

CSE 224:Fluid Mechanics

Chapter 5: Fluid Kinematics

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5.1 Introduction

- ☞ Fluid kinematics deals with the geometry of fluid motion in terms of displacement, velocity and acceleration without considering the forces causing the motion.
- ☞ After knowing the velocity, it becomes possible to calculate the pressure distribution and consequently, the force acting on the fluid can also be estimated.
- ☞ The methods by which the motion of a fluid may be described are; Lagrangian method and the Eulerian method.
- ☞ The Eulerian method is commonly used in fluid mechanics.
- ☞ In this chapter, the basic concepts related to fluid kinematics and the methods for determining velocity and acceleration are described.

5.2 Velocity of Fluid Particles

- ☞ The velocity of fluid flow is a function of space and time. Let ds be the distance travelled by a fluid particle in time dt in the space occupied by a fluid in motion.
- ☞ The velocity (V) of the fluid particle at any point is given by the following expression:

$$V = \lim_{dt \rightarrow 0} \frac{ds}{dt} \dots\dots\dots(5.1)$$

- ☞ The velocity is a **vector quantity** and thus, it has both magnitude as well as direction. The velocity V at any point in the fluid can be resolved into three components i.e. u , v and w in the three mutually perpendicular directions x , y and z respectively.
- ☞ If dx , dy and dz be the components of displacement ds in x , y and z directions, respectively, then the velocity components can be expressed as follows.

$$u = \lim_{dt \rightarrow 0} \frac{dx}{dt} = f_1(x, y, z, t) \dots\dots\dots(5.2 a)$$

$$v = \lim_{dt \rightarrow 0} \frac{dy}{dt} = f_2(x, y, z, t) \dots\dots\dots(5.2 b)$$

$$w = \lim_{dt \rightarrow 0} \frac{dz}{dt} = f_3(x, y, z, t) \dots\dots\dots(5.2 c)$$

- ☞ Thus, at a particular instant of time, the velocity components u , v and w vary at different points and each **individual particle has its own velocity** that varies both with respect to time and position.

5.2 Velocity of Fluid Particles..

☞ In vector form, the velocity of the fluid particle at any point can be represented as follows.

$$\vec{V} = ui + vj + wk \dots\dots\dots(5.3)$$

Where i, j and k are the unit vectors in the direction of coordinate axes.

☞ The magnitude of velocity (or the resultant velocity) is given by,

$$V = \sqrt{u^2 + v^2 + w^2} \dots\dots\dots(5.4)$$

Example 5.1: Find the velocity vector and its magnitude for the velocity components $u = (3xy - t)$, $v = (2yz + t + 1)$ and $w = (1 + 3ty)$ at point A (3, 2, 1)m at $t = 2$ s.

Solution

Let $u = (3xy - t)$, $v = (2yz + t + 1)$, $w = (1 + 3ty)$, $x = 3$ m, $y = 2$ m, $z = 1$ m and $t = 2$ s.

☞ Let \vec{V} be the velocity vector and V be its magnitude.

☞ The velocity components at point A (3, 2, 1) are calculated in the following expressions.

$$u = 3xy - t = 3 \times 3 \times 2 - 2 = 16 \text{ m/s}$$

$$v = 2yz + t + 1 = 2 \times 2 \times 1 + 2 + 1 = 7 \text{ m/s}$$

$$w = 1 + 3ty = 1 + 3 \times 2 \times 2 = 13 \text{ m/s}$$

$$\vec{V} = ui + vj + wk = 16i + 7j + 13k$$

$$V = \sqrt{u^2 + v^2 + w^2} = \sqrt{16^2 + 7^2 + 13^2} = 21.77 \text{ m/s}$$

5.3 Types of Fluid Flow

● The fluid flow may be of the following types classified as:

- 1) steady flow and unsteady flow,
- 2) uniform flow and non-uniform flow,
- 3) laminar flow and turbulent flow,
- 4) compressible flow and incompressible flow,
- 5) rotational flow and irrotational flow and,
- 6) one dimensional flow, two dimensional flow and three dimensional flow.

