Fundamentals of Mathematics I

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http://www.math.kent.edu/ebooks/10031/book.pdf

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Contents

Chapter 1

Arithmetic

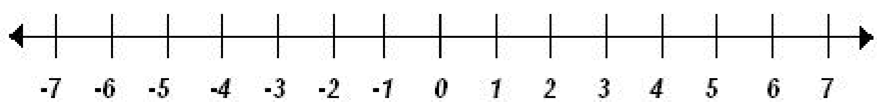
1.1 Real Numbers

As in all subjects, it is important in mathematics that when a word is used, an exact meaning needs to be properly understood. This is where we will begin.

When you were young an important skill was to be able to count your candy to make sure your sibling did not cheat you out of your share. These numbers can be listed: $\{1, 2, 3, 4, ...\}$. They are called **counting numbers** or **positive integers**. When you ran out of candy you needed another number 0. This set of numbers can be listed $\{0, 1, 2, 3, ...\}$. They are called **whole numbers** or **non-negative integers**. Note that we have used **set notation for our list. A set is just a collection of things. Each thing in the collection is called an element or member the set. When we describe a set by listing its elements, we enclose the list in curly braces, '{}'. In notation $\{1, 2, 3, ...\}$, the ellipsis, '...', means that the list goes on forever in the same pattern. So for example, we say that the number 23 is an element of the set of positive integers because it will occur on the list eventually. Using the language of sets, we say that 0 is an element of the non-negative integers but 0 is not an element of the positive integers. We also say that the set of non-negative integers contains the set of positive integers.**

As you grew older, you learned the importance of numbers in measurements. Most people check the temperature before they leave their home for the day. In the summer we often **estimate** to the nearest positive integer (choose the closest counting number). But in the winter we need numbers that represent when the temperature goes below zero. We can estimate the temperature to numbers in the set $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$. These numbers are called **integers**.

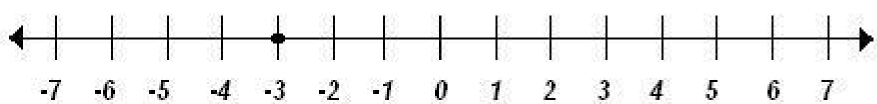
The **real numbers** are all of the numbers that can be represented on a number line. This includes the integers labeled on the number line below. (Note that the number line does not stop at -7 and 7 but continues on in both directions as represented by arrows on the ends.)



To **plot** a number on the number line place a solid circle or dot on the number line in the appropriate place.

Examples: Sets of Numbers & Number Line

EXAMPLE 1 *Plot on the number line the integer -3.*
Solution:



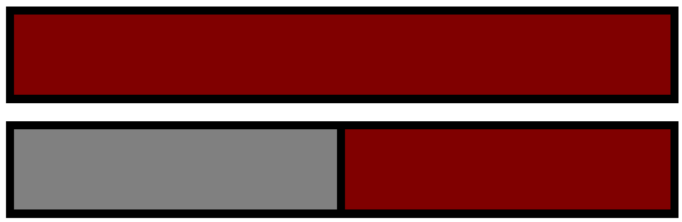
PRACTICE 2 *Plot on the number line the integer -5.*
Solution: Click here to check your answer.

Example 3 *Of which set(s) is 0 an element: integers, non-negative integers or positive integers?*
Solution: Since 0 is in the listings $\{0, 1, 2, 3, ...\}$ and $\{..., -2, -1, 0, 1, 2, ...\}$ but not in $\{1, 2, 3, ...\}$, it is an element of the integers and the non-negative integers.

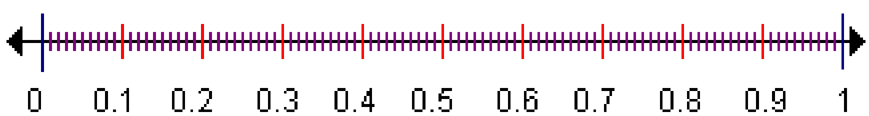
Practice 4 *Of which set(s) is 5 an element: integers, non-negative integers or positive integers?*
Solution: Click here to check your answer.

When it comes to sharing a pie or a candy bar we need numbers which represent a half, a third, or any partial amount that we need. A **fraction** is an integer divided by a nonzero integer. Any number that can be written as a fraction is called a **rational number**. For example, 3 is a rational number since $3 = 3 \div 1 = \frac{3}{1}$. All integers are rational numbers. Notice that a fraction is nothing more than a representation of a division problem. We will explore how to convert a decimal to a fraction and vice versa in section 1.9.

Consider the fraction $\frac{1}{2}$. One-half of the burgandy rectangle below is the gray portion in the next picture. It represents half of the burgandy rectangle. That is, 1 out of 2 pieces. Notice that the portions must be of equal size.



Rational numbers are real numbers which can be written as a fraction and therefore can be plotted on a number line. But there are other real numbers which cannot be rewritten as a fraction. In order to consider this, we will discuss decimals. Our number system is based on 10. You can understand this when you are dealing with the counting numbers. For example, 10 ones equals 1 ten, 10 tens equals 1 one-hundred and so on. When we consider a decimal, it is also based on 10. Consider the number line below where the red lines are the tenths, that is, the number line split up into ten equal size pieces between 0 and 1. The purple lines represent the hundredths; the segment from 0 to 1 on the number line is split up into one-hundred equal size pieces between 0 and 1.

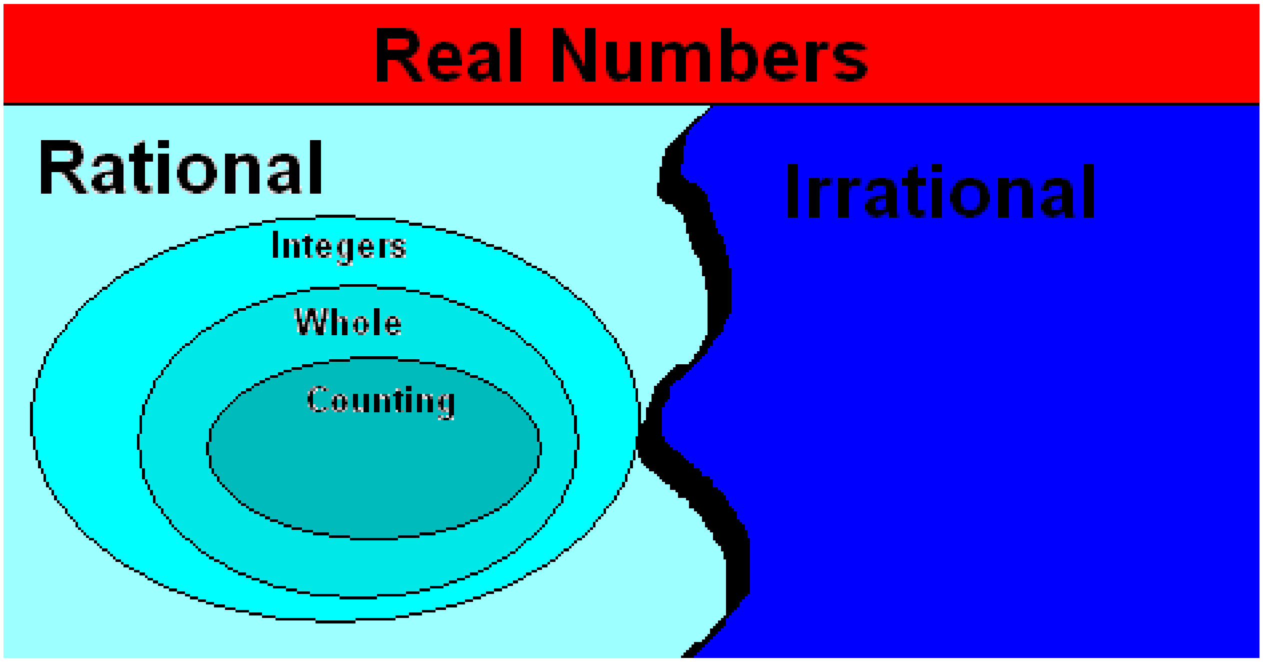


As in natural numbers these decimal places have place values. The first place to the right of the decimal is the tenths then the hundredths. Below are the place values to the millionths.

tens: ones: . : tenths: hundredths: thousandths: ten-thousandths: hundred-thousandths: millionths

The number 13.453 can be read "thirteen and four hundred fifty-three thousandths". Notice that after the decimal you read the number normally adding the ending place value after you state the number. (This can be read informally as "thirteen point four five three.) Also, the decimal is indicated with the word "and". The decimal 1.0034 would be "one and thirty-four ten-thousandths".

Real numbers that are not rational numbers are called **irrational numbers**. Decimals that do not terminate (end) or repeat represent **irrational numbers**. The set of all rational numbers together with the set of irrational numbers is called the set of **real numbers**. The diagram below shows the relationship between the sets of numbers discussed so far. Some examples of irrational numbers are $\sqrt{2}, \pi, \sqrt{6}$ (radicals will be discussed further in Section 1.10). There are infinitely many irrational numbers. The diagram below shows the terminology of the real numbers and their relationship to each other. All the sets in the diagram are real numbers. The colors indicate the separation between rational (shades of green) and irrational numbers (blue). All sets that are integers are in inside the oval labeled integers, while the whole numbers contain the counting numbers.



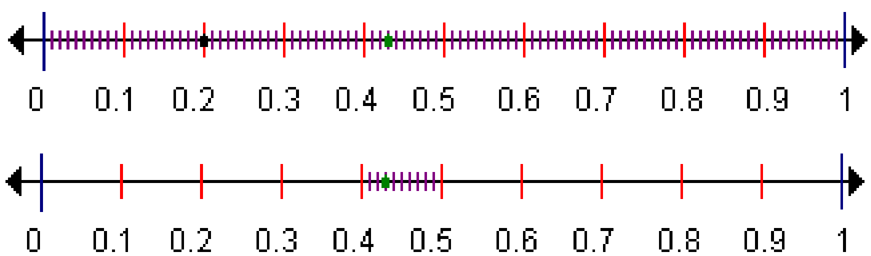
Examples: Decimals on the Number Line

EXAMPLE 5

a) Plot 0.2 on the number line with a black dot.

b) Plot 0.43 with a green dot.

Solution: For 0.2 we split the segment from 0 to 1 on the number line into ten equal pieces between 0 and 1 and then count over 2 since the digit 2 is located in the tenths place. For 0.43 we split the number line into one-hundred equal pieces between 0 and 1 and then count over 43 places since the digit 43 is located in the hundredths place. Alternatively, we can split up the number line into ten equal pieces between 0 and 1 then count over the four tenths. After this split the number line up into ten equal pieces between 0.4 and 0.5 and count over 3 places for the 3 hundredths.



PRACTICE 6

a) Plot 0.27 on the number line with a black dot.

b) Plot 0.8 with a green dot.

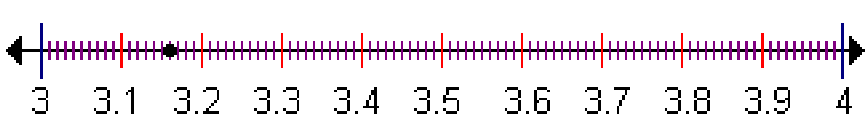
Solution: Click here to check your answer.

EXAMPLE 7

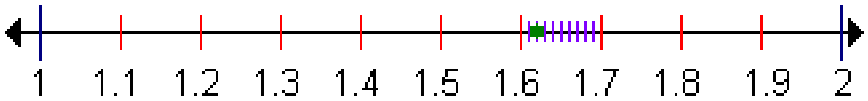
a) Plot 3.16 on the number line with a black dot.

b) Plot 1.62 with a green dot.

Solution: a) Using the first method described for 3.16, we split the number line between the integers 3 and 4 into one hundred equal pieces and then count over 16 since the digit 16 is located in the hundredths place.



b) Using the second method described for 1.62, we split the number line into ten equal pieces between 1 and 2 and then count over 6 places since the digit 6 is located in the tenths place. Then split the number line up into ten equal pieces between 0.6 and 0.7 and count over 2 places for the 2 hundredths.



PRACTICE 8

a) Plot 4.55 on the number line with a black dot.

b) Plot 7.18 with a green dot.

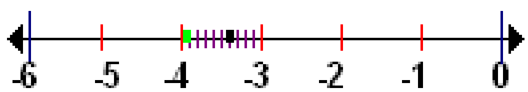
Solution: Click here to check your answer.

EXAMPLE 9

a) Plot -3.4 on the number line with a black dot.

b) Plot -3.93 with a green dot.

Solution: a) For -3.4, we split the number line between the integers -4 and -3 into one ten equal pieces and then count to the left (for negatives) 4 units since the digit 4 is located in the tenths place.
b) Using the second method, we place -3.93 between -3.9 and -4 approximating the location.



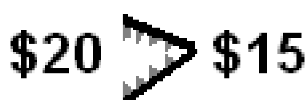
PRACTICE 10

a) Plot -5.9 on the number line with a black dot.

b) Plot -5.72 with a green dot.

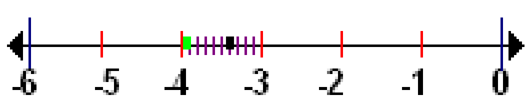
Solution: Click here to check your answer.

Often in real life we desire to know which is a larger amount. If there are 2 piles of cash on a table most people would compare and take the pile which has the greater value. Mathematically, we need some notation to represent that \$20 is greater than \$15. The sign we use is $>$ (greater than). We write, \$20 $>$ \$15. It is worth keeping in mind a little memory trick with these inequality signs. The thought being that the mouth always eats the larger number.



This rule holds even when the smaller number comes first. We know that 2 is less than 5 and we write $2 < 5$ where $<$ indicates "less than". In comparison we also have the possibility of equality which is denoted by $=$. There are two combinations that can also be used $\leq$ less than or equal to and $\geq$ greater than or equal to. This is applicable to our daily lives when we consider wanting "at least" what the neighbors have which would be the concept of $\geq$. Applications like this will be discussed later.

When some of the numbers that we are comparing might be negative, a question arises. For example, is -4 or -3 greater? If you owe \$4 and your friend owes \$3, you have the larger debt which means you have "less" money. So, $-4 < -3$. When comparing two real numbers the one that lies further to the left on the number line is always the lesser of the two. Consider comparing the two numbers in Example 9, -3.4 and -3.93.



Since -3.93 is further left than -3.4, we have that $-3.4 > -3.93$ or $-3.4 \geq -3.93$ are true. Similarly, if we reverse the order the following inequalities are true $-3.93 < -3.4$ or $-3.93 \leq -3.4$.

Examples: Inequalities

EXAMPLE 11 *State whether the following are true:*

a) $-5 < -4$

b) $4.23 < 4.2$

Solution:

a) True, because -5 is further left on the number line than -4.

b) False, because 4.23 is 0.03 units to the right of 4.2 making 4.2 the smaller number.

PRACTICE 12 *State whether the following are true:*

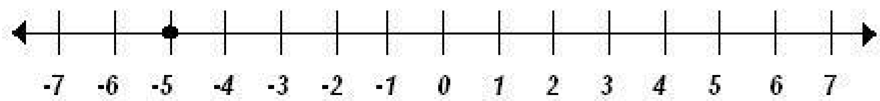
a) $-10 \geq -11$

b) $7.01 < 7.1$

Solution: Click here to check your answer.

Solutions to Practice Problems:

Practice 2



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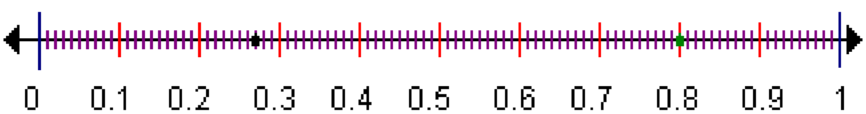
Practice 4

Since 5 is in the listings $\{0, 1, 2, 3, ...\}$, $\{..., -2, -1, 0, 1, 2, ...\}$ and $\{1, 2, 3, ...\}$, it is an element of the non-negative integers (whole numbers), the integers and the positive integers (or counting numbers). Back to Text

Practice 6



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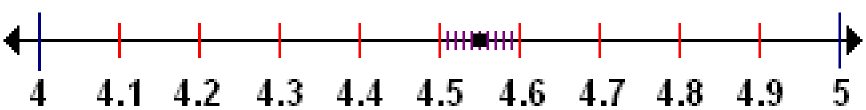
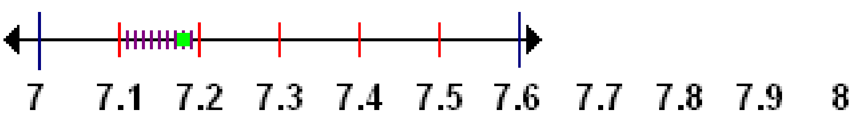
Practice 8





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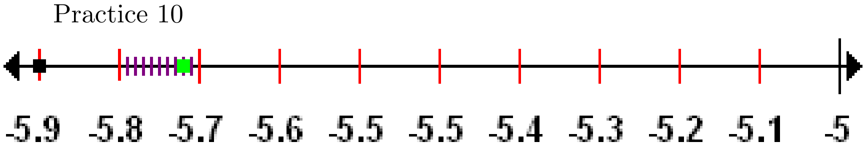
Practice 10



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Practice 12

Solution:

a) $-10 \geq -11$ is true since -11 is further left on the number line making it the smaller number.

b) $7.01 < 7.1$ is true since 7.01 is further left on the number line making it the smaller number.

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1.1.1 Exercises 1.1

Determine to which set or sets of numbers the following elements belong: irrational, rational, integers, whole numbers, positive integers. Click here to see examples.

1. -13
2. 50
3. $\frac{1}{2}$
4. -3.5
5. $\sqrt{15}$
6. 5.333

Plot the following numbers on the number line. Click here to see examples.

7. -9
8. 9
9. 0
10. -3.47
11. -1.23
12. -5.11

State whether the following are true: Click here to see examples.

13. $-4 \leq -4$
14. $-5 > -2$
15. $-20 < -12$
16. $30.5 > 30.05$
17. $-4 < -4$
18. $-71.24 > -71.2$

Click here to see the solutions.

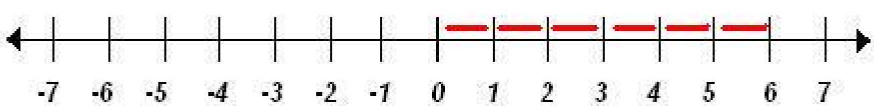
1.2 Addition

The concept of distance from a starting point regardless of direction is important. We often go to the closest gas station when we are low on gas. The **absolute value** of a number is the distance on the number line from zero to the number regardless of the sign of the number. The absolute value is denoted using vertical lines $|\#|$. For example, $|4| = 4$ since it is a distance of 4 on the number line from the starting point, 0. Similarly, $|-4| = 4$ since it is a distance of 4 from 0. Since absolute value can be thought of as the distance from 0 the resulting answer is a nonnegative number.

Examples: Absolute Value

EXAMPLE 1 *Calculate* $|6|$

Solution: $|6| = 6$ since 6 is six units from zero. This can be seen below by counting the units in red on the number line.



PRACTICE 2 *Calculate* $|-11|$

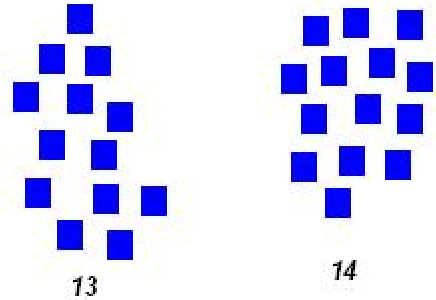
Solution: Click here to check your answer.

Notice that the absolute value only acts on a single number. You must do any arithmetic inside first.

We will build on this basic understanding of absolute value throughout this course.

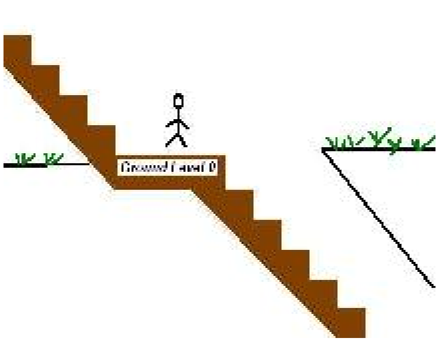
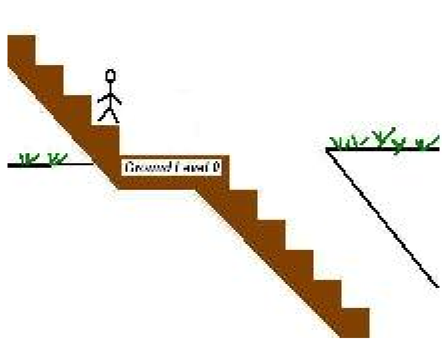
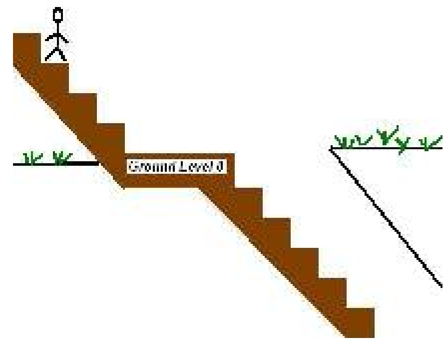
When adding non-negative integers there are many ways to consider the meaning behind adding. We will take a look at two models which will help us understand the meaning of addition for integers.

