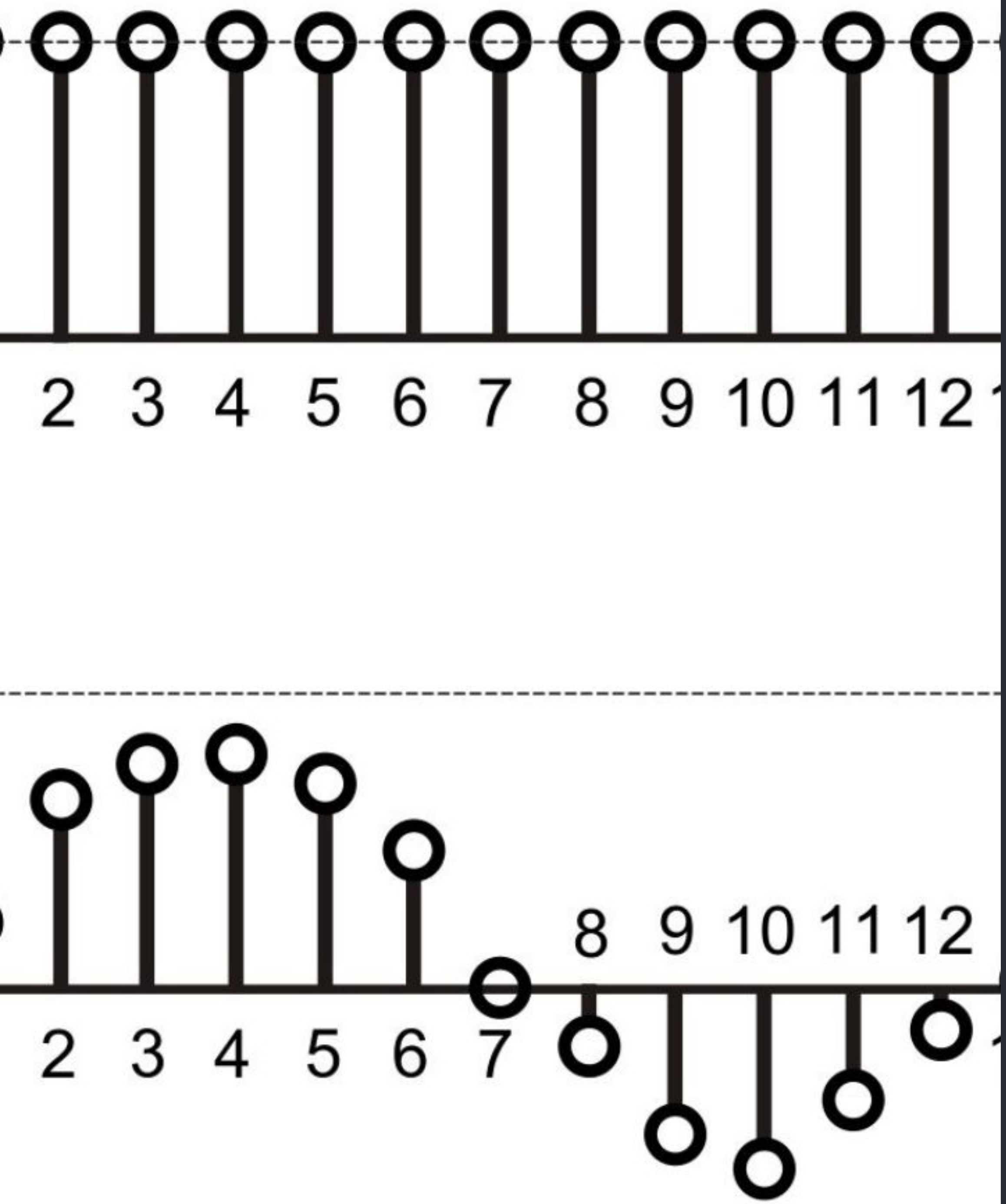


Circular Convolution

The Convolution Theorem of the Discrete Fourier Transform is a powerful tool widely used in signal processing applications. In this presentation, we will discuss circular convolution, its mathematical definition, and its significance in the field.

Praharsh Sharma

Convolution Theorem of DFT

The Convolution Theorem of the Discrete Fourier Transform (DFT) states that the circular convolution in the time domain is equivalent to the pointwise multiplication in the frequency domain. In mathematical terms, given two finite, discrete-time signals x[n] and h[n] of length N, and their DFTs:

$$x[n] \overset{\mathcal{DFT}}{\longleftrightarrow} X[k] \quad n, k \in \{0, \ldots, N-1\},$$

$$h[n] \overset{\mathcal{DFT}}{\longleftrightarrow} H[k] \quad n, k \in \{0, \ldots, N-1\},$$

Then, the multiplication of their DFTs corresponds to their circular convolution in the time domain:

The Convolution Theorem allows us to efficiently perform convolution operations using the fast Fourier transform algorithm, providing significant computational advantages. Understanding and applying the Convolution Theorem is crucial for various applications such as image processing, audio analysis, and communication systems.

Circular Convolution: Definition and Formula

Definition

Circular convolution is a type of signal processing operation used to combine two signals, x[n] and h[n]

⊛ symbol in the previous slide denotes the circular convolution.

