Indian Institute of Technology, Kanpur
Academic Semester: 2021-22 (I)

ME634: Advanced Computational Fluid Dynamics
Instructors: Prof. Anikesh Pal

MIDSEM REPORT
MULTIGRID SOLVER FOR POISSON'S EQUATION

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Contents

Problem Statement

Consider the PDE:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 50000 \cdot e^{[-50((1-x)^2+z^2)]} \cdot [100((1-x)^2 + z^2) - 2], \tag{1}$$

subjected to the following boundary conditions in x and z directions.

$$\phi(1, y, z) = 100(1 - z) + 500e^{-50z^2} \tag{2}$$

$$\phi(0, y, z) = 500e^{-50(1+z^2)} \tag{3}$$

$$\phi(x, y, 0) = 100x + 500e^{-50(1-x)^2} \tag{4}$$

$$\phi(x, y, 1) = 500e^{-50((1-x)^2+1)} \tag{5}$$

The y is treated periodically. The analytical solution of the PDE is given as:

$$\phi(x, y, z) = 500e^{-50((1-x)^2+z^2)} + 100x(1 - z) \tag{6}$$

$\Omega \in [0, 0, 0] - [1, 1, 1]$ for $41 \times 41 \times 41$ and $81 \times 81 \times 81$ grid points

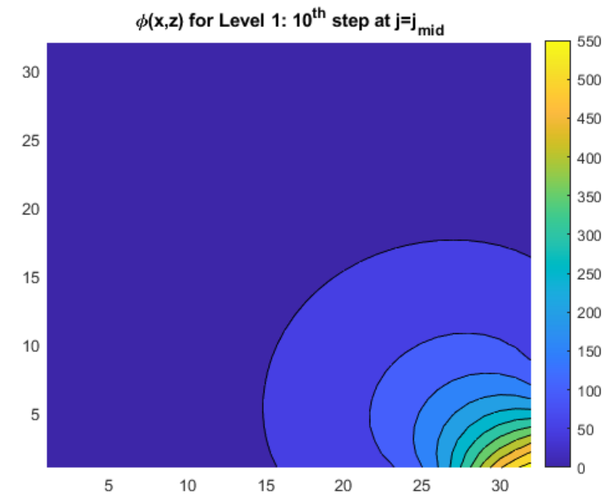
Solve numerically using Gauss-Seidel and report results including the number of V-cycles and plots for both, the error corrections and the final contours of the function.

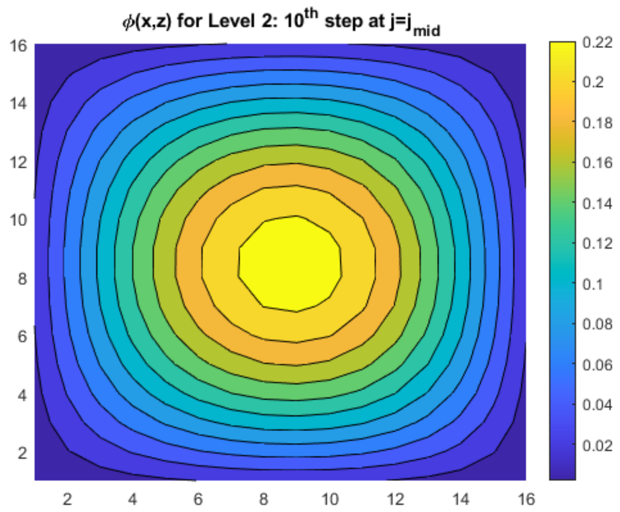
Plots of phi(j=j-midplane)

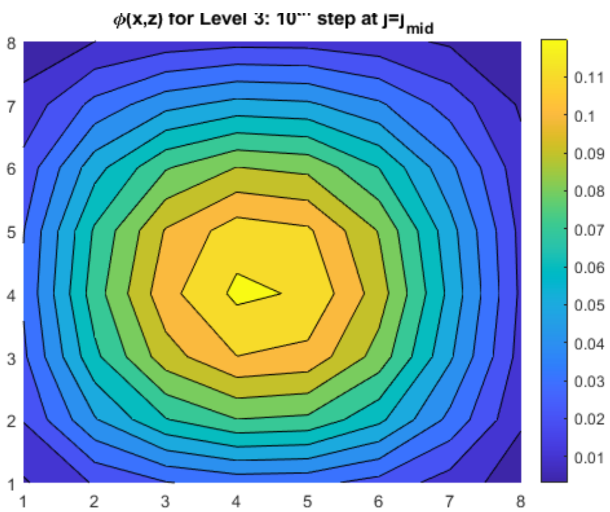
Note: x and y axes show ith and kth nodes