The first model is a simple counting example. If we are trying to calculate $13 + 14$, we can gather two sets of objects, one with 13 and one containing 14. Then count all the objects for the answer. (See picture below.)



If there are thirteen blue boxes in one corner and fourteen blue boxes in another corner altogether there are 27 blue boxes. The mathematical sentence which represents this problem is $13 + 14 = 27$.

Another way of considering addition of positive integers is by climbing steps. Consider taking one step and then two more steps, altogether you would take 3 steps. The mathematical sentence which represents this problem is $1 + 2 = 3$.

Even though the understanding of addition is extremely important, it is expected that you know the basic addition facts up to 10. If you need further practice on these try these websites:
http://www.slidermath.com/
http://www.ezschool.com/Games/Addition3.html

Examples: Addition of Non-negative Integers

Example 3 *Add.* $8 + 7 =$
Solution: $8 + 7 = 15$

Practice 4 *Add.* $6 + 8 =$
Solution: Click here to check your answer.

It is also important to be able to add larger numbers such as $394 + 78$. In this case we do not want to have to count boxes so a process becomes important. The first thing is that you are careful to add the correct places with each other. That is, we must consider place value when adding. Recall the place values listed below.

million: hundred-thousand: ten-thousand: thousand: hundred: ten: one: . : tenths: hundredths

Therefore, $1,234,567$ is read one million, two hundred thirty-four thousand, five hundred sixty-seven. Considering our problem $394+78$, 3 is in the hundreds column, 9 and 7 are in the tens column and 4 and 8 are in the ones column. Beginning in the ones column $4+8 = 12$ ones. Since we have 12 in the ones column, that is 1 ten and 2 ones, we add the one ten to the 9 and the 7 in the tens column. This gives us 17 tens. Again, we must add the 1 hundred in with the 3 hundred so $1+3 = 4$ hundred. Giving an answer $394 + 78 = 472$. As you can see this manner of thinking is not efficient. Typically, we line the columns up vertically.

$$\begin{array}{rr} & 11 \\ & 394 \\ + & 78 \\ \hline & 472 \end{array}$$

Notice that we place the 1's above the appropriate column.

Examples: Vertical Addition

Example 5 *Add* $8455 + 97$
Solution:

$$\begin{array}{rr} & 11 \\ & 8455 \\ + & 97 \\ \hline & 8552 \end{array}$$

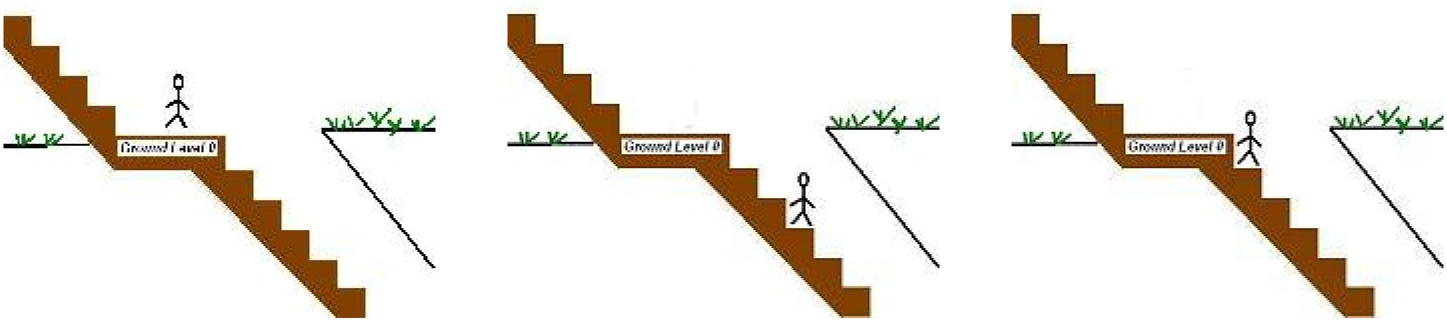
Practice 6 *Add* $42,062 + 391$
Solution: Click here to check your answer.

Example 7 *Add* $13.45 + 0.892$
Solution: In this problem we have decimals but it is worked the same as integer problems by adding the same units. It is often helpful to add in 0 which hold the place value without changing the value of the number. That is, $13.45+0.892 = 13.450+0.892$

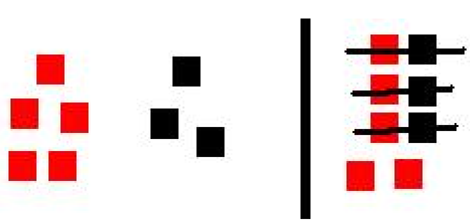
$$\begin{array}{rr} & 1\ 1 \\ & 13.450 \\ + & 0.892 \\ \hline & 14.342 \end{array}$$

Practice 8 *Add* $321.4 + 81.732$
Solution: Click here to check your answer.

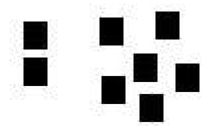
When we include all integers we must consider problems such as $-3 + 2$. We will initially consider the person climbing the stairs. Once again the person begins at ground level, 0. Negative three would indicate 3 steps down while 2 would indicate moving up two steps. As seen below, our stick person ends up one step below ground level which would correspond to -1. So $-3 + 2 = -1$.



Next consider the boxes when adding $5 + (-3)$. In order to view this you must think of black boxes being a negative and red boxes being a positive. If you match a black box and a red box they neutralize to make 0. That is, 3 red boxes neutralize the 3 black boxes leaving 2 red boxes which means $5 + (-3) = 2$.



Consider $-2 + (-6)$. This would be a set of 2 black boxes and 6 black boxes. There are no red boxes to neutralize so there are a total of 8 black boxes. So, $-2 + (-6) = -8$.



For further consideration of this go to
http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html
http://www.aaastudy.com/add65_x2.htm#section2

As before having to match up boxes or think about climbing up and downstairs can be time consuming so a set of rules can be helpful for adding $-50 + 27$. A generalization of what is occurring depends on the signs of the **addends** (the numbers being added). When the addends have different signs you subtract their absolute values. This gives you the number of "un-neutralized" boxes. The only thing left is to determine whether you have black or red boxes left. This is known by seeing which color of box had more when you started. In $-50 + 27$, the addends -50 and 27 have opposite signs so we subtract their absolute values $|-50| - |27| = 50 - 27 = 23$. But, since -50 has a larger absolute value than 27 the **sum** (the solution to an addition problem) will be negative -23, that is, $-50 + 27 = -23$.

In the case when you have the same signs $-20 + (-11)$ or $14 + 2$ we only have the same color boxes so there are no boxes to neutralize each other. Therefore, we just count how many we have altogether (add their absolute values) and denote the proper sign. For $-20 + (-11)$ we have 20 black boxes and 11 black boxes for a total of 31 black boxes so $-20 + (-11) = -31$. Similarly, $14 + 2$ we have 14 red boxes and 2 red boxes for a total of 16 red boxes giving a solution of $14 + 2 = 16$. A summary of this discussion is given below.

Adding Integers

1. Identify the addends.
   (a) For the same sign:
      i. Add the absolute value of the addends (ignore the signs)
      ii. Attach the common sign to your answer
   (b) For different signs:
      i. Subtract the absolute value of the addends (ignore the signs)
      ii. Attach the sign of the addend with the larger absolute value

Examples: Addition

EXAMPLE 9 $-140 + 90$
Solution:

Identify the addends	-140 and 90
Same sign or different	different signs
Subtract the absolute values	$140 - 90 = 50$
The largest absolute value	-140 has the largest absolute value
Attach the sign of addend with the largest absolute value	$-140 + 90 = -50$

PRACTICE 10 $-12 + 4$
Solution: Click here to check your answer.

Example 11 $-34 + (-55)$

Solution:

Identify the addends	-34 and -55
Same sign or different?	same signs
Add the absolute values	$34 + 55 = 89$
Attach the common sign of addends	$-34 + (-55) = -89$

Practice 12 $-52 + (-60)$

Solution: Click here to check your answer.

For more practice on addition of integers, click here.

Example 13 $-1.54 + (-3.2)$

Solution:

Identify the addends	-1.54 and -3.2
Same sign or different?	same signs
Add the absolute values	$1.54 + 3.2 = 4.74$
Attach the common sign of addends	$-1.54 + (-3.2) = -4.74$

Practice 14 $-20 + (-25.4)$

Solution: Click here to check your answer.

Click here for more practice on decimal addition.

Example 15 $|-8+5|$

Solution:

Since there is more than one number inside the absolute value we must add first	$-8 + 5$
Identify the addends	-8 and 5
The largest absolute value	-8 has the largest absolute value
Same sign or different	different signs
Subtract the absolute values	$8 - 5 = 3$
Attach the sign of addend with the largest absolute value	$-8 + 5 = -3$
Now take the absolute value	$|-8+5| = |-3| = 3$

Practice 16 $|-22 + (-17)|$

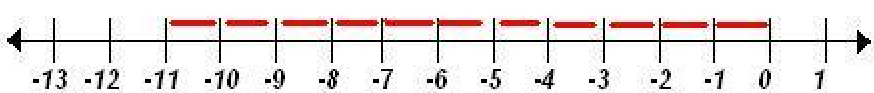
Solution: Click here to check your answer.

Notice that the absolute value only acts on a single number. You must do the arithmetic inside first.

Solutions to Practice Problems:

Practice 2

$|-11| = 11$ since -11 is 11 units from 0 (counting the units in red on the number line).



Back to Text

Practice 4

$6 + 8 = 14$ Back to Text

Practice 6

$$\begin{array}{r} 1 \\ 42062 \\ + \quad 391 \\ \hline 42453 \end{array}$$

Back to Text

Practice 8

$$\begin{array}{r} 1\ 1 \\ 321.400 \\ + \quad 81.732 \\ \hline 403.132 \end{array}$$

Back to Text

Practice 10
$-12 + 4 = -8$ since 4 red neutralize 4 black boxes leaving 8 black boxes. Back to Text

Practice 12
$-52 + (-60) = -112$ since the addends are the same we add $52 + 60 = 112$ and both signs are negative which makes the solution negative. Back to Text

Practice 14
$-20 + (-25.4) = -45.4$ since the signs are the same so we add and attach the common sign. Back to Text

Practice 16
Solution: $|-22 + (-17)|$ Determining the value of $-22 + -17$ first, note that the numbers have the same signs so we add their absolute values $22 + 17 = 39$ and attach the common sign -39. Therefore, $|-22 + (-17)| = |-39| = 39$ when we take the absolute value. Back to Text

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1.2.1 Exercises 1.2

Evaluate Click here to see examples.

1. $\|50\|$	2. $\|33\|$	3. $\|\frac{3}{7}\|$
4. $\|-3.5\|$	5. $\|-21\|$	6. $\|-55\|$

Add. Click here to see examples.

7. $-13 + 5$	8. $-3 + 10$	9. $59 + 88$
10. $36 + 89$	11. $104 + 1999$	12. $2357 + 549$
13. $-167 + (-755)$	14. $-382 + (-675)$	15. $22 + (-20)$
16. $39 + (-29)$	17. $-8 + 15$	18. $-7 + 12$
19. $\|12 + (-20)\|$	20. $\|33 + (-29)\|$	21. $\|-12.58 + (-78.8)\|$
22. $\|-253.2 + (-9.27)\|$	23. $\|509 + 3197\|$	24. $\|488 + 7923\|$

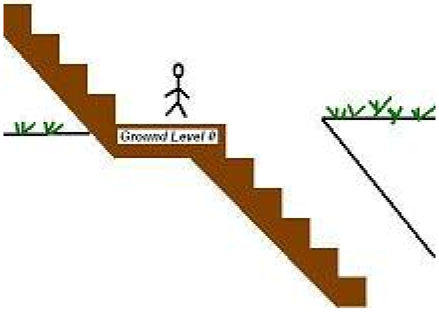
State whether the following are true: Click here to see examples.

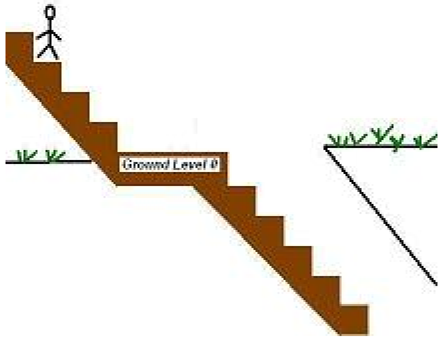
25. $\|-5 + 4\| > \|-5 + (-4)\|$	26. $\|-3 + (-2)\| \geq \|3 + 2\|$	27. $\|-12 + 15\| < \|15 - 12\|$
28. $\|-200 + 4\| \leq \|-200 + (-4)\|$	29. $100 + 3.1 \leq \|-100 + (-3.1)\|$	30. $\|-2 + 10\| > \|2 + (-10)\|$

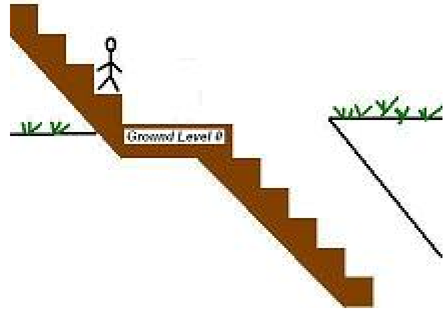
Click here to see the solutions.
Click here for more addition practice.

1.3 Subtraction

Let us begin with a simple example of $3 - 2$. Using the stairs application as in addition we would read this as “walk three steps up then down two steps”.







We must be able to extend this idea to larger numbers. Consider $1978 - 322$. Just as in addition we must be careful to line up place values always taking away the smaller absolute value. Again, a vertical subtraction is a good way to keep digits lined up.

$$\begin{array}{rr} & 1978 \\ - & 322 \\ \hline & 1656 \end{array}$$

Consider $1321 - 567$. When we line this up according to place values we see that we would like to take 7 away from 1 in the ones place. This cannot happen. Therefore, we need to borrow from the next column to the left, the tens. As in money, 1 ten-dollar bill is worth 10 one-dollar bills so it is that borrowing 1 ten equals 10 ones. We continue borrowing when necessary as seen below.

$$\begin{array}{rrrrr} & & & 1 & 11 \\ & 1 & 3 & \cancel{2} & \cancel{1} \\ - & & 5 & 6 & 7 \\ \hline & & & & 4 \end{array} \Rightarrow \begin{array}{rrrrr} & & 2 & 11 & 11 \\ & 1 & \cancel{3} & \cancel{2} & \cancel{1} \\ - & & 5 & 6 & 7 \\ \hline & & & 5 & 4 \end{array} \Rightarrow \begin{array}{rrrrr} & 0 & 12 & 11 & 11 \\ & \cancel{1} & \cancel{3} & \cancel{2} & \cancel{1} \\ - & & 5 & 6 & 7 \\ \hline & & 7 & 5 & 4 \end{array}$$

Examples: Vertical Subtraction

EXAMPLE 1 $\quad 13200 - 4154$

Solution: Notice that we have to borrow from 2 digits since there was a zero in the column from which we needed to borrow.

$$\begin{array}{rrrrrr} & & & 1 & 9 & 10 \\ & 1 & 3 & \cancel{2} & \cancel{0} & \cancel{0} \\ - & & 4 & 1 & 5 & 4 \\ \hline & & & 0 & 4 & 6 \end{array} \Rightarrow \begin{array}{rrrrrr} & 0 & 13 & 1 & 9 & 10 \\ & 1 & \cancel{3} & \cancel{2} & \cancel{0} & \cancel{0} \\ - & & 4 & 1 & 5 & 4 \\ \hline & & 9 & 0 & 4 & 6 \end{array}$$

PRACTICE 2 $\quad 4501 - 1728$

Solution: Click here to check your answer.

EXAMPLE 3 $\quad 83.05 - 2.121$

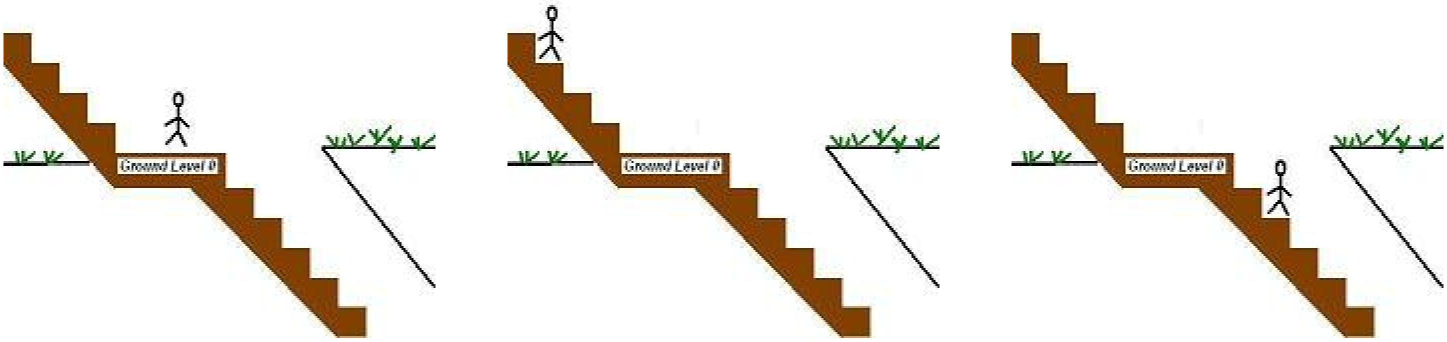
Solution: Decimal subtraction is handled the same way as integer subtraction by lining up place values. We also add in extra zeros without changing the value as we did in addition to help us in the subtraction. That is, $83.05-2.121 = 83.050-2.121$.

$$\begin{array}{rrrcrrr} & & 2 & & 10 & 4 & 10 \\ & 8 & \cancel{3} & . & \cancel{0} & \cancel{5} & \cancel{0} \\ - & & 2 & . & 1 & 2 & 1 \\ \hline & 8 & 0 & . & 9 & 2 & 9 \end{array}$$

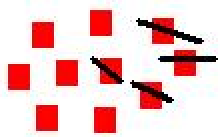
PRACTICE 4 $\quad 76.4 - 2.56$

Solution: Click here to check your answer.

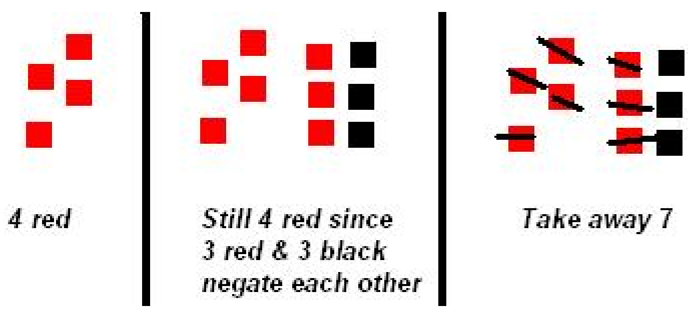
For a “nice” problem where the **minuend** (the first number in a subtraction problem) is greater than the **subtrahend** (the second number in a subtraction problem) we can use the rules we have been discussing. However, we need to know how to handle problems like $3 - 5$. This would read “walk up three steps then down 5 steps” which implies that you are going below ground level leaving you on step -2.



Now consider taking away boxes to comprehend the problem $10 - 4$. Using words with the box application this would read “ ten red (positive) boxes take away 4 red boxes”. We can see there are 6 red boxes remaining so that $10 - 4 = 6$.



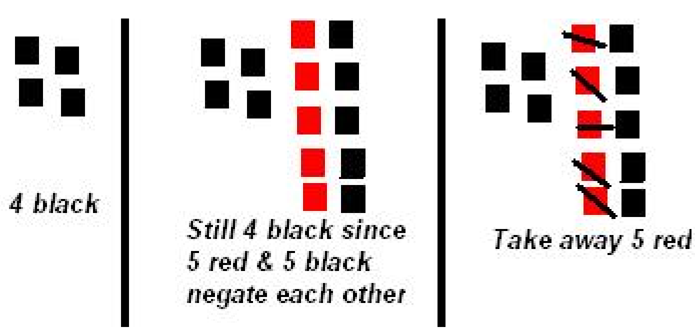
It is possible to use boxes when considering harder problems but a key thing that must be remembered is that a red and black box neutralize each other so it is as if we are adding nothing into our picture. Mathematically, it is as if we are adding zero, since adding zero to any number simply results in the number, (i.e., $5 + 0 = 5$). So, we can add as many pairs of red and black boxes without changing the problem. Consider the problem $4 - 7$. We need to add in enough pairs to remove 7 red boxes.