Math Formula

The mathematical formula for circular convolution can be expressed using the DFT.

$$x[n] \circledast h[n] = \sum_{m=0}^{N-1} x[m]h[(n-m)\%N],$$

where % denotes the modulo operation, i.e., 0%N=0; 1%N=1; N−1%N=N−1; N%N=0, etc.

Since both signals are of length N, it is called an N-point circular convolution.

Methods for Circular Convolution

1

Direct Method

Performing a DFT, point wise multiplication of the two DFT representations, and then an inverse DFT.

2

Concentric Circle Method

Take two concentric circles. Plot N samples of x1(n) on the circumference of the outer circle maintaining equal distance successive points in anti-clockwise direction.

Multiply corresponding samples on the two circles and add them to get output.

3

Matrix Multiplication Method

One of the given sequences is repeated via circular shift of one sample at a time to form a N X N matrix.

The other sequence is represented as column matrix.

The multiplication of two matrices give the result of circular convolution.

Why is Convolution Circular?

1 Periodic Boundary Conditions

Understand the underlying mathematics behind circular convolution and how periodic boundary conditions contribute to its circular nature.

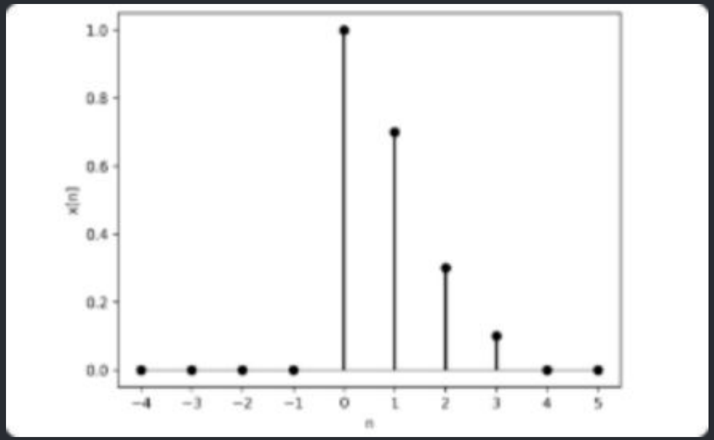
2 Relation to Cyclical Signals

Explore the concept of cyclical signals and their connection to circular convolution, shedding light on why it is fundamentally circular.

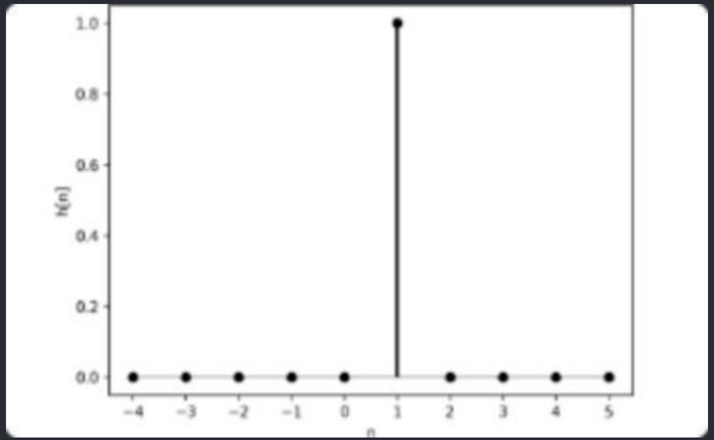
3 Implications for Signal Processing

Discover the practical significance of circular convolution and how its circular nature enables efficient analysis of time-varying signals.

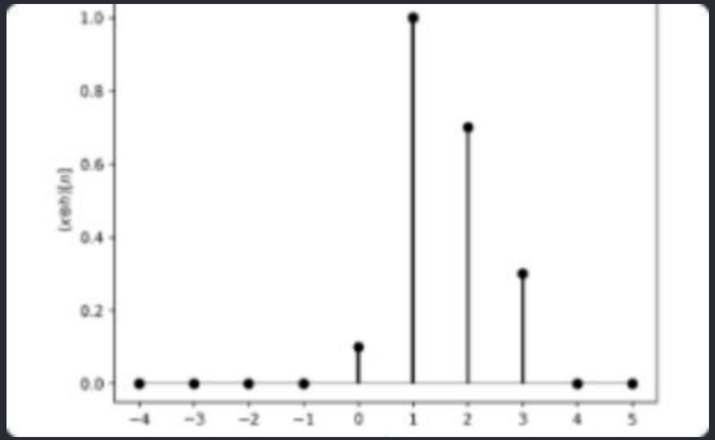
Illustrative Example



Signal 1: x[n]



Signal 2: h[n]



Circular Convolution

between x[n] and h[n]

Advantages and Disadvantages

Advantages of Circular Convolution

Uncover the advantages of circular convolution, such as efficient handling of cyclical data and its ability to preserve periodicity.

Drawbacks and Limitations

Explore the limitations of circular convolution, including its sensitivity to boundary effects and potential spectral leakage issues.

MATLAB Code and Conclusion

Performing circular convolution in MATLAB is a straightforward process that involves the fft function. This function computes the circular convolution of two sequences.

```
a = [2,3,5];

b = [7,11,13];

c = ifft(fft(a) .* fft(b));
```