32x32x32

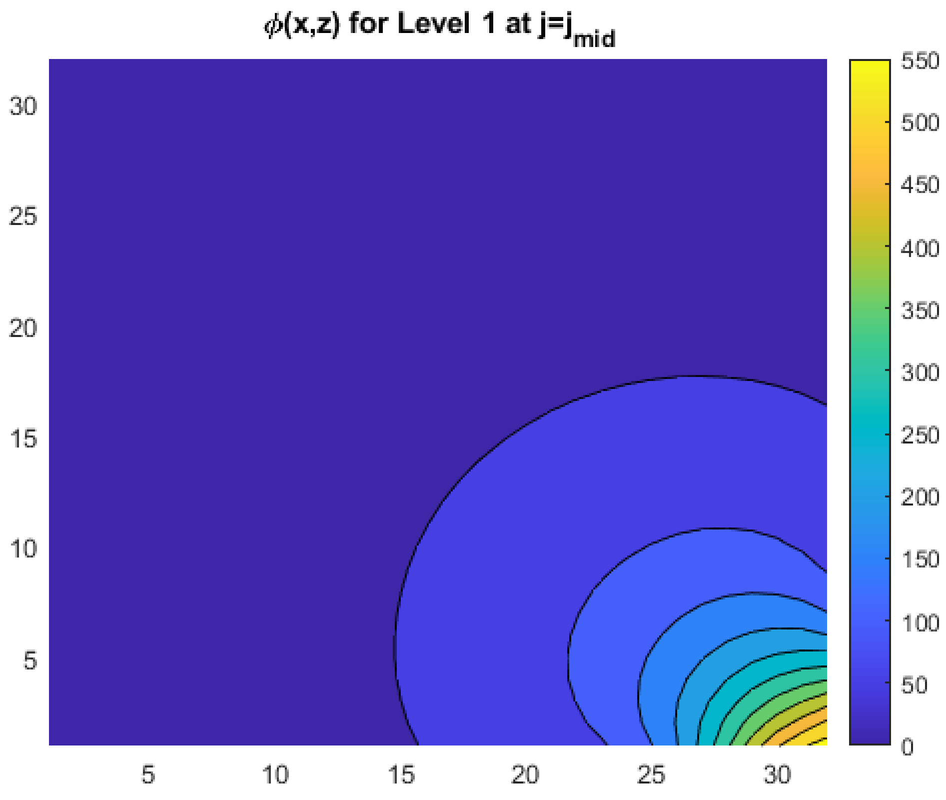
10th MG-iteration



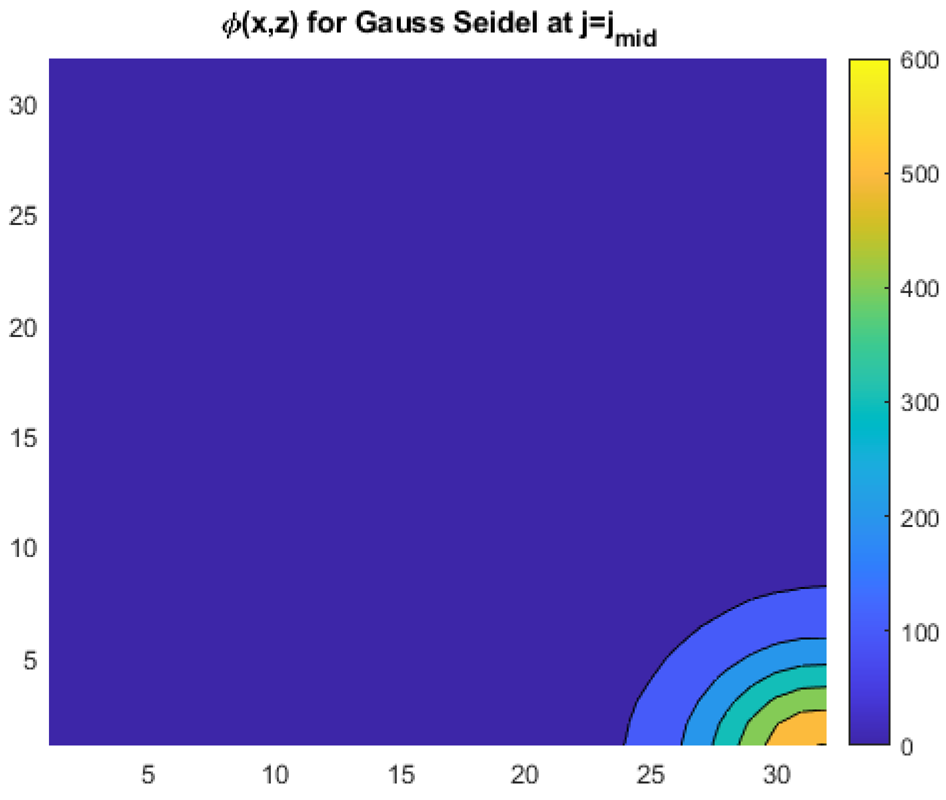




Final MG-iteration

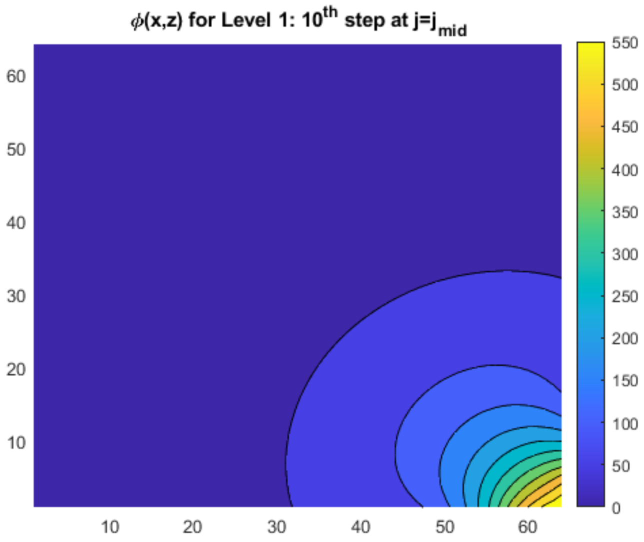


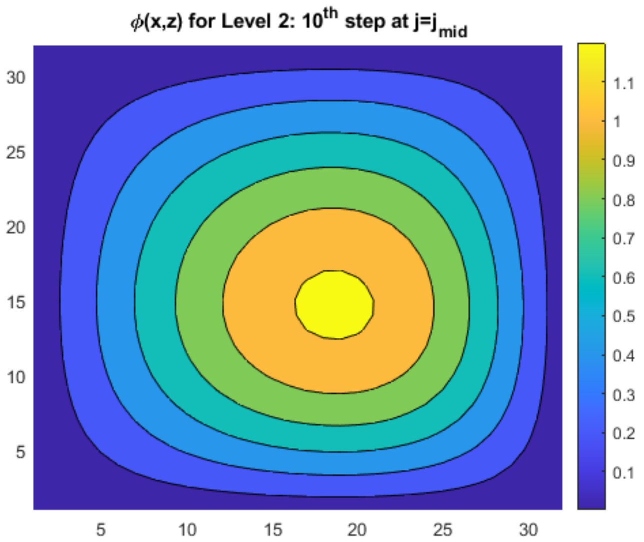