We see we are left with 3 black boxes so $4 - 7 = -3$.

Examples: Subtraction of Integers

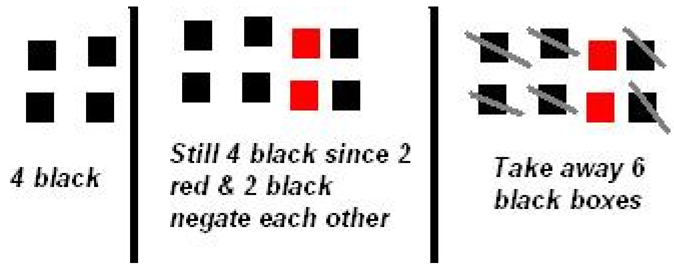
Example 5 $\quad -4 - 5$
Solution:



Therefore, $-4 - 5 = -9$

Practice 6 $\quad -3 - 7$
Solution: Click here to check your answer.

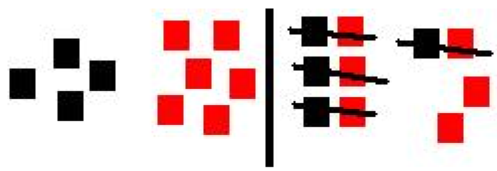
Example 7 $\quad -4 - (-6)$
Solution:



Therefore, $-4 - (-6) = 2$

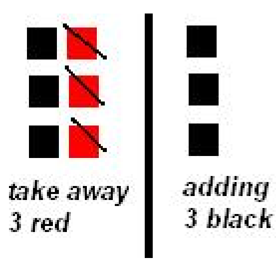
Practice 8 $\quad 3 - (-2)$
Solution: Click here to check your answer.

Compare Example 7, $-4 - (-6)$, with the problem $-4 + 6$.



We see that both $-4 - (-6)$ and $-4 + 6$ have a solution of 2. Notice that the first number -4 is left alone, we switched the subtraction to an addition and changed the sign of the second number, -6 to 6. Do you think this will always hold true? The answer is yes.

In the case above, we saw subtracting -6 is the same as adding 6. Let us consider another example. Is subtracting 3 the same as adding -3? Consider the picture below.



As you can see both sides end up with the same result. Although this does not prove "adding the opposite" always works, it does allow us to get an understanding concerning how this works so that we can generalize some rules for subtraction of integers.

Subtraction -

1. Identify the two numbers being subtracted
2. Leave the first number alone and add the opposite of the second number (If the second number was positive it should be negative. If it was negative it should be positive.)
3. Follow the rules of addition.

EXAMPLE 9 $-21 - 13$

Solution:

First number is left alone add the opposite	$-21 + (-13)$
Identify the addends	-21 and -13
Same sign or different	same signs
Add the absolute values	$21 + 13 = 34$
Attach the sign	$-21 - 13 = -21 + (-13) = -34$

PRACTICE 10 $-11 - 22$

Solution: Click here to check your answer.

EXAMPLE 11 $-1603 - (-128)$

Solution:

First number is left alone add the opposite	$-1603 + 128$
Identify the addends	-1603 and 128
Same sign or different	different signs
The largest absolute value	-1603 has the largest absolute value
Subtract the absolute values Be careful to	
subtract the smaller absolute value from the larger	$\begin{array}{rrrr} & 5 & 9 & 13 \\ 1 & \cancel{6} & \cancel{0} & \cancel{3} \\ - & 1 & 2 & 8 \\ \hline 1 & 4 & 7 & 5 \end{array}$
Attach the sign of addend with the largest absolute value	$-1603 + 128 = -1475$

PRACTICE 12 $-201 - (-454)$

Solution: Click here to check your answer.

EXAMPLE 13 $34 - 543$

Solution:

First number is left alone add the opposite	$34 + (-543)$
Identify the addends	34 and -543
Same sign or different	different signs
The largest absolute value	-543 has the largest absolute value
Subtract the absolute values Be careful to	
subtract the smaller absolute value from the larger	$\begin{array}{rrr} & 3 & 13 \\ 5 & \cancel{4} & \cancel{3} \\ - & 3 & 4 \\ \hline 5 & 0 & 9 \end{array}$
Attach the sign of addend with the largest absolute value	$34 - 543 = 34 + (-543) = -509$

PRACTICE 14 $-41 - 77$

Solution: Click here to check your answer.

EXAMPLE 15 $311 - (-729)$

Solution:

First number is left alone add the opposite	$311 + 729$
Identify the addends	311 and 729
Same sign or different	same signs
Add the absolute values	$\begin{array}{r} ^{1} \\ 311 \\ +\ 729 \\ \hline 1040 \end{array}$
Attach the sign	$311 - (-729) = 311 + 729 = 1040$

PRACTICE 16 $188 - 560$

Solution: Click here to check your answer.

EXAMPLE 17 $21.3 - 68.9$

Solution:

First number is left alone add the opposite	$21.3 + (-68.9)$
Identify the addends	21.3 and -68.9
Same sign or different?	different signs
Subtract the absolute values	$\begin{array}{r} 68.9 \\ -\ 21.3 \\ \hline 47.6 \end{array}$
Attach the sign	$21.3 - 68.9 = 21.3 + (-68.9) = -47.6$

PRACTICE 18 $15.4 - (-2.34)$

Solution: Click here to check your answer.

Eventually it will be critical that you become proficient with subtraction and no longer need to change the subtraction sign to addition. The idea to keep in mind is that the subtraction sign attaches itself to the number to the right. For example, $4 - 7 = -3$ since we are really looking at $4 + (-7)$.

Try these problems without changing the subtraction over to addition.

1. $5 - 9$
2. $-9 - 7$
3. $-10 - (-6)$
4. $8 - (-7)$

Click here for answers

More practice can be found online at http://www.aaastudy.com/sub65_x4.htm#section2.

Just as in addition when absolute value is involved you must wait to take the absolute value until you have just a single number to consider.

Examples: Subtraction with Absolute Value

EXAMPLE 19 $|15 - 27|$

Solution: $|15 - 27| = |15 + (-27)| = |-12| = 12$

Practice 20 $|-11-(-33)|$

Solution: Click here to check your answer.

Solutions to Practice Problems:

Practice 2

Solution:

$$\begin{array}{rrrrr} & & 4 & 9 & 11 \\ & 4 & \cancel{5} & \cancel{0} & \cancel{1} \\ - & 1 & 7 & 2 & 8 \\ \hline & & & 7 & 3 \end{array} \Rightarrow \begin{array}{rrrrr} & 3 & 14 & 9 & 11 \\ & \cancel{4} & \cancel{5} & \cancel{0} & \cancel{1} \\ - & 1 & 7 & 2 & 8 \\ \hline & 2 & 7 & 7 & 3 \end{array}$$

Back to Text

Practice 4

Solution:

$$\begin{array}{rrrcrr} & & & & & \\ & 7 & 6 & . & 4 & 0 \\ - & & 2 & . & 5 & 6 \\ \hline & & & & & \end{array} \Rightarrow \begin{array}{rrrcrr} & & & & 3 & 10 \\ & 7 & 6 & . & \cancel{4} & \cancel{0} \\ - & & 2 & . & 5 & 6 \\ \hline & & & & & 4 \end{array} \Rightarrow \begin{array}{rrrcrr} & & 5 & & 13 & 10 \\ & 7 & \cancel{6} & . & \cancel{4} & \cancel{0} \\ - & & 2 & . & 5 & 6 \\ \hline & 7 & 3 & . & 8 & 4 \end{array}$$

Back to Text

Practice 6

Solution: $-3-7=-10$ since we begin with 3 black boxes then put in 7 black and 7 red boxes (zeros out) then take away the 7 red boxes leaving 10 black boxes. Back to Text

Practice 8

Solution: $3-(-2)=5$ since we begin with 3 red boxes then put in 2 black and 2 red boxes (zeros out) then take away the 2 black boxes leaving 5 red boxes. Back to Text

Practice 10

Solution: $-11-22=-11+(-22)=-33$ Back to Text

Practice 12

Solution: $-201-(-454)=-201+454=253$ Back to Text

Practice 14

Solution: $-41-77=-41+(-77)=-118$ Back to Text

Practice 16

Solution: $188-560=188+(-560)=-372$ Back to Text

Practice 18

Solution: $15.4-(-2.34)=15.4+2.34=17.74$ Back to Text

Subtraction.

Solution:

1. -4
2. -16
3. -4
4. 15

Back to Text

Practice 20

Solution: $|-11-(-33)|=|-11+33|=|22|=22$ Back to Text

1.3.1 Exercises 1.3

Subtract. Click here to see examples.

1. $44 - 12$
2. $33 - 12$
3. $123 - 87$
4. $342 - 79$
5. $2100 - 321$
6. $1200 - 416$

Subtract. Click here to see examples.

7. $15 - 20$
8. $13 - 34$
9. $41 - 140$
10. $62 - 260$
11. $173 - (-547)$
12. $252 - (-798)$
13. $-54 - 12$
14. $-93 - 71$
15. $-6 - (-6)$
16. $-15 - 15$
17. $-43 - (-22)$
18. $-95 - (-64)$
19. $-232 - (-88)$
20. $-442 - (-87)$
21. $-3400 - 476$
22. $-8200 - 38.1$
23. $-0.33 - (-540)$
24. $-5.2 - (-7.63)$

State whether the following are true: Click here to see examples of Subtraction involving Absolute Value.

25. $|-5-4| > |-5-(-4)|$
26. $|-3-(-2)| \geq |3-2|$
27. $|-12-15| < |15-12|$
28. $|-200-4| \leq |-200-(-4)|$
29. $100 - 3.1 \leq |-100-(-3.1)|$
30. $|-2-10| > |2-(-10)|$

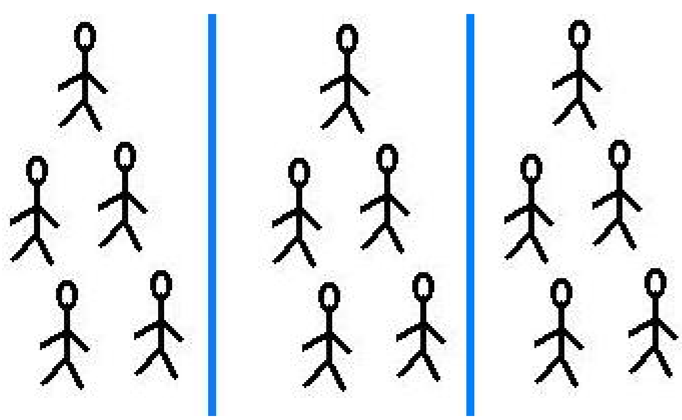
Vertical Addition.

31. $\begin{array}{rr} & 12 \\ + & -17 \\ \hline \end{array}$
32. $\begin{array}{rr} & -67 \\ + & -56 \\ \hline \end{array}$
33. $\begin{array}{rr} & 32 \\ + & -43 \\ \hline \end{array}$
34. $\begin{array}{rr} & 18 \\ + & -25 \\ \hline \end{array}$
35. $\begin{array}{rr} & -9 \\ + & 21 \\ \hline \end{array}$
36. $\begin{array}{rr} & -51 \\ + & 27 \\ \hline \end{array}$

Click here to see the solutions.

1.4 Multiplication

A helpful way to understand multiplication is through applications. For example, if a math class was split up into 3 groups of five students each how many students were in the class?



As you can see, 3 groups of 5 students is a total of 15 students. Multiplication is simply repeated addition. Mathematically we write this as $5 + 5 + 5 = 15$ or $3 \times 5 = 15$. Three and 5 in the multiplication sentence are known as **factors** of 15. That is, in a multiplication problem, the numbers being multiplied together are called **factors**.

Examples: Multiplication

EXAMPLE 1 $\quad 4 \times 3$

Solution: This is 4 groups of 3. The repeated addition number sentence is $3 + 3 + 3 + 3 = 12$.

Practice 2 3×7
Solution: Click here to check your answer.

Even though you can calculate the **product** (the answer to a multiplication problem) through repeated addition, it is important that you know all your multiplication facts through 10. Below you will see the multiplication table.

	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

To use this chart, find one factor in the first (red) row and the other factor in the first (red) column. Since multiplication is commutative it does not matter which factor is found in the first (red) row (see Section 1.11). Your answer is found where the row and column intersect. For example, to find 8×6 you follow down column 8 and across 6 and the point of intersection is the solution. Following the green on the chart below we see that $8 \times 6 = 48$.

	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Recognizing different signs for multiplication is necessary. For instance, * is used in computer science and for keying in multiplication on calculators on the computer. Often $\cdot$ is used to indicate multiplication. Multiplication is also understood when two numbers are in parentheses with no sign between them as in (2)(3).

We would not necessarily memorize a solution to $15 \cdot 3$. Also counting 15 groups of 3 is not very efficient. As in the larger addition and subtraction problems an algorithm (procedure) becomes the best solution. A vertical multiplication will be used. The simplest is to place on top the number with the most non-zero digits. Then multiply the ones column by all the digits above carrying where appropriate as when $3 \cdot 5 = 15$ has a carry over of 1 ten. That 1 ten must be added onto the product of the tens $3 \cdot 1 + 1 = 4$ tens.

$$\begin{array}{r} 15 \\ \times \quad 3 \\ \hline \end{array} \Longrightarrow \begin{array}{r} 1 \\ 15 \\ \times \quad 3 \\ \hline 5 \end{array} \Longrightarrow \begin{array}{r} 1 \\ 15 \\ \times \quad 3 \\ \hline 45 \end{array}$$

Now consider how we handle problems where both numbers have more than one non-zero digit. We will build on what we did in the last problem. Begin by first multiplying the ones digit by each digit in the other number.

$$\begin{array}{r} 426 \\ \times \quad 57 \\ \hline \end{array} \Longrightarrow \begin{array}{r} 4 \\ 426 \\ \times \quad 57 \\ \hline 2 \end{array} \Longrightarrow \begin{array}{r} 14 \\ 426 \\ \times \quad 57 \\ \hline 82 \end{array} \Longrightarrow \begin{array}{r} 14 \\ 426 \\ \times \quad 57 \\ \hline 2982 \end{array}$$

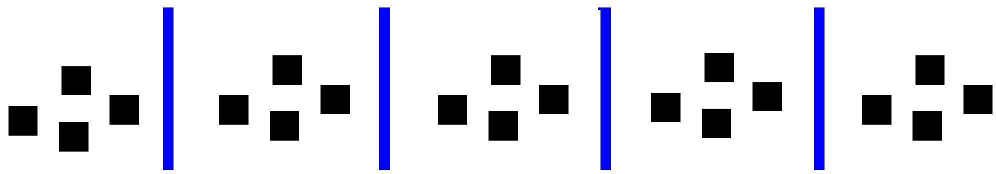
We then have to multiply the tens place by all the digits without losing the multiplication we already did.

$$\begin{array}{rr} & 426 \\ \times & 57 \\ \hline & 2982 \\ & 0 \end{array} \Longrightarrow \begin{array}{rr} & 3 \\ & 426 \\ \times & 57 \\ \hline & 2982 \\ & 00 \end{array} \Longrightarrow \begin{array}{rr} & 13 \\ & 426 \\ \times & 57 \\ \hline & 2982 \\ & 300 \end{array} \Longrightarrow \begin{array}{rr} & 13 \\ & 426 \\ \times & 57 \\ \hline & 2982 \\ & 21300 \end{array}$$

Finally, we add the columns.

$$\begin{array}{rr} & 426 \\ \times & 57 \\ \hline & 2982 \\ & 21300 \\ \hline & 24282 \end{array}$$

Now that we can handle multiplication problems of all non-negative integers we need to learn how to multiply all integers. Consider $5 \cdot -4$, that is, 5 groups of -4. We see that this results in a total of 20 negative (black) boxes. So $5 \cdot -4 = -20$. We could also use repeated addition for this problem $5 \cdot -4 = -4 + (-4) + (-4) + (-4) + (-4) = -20$.



We can generalize this to get the following rule.

negative $\cdot$ positive $=$ negative OR positive $\cdot$ negative $=$ negative

Consider another example, $-1 \cdot 3$. This is the same as $3 \cdot -1 = -1 + (-1) + (-1) = -3$. One way of thinking about $-1 \cdot 3$ is that $-1\cdot$ means the opposite of, that is, $-1 \cdot 3$ means the opposite of 3 which is -3. Now we can consider $-3 \cdot -4$. Since $-3 = -1 \cdot 3$, we can consider the opposite of $3 \cdot -4 = -4 + (-4) + (-4) = -12$. But the opposite of -12 is 12. A generalization of this also holds true.

negative $\cdot$ negative $=$ positive OR positive $\cdot$ positive $=$ positive

The algorithm we use to multiply integers is given below.

Multiplication $\times$, $(\ )(\ )$, $\cdot$,*

1. Multiply the numbers (ignoring the signs)
2. The answer is positive if they have the same signs.
3. The answer is negative if they have different signs.
4. Alternatively, count the amount of negative numbers. If there are an even number of negatives the answer is positive. If there are an odd number of negatives the answer is negative.

Examples: Multiplication of Integers

Example 3 $(-6)(4)$
Solution: $(-6)(4) = -24$ since $6 \cdot 4 = 24$ and negative $\cdot$ positive $=$ negative.

Practice 4 $(4)(-9)$
Solution: Click here to check your answer.

Example 5 $(-17)(-3)$
Solution: $(-17)(-3) = 51$ since $17 \cdot 3 = 51$ and negative $\cdot$ negative $=$ positive.

Practice 6 $(-26)(-9)$
Solution: Click here to check your answer.

Example 7 $(328)(-16)$
Solution: $(328)(-16) = 5248$ since negative $\cdot$ negative $=$ positive and

$$\begin{array}{rr} & {}^{14} \\ & 328 \\ \times & 16 \\ \hline & 1968 \\ & 3280 \\ \hline & 5248 \end{array}$$

.

Practice 8 $(-276)(23)$
Solution: Click here to check your answer.

Multiplication of decimals is not much harder than multiplication with integers. Consider the example $0.08 \cdot 0.005$. Notice, 0.08 has 2 digits to the right of the decimal while 0.005 has 3 digits. Altogether, there are $2 + 3 = 5$ digits to the right of the decimal. If $0.08 \cdot 0.005$ did not have decimals we would know that $8 \cdot 5 = 40$ but since there were 5 digits to the right of the decimal we move the decimal in 40 to the left 5 places which gives $0.08 \cdot 0.005 = 0.00040 = 0.0004$. (Explanations of why this works can be found in the Scientific Notation section of Fundamental Mathematics II.)

Examples: Multiplication with Decimals

Example 9 $0.006 \cdot 2 =$
Solution: $0.006 \cdot 2 = 0.012$ since $6 \cdot 2 = 12$ and there are $3 + 0 = 3$ digits to the right of the decimal in the problem.

Practice 10 $1.4 \cdot 3 =$
Solution: Click here to check your answer.

Example 11 $-2.003 * 0.0003 =$
Solution: $-2.003 * 0.0003 = -0.0006009$ since $2003 * 3 = 6009$ and there are $3 + 4 = 7$ digits to the right of the decimal in the problem.

Practice 12 $-1.0004 * -0.00002 =$
Solution: Click here to check your answer.

Example 13 $-2.523 \cdot -1.4 =$
Solution: $-2.523 \cdot -1.4 = 3.5322$ since $2523 \cdot 14 = 35,322$ and there are $3 + 1 = 4$ digits to the right of the decimal in the problem.

Practice 14 $5.016 \cdot -0.27 =$
Solution: Click here to check your answer.

Solutions to Practice Problems:
Practice 2
Solution: $3 \times 7 = 21$ Back to Text

Practice 4
Solution: $(4)(-9) = -36$ Back to Text

Practice 6
Solution: $(-26)(-9) = 234$ since $(26)(9) = 234$ and a negative times a negative equals a positive. Back to Text

Practice 8
Solution: $(-276)(23) = -6348$ since $(276)(23) = 6348$ and a negative times a positive equals a negative. Back to Text

Practice 10
Solution: $1.4 \cdot 3 = 4.2$ since $14 \cdot 3 = 42$ and there is one digit after the decimal. Back to Text

Practice 12
Solution: $-1.0004 * -0.00002 = 0.000020008$ since $10,004 * 2 = 20008$ and there are $4 + 5 = 9$ digits after the decimal. It is positive since a negative times a negative equals a positive. Back to Text

Practice 14
Solution: $5.016 \cdot -0.27 = -1.35432$ since $5016 \cdot 27 = 135432$ and there are $3 + 2 = 5$ digits after the decimal. It is negative because a positive times a negative equals a negative. Back to Text

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1.4.1 Exercises 1.4

Multiply. Click here to see examples.

