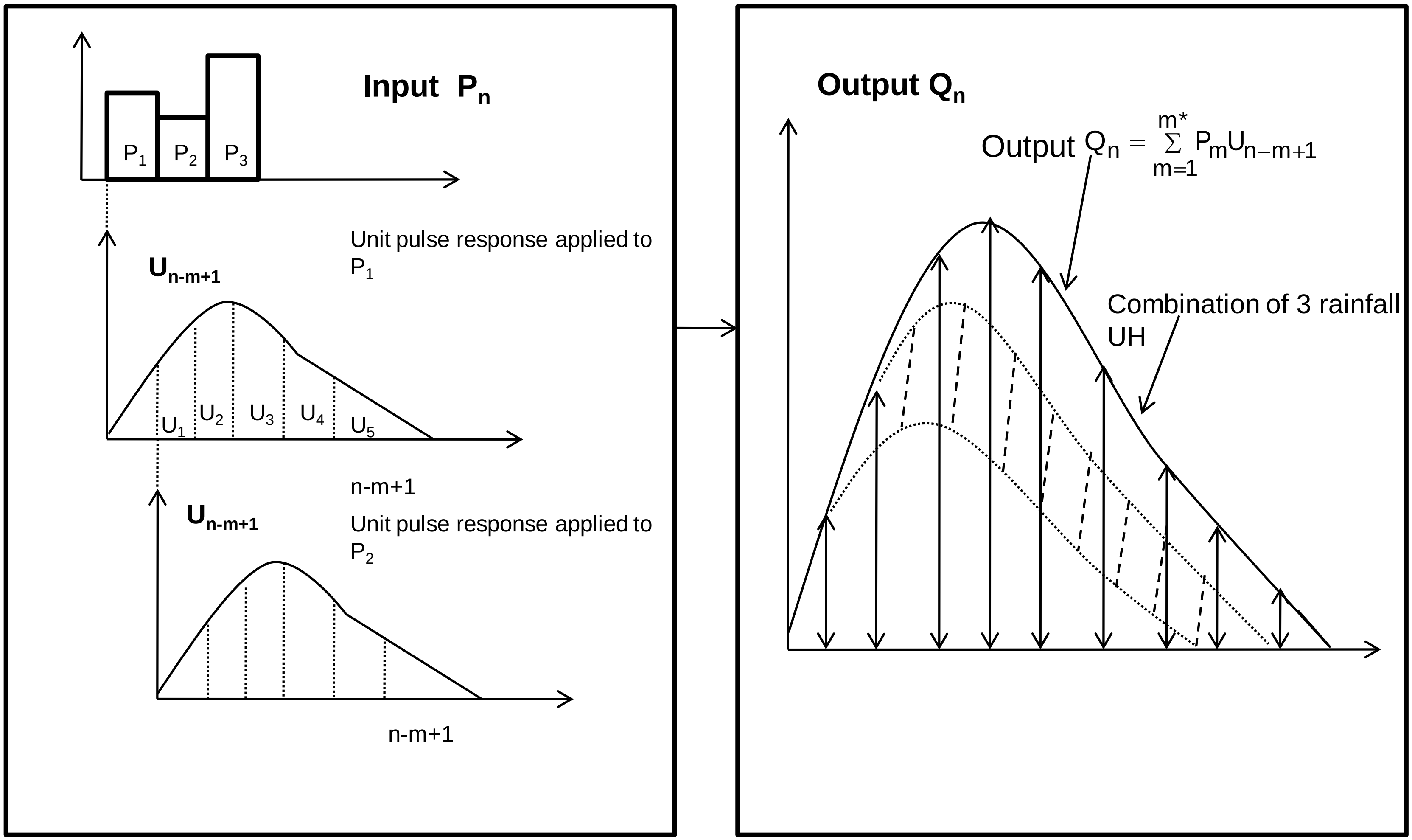
Where Q_n = Direct runoff

P_m = Excess rainfall

U_{n-m+1} = Unit hydrograph ordinates

Unit hydrograph

Contd....



The set of equations for discrete time convolution

$$Q_1 = P_1U_1$$

$$Q_2 = P_2U_1 + P_1U_2$$

$$Q_3 = P_3U_1 + P_2U_2 + P_1U_3$$

$$\vdots$$

$$Q_M = P_MU_1 + P_{M-1}U_2 + ... + P_1U_M$$

$$Q_{M+1} = 0 + P_MU_2 + ... + P_2U_M + P_1U_{M+1}$$

$$\vdots$$

$$Q_{N-1} = 0 + 0 + ... + 0 + 0 + ... + P_MU_{N-M} + P_{M-1}U_{N-M+1}$$

$$Q_N = 0 + 0 + ... + 0 + 0 + ... + 0 + P_{M-1}U_{N-M+1}$$

$$Q_n = \sum_{m=1}^{m^*} P_mU_{n-m+1}$$

$$n = 1, 2, \ldots, N$$

Example Problem Contd…

Solution

- The ERH and DRH in table have M=3 and N=11 pulses respectively.
- Hence, the number of pulses in the unit hydrograph is N-M+1=11-3+1=9.
- Substituting the ordinates of the ERH and DRH into the equations in table yields a set of 11 simultaneous equations

$$U_1 = \frac{Q_2 - P_2 U_1}{P_1} = \frac{1{,}928 - 1.93 x 404}{1.06} = 1{,}079 \text{ cfs/in}$$