5.3.1 Steady and Unsteady Flows

Steady flow:

- ☞ The flow in which the fluid characteristics like velocity (**V**), pressure (**p**), density (**ρ**), etc., do not change with **time** is called steady flow.
- ☞ However, these characteristics may be different at different points in the flowing fluid.
- ☞ Most of the practical engineering problems involve steady flow and thus, it can be analyzed easily.
- ☞ Mathematically, the steady flow at any point in the flowing fluid may be given by the following expressions.

$$\frac{\partial V}{\partial t} = 0, \frac{\partial p}{\partial t} = 0, \frac{\partial \rho}{\partial t} = 0, \text{ etc.}$$

5.3 Types of Fluid Flow....

Unsteady flow:

- ☞ The flow in which the fluid characteristics like velocity, pressure, density, etc., change with time is called unsteady flow (or transient flow).
- ☞ It is very difficult to analyze the unsteady flow except under special conditions e.g. Computational Fluid Dynamics (CFD), help in studying unsteady flows under controlled settings, Periodic flow.
- ☞ Mathematically, this type of flow at any point in the flowing fluid may be expressed as:

$$\frac{\partial V}{\partial t} \neq 0, \frac{\partial p}{\partial t} \neq 0, \frac{\partial \rho}{\partial t} \neq 0, \text{ etc.}$$

5.3.2 Uniform and Non-uniform Flows

Uniform flow: The flow in which the fluid velocity does not change with location over a specified region at any given time is called uniform flow. Mathematically, the uniform flow may be expressed as:

$$\frac{\partial V}{\partial s} = 0$$

s denotes any space coordinates

Non-uniform flow:

- ☞ The flow in which the fluid velocity at any given time changes with location over a specified region is called non-uniform flow.
 - ☞ Mathematically, the non-uniform flow may be expressed as:
- $$\frac{\partial V}{\partial s} \neq 0$$
- ☞ Therefore, for a non-uniform flow, the space rate of change of the flow parameters at any given time is not equal to zero.

5.3 Types of Fluid Flow....

5.3.3 Laminar and Turbulent Flows

Laminar flow (viscous flow or streamline flow)

- ☞ A laminar flow is characterized by a smooth flow of one layer (or lamina) of fluid over the adjacent layer.
- ☞ The fluid particles move in well-defined paths (**or streamlines**) and keep the same path at successive cross sections of the flow passage.
- ☞ Generally, **laminar flow occurs in highly viscous liquids** flow and in smooth pipes when the flow velocity is low.
- ☞ In this flow regime, **viscous forces are dominant, and inertial forces are relatively small.**

Turbulent flow

- ☞ A fluid motion is said to be turbulent when the fluid particles move in **a zigzag manner**, i.e., entirely in a disorderly manner.
- ☞ This causes rapid and continuous mixing of the fluid leading to momentum transfer when the flow occurs. **Eddies or vortices are formed in turbulent flow, and it causes energy losses.**
- ☞ More often turbulent flow occurs than laminar flow, **for examples, flow in water supply pipes, flow in natural streams, sewers, etc.**
- ☞ Reynolds number (Re) is defined as the ratio of inertia force to the viscous force. Laminar and turbulent flows are characterized based on Reynolds number. Flow through a pipe is laminar when $Re < 2000$, turbulent when $Re > 4000$ and transitional when Re lies between 2000 and 4000.

5.3 Types of Fluid Flow....

5.3.4 Compressible and Incompressible Flows

Compressible flow The flow in which the density of the fluid does not remain constant is called compressible flow. Thus for compressible flow, $\rho \neq \text{Constant}$.

Incompressible flow

- ☞ The flow in which the density remains constant is called incompressible flow.
- ☞ Thus, for incompressible flow, $\rho = \text{Constant}$. The densities of liquids are constant and thus, the flow of liquids for practical purposes can be considered as incompressible.
- ☞ Mach number (M) is defined as the square root of the ratio of the inertia force to the elastic force or the ratio of local flow velocity to the sonic velocity in the fluid.
- ☞ The compressibility effects are generally ignored for $M < 0.3$. Based on the Mach number, the flow may be subsonic flow ($M < 1$), sonic flow ($M = 1$), supersonic flow ($M > 1$) and hypersonic flow ($M \gg 1$).