Gauss-Siedel Solver

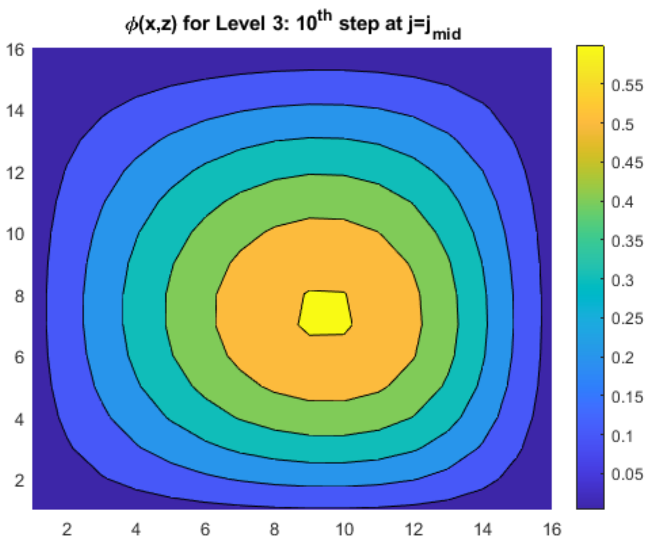


64x64x64

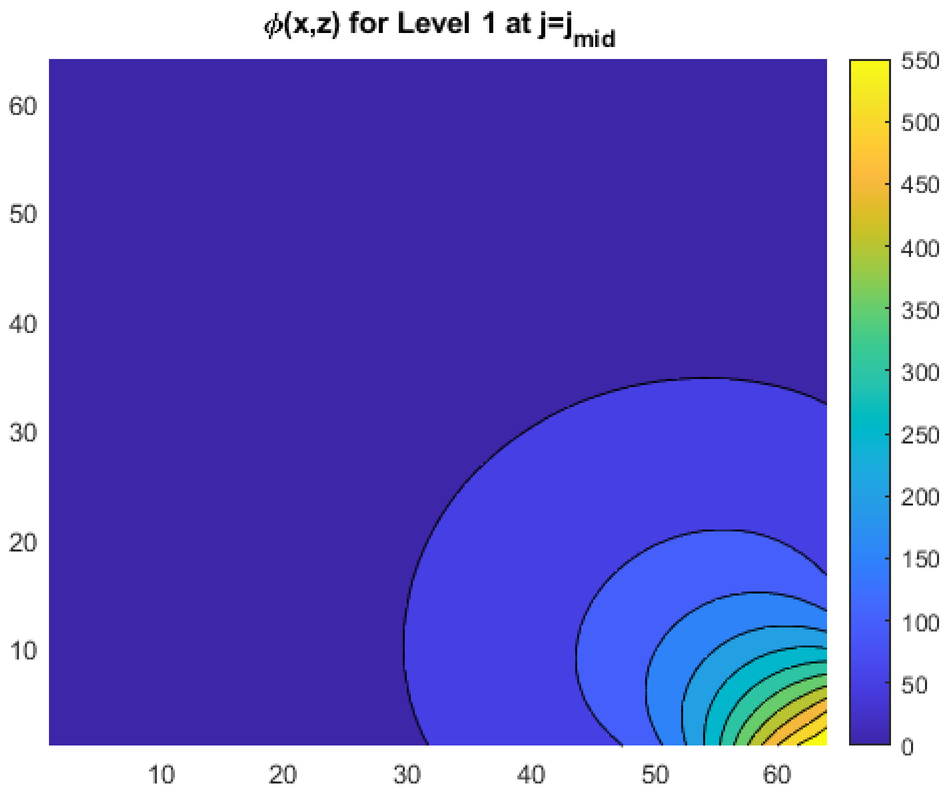
10^{th} MG-iteration



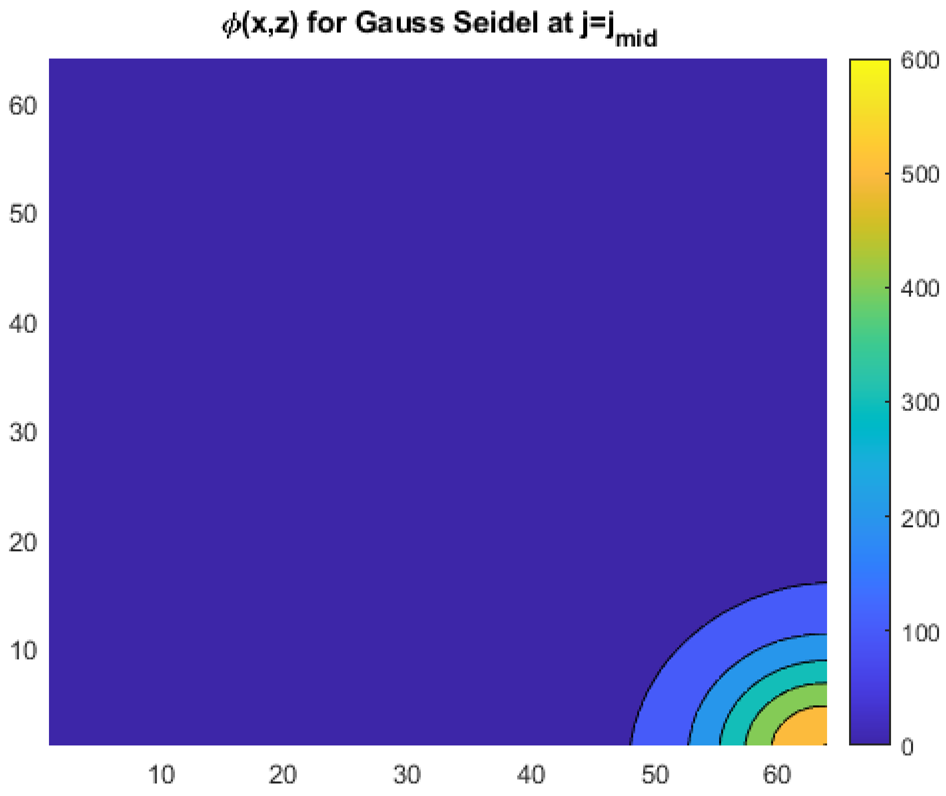