Similarly calculate for remaining ordinates and the final UH is tabulated below

n	1	2	3	4	5	6	7	8	9
U_n (cfs/in)	404	1,079	2,343	2,506	1,460	453	381	274	173

Chapter 6: Flood Hydrology **[7 hours]**

6.1 Design of flood and its frequency

Flood is relatively high stage of river flow which overtops the natural and artificial banks in any reach of a river as a result of unusual meteorological and climate combination. A flood magnitude used for the design of a structure in construction of its **safety, economy, life expectancy** and **probable damage** consideration is called **design flood.**

Causes of Flood:

(i) Natural

1. Continuous rainfall and cloud burst
2. Landslides: blocking of the river
3. Dam burst: sudden release of huge amount of water
4. Glacier Lake Outburst Flood(GLOF)
5. Synchronization of peak flow of rivers
6. Sea storm
7. Earthquake in sea (tsunami)

(ii) Human Intervention

1. Land use changes: deforestation, urbanization
2. Drainage congestion caused by uncoordinated activities
3. Structure failure: dam, embankment

Effects of Flood

1. Loss of life
2. Loss of property
3. Destruction of physical infrastructure
4. Disruption of social and economic development
5. Damage to agriculture
6. Damage to hydraulic structures-bridges, embankment
7. Effect on transportation
8. Damage to reservoirs and dams

Flood Prediction and Design Flood

Depending upon magnitude flood can be classified as:

(a) Ordinary Flood
(b) Frequency based Flood (FBF)
(c) Standard project Flood (SPF) : 40 to 60% of PMF
(d) Probable Maximum Flood (PMF)

6.2 Statistical methods of flood prediction

Flood Frequency Analysis:

The estimation of flood can be done by using empirical formula and unit hydrograph. Statistical methods are used for prediction of flood flows.

The values of annual maximum flood from a given catchment area for large number of successive years constitute a hydrologic data series called the ***annual series***. The data are then arranged in decreasing order of magnitude and probability P of each event being equaled or exceeded (plotting position) is calculated by the plotting position formula **(Weibull formula)**.

$$P = \frac{m}{N+1}$$

Where, m= order number of the event

N= total number of events in the data

The terms ***return period (T)*** **or** ***frequency*** and ***recurrence*** interval are used to denote the reciprocal of the annual probability exceedance.

6.2.2 Return period, Frequency and risk

Return period:

It is defined as the average interval of time T within which an event of given magnitude will be equaled or exceeded at least once.

Probability of occurrence of an event (P) with return period T is given by $\mathbf{P = \frac{1}{T}}$

Probability of not occurrence of event =1-P $= 1 - \frac{1}{T}$

Probability of not occurrence of event in n years (P_n) $\boldsymbol{= (1 - P)^n = \left(1 - \frac{1}{T}\right)^n}$

Probability of occurrence of event at least once in n years $\boldsymbol{= 1 - P_n = 1 - \left(1 - \frac{1}{T}\right)^n}$

Risk:

The probability of occurrence of event $(X \geq X_T)$ at least over a period of n successive years is called the risk (R). R represents probability of failure of a structure. That is, $\boldsymbol{R = 1 - \left(1 - \frac{1}{T}\right)^n}$

Frequency:

For discrete random variable, the number of occurrence of a variate is called frequency. When number of occurrence of a variate or the frequency is plotted against the variate as abscissa, a pattern of distribution is obtained which is called frequency distribution.

We have 75 observations of a variable with values between 2 and 12.

Variate (X)	Frequency(f)	Relative frequency (%)	Cumulative frequency (%)
2	1	1.3	1.3
3	3	4.0	5.3
4	7	9.3	14.6
5	15	20.0	34.7
6	20	26.7	61.4
7	13	17.4	78.8
8	8	10.6	89.4
9	4	5.3	94.7
10	2	2.7	97.4
11	1	1.3	98.7
12	1	1.3	100
Total	75	100	

In a flood series, the variations of flood magnitude with corresponding return period T calculated by equation $T = \frac{1}{P} = \frac{N+1}{m}$ forms the basic aspect of the flood frequency analysis. If a flood series of length N is arranged in descending order and each term is assigned a rank number m indicating the number of times the event is equaled or exceeded, and can be assigned a return period T.

If flood magnitude Q is plotted in Y-axis against return period T on the X-axis, the resulting plot is called a ***flood frequency plot***. It is also known as ***empirical probability distribution curve.***

Chow (1951) has given general equation of *hydrologic frequency analysis*:

$X_T = \overline{X} + K\,\sigma$

Where,

X_T = value of the variate X of random hydrologic series with return period T

$\overline{X}$ = mean of the variate

σ = standard deviation of the variate

K= frequency factor depends upon return period T

Various Measures in Statistics

The PDF of normal distribution is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]_{-\infty \le X \le \infty}$$

where, μ = population mean

σ = population standard deviation

If $Z = \frac{X-\mu}{\sigma}$ and $Z \rightarrow N\ (0, 1)$, it is called standard normal distribution.