5.3.5 One-dimensional, Two-dimensional and Three-dimensional Flows

One-dimensional flow

- ☞ The flow in which the parameter such as velocity is a function of time and has only one space coordinate is called one-dimensional flow.
- ☞ Thus, the flow parameters vary only in one direction and mathematically, it is expressed as:

$$V = f(x, t) \text{ or } u = f_1(x, t), v = 0 \text{ and } w = 0$$

5.3 Types of Fluid Flow....

- ☞ For a steady one-dimensional flow, the velocity is a function of one space coordinate only and the variation of velocities in other two directions is zero. Mathematically expressed as:

$$V = f(x) \text{ or } u = f_1(x), v = 0 \text{ and } w = 0$$

Two-dimensional flow

- ☞ The flow in which the parameter such as velocity is a function of **time and contains two space** coordinates is called two-dimensional flow.
- ☞ The flow is mathematically expressed as. $V = f(x, y, t) \text{ or } u = f_1(x, y, t), v = f_2(x, y, t) \text{ and } w = 0$
- ☞ For a steady two-dimensional flow, the velocity is a function of two space coordinates only and the variation of velocity in third direction is zero.
- ☞ Mathematically, it is expressed as. $V = f(x, y) \text{ or } u = f_1(x, y), v = f_2(x, y) \text{ and } w = 0$

Three-dimensional flow

- ☞ The flow in which the parameter such as velocity is a function of **time and contains three space coordinates** is called three-dimensional flow.
- ☞ The three-dimensional flow is mathematically expressed as:
 $V = f(x, y, z, t) \text{ or } u = f_1(x, y, z, t), v = f_2(x, y, z, t) \text{ and } w = f_3(x, y, z, t)$
- ☞ For a steady 3-D flow, the velocity is a function of three space co-ordinates, and it is mathematically expressed as. $V = f(x, y, z) \text{ or } u = f_1(x, y, z), v = f_2(x, y, z) \text{ and } w = f_3(x, y, z)$

5.3 Types of Fluid Flow....

5.3.6 Rotational and Irrotational Flows

Rotational flow : The flow in which the fluid particles rotate about their own axis.

Irrotational flow: The flow in which the fluid particles do not rotate about their own axis.

5.4 Description of fluid Flow Pattern (Flow Visualization)

- ☞ The fluid flow pattern may be described by means of streamlines, stream-tubes, pathlines, streaklines and timelines which are described subsequently.

1. Streamline

- ☞ A streamline may be defined as an imaginary line drawn through a flowing fluid in such a way that the tangent to it at any point gives the direction of the velocity of flow at that point.
- ☞ Thus, streamlines indicate the direction of motion of particles at each point.
- ☞ Since the streamlines are tangent to the velocity vector at every point in the flow field, there can be no flow across a streamline.
- ☞ Figure 5.1(a) illustrates streamlines in a two-dimensional flow field in which one of the streamlines passing through a point $A(x, y)$ is tangential to the velocity vector V at point A .
- ☞ Let ds be the distance travelled by a fluid particle along the streamline during the time interval dt . Here, dx and dy be the components of the displacement along x and y directions, respectively and u and v be the components of the velocity V along x and y directions, respectively.

5.4 Description of Fluid Flow Pattern (Flow Visualization)...

$$\frac{v}{u} = \tan \alpha = \frac{dy}{dx}$$

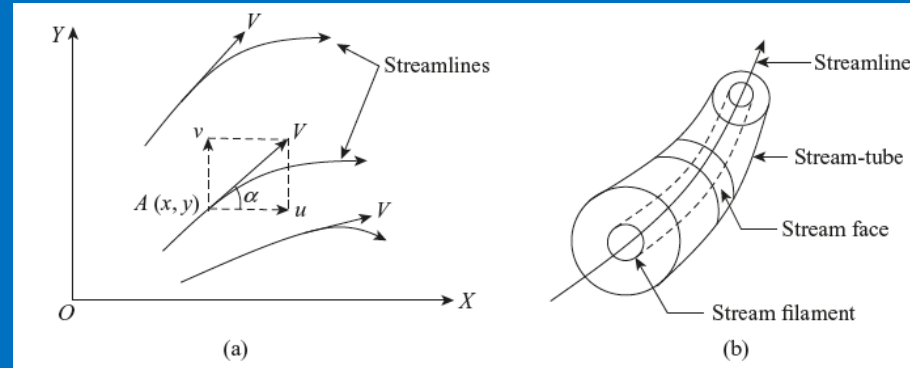


Figure 5.1 Streamlines and stream-tube

☞ Thus, the differential equation for streamlines in two-dimensional flow field is as follows.

$$\frac{dx}{u} = \frac{dy}{v} \dots\dots\dots (5.5)$$

☞ Similarly, the general differential equation for 3-D flow for streamlines as given as:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \dots\dots\dots (5.6)$$

☞ The streamlines for steady flow do not vary with time and it is constant for a given set of conditions. In unsteady flow, the streamline pattern would change from time to time.