Final MG-iteration

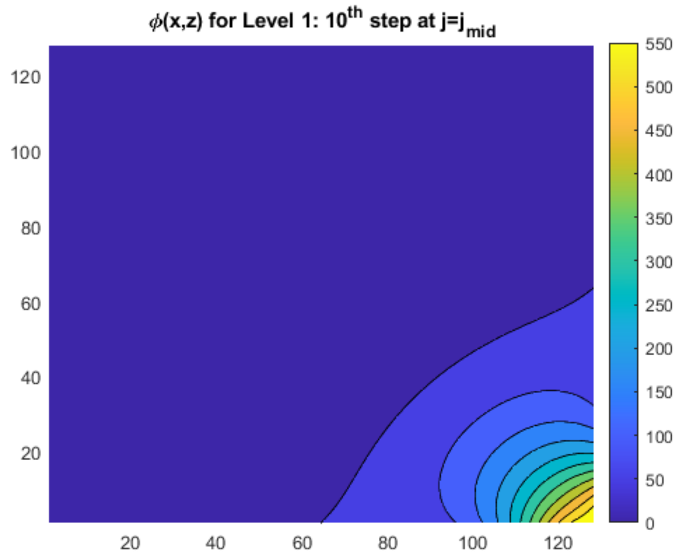


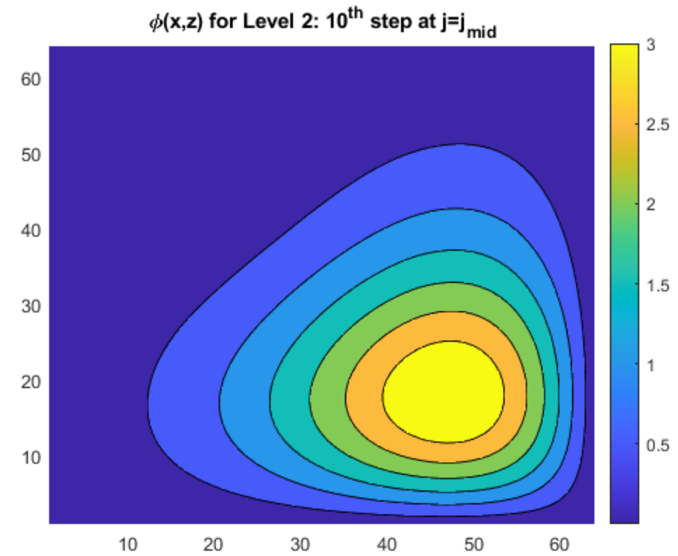
Gauss-Siedel Solver

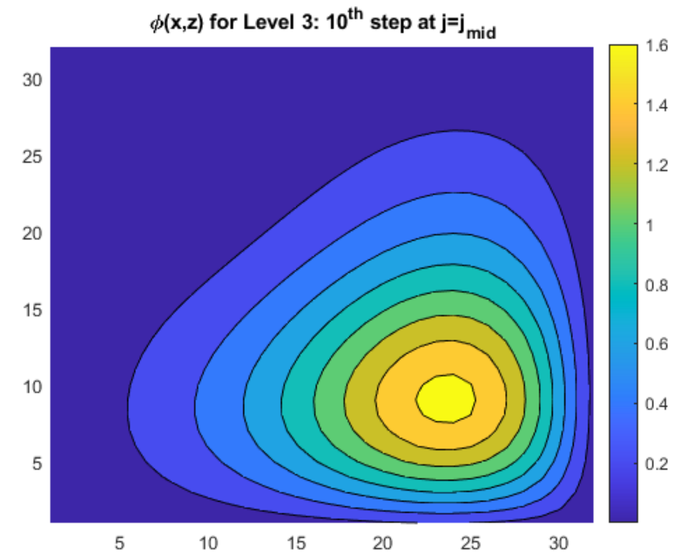


128x128x128

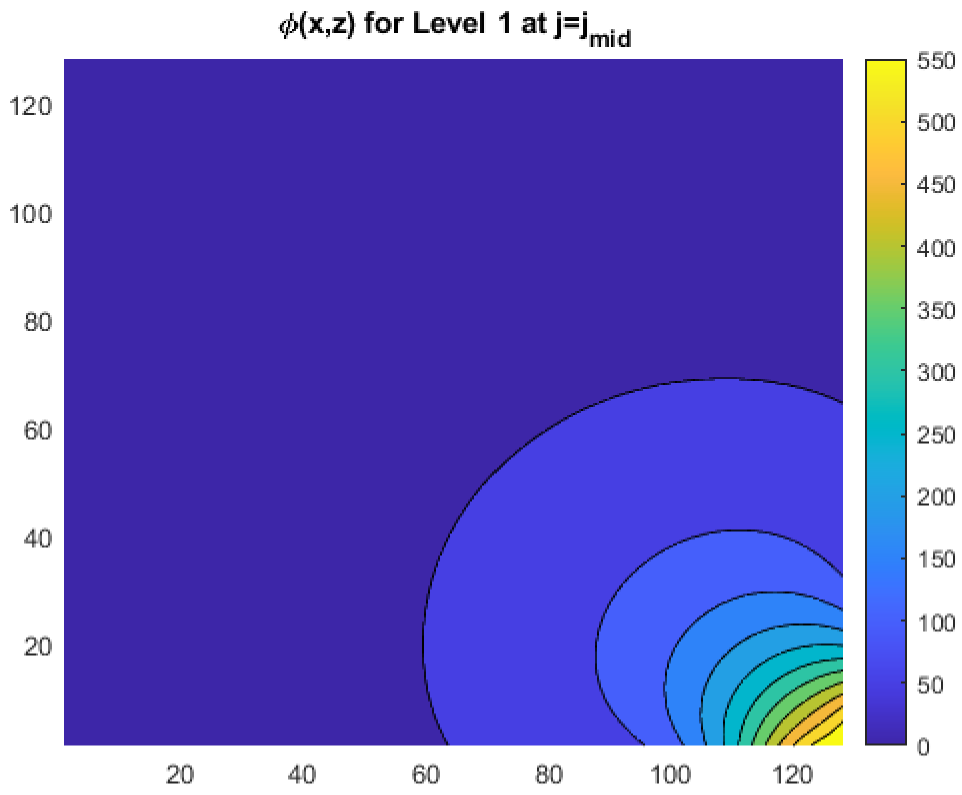