1. 3109×52	2. $2312 * 47$	3. -9×7
4. 8×-6	5. $-4 \cdot -6$	6. $-15 * 3$
7. $-28 \cdot -5$	8. $(20)(-37)$	9. $(-30)(42)$
10. $(-118)(39)$	11. $(216)(61)$	12. $-3179 \cdot -42$

Multiply. Click here to see examples.

13. $0.09 \cdot 0.0008$	14. $1.004 \cdot 0.00002$	15. $8.102 \cdot 0.007$
16. $-5.18 \cdot 5$	17. $-4.24 * 0.034$	18. $(-41.1)(-9.016)$

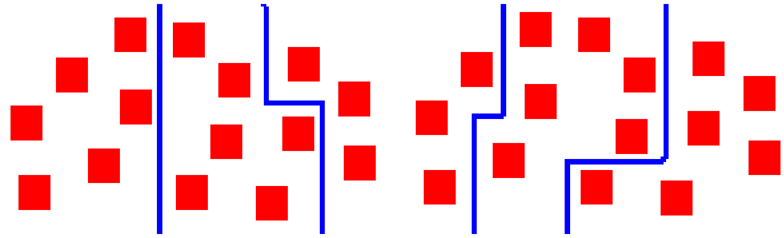
State whether the following are true:

19. $\lvert(-5)(4)\rvert > \lvert(-5)(-4)\rvert$	20. $\lvert(-3)(-2)\rvert \geq \lvert(3)(2)\rvert$	21. $\lvert(-2)(5)\rvert < \lvert(2)(-5)\rvert$

Click here to see the solutions.

1.5 Division

Division is often understood as the inverse operation of multiplication. That is, since $5 \cdot 6 = 30$ we know that $30 \div 6 = 5$ and $30 \div 5 = 6$. We can represent $30 \div 6$ by dividing 30 boxes into groups of 6 boxes each. The solution is then the number of groups which in this case is 5.



We can also understand division with negative integers through this by thinking of it as an inverse multiplication problem. Consider $-18 \div 3$. -18 is the **dividend**, 3 is the **divisor** and the solution is called the **quotient**. When thinking of division as the inverse of multiplication with what number would we need to fill in the triangle in order to make $\triangle \cdot 3 = -18$true? We know that $6 \cdot 3 = 18$ but the product is negative so using the rules negative $\cdot$ positive $=$ negative we deduce that the triangle must be -6. Therefore, $-18 \div 3 = -6$. Notice that division follows the same rule of signs as multiplication. This is true because of the relationship between multiplication and division.

Sign Rules for Division

negative $\div$ positive $=$ negative
positive $\div$ negative $=$ negative
negative $\div$ negative $=$ positive
positive $\div$ positive $=$ positive

Therefore, the general rules we follow for division are outlined below.

Division $\div$, /

1. Divide the absolute value of the numbers (ignoring the signs)
2. The answer is positive if they have the same signs.
3. The answer is negative if they have different signs.
4. Alternatively, count the amount of negative numbers. If there are an even number of negatives the answer is positive. If there are an odd number of negatives the answer is negative.

Use the rules above in the following examples.

Examples: Division of Integers

EXAMPLE 1 $\quad -24 \div -4$
Solution: $-24 \div -4 = 6$ since $24 \div 4 = 6$ $(6 \cdot 4 = 24)$ and negative $\div$ negative $=$ positive.

PRACTICE 2 $\quad 49 \div -7$
Solution: Click here to check your answer.

As in multiplication, in cases when there is a division involving numbers which cannot be calculated through the multiplication table, an algorithm is required: consider $74,635 \div 23$.

$$\begin{array}{rrl}
 & 3245 & \\
23 & \overline{)74635} & \text{23 goes into 74 three times} \\
 & \underline{69} & 23 \times 3 = 69 \\
 & 56 & \text{Subtract then bring down the 6 from the 74635} \\
 & \underline{46} & \text{23 goes into 56 two times; } 23 \times 2 = 46 \\
 & 103 & \text{Subtract then bring down the 3 from the 74635} \\
 & \underline{92} & \text{23 goes into 103 four times; } 23 \times 4 = 92 \\
 & 115 & \text{Subtract then bring down the 5 from the 74635} \\
 & \underline{115} & \text{23 goes into 115 five times; } 23 \times 5 = 115 \\
 & 0 & \text{Subtracting there is a remainder of zero}
\end{array}$$

Examples: Long Division

EXAMPLE 3 $\quad 9635 \div -41$

Solution: $9635 \div -41 = -235$ by the rules of signs of division and by long division.

$$\begin{array}{rrl}
 & 235 & \\
41 & \overline{)9635} & \text{41 goes into 96 two times} \\
 & \underline{82} & 41 \times 2 = 82 \\
 & 143 & \text{Subtract then bring down the 3 from the 9635} \\
 & \underline{123} & \text{41 goes into 143 three times; } 41 \times 3 = 123 \\
 & 205 & \text{Subtract then bring down the 5 from the 9635} \\
 & \underline{205} & \text{41 goes into 205 five times; } 41 \times 5 = 205 \\
 & 0 & \text{Subtracting there is a remainder of zero}
\end{array}$$

PRACTICE 4 $\quad -27,872 \div -32$

Solution: Click here to check your answer.

EXAMPLE 5 $\quad -3524 \div 6$

Solution: $-3524 \div 6 = -587r2$ where r indicates the remainder by long division and the sign rules for division. This can also be written as $-3524 \div 6 = -587\frac{2}{6} = -587\frac{1}{3}$. The fractional part of the answer is found by placing the remainder over the divisor. (Reducing it will be discussed in section 1.9.) When the remainder is 0 the fractional part is 0 so we do not write it.

$$\begin{array}{rrl}
 & 587 & \\
6 & \big|\overline{3524} & \text{6 goes into 35 five times} \\
 & \underline{30} & 6 \times 5 = 30 \\
 & 52 & \text{Subtract then bring down the 2 from the 3524} \\
 & \underline{48} & \text{6 goes into 52 eight times; } 6 \times 8 = 48 \\
 & 44 & \text{Subtract then bring down the 4 from the 3524} \\
 & \underline{42} & \text{6 goes into 42 seven times; } 6 \times 7 = 42 \\
 & 2 & \text{Subtracting there is a remainder of two}
\end{array}$$

PRACTICE 6 $4774 \div 9$

Solution: Click here to check your answer.

Next consider division involving decimals. We know that $0.3 \times 0.06 = 0.018$ from the previous section. Therefore, $0.018 \div 0.3 = 0.06$ and $0.018 \div 0.06 = 0.3$. When multiplying decimals, we first multiplied the factors, ignoring the decimal points. Then, at the end, we took into account the place values of the original numbers. We do the same for division. We will divide like normal and take into account the difference of the decimal places which are remaining. Consider, $0.024 \div 0.02$. $24 \div 2 = 12$ and 0.024 has 3 numbers to the right of the decimal while 0.02 has 2. Therefore, $0.024 \div 0.02 = 1.2$ since $3 - 2 = 1$ showing that we need 1 digit to the right of the decimal (alternatively, this is the same as moving the decimal one digit left). Let's look at another example, $30 \div 0.006$. We know that $30 \div 6 = 5$ but when calculating the digits needed to the right of the decimal we get $0 - 3 = -3$. When we get a negative answer this tells us that we need to add zeros (moving the decimal 3 digits right). Therefore, $30 \div 0.006 = 5000$. (Exploration of why this works can be found in Scientific Notation section of Fundamental Mathematics II.)

Examples: Decimal Division

EXAMPLE 7 $0.042 \div 2.1 =$

Solution: $42 \div 21 = 2$ and considering the digits to the right of the decimal $3 - 1 = 2$ we know $0.042 \div 2.1 = 0.02$.

PRACTICE 8 $1.44 \div 0.12 =$

Solution: Click here to check your answer.

EXAMPLE 9 $3.2 \div -0.00004 =$

Solution: $32 \div 4 = 8$ and considering the digits to the right of the decimal $1 - 5 = -4$ we add on 4 zeros to get $3.2 \div -0.00004 = -80,000$. The answer is negative since positive $\div$ negative = negative.

PRACTICE 10 $-12 \div 0.0003 =$

Solution: Click here to check your answer.

EXAMPLE 11 $0.81 \div 0.09 =$

Solution: $-81 \div -9 = 9$ and considering the digits to the right of the decimal $2 - 2 = 0$ we do not need to add any zeros nor do we need any digits to the right of the decimal. So, $-0.81 \div -0.09 = 9$. The answer is positive since negative $\div$ negative = positive.

PRACTICE 12 $0.56 \div -0.7 =$

Solution: Click here to check your answer.

Example 13 $4.3 \div 0.05 =$

Solution: 43 is not divisible by 5 but if we consider $4.30 \div 0.05 =$ we could do this problem. Notice that $4.3 = 4.30$ since we can always add zeros to the end when we are to the right of the decimal. $430 \div 5 = 86$ and considering the digits to the right of the decimal $4 - 4 = 0$ so that $4.3 \div 0.05 = 86$.

Practice 14 $2.3 \div 0.2 =$

Solution: Click here to check your answer.

While every possible case has not been considered for division, we have covered the main algorithm for division which is important for further study in mathematics. Now consider the calculator as a useful tool in division keeping in mind that caution must be used because it will estimate decimals that do not terminate quickly. When using a basic calculator we can do a division such as $247 \div 12 = 20.58\overline{3}$ where the line indicates 3 repeating forever. We can use this to get an exact answer involving fractions. Multiply the whole portion by the divisor $20 \times 12 = 240$. Then find the difference of the dividend and the product you just found $247 - 240 = 7$ which is the remainder. Therefore, $247 \div 12 = 20\frac{7}{12}$ which is the whole with the remainder over the divisor. This form of writing a quotient is known as an mixed number and will be explored further in Section 1.9.

Examples: Calculator Division

Example 15 *Divide and write as an improper fraction* $352 \div 7$

Solution: Using the calculator $352 \div 7 = 50.2857$. $50 \times 7 = 350$ and $352 - 350 = 2$. Therefore, $352 \div 7 = 50\frac{2}{7}$.

Practice 16 *Divide and write as an improper fraction* $537 \div 5$

Solution: Click here to check your answer.

Practice 2

Solution: $49 \div -7 = -7$ since $49 \div 7 = 7$ and positive $\cdot$ negative $=$ negative. Back to Text

Practice 4

Solution: $-27,872 \div -32 = 871$ by long division and negative $\cdot$ negative $=$ positive. Back to Text

Practice 6

Solution: $4774 \div 9 = 530r4 = 530\frac{4}{9}$. Back to Text

Practice 8

Solution: $1.44 \div 0.12 = 12$ since $144 \div 12 = 12$ and there are $2 - 2 = 0$ digits to the right of the decimal (that is, move the decimal 0 digits to the left). Back to Text

Practice 10

Solution: $-12 \div 0.0003 = -40000$ since $12 \div 3 = 4$ and there the decimal moves $0 - 4 = -4$ digits indicating 4 digits left of the decimal now (that is, move the decimal 4 digits right) adding zeros to hold the place values. The answer is negative since negative $\div$ positive $=$ negative. Back to Text

Practice 12

Solution: $0.56 \div -0.7 = -0.8$ since $56 \div 7 = 8$ and the decimal moves $2 - 1 = 1$ digits indicating 1 digit right of the decimal (that is, move the decimal 1 digit left) adding zeros to hold the place values. The answer is negative since positive $\div$ negative $=$ negative. Back to Text

Practice 14

Solution: 23 is not divisible by 2 but if we consider $2.30 \div 0.2 =$ we could do this problem. $230 \div 2 = 115$ and considering the digits to the right of the decimal $2-1 = 1$ (that is, move the decimal one place to the left) so that $2.3 \div 0.2 = 11.5$. Back to Text

Practice 16

Solution: Using the calculator $537 \div 5 = 107.4$. $107 \times 5 = 535$ and $537 - 535 = 2$. Therefore, $537 \div 5 = 107\frac{2}{5}$. Back to Text

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1.5.1 Exercise 1.5

Divide. Click here to see examples.

1. $54 \div 9$	2. $35 \div 5$	3. $-36 \div 6$
4. $-27 \div 3$	5. $-56 \div -7$	6. $-48 \div -8$

Use long division to find the quotient. Click here to see examples.

7. $52 \div -4$	8. $60 \div -10$	9. $-54 \div -2$
10. $-72 \div 3$	11. $-423 \div -3$	12. $-348 \div -6$
13. $-6288 \div 12$	14. $-13,503 \div 21$	15. $27,574 \div 34$

Divide. Click here to see examples.

16. $0.072 \div 0.9$	17. $-60.4882 \div -0.02$	18. $12.0003 \div -0.003$
19. $35.49 \div 0.0000007$	20. $-3.07 \div 0.05$	21. $-7.001 \div 0.2$

Find the quotient and write it as a mixed number. Click here to see examples.

22. $180 \div 7$	23. $413 \div 11$	24. $2119 \div 15$

Click here to see the solutions.

1.6 Exponents

Often in nature we find multiplication by the same number over and over. For example suppose a cell splits every minute. You begin with a single cell that splits so you then have 2 cells during the first minute. During the second minute both those cells split so now you have $2 \cdot 2$ cells. In the third minute all 4 cells would split giving you $2 \cdot 2 \cdot 2$ cells. And so it continues that after 10 minutes you would have $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ cells. This is a lot to write out so we use a notation to abbreviate it: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^{10}$ where 10 is the number of factors of 2. We read 2^{10} as two raised to the tenth power. In general,

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}$$

where n is called the **exponent** and a is called the **base**.

Examples: Exponents

EXAMPLE 1 3^4

Solution: $3^4 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{4 \text{ times}} = 9 \cdot 3 \cdot 3 = 27 \cdot 3 = 81$

PRACTICE 2 2^5

Solution: Click here to check your answer.

PRACTICE 3 10^3

Solution: Click here to check your answer.

PRACTICE 4 10^6

Solution: Click here to check your answer.

Do you notice a pattern with the zeros for the base 10 examples? Click Here.

This understanding of the ease of determining base 10 at large powers is used to simplify notation of large numbers. Consider the number $30,000,000 = 3 \times 10,000,000 = 3 \times 10^7$. This final way of writing thirty million as 3×10^7 is known as scientific notation. **Scientific notation** is a standardized method of writing a number which consists of the significant digits (the first number) which is between 1 (including 1) and 10 (not including 10) and is multiplied by a factor of base 10. Therefore, 3.12×10^5 is in scientific notation (the way you normally write a number) but 31.2×10^4 is not even though when we multiply them out and write them in standard notation they both equal $312,000$.

WARNING: Be careful when negatives are involved with exponents. The parentheses are important!
$(-3)^2 = -3 \cdot -3 = 9$ compared to $-3^2 = -3 \cdot 3 = -9$

The parentheses tell us that the negative is included as part of the base. If there are no parentheses then that negative is not part of the base. Consider the following examples.

Examples: Scientific Notation

EXAMPLE 5 *Write* 5.143×10^{12} *in standard notation.*
Solution: $5.143 \times 10^{12} = 5.143 \times 1,000,000,000,000 = 5,143,000,000,000$. Notice that this multiplication is the same as moving the decimal 12 units to the right.

PRACTICE 6 *Write* 7.3022×10^9 *in standard notation.*
Solution: Click here to check your answer.

EXAMPLE 7 *Write* $203,500,000$ *in scientific notation.*
Solution: First we identify the significant digits 2035 (these are the numbers to the left (and including) the last non-zero digit). The decimal must be placed after the first digit in order for it to be between 1 and 10. In our example, we have 2.035. Then multiply that number by the power of ten to hold the place value (a one is placed in the initial digit followed by zeros to hold the place value). Finally, we use the pattern notice earlier and count the zeros to identify the power. $203,500,000 = 2.035 \times 100,000,000 = 2.035 \times 10^8$.

PRACTICE 8 *Write* $61,000,000,000$ *in scientific notation.*
Solution: Click here to check your answer.

EXAMPLE 9 -5^2
Solution: $-5^2 = -\underbrace{5 \cdot 5}_{2\text{ times}} = -25$

PRACTICE 10 -2^4
Solution: Click here to check your answer.

EXAMPLE 11 $(-5)^2$
Solution: $(-5)^2 = \underbrace{-5 \cdot -5}_{2\text{ times}} = 25$

PRACTICE 12 $(-2)^4$
Solution: Click here to check your answer.

EXAMPLE 13 -2^3
Solution: $-2^3 = -\underbrace{2 \cdot 2 \cdot 2}_{3\text{ times}} = -8$

Practice 14 $(-2)^3$
Solution: Click here to check your answer.

Example 15 $(-1.3)^2$
Solution: $(-1.3)^2 = \underbrace{-1.3 \cdot -1.3}_{2 \text{ times}} = 1.69$

Practice 16 $(-.2)^3$
Solution: Click here to check your answer.

There are some special cases that are noteworthy in this section. Consider 2^0. We can see $2^0 = 1 \cdot 2^0 = 1$ since we can multiply by 1 without changing the solution but the zero exponent tells us to multiply by zero 2's. (This idea will be explored differently in a later course.) In general, $a^0 = 1$ for all nonzero real numbers a.

Another case is when we have zero as the base. Consider $0^3 = 0 \cdot 0 \cdot 0 = 0$. For all positive integer exponents, n, it is not hard to see that $0^n = 0$. The one that must be dealt with special is 0^0 which is undefined.

Consider 1^{43}. At first this problem may seem extreme but if we recall that no matter how many times you multiply 1 times itself you still get 1. Therefore this problem becomes simple $1^{43} = 1$. Similarly, a problem involving a base of -1 can look daunting but is simple when considering what happens when you multiply by a negative number. Consider the following list of problems with a negative one base and look for the pattern.
$(-1)^1 = -1$
$(-1)^2 = 1$
$(-1)^3 = -1$
$(-1)^4 = 1$
$(-1)^5 = -1$
$(-1)^6 = 1$
You should notice that these alternate from positive one to negative one. This is because a negative $\cdot$ positive $=$ negative and negative $\cdot$ negative $=$ positive. In general, we see that when the base is negative and there is a odd exponent the solution will be negative. Furthermore, when the base is negative and there is an even exponent then the solution will be positive. Therefore, $(-1)^{71} = -1$ since 71 is odd.

The following can be stated when n is a positive integer.

$a^0 = 1$ for $a \neq 0$
$0^n = 0$ for $n \neq 0$
0^0 is undefined
$1^n = 1$
$(-1)^n = -1$ for odd n
$(-1)^n = 1$ for even n

Practice 2
Solution: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$ Back to Text

Practice 3
Solution: $10^3 = 10 \cdot 10 \cdot 10 = 1000$ Back to Text

Practice 4
Solution: $10^6 = 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 1,000,000$ Back to Text

What is the pattern for base 10?
The exponent is equal to the number of zeros in the solution. Back to Text

Practice 6
Solution: $7.3022 \times 10^9 = 7.3022 \times 1,000,000,000 = 7,302,200,000$. Notice that this multiplication is the same as moving the decimal 9 units to the right. Back to Text

Practice 8
Solution: The significant digits in $61,000,000,000$ are 61. The appropriate decimal place is to the right of the first digit so

6.1. The decimal was moved 10 places so $61,000,000,000 = 6.1 \times 10^{10}$ in scientific notation. Back to Text

Practice 10
Solution: $-2^4 = -2 \cdot 2 \cdot 2 \cdot 2 = -16$ Back to Text

Practice 12
Solution: $(-2)^4 = (-2)(-2)(-2)(-2) = 16$ Back to Text

Practice 14
Solution: $(-2)^3 = (-2)(-2)(-2) = -8$ Back to Text

Practice 16
Solution: $(-0.2)^3 = (-0.2)(-0.2)(-0.2) = -0.008$ Back to Text

1.6.1 Exercises 1.6

Write in standard notation. Click here to see examples.

1. 3.2×10^3
2. 5.43×10^7
3. 9.021×10^6
4. 1×10^2
5. 4×10^5
6. 8.12×10^9

Write in scientific notation Click here to see examples.

7. $405,000$
8. 3200
9. $12,300,000$
10. $1,589,000$
11. $210,000,000$
12. $1,000$

Evaluate each exponential expression.Click here to see examples.

13. 4^3
14. 6^3
15. 2^6
16. 3^2
17. $(-1.1)^2$
18. $(-1.2)^2$
19. -1.1^2
20. -1.2^2
21. $(-3)^3$
22. $(-5)^3$
23. $(-3)^0$
24. $(-5)^1$
25. $(1)^{201}$
26. $(-1)^{445}$
27. -1^{28}
28. -10^6
29. 10^7
30. $(-10)^6$
31. $(-0.4)^3$
32. $(-0.1)^2$
33. 300^0
34. 0^0
35. 0^1
36. $(-1)^{352}$

Click here to see the solutions.