2. Stream-tube (Figure 5.1(b)).

☞ A stream-tube is a cylindrical passage or tube which may be imagined to form by a bundle of neighbouring streamlines through which the fluid flows.

☞ Since the stream-tube is bounded on all sides by streamlines, there can be no flow across the surface. Therefore, the flow can be only through the ends of a stream-tube.

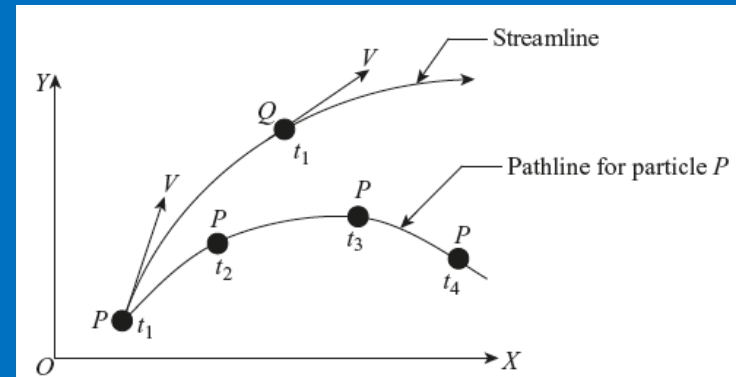
☞ The shape of a stream-tube changes from one instant to another due to change in the position of streamlines. However, in a steady flow it is fixed in space.

5.4 Description of Fluid Flow Pattern (Flow Visualization)...

3. Pathline:

- ☞ A pathline is the trace of the path of a single particle over a period of time. It shows the direction of the velocity of a fluid particle at successive instants of time.
- ☞ In a steady flow, the pathlines and streamlines are identical.
- ☞ However, in case of unsteady flow, the pathlines and streamlines are different as shown in Figure 5.2, a pathline can intersect itself at different times.
- ☞ The streamline shows the velocity vectors for particles P and Q at time t_1 . The particle P takes different positions at different times (t_2 , t_3 and t_4) and the line connecting these positions of P occupied at different instant of times signifies its Pathline.

Figure 5.2 Pathline and Streamline



4. Streakline:

- ☞ A line traced by a fluid particle passing through a fixed point in a flow field is known as a **streakline**. If a dye or a colour is injected into a flowing field, then the resulting trails of colour are known as streaklines.
- ☞ A line formed by the smoke particles (assume P_1 , P_2 and P_3) emanating from a fixed nozzle also forms a streakline as shown in Figure 5.3(a).

5.4 Description of Fluid Flow Pattern (Flow Visualization)...

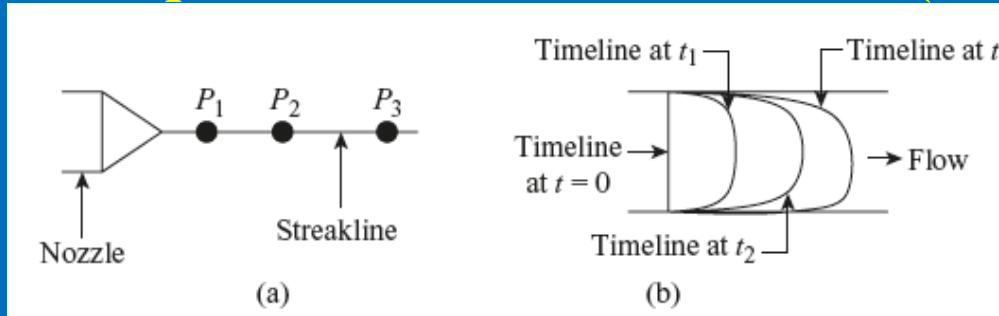


Figure 5.3 Streakline and Timelines

5. Timeline:

- ☞ A timeline is the line formed by a number of adjacent fluid particles in a flow field marked at a given instant.
- ☞ The timelines are used for examining the uniformity of a flow and it can also be used to study the deformation of a fluid under shear force. Here, the timelines generated in a water channel through the use of hydrogen bubble wire at different times are shown in Figure 5.3(b).

Example 5.2: The velocity vector in two different flow fields are given by the equations

$$(i) \vec{V} = 2xi - 2yj \quad (ii) \vec{V} = 2x^3i - 6x^2yj$$

Determine the equations of streamline in each case when it passes through a point A(3, 2).