Properties of normal distribution are:

- Bell Shaped
- Symmetric about mean
- Unbounded

The curve in which mean, median and mode value coincide is the normal curve. The normal or Gaussian frequency distribution is the most important in statistical theory.

Some commonly used distribution functions for prediction of extreme flood values are:

1. **Gumbel's extreme-value distribution**
2. **Log Pearson Type III distribution**
3. **Log Normal distribution**

6.2.5 Gumbel's Extreme Value Type I distribution:

This extreme value distribution was introduced by Gumbel (1941) and is commonly known as Gumbel's distribution. It is general practice to use extreme value type I distribution known as Gumbel's distribution to fit flood discharges of various rivers. It is one of the most widely used probability distribution functions for extreme values in hydrologic and meteorological studies for prediction of flood peaks, maximum rainfalls, maximum wind speed, etc.

Gumbel defined a flood as the largest of 365 daily flows and annual series of flood flows constitute a series of largest values of flows. According to his theory of extreme events, the probability of occurrence of an event equal to or larger than a value x_o is

$$P(X \ge x_O) = 1 - e^{-e^{-y}} \text{ ………………………….. (1)}$$

In which y is a dimensionless variable given by

$y = \alpha\ (x - a)$ $\quad$ $a = \bar{x} - 0.45005\ \sigma_x$ $\quad$ $\alpha = \frac{1.2825}{\sigma_x}$

Thus, $$y = \frac{1.285\,(x-\bar{x})}{\sigma_x} + 0.577 \;\dots\dots\dots\dots\; (2)$$

Where, $\bar{x}$ = mean and σ_x= standard deviation of the variate X.

In practice, it is the value of X for a given P that is required as and as such equation (1) is transposed as,

$$y_P = -\ln[ln(1-P)]$$

Noting that the return period $T = \frac{1}{P}$ and designating

y_P= the value of y, commonly called the reduced variate, for a given T

$$y_T = -\left[\ln.\ln\frac{T}{T-1}\right]$$

$$y_T = -\left[0.834 + 2.303 loglog\frac{T}{T-1}\right]$$

Now, rearranging the equation (2), the value of variate X with a return period T is

$$\mathbf{X_T = \bar{x} + K\,\sigma_X} \;\dots\dots\dots\dots\dots\; \mathbf{(3)}$$

Where, $$\mathbf{K = \frac{(y_T - 0.577)}{1.285}} \;\dots\dots\dots\; \mathbf{(4)}$$

Equation (3) and (4) constitute the basic Gumbel's equations and are applicable to an infinite sample size (i.e. $N\rightarrow \infty$)

Gumbel's Equation for Practical Use:

The equation (3) can be written as,

$$\mathbf{X_T = \bar{x} + K\,\sigma_{n-1}} \;\dots\dots\dots\; \mathbf{(5)}$$

Where, σ_{n-1} = standard deviation of the sample size N= $\sqrt{\frac{\Sigma(x-\bar{x})^2}{N-1}}$

K = frequency factor represented as

$$\mathbf{K= \frac{(y_T - \bar{y}_n)}{S_n}} \;\dots\dots\dots\dots\; \mathbf{(6)}$$

Where, $y_T = -\left[\ln.\ln\frac{T}{T-1}\right]$.................. **(7)**

Or, $y_T = -\left[0.834 + 2.303\log\log\frac{T}{T-1}\right]$

$\bar{y}_n$= reduced mean, a function of sample size N and is given in the table 1 and table 2 for

$N \to \infty$, $S_n \to 1.285$ and when $y \to \infty$, $\overline{y}_n \to 0.577$

Then, $\mathbf{K} = \frac{(y_T - 0.577)}{1.285}$

Chow has given a formula to find frequency factor for large data as:

$$\mathbf{K} = -\left[\mathbf{0.45 + 0.78\ln\left(\ln\frac{T}{T-1}\right)}\right]$$

Procedure to estimate the flood magnitude based on annual flood series.