1.7 Order of Operations

We have discussed all the basic operations that we use in arithmetic. It is time to put them all together. Of course, there is a special way to do this which is called the order of operations.

1. **P**arentheses (or any grouping symbol {braces}, [square brackets], |absolute value|)
   - Complete all operations inside the parentheses first following the order of operations
2. **E**xponents
3. **M**ultiplication & **D**ivision from left to right
4. **A**ddition & **S**ubtraction from left to right

Most of us remember these with the help of the sentence "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally" where the first letters match the order of operations. The best way to learn the order of operations is by example and lots of practice.

Examples: Order of Operations

Example 1 $2(3 \cdot 5 - 5^2)^2 - 6$

Solution:

$2(\,3 \cdot 5 - 5^2\,)^2 - 6$	inside parentheses first; the exponent is the first operation
$2(\,3 \cdot 5 - 25\,)^2 - 6$	no more exponents in parentheses; multiplication
$2(\,15 - 25\,)^2 - 6$	no more exponents, multiplication, division; subtract
$2(\,-10\,)^2 - 6$	all operations inside parentheses done; exponents
$2 \cdot 100 - 6$	no more parentheses, exponents; multiplication
$200 - 6$	subtraction is the only operation remaining
194	

Practice 2 $72 \div 3 - 3(4 - 6 * 3)$

Solution:

Click here to check your answer.

Example 3 $-21 \div 3 \cdot 5 - 4^2$

Solution:

$-21 \div 3 \cdot 5 - 4^2$	no parentheses; exponents
$-21 \div 3 \cdot 5 - 16$	no parentheses, exponents; multiplication & division left to right
$-7 \cdot 5 - 16$	no parentheses, exponents; multiplication & division left to right
$-35 - 16$	no parentheses, exponents; multiplication & division left to right
-51	subtraction $-35 + (-16)$

Practice 4 $-6 - 3(-4) \div (3 - 3^2)$

Solution: Click here to check your answer.

Example 5 $-2|7 - 11| - 2^3$

Solution:

$-2\|7 - 11\| - 2^3$	parentheses - subtraction in absolute value
$-2\|-4\| - 2^3$	parentheses - absolute value
$-2 \cdot 4 - 2^3$	no parentheses; exponents
$-2 \cdot 4 - 8$	no parentheses, exponents; multiplication
$-8 - 8$	subtraction
-16	

Practice 6 $(-2)^2 - 2^2|12 - 35 \div 5|$

Solution: Click here to check your answer.

Example 7 $9^0 - (2 \cdot 4 - 4 \cdot 3) \div 4$

Solution:

$9^0 - (2 \cdot 4 - 4 \cdot 3) \div 4$	parentheses - multiplication inside left to right
$9^0 - (8 - 12) \div 4$	parentheses - subtraction
$9^0 - (-4) \div 4$	no parentheses; exponents
$1 - (-4) \div 4$	no parentheses, exponents; division
$1 - (-1)$	subtraction
2	

Note that in the first step two multiplications were performed. This is permitted because of $2 \cdot 4$ and $4 \cdot 3$ are in two different terms since they are separated by a subtraction (or addition). (More will be studied on terms in section 2.1.) If ever in doubt whether an operation can be done simultaneously, the rule is do them separately.

Practice 8 $6 \div 3(-5) - (1 - 3^2)$

Solution: Click here to check your answer.

Practice 2

Solution:

$72 \div 3 - 3(4 - 6 * 3)$	parentheses - multiplication first
$72 \div 3 - 3(4 - 18)$	parentheses - subtraction
$72 \div 3 - 3(-14)$	parentheses - subtraction
$24 - (-42)$	no parentheses or exponents; multiplication & division left to right
66	subtraction is the only operation left

Back to Text

Practice 4

Solution:

$-6 - 3(-4) \div (3 - 3^2)$	in parentheses exponent
$-6 - 3(-4) \div (3 - 9)$	in parentheses exponent done so subtract
$-6 - 3(-4) \div (-6)$	no parentheses, exponents; multiplication & division left to right
$-6 - (-12) \div (-6)$	no parentheses, exponents; multiplication & division left to right
$-6 - 2$	no parentheses, exponents, multiplication & division; subtract
-8	

Back to Text

Practice 6

Solution:

$(-2)^2 - 2^2\lvert 12 - 35 \div 5\rvert$	inside absolute values no exponents; multiplication & division left to right
$(-2)^2 - 2^2\lvert 12 - 7\rvert$	inside absolute values no exponents, multiplication & division so subtract
$(-2)^2 - 2^2\lvert 5\rvert$	inside absolute values single number, take absolute value
$(-2)^2 - 2^2(5)$	no parentheses; exponents
$4 - 4(5)$	no parentheses, exponents; multiplication
$4 - 20$	subtraction only operation remaining
-16	

In the third step two exponents were calculated in the same step. This is permitted since they are separated by subtraction. For further explanation read the note following example 7. Back to Text

Practice 8

Solution:

$6 \div 3(-5) - (1 - 3^2)$	in parentheses exponents
$6 \div 3(-5) - (1 - 9)$	in parentheses no exponents, multiplication, division; subtract
$6 \div 3(-5) - (-8)$	no parentheses, exponents; multiplication & division left to right
$2(-5) - (-8)$	no parentheses, exponents; multiplication & division left to right
$-10 - (-8)$	subtraction is the only operation left
-2	

Back to Text

1.7.1 Exercises 1.7

Evaluate following order of operations. Click here to see examples.

1. $2 \cdot -3 + 5^2$
2. $5 \cdot 6 + 2^3 \div 4$
3. $7^2(-1)^{111} - \lvert -9 - 10\rvert$
4. $-3 \cdot 8 \div 6\lvert 7 + (-2)^3\rvert$
5. $-3 - 2 - 5 + 6$
6. $3 - 3(4^2 - 4)$
7. $4 \cdot 3 - 3 \cdot 5$
8. $(2^3 - 3) - 4 \cdot 5 \div 10$
9. $3(1 - 4)^2 - 4(-2)$
10. $(1 + 7) \cdot 6 - 3^0 \cdot 2$
11. $(0^0 + 6) - 3 \cdot 5 - 5$
12. $2(1 - 6) - 3(-2) + (-2)^2$
13. $(-0.3)^2 + 1$
14. $2.1 - 2 \cdot 0.3 - 0.3$
15. $0.1(3 - 5) - 1.1 + (-4.1)$

Click here to see the solutions.

1.8 Primes, Divisibility, Least Common Denominator, Greatest Common Factor

Before we continue further into our study of mathematics it is important to be able to work quickly with the factors of a positive integer. Throughout this section, we will be working only with positive integers. Recall that the positive integers are the counting numbers, $\{1, 2, 3, ...\}$. So whenever we talk about "a number" in this section, we will mean a counting number. Recall the factors in a multiplication problem are the numbers you are multiplying together. If a positive integer N can be written as a product of positive integers, $N = A \cdot B$, we say that each of the factors A and B are **factors of** N. For example, since $6 = 2 \cdot 3$, we say 2 is a factor of 6. Another way to say this is that 2 **divides** 6. Likewise, 3 is a factor of (or divides) 6. We say a positive integer greater than one is **prime** if its only positive integer factors are one and itself. The number 1 is neither prime nor composite. Otherwise, we call it **composite** (a composite number has factors other than one and itself). For example, since $2 \cdot 4 = 8$ we know 8 is a composite number. Notice that it only takes one set of factors not including 1 to make a number composite. Consider 2. Are there any positive factors other than 1 and 2 which we can multiply together to get 2? Since the answer to this question is no, we know that 2 is a prime number. In fact, we can write 8 as a product of prime factors, $8 = 2 \cdot 2 \cdot 2$. This is called the **prime factorization** of 8. The prime factorization of a number is the factorization of a number into the product of prime factors. This product is unique up to the rearrangement of factors. That is, $12 = 2 \cdot 2 \cdot 3 = 2 \cdot 3 \cdot 2$, the order may change, but for 12, the product will always contain two 2's and one 3.

Prime numbers are important to know. These are the building blocks of the other numbers, and hence, we often will look to a number's prime factorization to learn more about the number. We have already seen one prime number, 2, but this is just the beginning. The ten smallest prime numbers are $2, 3, 5, 7, 11, 13, 17, 19, 23, 29$. Each of the other numbers less than 30 has factors other than 1 and itself. These prime numbers are used to test for other prime numbers. Below are some helpful hints on determining factors.

Divisibility Tests for some Numbers

- A number is divisible by 2 if it is an even number (the last digit is $0, 2, 4, 6$ or 8).
- A number is divisible by 3 if the sum of its digits is divisible by 3 (ex: 3591 is divisible by 3 since $3 + 5 + 9 + 1 = 18$ is divisible by 3).
- A number is divisible by 5 if its final digit is a 0 or 5.
- A number is divisible by 9 if the sum of its digits is divisible by 9 (ex: 3591 is divisible by 9 since $3 + 5 + 9 + 1 = 18$ is divisible by 9).
- A number is divisible by 10 if its final digit is a 0.

Examples: Testing for Prime Numbers

EXAMPLE 1 *Is 237 prime or composite? Justify your answer.*
Solution: Elimination, beginning with the smallest prime number, is the best method to test for primes. 2 is not a factor of 237 since it is not even. $2 + 3 + 7 = 12$ which is divisible by 3 so 3 is a factor of 237. Therefore, 237 is composite since $237 = 3 \cdot 79$.

PRACTICE 2 *Is 91 prime or composite? Justify your answer.*
Solution: Click here to check your answer.

EXAMPLE 3 *Is 151 prime or composite? Justify your answer.*
Solution: 2 is not a factor of 151 since it is not even. $1 + 5 + 1 = 7$ which is not divisible by 3 so 3 is a not factor. (We do not need to check composite numbers. For example, if a number has 4 as a factor it would automatically have 2 as a factor since $2 \cdot 2 = 4$) 5 is not a factor since 151 does not end in 5 or 0. 7 is not a factor since when you divide $151 \div 7$ you have a remainder of 4. 11 is not a factor since when you divide $151 \div 11$ you have a remainder of 8. 13 is not a factor since when you $151 \div 13$ you have a remainder of 8. At this point we know that there will be no numbers less than or equal to 13 that can be a factor of 151 since $13 \cdot 13 = 169 > 151$. Therefore, 151 is prime. Note that if the square of a prime is greater than the number that we are testing, and we have been very careful to test all the primes smaller than that prime, then no larger prime is a factor either so we can stop testing and conclude that the number is prime.

Practice 4 *Is 79 prime or composite? Justify your answer.*
Solution: Click here to check your answer.

Example 5 *Find all the factors of 42 which are less than 15.*
Solution: As in determining composite or prime the best approach is to begin with 1 and test each number using the divisibility tests mentioned above when possible. A table will be used here just to convey the thought process to you in an orderly manner.

Factor?	Number	Reason	Factor?	Number	Reason
yes	1	always a factor	no	8	since 4 is not a factor
yes	2	42 is even	no	9	4+2=6 which is not divisible by 9
yes	3	4+2=6 which is divisible by 3	no	10	does not end with 0
no	4	$42 \div 4$ has a remainder of 2	no	11	$42 \div 11$ has a remainder of 9
no	5	does not end with 5 or 0	no	12	4 is not a factor so 12 cannot be
yes	6	2 and 3 are factors	no	13	$42 \div 13$ has a remainder of 3
yes	7	$42 \div 7$ has a remainder of 0	yes	14	$42 \div 14$ has a remainder of 0

Therefore the factors of 42 which are less than 15 are $\{1, 2, 3, 6, 7, 14\}$

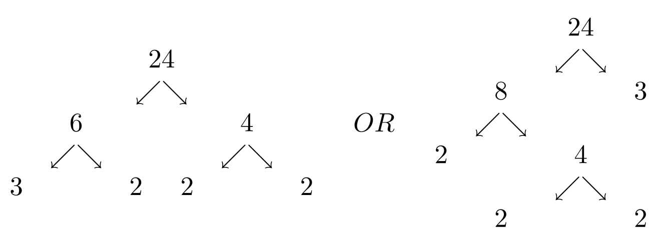
Practice 6 *Find all the factors of 36 which are less than 15.*
Solution: Click here to check your answer.

As stated earlier, prime factorizations are important for building other concepts. With this in mind, developing techniques to find the prime factorization is beneficial. Two methods will be described here.

The first method is through a factor tree. We begin by splitting the number into factors (other than 1) whose product is the number. We then split those two factors in the same manner. We continue this process until we have only prime factors.

Examples: Prime Factorization by Factor Trees

Example 7 *Find the prime factorization of 24.*
Solution:



Notice that the product of the numbers at the ends of the arrows is the number the arrows came from. For example, 3 and 2 are at the end of the arrows so $3 \cdot 2 = 6$ which is at the top of those two arrows. This must happen or you have done an incorrect split. Also, we end up with the same prime factorization with no respect to the initial factorization of the number. The final numbers of each stem make up the prime factorization of the number when multiplied. Therefore the prime factorization of 24 is $3 \cdot 2 \cdot 2 \cdot 2 = 2^3 \cdot 3$ since both 2 and 3 are primes. Note that prime factorizations are conventionally written with primes (raised to powers) in ascending order.

Practice 8 *Find the prime factorization of 18.*
Solution: Click here to check your answer.

Another method is using division by prime numbers. This process can take a lot longer than factor trees but it is a useful method to have in your thoughts in case you have difficulties. For this method you begin with the number and use a shorthand method of division where you use the previous answer to divide by another prime.

Examples: Prime Factorization by Division by Primes

EXAMPLE 9 *Find the prime factorization of 60 using division by primes.*
Solution:

$$\begin{array}{r} \\ \\ 30 \\ 2\overline{)60} \end{array} \Longrightarrow \begin{array}{r} \\ 15 \\ 2\overline{)30} \\ 2\overline{)60} \end{array} \Longrightarrow \begin{array}{r} 5 \\ 3\overline{)15} \\ 2\overline{)30} \\ 2\overline{)60} \end{array}$$

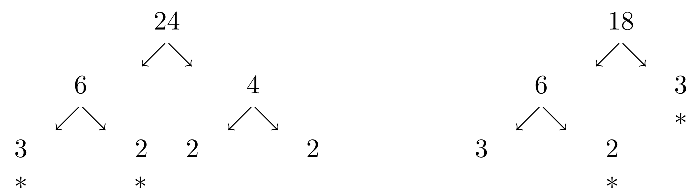
Therefore, $60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$ is the prime factorization. It is worth noting that the order you divide by the primes does not matter. The prime factorization is unique to each number.

WARNING: In this method you must only divide by prime numbers!

PRACTICE 10 *Find the prime factorization of 54 using division by primes.*
Solution: Click here to check your answer.

The **greatest common factor** of two numbers is the largest factor they have in common. For example, the greatest common factor of 8 and 12 is 4 because it is the largest factor which belongs to both numbers. The greatest common factor of 5 and 12 is 1 since they do not share any factors except for 1. In this case when the only common factor is 1 the numbers are called **relatively prime**. For small numbers the greatest common factor can usually be identified quickly once you have practiced finding them. However for larger numbers knowing a method can be useful.

The method described here takes advantage of the prime factorizations of the numbers. Let's begin with the two numbers whose prime factorizations we already know, 24 and 18. Below you see their factor trees with stars below the primes which can be paired up. If we multiply the starred primes in each number we see that in both cases we get $3 \cdot 2 = 6$ so 6 is the greatest common factor of 24 and 18.



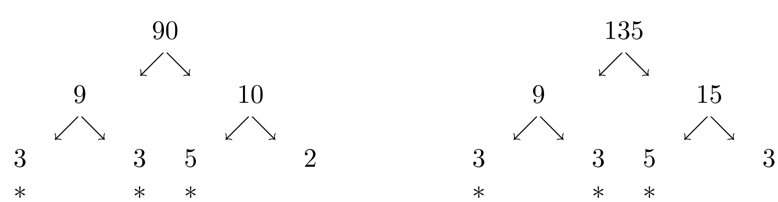
The following summarizes the process described above.

Greatest Common Factor (GCF)

1. Find the prime factorizations of each number.
   - To find the prime factorization one method is a factor tree where you begin with any two factors and proceed dividing the numbers until you reach all prime factors.
2. Star factors which are shared. (see above)
3. Then multiply the starred factors of either number. (This is the GCF)

Examples: Greatest Common Factor

EXAMPLE 11 *Find the greatest common factor of 90 and 135.*
Solution:



Therefore, $3 \cdot 3 \cdot 5 = 45$.

Practice 12 *Find the greatest common factor of* 14 *and* 42.
Solution: Click here to check your answer.

Example 13 *Find the greatest common factor of* 13 *and* 10.
Solution:

$$13 \qquad\qquad \begin{array}{ccc} & 10 & \\ & \swarrow\ \searrow & \\ 2 & & 5 \end{array}$$

Notice that since 13 is prime so there are no factors other than 1 and 13 which we will not represent on a factor tree. Because there are no shared prime factors, other than one, 13 and 10 are relatively prime with a greatest common factor of 1.

Practice 14 *Find the greatest common factor of* 15 *and* 14.
Solution: Click here to check your answer.

The least common multiple is also an important concept which will be used throughout your mathematical studies. A **multiple** of a number is the product of that number and a non-zero integer. For example, multiples of 4 would be $4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...$ since $4 \cdot 1 = 4$, $4 \cdot 2 = 8$, $4 \cdot 3 = 12$ and so on. The **least common multiple** of two numbers is the smallest multiple which is shared by both numbers. Consider the least common multiple of 4 and 10. We saw multiples of 4 are $4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...$ and multiples of 10 are $10, 20, 30, 40, 50, ...$ Comparing these lists we see that the smallest number that is a multiple of both 4 and 10 is 40, so this is the least common multiple. This method of comparing lists of multiples can always be used but sometimes this can be time consuming. Another method involves using prime factorizations.

To find the least common multiple of 24 and 36, first find the prime factorization and star the shared factors as in finding the greatest common factor. To find the least common multiple through prime factorizations we multiply all the prime factors together with the exception of only counting the shared starred factors once. This is the same if you just start with one of the numbers and multiply it by the un-starred numbers of the other number.

$$\begin{array}{ccccccc} & & & 24 & & & \\ & & & \swarrow\ \searrow & & & \\ & 6 & & & & 4 & \\ & \swarrow\ \searrow & & & & \swarrow\ \searrow & \\ 3 & & 2 & & 2 & & 2 \\ * & & * & & * & & \end{array} \qquad \begin{array}{ccccccc} & & & 36 & & & \\ & & & \swarrow\ \searrow & & & \\ & 6 & & & & 6 & \\ & \swarrow\ \searrow & & & & \swarrow\ \searrow & \\ 3 & & 2 & & 2 & & 3 \\ * & & * & & * & & \end{array}$$

So the least common multiple is $24 \cdot 3 = 72$ or $36 \cdot 2 = 72$.

Least Common Multiple (LCM)

1. Find the prime factorizations of each number.
   - To find the prime factorization one method is a factor tree where you begin with any two factors and proceed by dividing the numbers until all the ends are prime factors.
2. Star factors which are shared.
3. Then multiply the un-starred factors of one of the numbers by the other number. (This is the LCD)

Examples: Least Common Multiple

Example 15 *Find the least common multiple of* 15 *and* 18.
Solution:

15 → 3, 5
3 *

18 → 3, 6
6 → 2, 3
3 *

Therefore, the least common multiple is $15 \cdot 3 \cdot 2 = 90$ or $18 \cdot 5 = 90$.

PRACTICE 16 *Find the least common multiple of* 12 *and* 16.
Solution: Click here to check your answer.

EXAMPLE 17 *Find the least common multiple of* 14 *and* 7.
Solution:

14 → 2, 7
7 *

7
*

Since 7 is prime we do not need to use the factor tree. The least common multiple is $14 \cdot 1 = 14$ or $7 \cdot 2 = 14$.

PRACTICE 18 *Find the least common multiple of* 3 *and* 10.
Solution: Click here to check your answer.