Solution

$$(i) \vec{V} = 2xi - 2yj$$

$$u = 2x \text{ and } v = -2y$$

since

$$\frac{dx}{u} = \frac{dy}{v} \quad [\text{Streamline equation}]$$

$$\frac{dx}{2x} = \frac{dy}{-2y}$$

Examples/Solution..

$$\int \frac{dx}{x} = -\int \frac{dy}{y}$$

$$\ln x = -\ln y + \ln C$$

$$\ln x + \ln y = \ln C$$

$$\ln(xy) = \ln C$$

$$xy = C \dots\dots\dots (i)$$

The streamline passes through point A(3, 2) and thus, it must satisfy the expression (i).

$$3 \times 2 = C \text{ or } C = 6$$

$$xy = 6$$

Therefore, the required streamline equation is:

(ii) Compute for this question

5.5 Acceleration of A fluid Particle

- ☞ The rate of change of velocity with respect to time is called acceleration.
- ☞ At any instant of time, each fluid particle has its own velocity and acceleration that varies with respect to time and position.
- ☞ The fluid motion is described by two methods, namely **Lagrangian method** and **Eulerian method**. Generally, the **Eulerian method** is used in fluid mechanics.

5.5.1 Lagrangian Method

- ☞ In this method, a **single particle** is followed over the flow field during its course of motion by a **moving rectangular coordinate system** and its behaviour is observed.
- ☞ Let the initial coordinate of a fluid particle be **a**, **b** and **c** which change to **x**, **y** and **z** after time interval **t**. The position of the fluid particle can be expressed as given below.

$$x = f_1(a, b, c, t), y = f_2(a, b, c, t), z = f_3(a, b, c, t) \dots\dots\dots(5.7)$$

- ☞ From the above equations, the velocity and acceleration components of the fluid particles can be obtained by taking derivatives with respect to time.
- ☞ The velocity components can be obtained by the following expression.

$$u = \frac{dx}{dt}, v = \frac{dy}{dt}, w = \frac{dz}{dt} \dots\dots\dots(5.8)$$

- ☞ The acceleration components can be obtained by the following expression.

$$a_x = \frac{du}{dt} = \frac{d^2x}{dt^2}, a_y = \frac{dv}{dt} = \frac{d^2y}{dt^2}, a_z = \frac{dw}{dt} = \frac{d^2z}{dt^2} \dots\dots\dots(5.9)$$

5.5 Acceleration of A fluid Particle...

- ☞ The **magnitude** of velocity (or the **resultant** velocity) is given by the following expression.

$$V = \sqrt{u^2 + v^2 + w^2} \dots\dots\dots(5.10)$$

- ☞ The magnitude of acceleration (or the resultant acceleration) is given by Eq. 5.11.

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} \dots\dots\dots(5.11)$$

- ☞ Similarly, other quantities, such as pressure, density, etc., can be determined. However, the motion of single fluid particle is not sufficient to describe the entire flow field.
- ☞ Moreover, the solution of equations of motion is difficult due to its non-linear nature and therefore, this method is rarely used.

5.5.2 Eulerian Method

- ☞ In this method, a **finite volume** called **control volume** (or **flow domain**) is defined through which the fluid flows **in** and **out**.
- ☞ The motion of fluid is specified by velocity components expressed as functions of space and time in the control volume. Thus, Eulerian method does not follow an individual particle.
- ☞ Let V be the **resultant velocity at any point in a fluid flow** with u , v and w being its components in **x**, **y** and **z** directions, respectively. Thus, the velocity components can be mathematically expressed as in Eq. 5.12.

$$u = f_1(x, y, z, t), v = f_2(x, y, z, t), w = f_3(x, y, z, t) \dots\dots\dots(5.12)$$

5.5.2 Eulerian Method.....

☞ Let **a** be the resultant acceleration at any point in a fluid flow with **a_x**, **a_y** and **a_z** being its components in **x**, **y** and **z** directions, respectively.

☞ The components of acceleration of the fluid particles can be worked out by partial differentiation. Therefore, for **x** component of acceleration, we have the following expressions.