1. Assemble the discharge data and note the sample size N. Here, the annual flood value is the variate X. Find $\overline{x}$ and σ_{n-1} for the given data
2. Use table values and determine $\overline{y}_n$ and S_n appropriate to given N
3. Find y_T for a given T by using equation (7)
4. Find K by using equation (6)
5. Determine the required X_T by equation (5)

Table 1: Reduced mean $\overline{y}_n$ in Gumbel's extreme value distribution

N= sample size

N	0	1	2	3	4	5	6	7	8	9
10	0.4952	0.4996	0.5035	0.5070	0.5100	0.5128	0.5157	0.5181	0.5202	0.5220
20	0.5236	0.5252	0.5268	0.5283	0.5296	0.5309	0.5320	0.5332	0.5343	0.5353
30	0.5362	0.5371	0.5380	0.5388	0.5396	0.5402	0.5410	0.5418	0.5424	0.5430
40	0.5436	0.5442	0.5448	0.5453	0.5458	0.5463	0.5468	0.5473	0.5477	0.5481
50	0.5485	0.5489	0.5493	0.5497	0.5501	0.5504	0.5508	0.5511	0.5515	0.5518
60	0.5521	0.5524	0.5527	0.5530	0.5533	0.5535	0.5538	0.5540	0.5543	0.5545
70	0.5548	0.5550	0.5552	0.5555	0.5557	0.5559	0.5561	0.5563	0.5565	0.5567
80	0.5569	0.5570	0.5572	0.5574	0.5576	0.5578	0.5580	0.5581	0.5583	0.5585
90	0.5586	0.5587	0.5589	0.5591	0.5592	0.5593	0.5595	0.5596	0.5598	0.5599
100	0.5600									

Table 2: Reduced standard Deviation S_n in Gumbel's extreme value distribution

N= sample size

N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	0.9496	0.9676	0.9833	0.9971	1.0095	1.0206	1.0316	1.0411	1.0493	1.0565
20	1.0628	1.0696	1.0754	1.0811	1.0864	1.0915	1.0961	1.1004	1.1047	1.1086
30	1.1124	1.1159	1.1193	1.1226	1.1255	1.1285	1.1313	1.1339	1.1363	1.1388
40	1.1413	1.1436	1.1458	1.1480	1.1499	1.1519	1.1538	1.1557	1.1574	1.1590
50	1.1607	1.1623	1.1638	1.1658	1.1667	1.1681	1.1696	1.1708	1.1721	1.1734
60	1.1747	1.1759	1.1770	1.1782	1.1793	1.1803	1.1814	1.1824	1.1834	1.1844
70	1.1854	1.1863	1.1873	1.1881	1.1890	1.1898	1.1906	1.1915	1.1923	1.1930
80	1.1938	1.1945	1.1953	1.1959	1.1967	1.1973	1.1980	1.1987	1.1994	1.2001
90	1.2007	1.2013	1.2020	1.2026	1.2032	1.2038	1.2044	1.2049	1.2055	1.2060
100	1.2065									

Example: The annual maximum recorded floods in the river for the period 1971-1997 is given below. Verify if the Gumbel's extreme value distribution fit in the recorded values. Estimate the flood discharge with recurrence interval of (i) 100 years.

Year	1971	1972	1973	1974	1975	1976	1977	1978	1979
Max Q(m3/s)	2947	3521	2399	4124	3496	2947	5060	4903	3757
Year	1980	1981	1982	1983	1984	1985	1986	1987	1988
Max Q(m3/s)	4798	4290	4652	5050	6900	4366	3380	7826	3320
Year	1989	1990	1991	1992	1993	1994	1995	1996	1997
Max Q(m3/s)	6599	3700	4175	2988	2709	3873	4593	6761	1971

Solution: Arrange the flood discharge in the descending order and plotting position of recurrence interval Tp for each of the discharge is obtained as, $T_P = \frac{N+1}{m} = \frac{27+1}{m} = \frac{28}{m}$

Where, m= order number. The discharge magnitude is plotted against the corresponding T_P on Gumbel extreme probability paper.

Order number (m)	Flood discharge Q (m^3/s)	$T_P = \frac{N+1}{m} = \frac{28}{m}$
1	7826	28.00
2	6900	14.00
3	6761	9.33

4	6599	7.00
5	5060	5.60
6	5050	4.67
7	4903	4.00
8	4798	3.50
9	4652	3.11
10	4593	2.80
11	4366	2.55
12	4290	2.33
13	4175	2.15
14	4124	2.00
15	3873	1.87
16	3757	1.75
17	3700	1.65
18	3521	1.56
19	3496	1.47
20	3380	1.40
21	3320	1.33
22	2988	1.27
23	2947	1.21
24	2947	1.17
25	2709	1.12
26	2399	1.08
27	1971	1.04

N= 27 $\overline{x} = 4263\ m^3/s$ $\sigma_{n-1} = 1432\ m^3/s$

From Table 1 and Table 2, for N= 27, $\overline{y}_n = 0.5332$ and $S_n = 1.1004$

$$y_T = -\left[\ln.\ln\frac{T}{T-1}\right]$$

$$y_{100} = -\left[\ln.\ln\frac{100}{99}\right] = 4.600$$

$$K = \frac{(y_T - \overline{y}_n)}{S_n} = \frac{(4.600 - 0.5332)}{1.1004} = 3.695$$

$X_T = \overline{x} + K\,\sigma_{n-1} = 4263 + 3.695 \times 1432 = 9554.24\ m^3/s$

6.2.4 Log Pearson Type III distribution:

This distribution is extensively used in US for projects sponsored by US for frequency analysis of annual maximum floods. In this method, the variate is first transformed into logarithmic form base 10.