Practice 4
Solution: 91 is not even so it is not divisible by 2 which rules out any even number. $9 + 1 = 10$ which is not divisible by 3 so it is not divisible by 3. It is not divisible by 5 since it does not end in 0 or 5. $91 \div 7$ has a remainder of 0 so it is divisible by 7. Therefore, 91 is composite since $7 \cdot 13 = 91$. Back to Text

Practice 2
Solution: 79 is not even so it is not divisible by 2 which rules out any even number. $7 + 9 = 16$ which is not divisible by 3 so it is not divisible by 3. It is not divisible by 5 since it does not end in 0 or 5. $79 \div 7$ has a remainder of 2 so it is not divisible by 7. 11 does not divide 79 and $11^2 = 121 > 79$ Therefore, 79 is prime since there are no prime numbers whose squares are smaller than 79 that divide it. Back to Text

Practice 6
Solution: The various factors of 36 come from $1 \cdot 36 = 36$, $2 \cdot 18 = 36$, $3 \cdot 12 = 36$, $4 \cdot 9 = 36$, 5 is not a factor of 36, and $6 \cdot 6 = 36$. At this point we are confident we have listed all the factorizations of 36 since the multiplication sentences will just be the reverse order of the ones we already listed. (This will happen in all factorizations.) Therefore the factors of 36 which are less than 15 are $\{1, 2, 3, 4, 6, 9, 12\}$ Back to Text

Practice 8
Solution:

18 → 6, 3
6 → 3, 2

Therefore, the prime factorization of 18 is $18 = 3 \cdot 2 \cdot 3 = 2 \cdot 3^2$ Back to Text

Practice 10

Solution:

$$\begin{array}{r} \\ \\ 30 \\ 2|\overline{54} \end{array} \Longrightarrow \begin{array}{r} \\ 9 \\ 3|\overline{27} \\ 2|\overline{54} \end{array} \Longrightarrow \begin{array}{r} 3 \\ 3|\overline{\ 9} \\ 3|\overline{27} \\ 2|\overline{54} \end{array}$$

Therefore, $54 = 2 \cdot 3 \cdot 3 \cdot 3 = 2 \cdot 3^3$ is the prime factorization.

Back to Text

Practice 12

Solution:

$$\begin{array}{ccc} & 14 & \\ & \swarrow\searrow & \\ 2 & & 7 \\ * & & * \end{array} \qquad \begin{array}{ccccc} & & & 42 & \\ & & & \swarrow\searrow & \\ & 6 & & & 7 \\ & \swarrow\searrow & & & * \\ 2 & & 3 & & \\ * & & & & \end{array}$$

Therefore, $2 \cdot 7 = 14$.

Back to Text

Practice 14

Solution:

$$\begin{array}{ccc} & 15 & \\ & \swarrow\searrow & \\ 3 & & 5 \end{array} \qquad \begin{array}{ccc} & 14 & \\ & \swarrow\searrow & \\ 2 & & 7 \end{array}$$

Since there are no prime factors in common, 15 and 14 are relatively prime with a GCF of 1.

Back to Text

Practice 16

Solution:

$$\begin{array}{ccccc} & 12 & & & \\ & \swarrow\searrow & & & \\ 3 & & & 4 & \\ & & & \swarrow\searrow & \\ & 2 & & & 2 \\ & * & & & * \end{array} \qquad \begin{array}{ccccccc} & & & 16 & & & \\ & & & \swarrow\searrow & & & \\ & 4 & & & & 4 & \\ & \swarrow\searrow & & & & \swarrow\searrow & \\ 2 & & 2 & & 2 & & 2 \\ * & & * & & & & \end{array}$$

Therefore, the least common multiple is $12 \cdot 2 \cdot 2 = 48$ or $16 \cdot 3 = 48$.

Back to Text

Practice 18

Solution:

$$3 \qquad \begin{array}{ccc} & 10 & \\ & \swarrow\searrow & \\ 2 & & 5 \end{array}$$

Since 3 is prime we do not need to use the factor tree. The least common multiple is $3 \cdot 10 = 30$ or $10 \cdot 3 = 30$.

Back to Text

1.8.1 Exercises 1.8

Determine whether the number is composite or prime. Click here to see examples.

1. 6	2. 29	3. 37
4. 39	5. 51	6. 91

Find all the factors of the numbers which are less than 15. Click here to see examples.

7. 30	8. 66	9. 58
10. 24	11. 54	12. 25

Find the prime factorizations of the following numbers. Click here to see examples.

13. 15	14. 90	15. 10
16. 16	17. 12	18. 24

Find the greatest common factors of the following numbers. Click here to see examples.

19. 90, 15	20. 12, 30	21. 10, 12
22. 16, 24	23. 3, 11	24. 2, 16

Find the least common multiple of the following numbers. Click here to see examples.

25. 90, 15	26. 12, 30	27. 10, 12
28. 16, 24	29. 3, 11	30. 2, 16

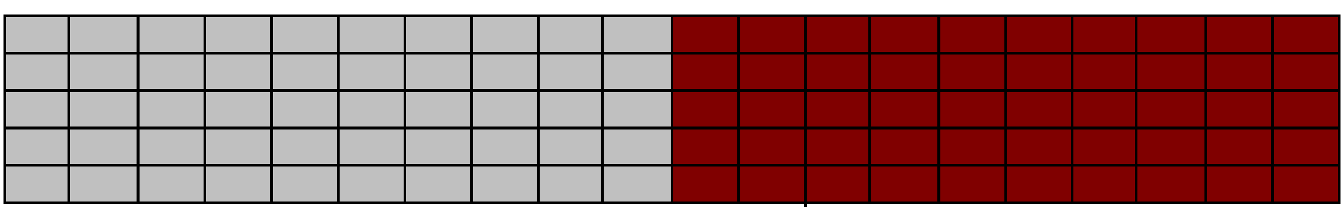
Click here to see the solutions.

1.9 Fractions and Percents

Since a fraction is just a division we can perform a division and get a representation of the number. Consider $\frac{1}{2} = 1 \div 2 = 1.0 \div 2 = 0.5$ since $10 \div 2 = 5$ and $1 - 0 = 1$ decimal in the final answer. Or as seen below, we just bring the decimal point straight up and rewrite 1 as 1.0.

$$\frac{1}{2} = \quad \begin{array}{r} 0.5 \\ 2|\overline{1.0} \\ \underline{10} \\ 0 \end{array}$$

We can also think of one-half of one-hundred. Splitting the burgundy rectangle into one hundred pieces half of the one hundred pieces would be 50 pieces. This is the percentage of the rectangle that is gray. A **percentage** is the amount out of 100. In our case that is 50%.



Converting fractions to decimals and decimals to percents and all other combinations are important skills which are shown in the following examples.

Examples: Converting Fractions, Decimals, & Percents

Example 1 *Convert $\frac{4}{5}$ to a decimal and a percent.*

Solution:

0.8	the top goes in the box and add zeros to the right of the decimal as needed.
$5|\overline{4.0}$	Bring the decimal point up.
$\underline{40}$	divide
0	

Or, $4 \div 5 = 4.0 \div 5 = 0.8$ since $40 \div 5 = 8$ and there are $1 - 0 = 1$ decimal places. There are several ways to get the percentage but it is always out of one-hundred. The easiest way is to take $0.8 \cdot 100 = 80\%$. Therefore, $\frac{4}{5} = 0.8 = 80\%$.

Practice 2 *Convert $\frac{7}{10}$ to a decimal and a percent.*
Solution: Click here to check your answer.

Example 3 *Convert 0.3 to a fraction and a percent.*
Solution: As in the previous example, just multiply .3 by 100, that is, $.3 \times 100 = 30\%$. 30% means 30 out of 100 which translates into $\frac{30}{100}$ (note that this is not simplified but we will put this off until later in this section).

Practice 4 *Convert 0.55 to a fraction and a percent.*
Solution: Click here to check your answer.

Example 5 *Convert 40% to a fraction and a decimal.*
Solution: 40% means 40 out of 100 which translates to $\frac{40}{100}$ (which is not simplified). Now to get the decimal we can always divide since $\frac{40}{100} = 40 \div 100$.

$$\begin{array}{r} 0.4 \\ 100\overline{)40.0} \\ \underline{400} \\ 0 \end{array}$$

Therefore the decimal is 0.4. Notice that all we did was divide the percent by 100 which just moves the decimal 2 places to the left.

Practice 6 *Convert 35% to a fraction and a decimal.*
Solution: Click here to check your answer.

Example 7 *Convert $\frac{4}{25}\%$ to a fraction and a decimal.*
Solution: First, change $\frac{4}{25}$ to a decimal. $\frac{4}{25} = 4 \div 25 = 4.00 \div 25 = 0.16$ since $400 \div 25 = 16$ and there are $2 - 0 = 2$ decimal places. Therefore, $\frac{4}{25}\% = 0.16\% = \frac{0.16}{100} = 0.16 \div 100 = 0.1600 \div 100 = 0.0016$. Translating this into a fraction not containing a decimal we have $0.0016 = \frac{16}{10000}$. Note that the number of decimals places to the right is equal to the number of zeroes in the denominator. This is explained in Examples: Equivalent Fractions.

Practice 8 *Convert $\frac{2}{5}\%$ to a fraction and a decimal.*
Solution: Click here to check your answer.

We have been working with decimals when introducing the operations. So we will not discuss this further. But we will take advantage of the skills already learned when dealing with percentages. When you are performing operations with percentages the first rule to remember is **convert the percent to a decimal**. For example, multiply $46 \cdot 20\%$. First you should convert the 20% to a decimal, $20 \div 100 = 0.20 = 0.2$. Then, continue the operation $46 \cdot 20\% = 46 \cdot 0.2 = 9.2$.

Examples: Order of Operations with Percents

Example 9 $12 \cdot 163\% + 31.2$
Solution: First, $163\% = 1.63$. Then, $12 \cdot 163\% + 31.2 = 12 \cdot 1.63 + 31.2 = 19.56 + 31.2 = 50.76$.

Practice 10 $111 - 25\% \cdot 150$
Solution: Click here to check your answer.

While most percentages can be dealt with by converting to a decimal, fractions often must be left as fractions. The reason is that when we are using fractions we are dealing with exact amounts. Consider a simple fraction like $\frac{1}{3}$. If we try to convert this to a decimal by long division we get $\frac{1}{3} = 0.\overline{3}$ where the line over the 3 indicates that it is repeated forever. That is, $\frac{1}{3} \neq 0.3333$ so it cannot be expressed exactly. Therefore, learning to work with fractions is critical to your success in mathematics.

There is always more than one way to express a fraction. Fractions that have the same value are called equivalent fractions. Consider $\frac{2}{3}$ and $\frac{4}{6}$. If we shade in 2 pieces of a bar split into three equal pieces this will be the same as shading 4 pieces of bar split into six equal pieces. Consider if we split each of these 6 pieces into thirds. We then have 18 pieces, this is calculated by $6 \cdot 3 = 18$ which then means we have $4 \cdot 3 = 12$ pieces shaded. So,

$$\frac{2}{3} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{4}{6} = \frac{4 \cdot 3}{6 \cdot 3} = \frac{12}{18}$$

In fact, we can multiply the numerator and the denominator by any non-zero real number and have an equivalent fraction. This is true since you are just multiplying by a form of one $1 = \frac{2}{2} = \frac{3}{3}$. (Multiplication of fractions will be discussed in detail later in this section.)

Examples: Equivalent Fractions

EXAMPLE 11 *Write an equivalent fraction for $\frac{2}{5}$ with a denominator of 35.*
Solution: Since $5 \cdot 7 = 35$, we need to multiply the numerator by the same number. $\frac{2 \cdot 7}{5 \cdot 7} = \frac{14}{35}$. Therefore the equivalent fraction is $\frac{14}{35}$.

PRACTICE 12 *Write an equivalent fraction for $\frac{3}{8}$ with a denominator of 48.*
Solution: Click here to check your answer.

EXAMPLE 13 *Write an equivalent fraction for $\frac{0.16}{100}$ to eliminate the decimal.*
Solution: Since 0.16 has two decimal places we can multiply this number by 100 to get an integer. $0.16 \cdot 100 = 16$, we need to multiply the denominator by the same number. $\frac{(0.16)(100)}{(100)(100)} = \frac{16}{10000}$. Therefore the equivalent fraction is $\frac{16}{10000}$.

PRACTICE 14 *Write an equivalent fraction for $\frac{0.43}{100}$ to eliminate the decimal.*
Solution: Click here to check your answer.

EXAMPLE 15 *Write an equivalent fraction for $-\frac{2.2}{1.147}$ to eliminate the decimal.*
Solution: Since 1.147 has three decimal places which is more than the one decimal place in 2.2 we can multiply these numbers by 1000 to get an integer in both cases. $-\frac{(2.2)(1000)}{(1.147)(1000)} = -\frac{2200}{1147}$. Therefore the equivalent fraction is $-\frac{2.2}{1.147} = -\frac{2200}{1147}$ (noting the negative does not change any computations here).

PRACTICE 16 *Write an equivalent fraction for $\frac{3}{5.1}$ to eliminate the decimal.*
Solution: Click here to check your answer.

EXAMPLE 17 *Write an equivalent fraction for $\frac{9}{24}$ with a denominator of 8.*
Solution: $\frac{9}{24} = \frac{3 \cdot 3}{8 \cdot 3} = \frac{3}{8}$. This is just reverse of the way we were thinking before. We are "getting rid" of the common factor or dividing it out. (We are dividing by 1 which does not change the fraction. Division involving fractions will be discussed in detail later in this section.) Therefore the equivalent fraction is $\frac{3}{8}$.

PRACTICE 18 *Write an equivalent fraction for $\frac{40}{50}$ with a denominator of 5.*
Solution: Click here to check your answer.

In last two examples we put the fractions into simplified form. This is the case when the numerator and the denominator share no common factors other than one. This is the typical way we express our answers.

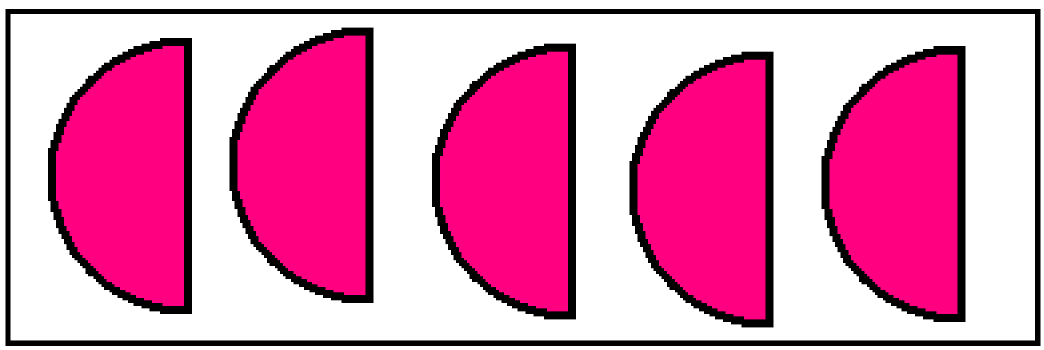
EXAMPLE 19 *Write $-\frac{12}{36}$ in simplified form.*
Solution: $-\frac{12}{36} = -\frac{2 \cdot 6}{6 \cdot 6} = -\frac{2}{6} = -\frac{1 \cdot 2}{3 \cdot 2} = -\frac{1}{3}$ or $-\frac{12}{36} = -\frac{1 \cdot 12}{3 \cdot 12} = -\frac{1}{3}$ if you use the greatest common factor. Either way the simplified form is $-\frac{1}{3}$ (noting the negative does not change any computations here).

Practice 20 *Write $\frac{54}{72}$ in simplified form.*
Solution: Click here to check your answer.

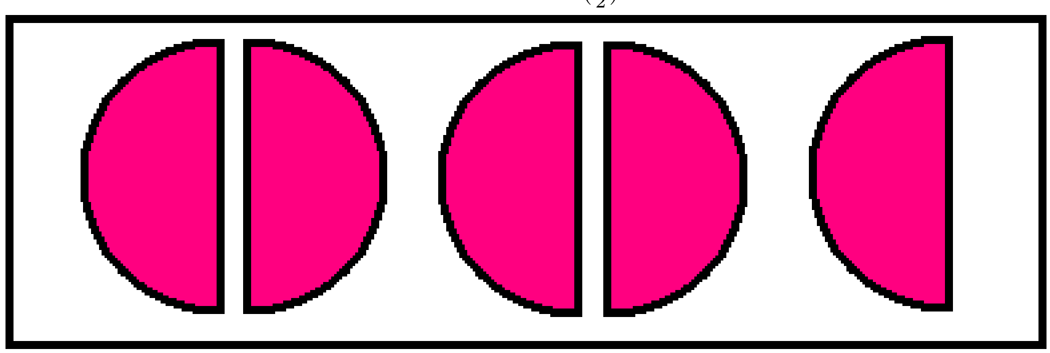
Example 21 *Write $\frac{70}{28}$ in simplified form.*
Solution: $\frac{70}{28} = \frac{5 \cdot 14}{2 \cdot 14} = \frac{5}{2}$.

Practice 22 *Write $-\frac{30}{45}$ in simplified form.*
Solution: Click here to check your answer.

The answer to the last example is called an improper fraction. A fraction is **improper** when the numerator is larger than the denominator. We can convert this to a mixed number by considering how many whole "circles" we have. There are two groups of 2's (denominator) in 5 (numerator) with one left over. That is, $\frac{5}{2} = 2\frac{1}{2}$. Look at the picture below to see that five halves make two whole circles with one half left over.



Five Halves $(\frac{5}{2})$



By rearranging, Two Wholes and One Half $(2\frac{1}{2})$

Fractions are just another way to write division. This understanding helps us calculate a mixed number. For example, $\frac{5}{2} = 5 \div 2$ which can be determined by the picture to equal $2\frac{1}{2}$. Consider $\frac{32}{5} = 32 \div 5$. If we divide this problem we see that 5 divides into 32 six times with 2 remaining $(32 = 6 \cdot 5 + 2)$. This indicates that $\frac{32}{5} = 6\frac{2}{5}$. Going from a mixed number to improper fraction is somewhat easier since we are only calculating the numerator. Writing $6\frac{2}{5}$ as an improper fraction can be done by keeping the same denominator and finding the numerator by multiplying the whole number times the denominator then adding on the numerator (numerator $= 6 \cdot 5 + 2 = 32$). That is, $6\frac{2}{5} = \frac{32}{5}$.

Examples: Mixed Numbers and Improper Fractions

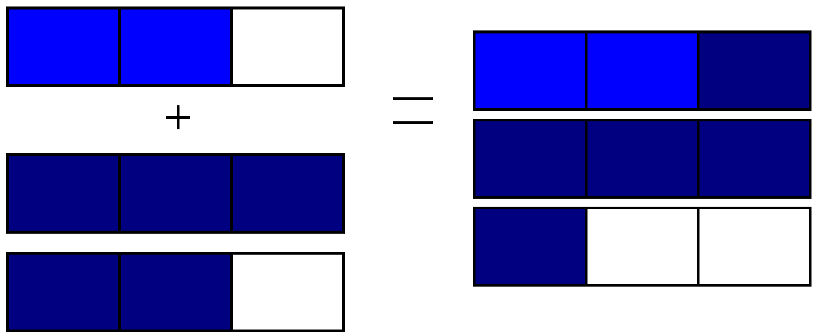
Example 23 *Write $\frac{-60}{7}$ as a mixed number.*
Solution: 7 divides into 60 eight times with 4 remaining $(8 \cdot 7 + 4 = 60)$. Therefore, $\frac{-60}{7} = -8\frac{4}{7}$ where we include the negative sign since it is a negative number.

Practice 24 *Write $\frac{47}{3}$ as a mixed number.*
Solution: Click here to check your answer.

Example 25 *Write $-11\frac{3}{8}$ as an improper fraction.*
Solution: $8 \cdot 11 + 3 = 91$ so $-11\frac{3}{8} = -\frac{91}{8}$ where we include the negative sign since it is a negative number.

Practice 26 *Write $-17\frac{1}{3}$ as an improper fraction.*
Solution: Click here to check your answer.

Consider adding $\frac{2}{3} + \frac{5}{3}$.



Grouping together we see that there are 7 thirds ($\frac{7}{3}$) or 2 wholes and 1 third ($2\frac{1}{3}$). Notice when we have common denominators we can just add the numerators. Likewise, for subtraction with common denominators we can just subtract the numerators and keep the common denominator. Always be careful to simplify.

Examples: Adding & Subtracting with Like Denominators

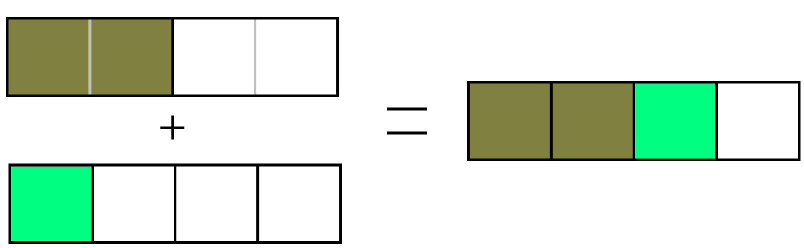
Example 27 *Add and simplify $\frac{3}{8} + \frac{7}{8}$.*
Solution: $\frac{3}{8} + \frac{7}{8} = \frac{3+7}{8} = \frac{10}{8}$ by adding the numerators since we have a common denominator. $\frac{10}{8} = \frac{2 \cdot 5}{2 \cdot 4} = \frac{5}{4}$ when dividing out the common factor of 2 from top and bottom. You may want to change the improper fraction to a mixed number $\frac{5}{4} = 1\frac{1}{4}$.

Practice 28 *Add and simplify $\frac{4}{9} + \frac{8}{9}$*
Solution: Click here to check your answer.