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz + \frac{\partial u}{\partial t} dt$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} + \frac{\partial u}{\partial z} \frac{dz}{dt} + \frac{\partial u}{\partial t} \frac{dt}{dt}$$

But

$$\frac{dx}{dt} = u, \quad \frac{dy}{dt} = v, \quad \frac{dz}{dt} = w$$

$$\therefore a_x = \frac{du}{dt} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \dots\dots\dots(5.13a)$$

☞ Similarly, the y and z components of acceleration are given in the Eqns. 5.13 b, c.

$$a_y = \frac{dv}{dt} = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \dots\dots\dots(5.13b)$$

$$a_z = \frac{dw}{dt} = u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} \dots\dots\dots(5.13c)$$

☞ Acceleration vector is given by Eq. 5.14,

$$\vec{a} = a_x i + a_y j + a_z k \dots\dots\dots(5.14)$$

5.5.2 Eulerian Method.....

☞ The magnitude of acceleration (resultant acceleration) is given by Eq. 5.15.

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} \dots\dots\dots(5.15)$$

☞ **Local acceleration:** It is defined as the rate of change of velocity of the fluid particles with respect to time at a given point in a fluid flow. In Eqns. 5.13 (a), 5.13(b) and 5.13(c), the following given expressions are called **local acceleration or temporal acceleration**.

$$\frac{\partial u}{\partial t}; \frac{\partial v}{\partial t}; \frac{\partial w}{\partial t} \dots\dots\dots(5.16)$$

☞ **Convective acceleration:** It is defined as the rate of change of velocity due to the **change in position** of the fluid particles in a flow field. In Eqns. 5.13(a), 5.13(b) and 5.13(c), the following given expressions are called convective acceleration or advective acceleration.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}, u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}, u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \dots\dots\dots(5.17)$$

☞ The **total acceleration**, i.e., the sum of local acceleration and convective acceleration of the fluid particle is called **material or substantial acceleration**. In a steady flow, the local acceleration is zero, **since the velocity at any point is invariant with time**. However, in uniform flow, the convective acceleration is zero, **since the velocity components are not the functions of space coordinates**.

☞ In **steady and uniform** flow, both the local and convective acceleration are zero and hence, there exists no total acceleration.

Examples & Exercises

Exercise 5.1

Find the velocity and acceleration components at point $A(1, 2, 3)$ m and at $t = 2$ s for the fluid flow described by the velocity vector $\vec{V} = 2x^3i - 5x^2yj + 3tk$

Example 5.3

Find the velocity and acceleration components of a fluid particle at position $\vec{r}(x, y, z) = 2i + j + 3k$, when $t = 1.5$ s for the fluid flow described by the velocity vector

Solution

$$\vec{V}(x, y, z, t) = 5xyi + 3x^2j + 2(t^2x + z)k.$$

Let $x = 2, y = 1, z = 3, t = 1.5, u = 5xy, v = 3x^2$ and $w = 2(t^2x + z)$.

☞ Let \vec{V} be the velocity vector, V be the resultant velocity, \vec{a} be the acceleration vector and a be the resultant acceleration.

☞ The velocity components at point $(2, 1, 3)$ and at time $t = 1.5$ s are calculated in the below expressions.

$$V = \sqrt{u^2 + v^2 + w^2} = \sqrt{10^2 + 12^2 + 15^2} = 21.66 \text{ units}$$

$$u = 5xy = 5 \times 2 \times 1 = 10 \text{ units}$$

$$v = 3x^2 = 3 \times 2^2 = 12 \text{ units}$$

$$w = 2(t^2x + z) = 2(1.5^2 \times 2 + 3) = 15 \text{ units}$$

$$\vec{V} = ui + vj + wk = 10i + 12j + 15k$$

$$a_x = \frac{du}{dt} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

Acceleration Components

Now $\frac{\partial u}{\partial x} = \frac{\partial(5xy)}{\partial x} = 5y, \frac{\partial u}{\partial y} = \frac{\partial(5xy)}{\partial y} = 5x, \frac{\partial u}{\partial z} = \frac{\partial(5xy)}{\partial z} = 0, \frac{\partial u}{\partial t} = \frac{\partial(5xy)}{\partial t} = 0$

Examples & Exercises..

Solution..