10^{th} MG-iteration



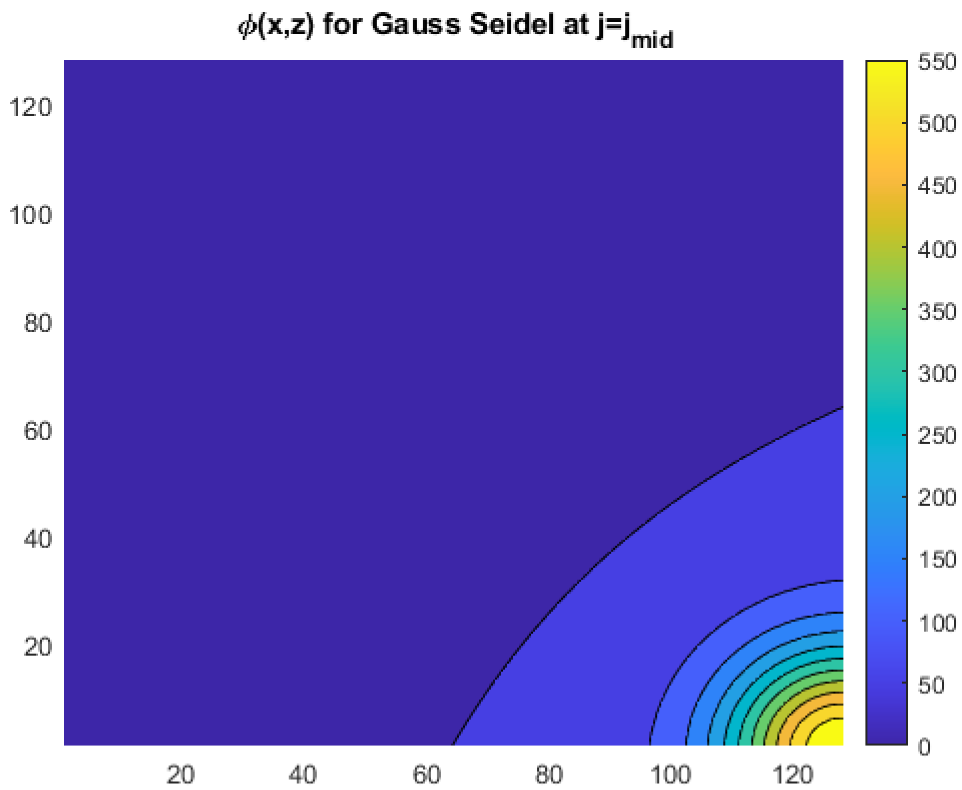




Final MG-iteration



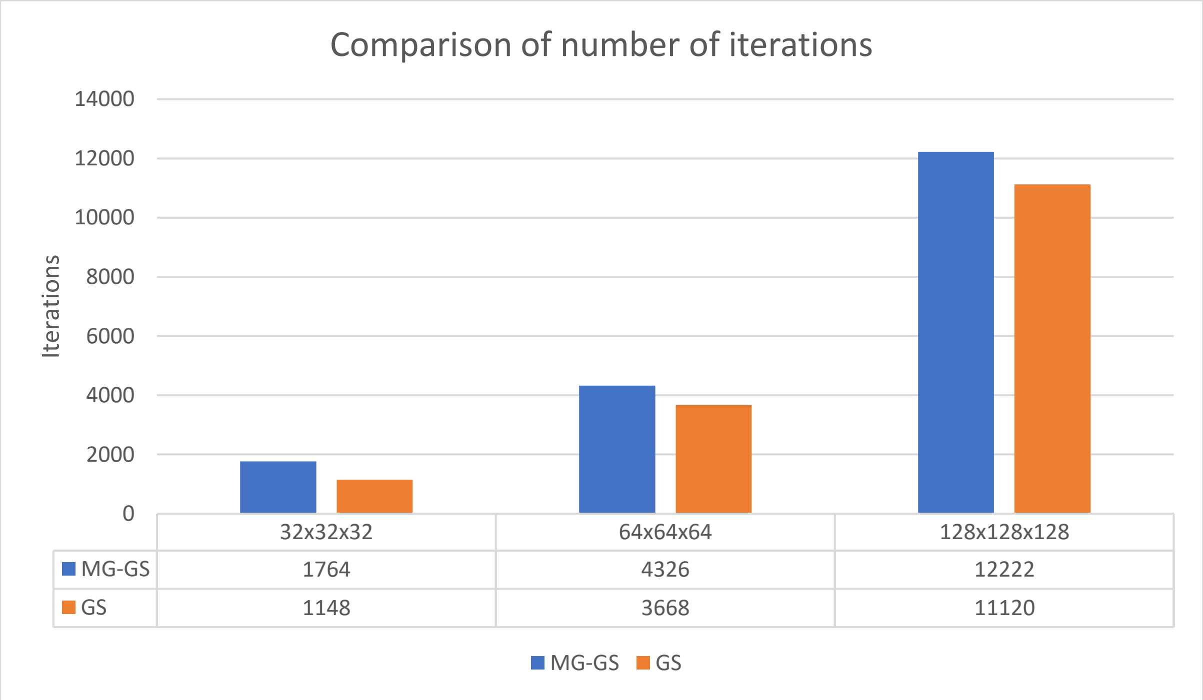
Gauss-Siedel Solver



Comparing GS and MG solver

GS iterations within one MG iteration = 42	Convergent GS iterations for tol = 1.0e-4
<u>MG-32</u>	<u>GS-32</u>
Number of iterations 42	Number of iterations = 1148
res = 9.0977e-07	res = 9.9975e-05
<u>MG-64</u>	<u>GS-64</u>
Number of iterations 103	Number of iterations = 3668
res = 9.7856e-07	res = 9.9868e-05
<u>MG-128</u>	<u>GS-128</u>
Number of iterations 291	Number of iterations = 11120
res = 9.7827e-07	res = 9.9998e-05