Example 29 *Subtract and simplify $\frac{5}{6} - 1\frac{1}{6}$.*
Solution: First change the mixed number to an improper fraction: $1\frac{1}{6} = \frac{7}{6}$, and then subtract $\frac{5}{6} - \frac{7}{6} = \frac{5-7}{6} = \frac{-2}{6}$. Finally, simplify $\frac{-2}{6} = \frac{2 \cdot (-1)}{2 \cdot 3} = \frac{-1}{3}$.

Practice 30 *Subtract and simplify $1\frac{5}{7} - 2\frac{2}{7}$.*
Solution: Click here to check your answer.

Up to this point we have been discussing addition and subtraction of fractions with the same denominator. We need to consider how to add fractions when the denominators are not the same. Consider adding $\frac{1}{2} + \frac{1}{4}$. Before we can add these we need to have a common denominator. In the picture below, we see that the bar representing $\frac{1}{2}$ can be properly split into fourths.



Algebraically, $\frac{1}{2} = \frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4}$ so $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}$. We could have also used, $\frac{1 \cdot 4}{2 \cdot 4} + \frac{1 \cdot 2}{4 \cdot 2} = \frac{4}{8} + \frac{2}{8} = \frac{6}{8} = \frac{3}{4}$. So what is the best denominator to use for addition and subtraction of fractions? If the fractions are in lowest terms, the answer is the least common multiple of the denominators. We call this the Least Common Denominator (LCD). If we choose another common denominator we are guaranteed to have to simplify our answer.

Examples: Addition & Subtraction with Different Denominators

EXAMPLE 31 $\frac{2}{15} - \frac{1}{6}$

Solution: Get equivalent fractions with the LCD as the denominator. Use factor trees to write out the prime factorizations of both numbers, identifying the common factors (in red below, starred previously) and multiplying by the unshared factors (shown in green).

$$\frac{2}{15} - \frac{1}{6} = \frac{2}{3 \cdot 5} - \frac{1}{2 \cdot 3} = \frac{2 \cdot 2}{3 \cdot 5 \cdot 2} - \frac{1 \cdot 5}{2 \cdot 3 \cdot 5} = \frac{4}{30} - \frac{5}{30} = \frac{4-5}{30} = -\frac{1}{30}$$

PRACTICE 32 $\frac{5}{12} - \frac{3}{16}$

Solution: Click here to check your answer.

EXAMPLE 33 $\frac{1}{12} + \frac{1}{4}$

Solution:

$$\frac{1}{12} + \frac{1}{4} = \frac{1}{3 \cdot 4} + \frac{1}{4} = \frac{1}{3 \cdot 4} + \frac{1 \cdot 3}{4 \cdot 3} = \frac{1}{12} + \frac{3}{12} = \frac{4}{12} = \frac{1}{3}$$

PRACTICE 34 $\frac{1}{5} - \frac{3}{10}$

Solution: Click here to check your answer.

EXAMPLE 35 $2\frac{1}{2} + \frac{2}{5}$

Solution: First change the mixed number to an improper fraction and then continue as before. $2\frac{1}{2} = \frac{2 \cdot 2 + 1}{2} = \frac{5}{2}$. There are no shared factors in the denominators so

$$2\frac{1}{2} + \frac{2}{5} = \frac{5}{2} + \frac{2}{5} = \frac{5 \cdot 5}{2 \cdot 5} + \frac{2 \cdot 2}{5 \cdot 2} = \frac{25}{10} + \frac{4}{10} = \frac{29}{10} = 2\frac{9}{10}.$$

PRACTICE 36 $3\frac{3}{4} + 1\frac{2}{3}$

Solution: Click here to check your answer.

EXAMPLE 37 $-1\frac{1}{3} - 3$

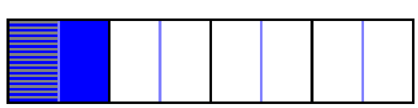
Solution: First change the mixed number to an improper fraction and then continue as before. $-1\frac{1}{3} = -\frac{1 \cdot 3 + 1}{3} = -\frac{4}{3}$. Next, think of 3 as a fraction, $3 = \frac{3}{1}$. There are no shared factors in the denominators so

$$-1\frac{1}{3} - 3 = -\frac{4}{3} - \frac{3}{1} = -\frac{4}{3} - \frac{3 \cdot 3}{1 \cdot 3} = -\frac{4}{3} - \frac{9}{3} = \frac{-4-9}{3} = \frac{-13}{3} = -4\frac{1}{3}.$$

PRACTICE 38 $2 - 4\frac{1}{10}$

Solution: Click here to check your answer.

Multiplication of fractions does not require a common denominator since it is just repeated addition. For example, $3 \cdot \frac{2}{9} = \frac{2}{9} + \frac{2}{9} + \frac{2}{9} = \frac{6}{9} = \frac{2}{3}$. Notice that the numerator is derived from $2 + 2 + 2 = 3 \cdot 2$. That is, the numerator is equal to the multiplication of the numerators of the original numbers when considering that $3 = \frac{3}{1}$. Similarly, consider $\frac{1}{2} \cdot \frac{1}{4}$ which is one-half of $\frac{1}{4}$.



As seen in the picture one-half of $\frac{1}{4}$ is $\frac{1}{8}$ noting that $2 \cdot 4 = 8$. This indicates that to multiply fractions the process to follow is to multiply the numerators to find the numerator and multiply the denominators to find the denominator. For example, $\frac{2}{5} \cdot \frac{3}{7} = \frac{2 \cdot 3}{5 \cdot 7} = \frac{6}{35}$.

In the previous example, the simplification was relatively easy, but consider, $\frac{12}{16} \cdot \frac{8}{15} = \frac{12 \cdot 8}{16 \cdot 15} = \frac{96}{240}$. This takes a little more effort if we multiply the numerator and denominator and then simplify. But we can start by factoring $\frac{12 \cdot 8}{16 \cdot 15} = \frac{\cancel{3} \cdot 4 \cdot \cancel{8}}{2 \cdot \cancel{8} \cdot \cancel{3} \cdot 5} = \frac{4}{2 \cdot 5} = \frac{\cancel{2} \cdot 2}{\cancel{2} \cdot 5} = \frac{2}{5}$. Notice, when we recognize a common factor we use that even if it is not prime.

Examples: Multiplication of Fractions

EXAMPLE 39 $\quad \frac{-30}{49} \cdot \frac{28}{20}$

Solution:

$$\frac{-30}{49} \cdot \frac{28}{20} = -\frac{\cancel{5} \cdot 3 \cdot 2 \cdot \cancel{7} \cdot \cancel{4}}{\cancel{7} \cdot 7 \cdot \cancel{5} \cdot \cancel{4}} = -\frac{6}{7}$$

(Note that the rules for multiplication involving negative numbers is the same as discussed in section 1.4.)

PRACTICE 40 $\quad \frac{-6}{45} \cdot \frac{-30}{12}$

Solution: Click here to check your answer.

EXAMPLE 41 $\quad -2\frac{2}{7} \cdot 14$

Solution: $-2\frac{2}{7} \cdot 14 = -\frac{16}{7} \cdot \frac{14}{1} = -\frac{16 \cdot 14}{7 \cdot 1} = -\frac{16 \cdot \cancel{7} \cdot 2}{\cancel{7} \cdot 1} = -\frac{32}{1} = -32$

PRACTICE 42 $\quad -6 \cdot -3\frac{3}{4}$

Solution: Click here to check your answer.

Before we discuss division of fractions, we must first be able to find the reciprocal of a number. The **reciprocal** of a number is 1 divided by the number. Let's look at some examples. The reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$. We can check that $1 \div \frac{2}{5} = \frac{5}{2}$ by multiplying: $\frac{2}{5} \cdot \frac{5}{2} = \frac{2 \cdot 5}{5 \cdot 2} = \frac{10}{10} = 1$. The reciprocal of $-\frac{1}{3}$ is $-\frac{3}{1} = -3$. The reciprocal of $4 = \frac{4}{1}$ is $\frac{1}{4}$.

If you have 10 m&m's and you divide them into 2 equal shares our standard verbiage is to ask our friend "Do you want half?". This is correct when we consider the connection of division and multiplication being inverse operations. Consider $10 \div 2 = 10 \cdot \frac{1}{2} = \frac{10}{1} \cdot \frac{1}{2} = \frac{10}{2} = \frac{\cancel{2} \cdot 5}{\cancel{2} \cdot 1} = 5$. Notice that division is the same as multiplication by the reciprocal.

$$\frac{3}{5} \div 2 = \frac{3}{5} \div \frac{2}{1} = \frac{3}{5} \cdot \frac{1}{2} = \frac{3}{10}$$

The process:

1. leave the first number unchanged
2. change the division sign to a multiplication sign
3. write the reciprocal of the second number.
4. multiply

In short, leave the first number alone, flip the second number and then multiply.

Examples: Division of Fractions

EXAMPLE 43 $\quad \frac{4}{5} \div \frac{-3}{10}$

Solution:

$$\frac{4}{5} \div \frac{-3}{10} = \frac{4}{5} \cdot \frac{-10}{3} = -\frac{4 \cdot 10}{5 \cdot 3} = -\frac{4 \cdot \cancel{5} \cdot 2}{\cancel{5} \cdot 3} = -\frac{8}{3}$$

(Note that the rules for division involving negative numbers is the same as discussed in section 1.5.)

PRACTICE 44 $\quad \frac{12}{21} \div \frac{-6}{35}$

Solution: Click here to check your answer.

Example 45 $\frac{5}{7} \div 3\frac{4}{7}$

Solution: $\frac{5}{7} \div 3\frac{4}{7} = \frac{5}{7} \div \frac{25}{7} = \frac{5}{7} \cdot \frac{7}{25} = \frac{5 \cdot 7}{7 \cdot 25} = \frac{\cancel{5} \cdot \cancel{7}}{\cancel{7} \cdot \cancel{5} \cdot 5} = \frac{1}{5}$

Practice 46 $4\frac{1}{3} \div 26$

Solution: Click here to check your answer.

Example 47 $-3\frac{1}{5} \div -4$

Solution: $-3\frac{1}{5} \div -4 = -\frac{16}{5} \div \frac{-4}{1} = -\frac{16}{5} \cdot \frac{-1}{4} = \frac{16 \cdot 1}{5 \cdot 4} = \frac{4 \cdot \cancel{4} \cdot 1}{5 \cdot \cancel{4}} = \frac{4}{5}$

Practice 48 $-5 \div \frac{5}{7}$

Solution: Click here to check your answer.

The order of operations must be used when working with fractions. We apply the order of operations in the same manner as we did previously.

Examples: Order of Operations with Fractions

Example 49 $\frac{1}{5} - \frac{2}{3} + \frac{1}{6}$

Solution: Working left to right with only 2 fractions at a time, $\frac{1}{5} - \frac{2}{3} + \frac{1}{6} = \frac{1 \cdot 3}{5 \cdot 3} - \frac{2 \cdot 5}{3 \cdot 5} + \frac{1}{6} = \frac{3}{15} - \frac{10}{15} + \frac{1}{6} = -\frac{7}{15} + \frac{1}{6} = -\frac{7 \cdot 2}{3 \cdot 5 \cdot 2} + \frac{1 \cdot 5}{3 \cdot 2 \cdot 5} = \frac{-14}{30} + \frac{5}{30} = -\frac{9}{30} = -\frac{3 \cdot \cancel{3}}{\cancel{3} \cdot 10} = -\frac{3}{10}$

Practice 50 $\frac{-1}{4} - \frac{2}{5} + \frac{3}{10}$

Solution: Click here to check your answer.

Example 51 $\frac{7}{12} - \frac{14}{15} \cdot \frac{5}{8}$

Solution: Multiplication before subtraction, $\frac{7}{12} - \frac{14}{15} \cdot \frac{5}{8} = \frac{7}{12} - \frac{\cancel{2} \cdot 7}{3 \cdot \cancel{5}} \cdot \frac{\cancel{5}}{\cancel{2} \cdot 4} = \frac{7}{12} - \frac{7}{12} = \frac{0}{12} = 0$.

Practice 52 $\frac{15}{22} \div \frac{4}{11} - \frac{3}{8}$

Solution: Click here to check your answer.

Solutions to Practice Problems:

Practice 2

Solution: $7 \div 10 = 7.0 \div 10 = 0.7$ since $70 \div 10 = 7$ and there are $1 - 0 = 1$ decimal places. $0.7 \cdot 100 = 70\%$. Therefore, $\frac{7}{10} = 0.7 = 70\%$. Back to Text

Practice 4

Solution: $0.55 \times 100 = 55\%$. 55% means 55 out of 100 which translates into $\frac{55}{100}$ (note that this is not simplified but we will put this off until later in this section). Back to Text

Practice 6

Solution: 35% means 35 out of 100 which translates to $\frac{35}{100}$ (which is not simplified). Now to get the decimal we can always divide since $\frac{35}{100} = 35 \div 100 = 35.00 \div 100 = 0.35$ since $3500 \div 100 = 35$ and $2 - 0 = 2$ decimal places. Back to Text

Practice 8

Solution: First, change $\frac{2}{5}$ to a decimal. $\frac{2}{5} = 2 \div 5 = 2.0 \div 5 = 0.4$ since $20 \div 5 = 4$ and there are $1 - 0 = 1$ decimal places. Therefore, $\frac{2}{5}\% = 0.4\% = \frac{0.4}{100} = 0.4 \div 100 = 0.400 \div 100 = 0.004$. Translating this into a fraction not containing a decimal we have $0.004 = \frac{4}{1000}$. Back to Text

Practice 10

Solution: First, $25\% = 0.25$. Then, $111 - 25\% \cdot 150 = 111 - 0.25 \cdot 150 = 111 - 37.5 = 73.5$. Back to Text

Practice 12

Since $8 \cdot 6 = 48$, we need to multiply the numerator by the same number. $\frac{3 \cdot 6}{8 \cdot 6} = \frac{18}{48}$. Therefore the equivalent fraction is $\frac{18}{48}$.

Back to Text

Practice 14
Solution: Since 0.43 has two decimal places we can multiply this number by 100 to get an integer. $0.43 \cdot 100 = 43$, we need to multiply the denominator by the same number. $\frac{0.43 \cdot 100}{100 \cdot 100} = \frac{43}{1000}$. Therefore the equivalent fraction is $\frac{43}{1000}$. Back to Text

Practice 16
Solution: Since 5.1 has one decimal place we can multiply this number by 10 to get an integer in both cases. $\frac{3 \cdot 10}{5.1 \cdot 10} = \frac{30}{51}$. Therefore the equivalent fraction is $\frac{30}{51}$. Back to Text

Practice 18
Solution: $\frac{40}{50} = \frac{4 \cdot 10}{5 \cdot 10} = \frac{4}{5}$. Therefore the equivalent fraction is $\frac{4}{5}$. Back to Text

Practice 20
Solution: $\frac{54}{72} = \frac{6 \cdot 9}{8 \cdot 9} = \frac{6}{8} = \frac{2 \cdot 3}{2 \cdot 4} = \frac{3}{4}$ or $\frac{54}{72} = \frac{3 \cdot 18}{4 \cdot 18} = \frac{3}{4}$ if you use the greatest common factor. Either way the simplified form is $\frac{3}{4}$. Back to Text

Practice 22
Solution: $-\frac{30}{45} = -\frac{15 \cdot 2}{15 \cdot 3} = -\frac{2}{3}$. Back to Text

Practice 24
Solution: 3 divides into 47 fifteen times with 2 remaining $(15 \cdot 3 + 2 = 47)$. Therefore, $\frac{47}{3} = 15\frac{2}{3}$. Back to Text

Practice 26
Solution: $17 \cdot 3 + 1 = 52$ so $-17\frac{1}{3} = -\frac{52}{3}$. Back to Text

Practice 28
Solution: $\frac{4}{9} + \frac{8}{9} = \frac{4+8}{9} = \frac{12}{9} = \frac{4 \cdot \cancel{3}}{3 \cdot \cancel{3}} = \frac{4}{3} = 1\frac{1}{3}$. Back to Text

Practice 30
Solution: Changing both to numbers to improper fractions we get,

$$1\frac{5}{7} - 2\frac{2}{7} = \frac{12}{7} - \frac{16}{7} = \frac{12 - 16}{7} = \frac{-4}{7}.$$

Back to Text

Practice 32
Solution:

$$\frac{5}{12} - \frac{3}{16} = \frac{5}{3 \cdot 4} - \frac{3}{4 \cdot 4} = \frac{5 \cdot 4}{3 \cdot 4 \cdot 4} - \frac{3 \cdot 3}{4 \cdot 4 \cdot 3} = \frac{20}{48} - \frac{9}{48} = \frac{11}{48}$$

Back to Text

Practice 34
Solution:

$$\frac{1}{5} - \frac{3}{10} = \frac{1}{5} - \frac{3}{2 \cdot 5} = \frac{1 \cdot 2}{5 \cdot 2} - \frac{3}{2 \cdot 5} = \frac{2}{10} - \frac{3}{10} = \frac{2-3}{10} = -\frac{1}{10}$$

Back to Text

Practice 36
Solution:

$$3\frac{3}{4} + 1\frac{2}{3} = \frac{15}{4} + \frac{5}{3} = \frac{15 \cdot 3}{4 \cdot 3} + \frac{5 \cdot 4}{3 \cdot 4} = \frac{45}{12} + \frac{20}{12} = \frac{65}{12} = 5\frac{5}{12}$$

Back to Text

Practice 38
Solution:

$$2 - 4\frac{1}{10} = \frac{2}{1} - \frac{41}{10} = \frac{2 \cdot 10}{1 \cdot 10} - \frac{41}{10} = \frac{20}{10} - \frac{41}{10} = \frac{20 - 41}{10} = \frac{-21}{10} = -2\frac{1}{10}$$

Back to Text

Practice 40

Solution: $-\frac{6}{45}\cdot -\frac{30}{12} = \frac{6\cdot 30}{45\cdot 12} = \frac{\cancel{6}\cdot\cancel{15}\cdot\cancel{2}}{3\cancel{15}\cdot\cancel{2}\cdot\cancel{6}} = \frac{1}{3}$ Back to Text

Practice 42

Solution: $-6\cdot -3\frac{3}{4} = -\frac{6}{1}\cdot -\frac{15}{4} = \frac{6\cdot 15}{1\cdot 4} = \frac{\cancel{2}\cdot 3\cdot 15}{1\cdot\cancel{2}\cdot 2} = \frac{45}{2} = 22\frac{1}{2}$ Back to Text

Practice 44

Solution: $\frac{12}{21}\div\frac{-6}{35} = \frac{12}{21}\cdot\frac{-35}{6} = -\frac{12\cdot 35}{21\cdot 6} = -\frac{2\cdot\cancel{6}\cdot\cancel{7}\cdot 5}{\cancel{7}\cdot 3\cdot\cancel{6}} = -\frac{10}{3}$ Back to Text

Practice 46

Solution: $4\frac{1}{3}\div 26 = \frac{13}{3}\div\frac{26}{1} = \frac{13}{3}\cdot\frac{1}{26} = \frac{13\cdot 1}{3\cdot 26} = \frac{\cancel{13}\cdot 1}{3\cdot 2\cdot\cancel{13}} = \frac{1}{6}$ Back to Text

Practice 48

Solution: $-5\div\frac{5}{7} = -\frac{5}{1}\div\frac{5}{7} = -\frac{5}{1}\cdot\frac{7}{5} = -\frac{\cancel{5}\cdot 7}{1\cdot\cancel{5}} = \frac{-7}{1} = -7$ Back to Text

Practice 50

Solution: Addition and subtraction only so we do this left to right. $\frac{-1}{4} - \frac{2}{5} + \frac{3}{10} = \frac{-1\cdot 5}{4\cdot 5} - \frac{2\cdot 4}{5\cdot 4} + \frac{3}{10} = \frac{-5}{20} - \frac{8}{20} + \frac{3}{10} = -\frac{13}{20} + \frac{3}{10} = -\frac{13}{20} + \frac{3\cdot 2}{10\cdot 2} = -\frac{13}{20} + \frac{6}{20} = -\frac{7}{20}$ Back to Text

Practice 52

Solution: Division must be done before subtraction. $\frac{15}{22}\div\frac{4}{11} - \frac{3}{8} = \frac{15}{22}\cdot\frac{11}{4} - \frac{3}{8} = \frac{15}{2\cdot\cancel{11}}\cdot\frac{\cancel{11}}{4} - \frac{3}{8} = \frac{15}{8} - \frac{3}{8} = \frac{12}{8} = \frac{\cancel{4}\cdot 3}{\cancel{4}\cdot 2} = \frac{3}{2} = 1\frac{1}{2}$
Back to Text

1.9.1 Exercises 1.9

Show the equivalences for decimal, percent, and fraction of the following numbers. Click here to see examples.