Steps:

1. First transform all the X values i.e. variate of random hydrologic series into logarithmic form (base 10).
2. For this Y series recurrence interval T is given by

   $Y_T = \overline{Y} + K_T\, \sigma_Y$

3. Find σ_Y = Standard deviation = $\sqrt{\frac{\Sigma(Y_i - \overline{Y})^2}{N-1}}$
4. Calculate the coefficient of Skewness

$$C_S = \frac{N\sum(Y_i - \overline{Y})^3}{(N-1)(N-2)(\sigma_Y)^3}$$

5. Obtain the variation of $K_T = f\,(C_S, T)$ from the table for Log Pearson type III Distribution or by using following formula and steps:

$$P = \frac{1}{T}$$

$$w = \sqrt{\ln\left(\frac{1}{P^2}\right)}$$

$$Z = w - \frac{(2.51557 + 0.8028\, w + 0.010328\, w^2)}{(1 + 1.432788\, w + 0.189269\, w^2 + 0.001308\, w^3)}$$

$$K_T = Z + (Z^2 - 1)\,\frac{C_S}{6} + (Z^3 - 6Z)\left(\frac{C_S}{6}\right)^2 - (Z^2 - 1)\left(\frac{C_S}{6}\right)^3 + Z\left(\frac{C_S}{6}\right)^4 + \frac{1}{3}\left(\frac{C_S}{6}\right)^5$$

$$Y_T = \overline{Y} + \sigma_Y\, K_T$$

Take antilog of Y_T i.e. $X = 10^{Y_T}$

Table 3: $K_T = f\,(C_s, T)$ for use in Log Pearson III distribution

Coefficient of skew, C_S	Recurrence interval T in years						
	2	10	25	50	100	200	1000
3.0	-0.396	1.18	2.278	3.152	4.051	4.970	7.250
2.5	-0.360	1.250	2.262	3.048	3.845	4.652	6.600
2.2	-0.330	1.284	2.240	2.970	3.705	4.444	6.200
2.0	-0.307	1.302	2.219	2.912	3.605	4.298	5.910
1.8	-0.282	1.318	2.193	2.848	3.499	4.147	5.660
1.6	-0.254	1.329	2.163	2.780	3.388	3.990	5.390
1.4	-0.225	1.337	2.128	2.706	3.271	3.828	5.110

1.2	-0.195	1.340	2.087	2.626	3.149	3.661	4.820
1.0	-0.164	1.340	2.043	2.542	3.022	3.489	4.540
0.9	-0.148	1.339	2.018	2.498	2.957	3.401	4.395
0.8	-0.132	1.336	1.998	2.453	2.891	3.312	4.250
0.7	-0.116	1.333	1.967	2.407	2.824	3.223	4.105
0.6	-0.099	1.328	1.939	2.359	2.755	3.132	3.960
0.5	-0.083	1.323	1.910	2.311	2.686	3.041	3.815
0.4	-0.066	1.317	1.880	2.261	2.615	2.949	3.670
0.3	-0.050	1.309	1.849	2.211	2.544	2.856	3.525
0.2	-0.033	1.301	1.818	2.159	2.472	2.763	3.380
0.1	-0.017	1.292	1.785	2.107	2.400	2.670	3.235
0.0	**0.000**	**1.282**	**1.751**	**2.054**	**2.326**	**2.576**	**3.090**
-0.1	0.017	1.270	1.716	2.000	2.252	2.482	2.950
-0.2	0.033	1.258	1.680	1.945	2.178	2.388	2.810
-0.3	0.050	1.245	1.643	1.890	2.104	2.294	2.675
-0.4	0.066	1.231	1.606	1.834	2.029	2.201	2.540
-0.5	0.083	1.216	1.567	1.777	1.955	2.108	2.400
-0.6	0.099	1.200	1.528	1.720	1.880	2.016	2.275
-0.7	0.116	1.183	1.488	1.663	1.806	1.926	2.150
-0.8	0.132	1.166	1.448	1.606	1.733	1.837	2.035
-0.9	0.148	1.147	1.407	1.549	1.660	1.749	1.910
-1.0	0.164	1.128	1.366	1.492	1.588	1.664	1.880
-1.4	0.225	1.041	1.198	1.270	1.318	1.351	1.465
-1.8	0.282	0.945	1.035	1.069	1.087	1.097	1.130
-2.2	0.330	0.844	0.888	0.900	0.905	0.907	0.910
-3.0	0.396	0.660	0.666	0.666	0.667	0.667	0.668

[Note: C_s =0 corresponds to log normal distribution]

Ex: The following are the annual peak flow data in m^3/s of Bagmati river from 1987-2006 are given below. Compute flood magnitude with 25 year return period using lognormal and Log Pearson type III distribution. Obtain frequency factor value using (a) table and (b) formula.