Since

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

Thus

$$a_x = 5xy(5y) + (3x^2)(5x) + 2(t^2x + z)(0) + 0 = 25xy^2 + 15x^3$$

The acceleration in x-direction at point (2, 1, 3) and at $t = 1.5$ is derived as given below.

$$a_x = 25xy^2 + 15x^3 = 25 \times 2 \times 1^2 + 15 \times 2^3 = 170 \text{ units}$$

$$a_y = \frac{dv}{dt} = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

Now

$$\frac{\partial v}{\partial x} = \frac{\partial(3x^2)}{\partial x} = 6x, \quad \frac{\partial v}{\partial y} = \frac{\partial(3x^2)}{\partial y} = 0, \quad \frac{\partial v}{\partial z} = \frac{\partial(3x^2)}{\partial z} = 0, \quad \frac{\partial v}{\partial t} = \frac{\partial(3x^2)}{\partial t} = 0$$

Since

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

Thus

$$a_y = 5xy(6x) + (3x^2)(0) + 2(t^2x + z)(0) + 0 = 30x^2y$$

The acceleration in y-direction at point (2, 1, 3) and at $t = 1.5$ is derived as given below.

$$a_y = 30x^2y = 30 \times 2^2 \times 1 = 120 \text{ units}$$

Examples & Exercises..

Solution..

$$a_z = \frac{dw}{dt} = u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$\frac{\partial w}{\partial x} = \frac{\partial[2(t^2x + z)]}{\partial x} = 2t^2; \quad \frac{\partial w}{\partial y} = \frac{\partial(3t)}{\partial y} = 0; \quad \frac{\partial w}{\partial z} = \frac{\partial[2(t^2x + z)]}{\partial z} = 2; \quad \frac{\partial w}{\partial t} = \frac{\partial[2(t^2x + z)]}{\partial t} = 4tx$$

$$a_z = u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$a_z = 5xy(2t^2) + (3x^2)(0) + 2(t^2x + z)(2) + 4tx$$

$$\therefore a_z = 10xyt^2 + 4(t^2x + z) + 4tx$$

The acceleration in z-direction at point (2, 1, 3) and at $t = 1.5$ is derived as given below.

$$a_z = 10 \times 2 \times 1 \times 1.5^2 + 4(1.5^2 \times 2 + 3) + 4 \times 1.5 \times 2 = 87 \text{ units}$$

$$\vec{a} = a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k} = \mathbf{170i + 120j + 87k}$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{170^2 + 120^2 + 87^2} = \mathbf{225.54 \text{ units}}$$

5.6 Rate of Flow (Discharge)

- ☞ The rate of flow (or discharge) is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel.
- ☞ Generally, the value of rate of flow is denoted by Q .
- ☞ Let A be the area of cross section of the pipe and V be the average velocity of the liquid flowing through the pipe. The discharge value is given by Eq. 5.18. $\boxed{Q = AV}$ (5.18)
- ☞ The value of discharge is measured in m^3/s (cumecs) and it may also be measured in litres per second (l / s) and $1\text{m}^3/\text{s} = 1000 \text{ l} / \text{s}$.

5.7 Continuity Equation

- ☞ The equation based on the principle of conservation of mass is called the continuity equation.
- ☞ According to continuity equation, the mass of a fluid passing through different sections of a pipe is the same if no fluid is added or removed from it.
- ☞ Consider section 1–1 and section 2–2 of a pipe as shown in Figure 5.4.
- ☞ Let ρ_1 be the density of the fluid at section 1-1, A_1 be the area of the pipe at section 1–1, V_1 be the velocity of the fluid at section 1–1 and ρ_2, A_2, V_2 be the corresponding values at section 2–2.
- ☞ The mass flow rate at section 1–1 is given by,

$$m_1 = \text{Density} \times \text{Discharge} = \rho_1 \times A_1 V_1$$

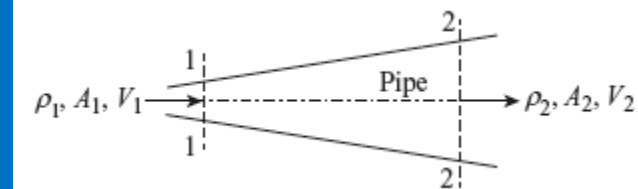


Fig. 5.4 Fluid flow through a pipe

5.7 Continuity Equation.....

☞ The mass flow rate at section 2–2 is given by, $m_2 = \rho_2 \times A_2 V_2$

☞ According to the law of conservation of mass $m_1 = m_2$, we obtain equation 5.19.

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \dots\dots\dots(5.19)$$

☞ Equation (5.19) is called continuity equation and it is applicable to compressible as well as incompressible fluids. For incompressible fluids, $\rho_1 = \rho_2$ and thus, the continuity eq.5.19 is expressed as in Eq. 5.20.

$$A_1 V_1 = A_2 V_2 \dots\dots\dots(5.20)$$

☞ The Equation (5.20) is applicable to a steady one-dimensional flow of incompressible fluid.