Grid Size	MG solver		GS-iterations	% decrease by using MG
	V-cycles	Total		
32x32x32	42	1764	1148	26%
64x64x64	103	4326	3668	35%
128x128x128	291	12222	11120	32%



Appendix

main.m

```
% Pragya Patel
% 17807477
% Multigrid Solver: Main

%% Initialize
tol = 1.0e-4;
Lx = 1; Ly = 1; Lz = 1;

% Grid size
N = 32;
Nx1 = N; Ny1 = N; Nz1 = N;

%
nxp21 = Nx1+2; nyp21 = Ny1+2; nzp21 = Nz1+2;
Nx2 = Nx1/2; Ny2 = Ny1/2; Nz2 = Nz1/2;
Nx3 = Nx2/2; Ny3 = Ny2/2; Nz3 = Nz2/2;
phi1 = zeros(nxp21,nyp21,nzp21);

%% Coefficient matrices
C1 = coeff_uni(Nx1,Ny1,Nz1,Lx,Ly,Lz);
C2 = coeff_uni(Nx2,Ny2,Nz2,Lx,Ly,Lz);
C3 = coeff_uni(Nx3,Ny3,Nz3,Lx,Ly,Lz);
g = pderhs([Nx1,Ny1,Nz1,Lx,Ly,Lz]);

%% Execution
% Level 1
phi1 = GS(phi1,C1,g,10);
phi0 = phi1;

% V-cycle begins
tmax = 200;
for t = 1:tmax
    phi1 = V3(C1,C2,C3,g,phi0,t);
    res = L2norm(phi0,phi1);
    if res < tol
        disp(['Number of iterations ', num2str(t)])
        disp(['res = ', num2str(res)])
        break
    else
        phi0 = phi1;
    end
end

%% Plot
midplane(phi1,Nx1,Ny1,Nz1,'Level 1')
```

V3.m

```
% Pragya Patel
% 17807477
% Multigrid Solver: V-cycle upto the 3rd level
% Functions used:
%   resi.m, updatebcr.m, restrict.m,
%   updatebcp.m, GS.m and prolong.m

function phi1 = V3(C1,C2,C3,g,phi0,iter)
% This function represents the multigrid accelerator
% upto 3-levels of the V-cycle (Total GS within = 10+10+4+8+10 =
42)
%
% Inputs
%   finest coefficient matrix (C1), level 2 coefficient matrix
(C2),
%   level 3 coefficient matrix (C3), rhs
% Output
%   x1(final)

s = C1.s; Nx1=s(1); Ny1=s(2); Nz1=s(3);

% V-cycle begins
phi1 = GS(phi0,C1,g,10);

% DOWNSWEEP
% 1 to 2
res = resi(phi1,C1,g);
res = updatebcr(res);
rhs2 = restrict(res);
rhs2 = updatebcr(rhs2);

% Level 2
phi2 = zeros(Nx1/2+2,Ny1/2+2,Nz1/2+2);
phi2 = GS(phi2,C2,rhs2,10);

% 2 to 3
res = resi(phi2,C2,rhs2);
res = updatebcr(res);
rhs3 = restrict(res);
rhs3 = updatebcr(rhs3);

% Level 3
phi3 = zeros(Nx1/4+2,Ny1/4+2,Nz1/4+2);
phi3 = GS(phi3,C3,rhs3,4);

% UPSWEEP
% 3 to 2
e = prolong(phi3);
e = updatebcr(e);
```

```matlab
phi2 = phi2 + e;
phi2 = GS(phi2,C2,rhs2,8);

% 2 to 1
e = prolong(phi2);
e = updatebcr(e);
phi1 = phi1 + e;
phi1 = GS(phi1,C1,g,10);

if iter == 10 % || iter == itermax
    midplane(phi3,Nx1/4,Ny1/4,Nz1/4,['Level 3: ' num2str(iter)
'^{th} step'])
    midplane(phi2,Nx1/2,Ny1/2,Nz1/2,['Level 2: ' num2str(iter)
'^{th} step'])
    midplane(phi1,Nx1,Ny1,Nz1,['Level 1: ' num2str(iter) '^{th}
step'])
end
end
```