Year	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
Q(m^3/s)	2890	2670	4410	5620	3090	2100	16000	3700	3500	3060
Year	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Q(m^3/s)	3300	4660	5080	6200	3140	8000	3380	6850	5580	3500

Solution:

Log transformed data to log base 10 ($Y = \log_{10} X$) are:

Q (m^3/s) (X)	Y= $\log_{10}$(X)	$(Y_i - \overline{Y})$	$(Y_i - \overline{Y})^2$	$(Y_i - \overline{Y})^3$
2890	3.461	-0.171	0.0292	-0.0050
2670	3.427	-0.205	0.0420	-0.0086
4410	3.644	0.012	0.0001	0.0000
5620	3.750	0.118	0.0139	0.0016
3090	3.490	-0.142	0.0202	-0.0029
2100	3.322	-0.31	0.0961	-0.0298
16000	4.204	0.572	0.3272	0.1871
3700	3.568	-0.064	0.0041	-0.0003
3500	3.544	-0.088	0.0077	-0.0007
3060	3.486	-0.146	0.0213	-0.0031
3300	3.519	-0.113	0.0128	-0.0014
4660	3.668	0.036	0.0013	0.0000
5080	3.706	0.074	0.0055	0.0004
6200	3.792	0.16	0.0256	0.0041
3140	3.497	-0.135	0.0182	-0.0025
8000	3.903	0.271	0.0734	0.0199
3380	3.529	-0.103	0.0106	-0.0011
6850	3.836	0.204	0.0416	0.0085
5580	3.747	0.115	0.0132	0.0015
3500	3.544	-0.088	0.0077	-0.0007
Average $\overline{Y}$	**3.632**	**-** **Sum**	**0.772**	**Sum = 0.1673**

1. Find σ_Y = Standard deviation = $\sqrt{\frac{\sum(\mathbf{Y}-\overline{\mathbf{Y}})^2}{\mathbf{N-1}}} = \sqrt{\frac{\mathbf{0.772}}{\mathbf{20-1}}} = 0.202$

2. Calculate the coefficient of Skewness

$$C_S = \frac{N\sum(Y-\overline{Y})^3}{(N-1)(N-2)(\sigma_Y)^3} = \frac{20 \times 0.1673}{(20-1)(20-2)(0.202)^3} = 1.2$$

(a) Using Table 3

Log Normal Case:

For log normal distribution $C_s = 0$

Average $\overline{\mathbf{Y}} = 3.632$

For T= 25 years return Period Cs = 0, $K_T = 1.751$

$Y_T = \overline{Y} + \sigma K_T = 3.632 + 0.202 \times 1.751 = 3.985$

Then, Flood of 25 year return period $(\mathbf{X_{25}}) = 10^{3.985} = 9660.50\ m^3/s$

Log Pearson III Case:

For T-25 year return period and $C_s = 1.2$, $K_T = 2.087$

$Y_T = \overline{Y} + \sigma K_T = 3.632 + 0.202 \times 2.087 = 4.053$

Then, Flood of 25 year return period $(\mathbf{X_{25}}) = 10^{4.053} = 11{,}297.95\ m^3/s$

(b) Using Formula

Log Normal Case:

$$P = \frac{1}{T} = \frac{1}{25} = 0.04$$

$$w = \sqrt{\ln\left(\frac{1}{P^2}\right)} = \sqrt{\ln\left(\frac{1}{0.04^2}\right)} = 2.537$$

$$Z = w - \frac{(2.51557 + 0.8028\,w + 0.010328\,w^2)}{(1 + 1.432788\,w + 0.189269\,w^2 + 0.001308\,w^3)}$$

$$Z = 2.537 - \frac{(2.51557 + 0.8028\,w + 0.010328\,x\,2.537^2)}{(1 + 1.432788\,x\,2.537 + 0.189269x\,2.537^2 + 0.001308x\,2.537^3)}$$

$Z = 1.757 = K_T$ (In log normal case, $K_T = Z$)

$Y_T = \overline{Y} + \sigma K_T = 3.632 + 0.202 \times 1.757 = 3.985$

Then, Flood of 25 year return period $(\mathbf{X_{25}}) = 10^{3.985} = 9660.50\ m^3/s$

Log Pearson Type III distribution

$Z = 1.757$

$$K = \frac{C_S}{6} = \frac{1.2}{6} = 0.2$$

$$K_T = Z + (Z^2 - 1)\,k + (Z^3 - 6Z)\,(k)^2 - (Z^2 - 1)\,(k)^3 + Z\,(k)^4 + \frac{1}{3}(k)^5$$

$$K_T = 1.575 + (1.575^2 - 1)\, 0.2 \; + (1.575^3 - 6x1.575)\, (0.2)^2 - (1.575^2 - 1)\, (0.2)^3 + 1.575\, (0.2)^4 + \frac{1}{3}(0.2)^5$$

$K_T = 2.09$

$Y_T = \overline{Y} + \sigma\, K_T = 3.632 + 0.202 \text{ x } 2.09 = 4.05$

Then, Flood of 25 year return period **(X_{25}) =**$10^{4.05}$ = 11, 220.00 m^3/s

Design Flood

A design flood is the flood adopted for design of structure after careful consideration of economic and hydrologic factors. As the magnitude of the design flood increases, the capital cost of the structure also increases but the probability of annual damages will decrease. The most economical design flood can be found after studying the various alternatives. Several design floods can be adopted in single structure.