Example 5.4: A pipe 1 m in diameter carrying water at a velocity of 4 m/s is branched into two pipes. The first branch is 0.6 m in diameter and it carries one-third of the water flow. If water flows in the second branch with a velocity of 3 m/s, then determine the flow velocity in the first branch pipe and the diameter of the second branch.

Solution Let $d = 1$ m, $V = 4$ m/s, $d_1 = 0.6$ m, $Q_1 = (Q/3)$ m³/s and $V_2 = 3$ m/s.

• Let Q be the total discharge, V_1 be the water velocity in the first branched pipe and d_2 be the diameter of the second branched pipe.

$$Q = AV = \frac{\pi}{4} d^2 \times V = \frac{\pi}{4} \times 1^2 \times 4 = 3.1416 \text{ m}^3/\text{s}$$

$$Q_1 = \frac{Q}{3} = \frac{3.1416}{3} = 1.0472 \text{ m}^3/\text{s}$$

$$V_1 = \frac{Q_1}{A_1} = \frac{Q_1}{(\pi/4)d_1^2} = \frac{1.0472}{(\pi/4) \times 0.6^2} = 3.704 \text{ m/s}$$

5.7 Continuity Equation.....

$$Q_2 = Q - Q_1 = 3.1416 - 1.0472 = 2.0944 \text{ m}^3/\text{s}$$

$$A_2 = \frac{Q_2}{V_2} = \frac{2.0944}{3} = 0.6981 \text{ m}^2$$

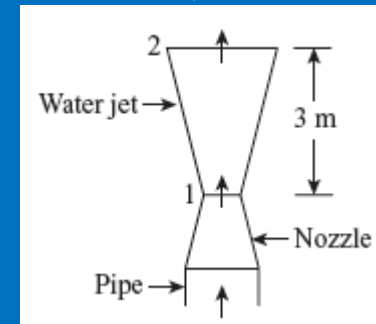
Thus

$$\frac{\pi}{4} d_2^2 = 0.6981$$

$$\therefore d_2 = \sqrt{\frac{4 \times 0.6981}{\pi}} = 0.9428 \text{ m}$$

Exercises

1. A water pipe of enlarging cross section has diameters 0.4 m and 1.2 m at sections 1-1 and 2-2, respectively. If the average flow velocity is 2 m/s at section 1-1, then find the velocity at section 2-2. Also determine the discharge and the mass flow rate of water.
2. A 10 mm water jet leaves the tip of the nozzle fitted at the end of a pipe with 10 m/s velocity in the vertically upward direction. If there is no energy loss and the jet remains circular, then determine its diameter at a point 3 m above the nozzle tip. (Refer to figure given).



3. A cylindrical container of radius 3 m and height 9 m is to be filled completely with water by a number of pipes in 50 minutes. Determine the required water inflow of the container in m^3/s and also determine the number of pipes required if the container is to be filled by 5 cm diameter water pipes in which water flows with a velocity of 5 m/s.

Exercises.....

4. For the following flows find the equation of the streamline passing through (1, 1).

$$(i) V = 3xi - 3yj$$

$$(ii) V = -y^2i - 6xj$$

5. For the following velocity vectors determine the magnitude of velocity at: A ($x = 2, y = -3, z = 1, t = 2$)

$$(i) V = (10t + xy)i + (-yz - 10t)j + (-yz + z^2/2)k$$

$$(i) V = 4xi + (-4y + 3t)j$$

6. Determine which of the velocity component sets given below satisfy the equation of continuity:

$$(a) u = cx, \quad v = -cy$$

$$(b) u = -cx/y, \quad v = c \ln xy$$

$$(c) u = A \sin xy, v = -A \sin xy \quad (d) u = x + y, v = x - y$$

$$(e) u = 2x^2 + zy, v = -2xy + 3y^3 + 3zy, w = -\frac{3}{2}z^2 - 2xy - 6yz$$

7. A steady two-dimensional flow has the following velocity field:

$$u = 2x + 3y - 5$$

$$v = 5x - 2y - 9$$

Determine the acceleration at the point (1, 1).

Exercises...

8. In a flow field, if $\vec{V} = 2x^3i - 5x^2yj - x^2zk$ is the velocity vector, then find whether it is a possible case of steady incompressible flow. If so, then determine the velocity and acceleration of fluid particle at (3, 1, 3).

End of the Lecture