6.3 Flood prediction by Rational and Empirical Method

Methods in estimating design flood

1. Increasing the observed maximum flood by certain percentage
2. For un-gauged basin
    - Rational method
    - Empirical method
3. For gauged basin
    - Unit hydrograph technique
    - Flood frequency analysis

Rational Method:

The idea behind this method is that if a rainfall of I intensity begins instantaneously and continues indefinitely, the rate of runoff will increase until the time of concentration (t_c), when all of the basin is contributing to flow at the outlet.

After t_c the runoff becomes constant for the period of rainfall excess ($t-t_c$). After the cessation of rain, the runoff recedes gradually to become zero at time t_c from the end of the peak.

The product of rainfall intensity I and basin area (A) is the inflow rate for the system. The peak discharge is given by:

In SI unit, $Q_p = CiA$

Where, Q_p = peak discharge in m^3/s

C= runoff coefficient = ratio of runoff to rainfall (Varies from 0 to 1)

I= intensity of rain fall in mm/hour

A= area of the basin in km^2

Rational method is used for catchment up to $50km^2$.

$$Q_p = CiA = C \times \frac{i}{1000 \times 3600} \times A \times 10^6 = \frac{C\,i\,A}{3.6} = 0.278\,CiA$$

Assumptions:

1. The computed peak rate of runoff at the outlet is function of the average rainfall rate during tc.
2. T_c employed is the time for runoff to become established and flow from the remotest part of the basin to the outlet
3. Rainfall intensity is constant throughout the storm duration

   Values of C given in the table

S.N.	Type of Basin	Value of C
1	Rocky and permeable	0.8-1.0
2	Slightly impermeable, bare	0.6-0.8
3	Cultivated or covered with vegetation	0.4-0.6
4	Cultivated absorbent soil	0.3-0.4
5	Sandy Soil	0.2-0.3
6	Heavy Forest	0.1-0.3

For non-homogenous basin, divide into sub-basins, get C for each sub basin and compute weighted average of C as $C = \frac{\sum C_i A_i}{A}$

$$C = \frac{\sum C_1A_1 + C_2A_2 + C_3A_3 + C_4A_{4.} \dots \dots}{A_1 + A_1 + A_1 + \cdots \dots..}$$

In the absence of data of rainfall intensity, I shall be estimated by

$$i = \frac{KT^a}{(t_c + b)^n}$$

Where, T = return period = $\frac{1}{P}$, where, P = probability of exceedence

t_c= time of concentration (in minutes)

K, a, b, n are constants which are to be defined for the particular site.

For Nepal, K=5.92, a=0.162, b= 0.5, and n=1.013

Kirpich formula to estimate t_c in minutes is given as

$$t_c = 0.019478\, L^{0.77} S^{0.385} \text{ (minutes)}$$

Where, L = maximum length of travel of water in (m)

S= Slope = $\frac{H}{L}$, H = difference of elevation between the remotest point of basin and outlet (m)

6.3 Flood Prediction by Rational and Empirical methods:

All regional formulas are based on statistical correlation of the observed peak and important catchment properties.

Qp= f (A) where, Qp= peak discharge and A= Area

Empirical formula shall be used only when a more accurate method for flood prediction cannot be applied because of lack of data. For flood prediction in un-gauged basins of Nepal, the empirical formulas as given below may be used with caution and proper justification.

A. Modified Dicken's method

Dicken's formula is given as: $Q_T = C_T\, A^{0.75}$ (m^3/s)

Where, A= basin area in km2, C_T= Modified Dicken's constant for Himalayan rivers. CT can be computed as, $C_T = 2.342 \log(0.6T) \log\left(\frac{1185}{p}\right) + 4$

$$p = 100 \left(\frac{a+6}{A+a}\right)$$

Where, a = perpetual snow area in km^2 and T is the return period in years.

B. Snyder's Method

For un-gauged river Snyder's method may be used to estimate flood by deriving a synthetic unit hydrograph based on known physical characteristics of the basin. The peak discharge is given as,

$Q_{PR} = q_{PR}\ C_A\, A\, R$ (m^3/s)

Where, Q_{PR} is he peak discharge per sq km of the drainage area due to 1cm of effective rainfall for duration of t_r in m^3/s/sq.km.

C_A= areal reduction factor

R= Rainfall (cm)

q_{PR} is computed as, $q_{PR} = 2.78\ \frac{C_P}{t_{PR}}$

Where, C_p = coefficient = 0.62

t_{PR}= lagged time in hour for rainfall duration t_R

t_{PR} is calculated as $t_{PR} = t_{pr} + 0.25\,(t_R - t_r)$

Where, t_r is the standard duration of effective rainfall in hours given as,

$$t_r = \frac{t_{pr}}{5.5}$$

t_{pr} = lagged time from the midpoint of effective rainfall of duration t_r to the peak of unit hydrograph computed as,

$$t_{pr} = 0.75\, C_t\, (L\, L_C)^{0.3}$$

Where, C_t = coefficient =1.5

L= length of the stream from station to U/S limit of drainage area (km)

L_C = distance along the main stream from the basin outlet to the point on the stream nearest to the centroid of the basin (km).

C. B.D. Richard's Method

This method has been comprehensively used in Mahakali Irrigation Project in western Nepal. The formula is given as,

Q= 0.222 AIF

Where, A = basin area in km^2

I = Rainfall intensity

F= Areal reduction Factor given by formula as:

F= 1.09352- 0.6628 ln (A)

The value of I shall be estimated by iterative procedure with initial assumed value of tc in hours. After that following computation be performed in sequence:

$$D = 1.102\,\frac{L^2}{S}F$$

$$R_{TC} = 0.22127\, R_T\, (T_C)^{0.476577}$$

$$I = \frac{R_{TC}}{T_C}$$

$$K_R = 0.65\, I\, (T_C + 1)$$

$$C_{KR} = \frac{0.95632}{(K_R)^{1.4806}}$$

$$T_{C3} = D\, C_{KR}$$

$$T_{C2} = \left(\frac{T_{C3}}{0.585378}\right)^{\frac{1}{2.17608}}$$

Where, L= length of basin in km

S= basin slope

R_{TC} is 24 hour rainfall in mm for return period T

T_{C2}= Second estimate of time of concentration

The iterations shall be repeated with $T_C = T_{C2}$ till the difference between the assumed T_c and the resulting second estimate T_{C2} is less than 5%.

D. Fuller's method

Used for un-gauged basins of Nepal for comparison purposes. In this method, maximum instantaneous flood discharge Q_{max} in m^3/s shall be estimated as,

$$Q_{max} = Q_T\left[1 + 2\left(\frac{A}{2.59}\right)^{-0.3}\right]$$

Where, Q_T = maximum 24 hour flood with frequency once in T years in m^3/s

A= basin area in sq. km.

$$Q_T = Q_{av}(1 + 0.8\log T)$$

Where, Q_{av}= yearly average 24 hour flood over number of years (m^3/s)

$$Q_{av} = C_f A^{0.8}$$

Where, C_f = Fuller's coefficient (0.18 to 1.88). For Nepal C_f= 1.03 may be taken.

E. Horton's Formula

$$q_{tr} = 71.2\,\frac{T^{0.25}}{A^{0.5}} \quad (m^3/s/sq.km)$$

Where, A= drainage area in sq. km.

T= Return period in Year

F. WECS Formula

Water and Energy commission Secretariat (WECS) developed empirical formula. The formula for 2 year return period is

$$Q_2 = 1.8767\,(A_{3000} + 1)^{0.8783}$$

The formula for 100 year return period is

$$Q_{100} = 14.63\,(A_{3000} + 1)^{0.7342}$$

Where, A_{3000} = Basin area km^2 below 3000m elevation.

For other return period,
$Q_T = \exp(\ln Q_2 + S\,\sigma)$

Where,

Q_T = Flood of T year return period (m^3/s), S= standard normal variate.

σ = Parameter computed = $\mathbf{\ln \frac{\left(\frac{Q_{100}}{Q_2}\right)}{2.326}}$

WECS Table: Value of T and S

T (Return period) (Years)	S
2	0
5	0.842
10	1.282
25	1.645
50	2.054
100	2.326

Example: The basin area of a river is 653km^2. Compute flood of 50 year return period using modified Dicken's method and WECS method.

Solution:
Basin area (A) = 653 km^2
Return Period =50 years
Flood of 50year return period (Q_{50}) =?

(a) Using Modified Dicken's method

$p = 100\left(\frac{a+6}{A+a}\right)$ where, a= perpetual snow area km^2

$$= 100\left(\frac{0+6}{653+0}\right) = 0.918$$

$$C_T = 2.342\log(0.6T)\log\left(\frac{1185}{p}\right) + 4$$

$$= 2.342\log(0.6x50)\log\left(\frac{1185}{0.918}\right) + 4 = \text{`}14.8$$

$Q_{50} = C_T\, A^{0.75}$ (m^3/s)

$= 14.8 \times 653^{0.75}$ (m^3/s) = 1912(m^3/s)

(b) Using WECS method

$Q_2 = 1.8767\,(A_{3000} + 1)^{0.8783}$

$= 1.8767\,(653 + 1)^{0.8783}$ =558 (m^3/s)

$Q_{100} = 14.63\,(A_{3000} + 1)^{0.7342}$

$= 14.63\,(653 + 1)^{0.7342}$=1708 ($m^3$/s)

$$\sigma = \ln \frac{\left(\frac{Q_{100}}{Q_2}\right)}{2.326} = \ln \frac{\left(\frac{1708}{558}\right)}{2.326} = 0.48$$

For 50year return period, S= 2.054 (from WECS table)

For 5o year return period,

$Q_T = \exp\,(\ln Q_2 + S\,\sigma)$

Or, $Q_{50} = \exp\,(\ln 558 + 2.054 \times 0.48)$= 1496 m^3/s