1. 0.5%
2. 27%
3. 0.05
4. 1.12
5. $\frac{2}{25}$
6. $\frac{1}{40}$
7. $\frac{3}{10}$
8. $\frac{3}{10}\%$
9. $\frac{7}{20}\%$

Evaluate the expression being careful to follow order of operations. Click here to see examples.

10. $32\% \div 8(10-3^2)$
11. $2^3 \cdot 20\%$
12. $(2.1+5.3)15\%$

Write an equivalent fraction with 48 as the denominator. Click here to see examples.

13. $\frac{5}{12}$
14. $-\frac{1}{6}$
15. $\frac{2}{3}$

Write an equivalent fraction which eliminates the decimal. Click here to see examples.

16. $\frac{0.3}{10}$
17. $\frac{1.2}{100}$
18. $-\frac{1.1}{1000}$
19. $\frac{2.3}{5}$
20. $-\frac{2}{1.67}$
21. $\frac{1.32}{0.5}$

Write each fraction in simplified form. Click here to see examples.

22. $\frac{8}{20}$
23. $-\frac{9}{63}$
24. $-\frac{90}{162}$
25. $\frac{24}{88}$
26. $\frac{60}{36}$
27. $-\frac{144}{64}$

Write as a mixed number. Click here to see examples.

28. $-\frac{27}{7}$
29. $\frac{35}{6}$
30. $-\frac{309}{10}$

Write as an improper fraction. Click here to see examples.

31. $2\frac{5}{8}$
32. $-3\frac{4}{7}$
33. $-12\frac{3}{5}$

Add or subtract being careful you put your answer into simplified form. Click here to see examples.

34. $\frac{3}{10}+\frac{2}{10}$
35. $\frac{1}{4}-4\frac{3}{4}$
36. $-4\frac{2}{5}-7\frac{3}{5}$

Add or subtract being careful you put your answer into simplified form. Click here to see examples.

37. $\frac{2}{7}-\frac{1}{4}$
38. $\frac{5}{14}+\frac{5}{6}$
39. $1\frac{2}{5}-1\frac{1}{15}$
40. $-\frac{1}{3}-1\frac{1}{4}$
41. $-\frac{7}{20}+2\frac{1}{15}$
42. $4-\frac{7}{11}$

Multiply being careful you put your answer into simplified form. Click here to see examples.

43. $\frac{15}{6}\cdot\frac{8}{14}$
44. $-16\cdot\frac{3}{32}$
45. $1\frac{1}{4}\cdot -3\frac{1}{5}$
46. $-3\cdot -3\frac{2}{3}$
47. $-\frac{8}{20}\cdot\frac{10}{12}$
48. $2\frac{1}{3}\cdot 3\frac{3}{7}$

Divide being careful you put your answer into simplified form. Click here to see examples.

49. $\frac{1}{2}\div\frac{1}{8}$
50. $-\frac{6}{18}\div\frac{4}{15}$
51. $-1\frac{4}{5}\div -2\frac{3}{10}$
52. $3\frac{3}{5}\div -2$
53. $4\div 1\frac{5}{11}$
54. $\frac{9}{20}\div 3$

Evaluate each expression being careful you put your answer into simplified form. Click here to see examples.

55. $\frac{1}{4}+\frac{5}{6}-\frac{1}{2}$
56. $-\frac{1}{3}+\frac{6}{15}-\frac{2}{5}$
57. $\left(-\frac{1}{4}\right)^2-\frac{3}{8}$
58. $\frac{1}{3}\cdot\frac{1}{2}-\frac{1}{2}\cdot\frac{1}{5}$
59. $3\left(\frac{1}{5}-2\right)$
60. $2\cdot\frac{1}{4}+1\frac{2}{3}$

Click here to see the solutions.

1.10 Introduction to Radicals

We have seen the relationship between addition and subtraction where we know since $5+7=12$ then $12-7=5$. Similarly for multiplication and division we know that since $3\times 4=12$ then $12\div 4=3$. We also studied exponents where $3^2=9$ and $2^3=8$ but up to this point we have not discussed the operation that is related to this. That is, we need an operation that when we perform it, it will take 9 to 3 and 8 to 2. This operation is a radical. We have that $\sqrt{9}=3$ (the square root of 9) since $3^2=9$ and the $\sqrt[3]{8}=2$ (the cube root of 8) since $2^3=8$. To translate from a radical to an exponent or the other way we use the following:

$$a^n=b \quad \Leftrightarrow \quad \sqrt[n]{b}=a$$

The index (which becomes the exponent when translating) is the number of times you multiply the number by itself to get radicand.

$$\sqrt[index]{radicand}$$

If no index is written it is understood to be a 2. For example, $\sqrt{49}=\sqrt[2]{49}=7$ since $7^2=49$. When the index is even, the radical always stands for the positive root. For example, even though $(-7)^2=49$, we always have $\sqrt{49}=7$. If we want the negative square root of 49, we write $-\sqrt{49}=-7$.

Examples: Radicals

EXAMPLE 1 $\sqrt{121}$
Solution: $\sqrt{121}=11$ since $11^2=121$

PRACTICE 2 $\sqrt{64}$
Solution: Click here to check your answer.

EXAMPLE 3 $\sqrt[4]{16}$
Solution: $\sqrt[4]{16}=2$ since $2^4=16$

PRACTICE 4 $\sqrt[3]{125}$
Solution: Click here to check your answer.

Consider $\sqrt{-16}$. We need to find the number that when we multiply it by itself it equals -16. If we try -4, we get $-4\cdot -4=16$. If we try 4, we get $4\cdot 4=16$. In both cases we end up with a positive number. In fact, since the product of an even number of negative factors is positive, when we have an even index with a negative radicand, it is not a real number. Therefore, $\sqrt{-16}$ is not a real number. Likewise, $\sqrt[4]{-16}$ is not a real number. In general, a negative number under an even indexed radical is not a real number.

However, $\sqrt[3]{-27}$ is a real number. Consider $-3\cdot -3\cdot -3=-27$ which means $\sqrt[3]{-27}=-3$. This is possible in the real numbers because of the index being odd.

Examples: Radicals with Negative Radicands

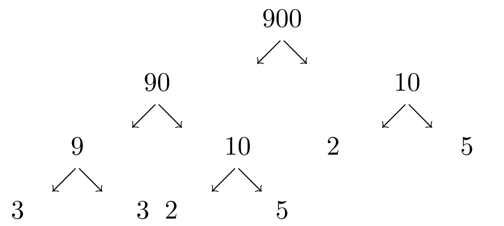
EXAMPLE 5 $\sqrt[5]{-32}$
Solution: $\sqrt[5]{-32}=-2$ since $(-2)^5=-32$

PRACTICE 6 $\sqrt[3]{-64}$
Solution: Click here to check your answer.

EXAMPLE 7 $\sqrt[4]{-81}$
Solution: This is not a real number since we have a negative radicand (-81) under an even indexed (4) radical.

PRACTICE 8 $\sqrt{-64}$
Solution: Click here to check your answer.

For larger numbers we need a procedure. Consider $\sqrt{900}$. The index tells us how many times we need a factor repeated. In this case, we need a factor repeated twice since the index is 2. To take advantage of this grouping, find the prime factorization of the number.



We see that the prime factorization of $900 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5$ which with rearranging we can get $900 = \underline{2 \cdot 3 \cdot 5} \cdot \underline{2 \cdot 3 \cdot 5} = 30 \cdot 30$. Therefore, $\sqrt{900} = 30$.

Examples: Large Radicals

Example 9 $\sqrt[3]{42,875}$

Solution: $42,875 = 5 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 7$ and since we must have three of the same factors since our radical has index three we have $\sqrt[3]{42,875} = 5 \cdot 7 = 35$.

Practice 10 $\sqrt{3,136}$

Solution: Click here to check your answer.

Consider $\sqrt{50}$. The best we can do is an estimation if we do not want to have a radical involved since the prime factorization ($50 = 2 \cdot 5 \cdot 5$) has an unpaired 2. Since 50 is close to 49, $\sqrt{50}$ is close to $\sqrt{49} = 7$. Even if the radicand is not close to a perfect power, we can find whole numbers above and below the radical.

Examples: Estimating a Radical

Example 11 *Estimate* $\sqrt{72}$.

Solution: Since $8^2 = 64 < 72 < 81 = 9^2$, we have $8 = \sqrt{64} < \sqrt{72} < \sqrt{81} = 9$.

Practice 12 *Estimate* $\sqrt[3]{39}$.

Solution: Click here to check your answer.

Solutions to Practice Problems:

Practice 2

Solution: $\sqrt{64} = 8$ since $8^2 = 64$ Back to Text

Practice 4

Solution: $\sqrt[3]{125} = 5$ since $5^3 = 125$ Back to Text

Practice 6

Solution: $\sqrt[3]{-64} = -4$ since $(-4)^3 = -64$. Back to Text

Practice 8

Solution: $\sqrt{-64}$ is not a real number since we have a negative number (-64) under an even indexed (2) radical.
Back to Text

Practice 10

Solution: $\sqrt{3,136} = 2 \cdot 2 \cdot 2 \cdot 7 = 56$ since we are looking for pairs of factors and the prime factorization is $3136 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7$
Back to Text

Practice 12

Solution: Since $3^3 = 27 < 39 < 64 = 4^3$, we have $3 = \sqrt[3]{27} < \sqrt[3]{39} < \sqrt[3]{64} = 4$. Back to Text

1.10.1 Exercises 1.10

Evaluate. Click here to see examples.

1. $\sqrt{81}$
2. $\sqrt[4]{81}$
3. $\sqrt[5]{32}$

Evaluate. Click here to see examples.

4. $\sqrt[3]{-8}$
5. $\sqrt{-49}$
6. $\sqrt[4]{-16}$

Evaluate. Click here to see examples.

7. $\sqrt{144}$
8. $\sqrt{3969}$
9. $\sqrt{-900}$
10. $\sqrt[3]{-216}$
11. $\sqrt[4]{625}$
12. $\sqrt[5]{-7776}$

Estimate. Click here to see examples.

13. $\sqrt{104}$
14. $\sqrt{55}$
15. $\sqrt[3]{-33}$

Click here to see the solutions.

1.11 Properties of Real Numbers

There are several properties of real numbers that you use frequently. For example, you know that it does not matter if you add $5+3$ or $3+5$; both will equal 8. In this section we will formalize several of these properties.

The Commutative Property of Addition is the example that was just given. When you commute to work or school you physically change the place where you are located. This is the same thing for the commutative property, the numbers physically change places.

Commutative Property of Addition:

$$a+b=b+a$$

Commutative Property of Multiplication:

$$a\cdot b=b\cdot a$$

We know $5+3=3+5$ because of the commutative property of addition. Similarly, $5\cdot 3=3\cdot 5$ by the commutative property of multiplication. But what about subtraction and division? Consider that $3-5=-2$ while $5-3=2$ so $3-5\neq 5-3$. Likewise, $10\div 2=5$ while $2\div 10=\frac{1}{5}$ which are not equal. So, the Commutative Property holds for neither subtraction nor division.

Another property we often use when we are adding three or more numbers is the idea that we can add in any order we like without changing the answer. That is, adding $2+3$ first then add 4, $(2+3)+4$, is the same as adding $3+4$ then add 2, $2+(3+4)$. This is known as the associative property of addition. When you associate with people, these are the people with whom you are grouped at the present time. If you change those with whom you associate you are changing your grouping. This is the same mathematically when we are using the associative property. It changes the numbers that are being grouped.

Associative Property of Addition:

$$(a+b)+c=a+(b+c)$$

Associative Property of Multiplication:

$$(a\cdot b)\cdot c=a\cdot(b\cdot c)$$

Again, these properties do not hold for subtraction and division. Consider that $(10-3)-2=7-2=5$ while $10-(3-2)=10-1=9$ which are not equal. Similarly, $(12\div 3)\div 2=4\div 2=2$ but $12\div(3\div 2)=12\div\frac{3}{2}=12\cdot\frac{2}{3}=\frac{12\cdot 2}{3}=\frac{\cancel{3}\cdot 4\cdot 2}{\cancel{3}}=8$ which are not equal.

Suppose that your boss tells you to go get 3 donuts for everybody in the accounting (4 people) and sales (5 people) departments. You have 2 ways to calculate how many donuts you need. First, you could calculate a total of 9 people in both departments for which each get 3 donuts, $3(4+5)$. Or, you could calculate how many donuts accounting $(3\cdot 4)$ and how many sales, $3 \cdot 5$, will get and then add them together, $3 \cdot 4 + 3 \cdot 5$. Both ways of calculating will get you the correct amount of donuts. Therefore, $3(4+5) = 3 \cdot 4 + 3 \cdot 5$. This is known as the Distributive Property of Multiplication over Addition (in our example you are distributing donuts).

Distributive Property:

$$a(b+c) = a \cdot b + a \cdot c$$
$$a(b-c) = a \cdot b - a \cdot c$$

Notice that we only distribute over addition and subtraction.

Examples: Identifying Properties

EXAMPLE 1 *State the property used:*

1. $5 + 4 = 4 + 5$
2. $(2+3) + 7 = 2 + (3+7)$
3. $(2 \cdot 3) \cdot 7 = (3 \cdot 2) \cdot 7$
4. $(2+3) \cdot 7 = 7 \cdot (2+3)$
5. $2(7+3) = 2 \cdot 7 + 2 \cdot 3$

Solution:

1. $5 + 4 = 4 + 5$ is the commutative property of addition since the 4 and 5 changed places.
2. $(2+3) + 7 = 2 + (3+7)$ is the associative property of addition since 2 and 3 are grouped together on the left side and 3 and 7 are grouped on the right side. Note also that addition is the only operation.
3. $(2 \cdot 3) \cdot 7 = (3 \cdot 2) \cdot 7$ is the commutative property of multiplication since the grouping did not change but the 2 and 3 changed places. Multiplication is the only operation in this problem.
4. $(2+3) \cdot 7 = 7 \cdot (2+3)$ is commutative property of multiplication since the group $(2+3)$ changes places with 7 around multiplication.
5. $2(7+3) = 2 \cdot 7 + 2 \cdot 3$ is the distributive property since the 2 is distributed across addition.

PRACTICE 2 *State the property used:*

1. $(3 \cdot 9) \cdot 2 = 3 \cdot (9 \cdot 2)$
2. $(2+3) \cdot 6 = (3+2) \cdot 6$
3. $(5 \cdot 6) + 3 = 3 + (5 \cdot 6)$
4. $2(4-1) = 2 \cdot 4 - 2 \cdot 1$
5. $7 + (5+3) = (7+5) + 3$

Solution: Click here to check your answer.

Zero has an important role in addition. We can add 0 to any number and get the number back unchanged. Zero is known as the additive identity. One way of thinking about this property is that you can identify yourself in the mirror because you get back your image. In this thought the mirror is the 0 since $7+0 = 7$ (7 looks into the mirror, 0, and sees itself 7). Similarly, sometimes we have to turn around (go the inverse direction) to see the mirror. This is known as the additive inverse. The goal is to get back to the mirror, 0. We have to ask ourselves, what can we add to 7 to get it back to 0. If we think back to the rule of addition we need to subtract $7 - 7 = 0$, but we must be adding. That is, the original signs must have been different. Alternatively, change the problem over to addition by adding the opposite, $7 + (-7) = 0$. Therefore, -7 is the additive inverse of 7.

Additive Identity:

$$a + 0 = 0 + a = a$$

Additive Inverse:

$$a + (-a) = (-a) + a = 0$$

Similarly, in multiplication 1 has this important role of being the identity (the mirror). One is the multiplicative identity, $7 \cdot 1 = 1 \cdot 7 = 7$. Consider what needs to be done to 7 to get to the multiplicative identity, 1. We know $7 \div 7 = 1$ but again we need multiplication. Change this problem over to multiplication by leaving the first alone multiply and flip, $7 \div 7 = 7 \cdot \frac{1}{7} = 1$. The reciprocal, $\frac{1}{7}$, is the multiplicative inverse of 7.

Multiplicative Identity:

$$a \cdot 1 = 1 \cdot a = a$$

Multiplicative Inverse:

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1 \quad a \neq 0$$

Notice that we do not discuss these properties for subtraction and division. However, we only need to change subtraction over to addition and division to multiplication. However, it should be noted that 0 does not have a multiplicative inverse.

Examples: Identities and Inverses

EXAMPLE 3

1. *What is the additive inverse of -5?*
2. *What is the multiplicative inverse of -5?*

Solution:

1. 5 is the additive inverse of -5 since $-5 + 5 = 0$.
2. $-\frac{1}{5}$ is the multiplicative inverse of -5 since it is the reciprocal and $-\frac{1}{5} \cdot -5 = 1$.

PRACTICE 4

1. *What is the additive inverse of $\frac{3}{7}$?*
2. *What is the multiplicative of $\frac{3}{7}$?*

Solution: Click here to check your answer.

EXAMPLE 5 *Identify which property is being used.*

1. $5 \cdot 1 = 5$
2. $0 + (3 + 2) = (3 + 2)$
3. $\frac{1}{3} \cdot 3 = 1$
4. $-\frac{4}{5} + \frac{4}{5} = 0$

Solution:

1. $5 \cdot 1 = 5$ is multiplicative identity.

2. $0 + (3 + 2) = (3 + 2)$ is additive identity.
3. $\frac{1}{3} \cdot 3 = 1$ is multiplicative inverse.
4. $-\frac{4}{5} + \frac{4}{5} = 0$ is additive inverse.

Practice 6 *Identify which property is being used.*

1. $2 + 0 = 2$
2. $(-1)(-1) = 1$
3. $-0.4 + 0.4 = 0$
4. $\frac{-2}{3} \cdot \frac{-3}{2} = 1$

Solution: Click here to check your answer.

Solutions to Practice Problems:
Practice 2
Solution:

1. $(3 \cdot 9) \cdot 2 = 3 \cdot (9 \cdot 2)$ is the associative property of multiplication since 3 and 9 are grouped together on the left side but 9 and 2 are grouped together on the right side with multiplication being the operation.
2. $(2 + 3) \cdot 6 = (3 + 2) \cdot 6$ is the commutative property of addition since 2 and 3 change places around addition.
3. $(5 \cdot 6) + 3 = 3 + (5 \cdot 6)$ is the commutative property of addition since the group $(5 \cdot 6)$ changes places with the 3 around addition.
4. $2(4 - 1) = 2 \cdot 4 - 2 \cdot 1$ is the distributive property since 2 is distributed across subtraction.
5. $7 + (5 + 3) = (7 + 5) + 3$ is the associative property of addition since 5 and 3 are grouped together on the left side but 7 and 5 are grouped together on the right side with addition being the operation.

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Practice 4
Solution:

1. $-\frac{3}{7}$ is the additive inverse of $\frac{3}{7}$ since $-\frac{3}{7} + \frac{3}{7} = 0$
2. $\frac{7}{3}$ is the multiplicative of $\frac{3}{7}$ since it is the reciprocal and $\frac{7}{3} \cdot \frac{3}{7} = 1$

Back to Text

Practice 6
Solution:

1. $2 + 0 = 2$ is additive identity.
2. $(-1)(-1) = 1$ is multiplicative inverse.
3. $-0.4 + 0.4 = 0$ is additive inverse.
4. $\frac{-2}{3} \cdot \frac{-3}{2} = 1$ is multiplicative inverse.

Back to Text

1.11.1 Exercises 1.11

Identify the property. Click here to see examples.

1. $(5+3)\cdot 2=(3+5)\cdot 2$
2. $(5+3)\cdot 2=2\cdot(5+3)$
3. $2\cdot(5+3)=2\cdot 5+2\cdot 3$
4. $(7+2)+4=7+(2+4)$
5. $(7+2)+4=4+(7+2)$
6. $3\cdot(4\cdot 5)=(3\cdot 4)\cdot 5$

State the additive inverse of each number. Click here to see examples.

7. 23
8. 0
9. $-5\frac{3}{4}$

State the multiplicative inverse of each number. Click here to see examples.

10. 12
11. 1
12. $-\frac{4}{7}$

Identify the property. Click here to see examples.

13. $(4+3)\cdot 1=(4+3)$
14. $9\cdot\frac{1}{9}=1$
15. $\frac{7}{12}+\frac{-7}{12}=0$

Identify the property.

16. $(-3+3)+0=(-3+3)$
17. $5(6-3)=5\cdot 6-5\cdot 3$
18. $-\frac{1}{4}(-4\cdot 1)=-\frac{1}{4}(-4)$
19. $(1\cdot 3)+4=(3\cdot 1)+4$
20. $(-2)(0+7)=(0+7)(-2)$
21. $(-3+3)+5=-3+(3+5)$

Click here to see the solutions.

Chapter 2

Basic Algebra

2.1 Combining Like Terms

In every day life we often find repetition. A simple thing like going to the grocery store and deciding if you want 3 or 4 packs of gum which cost \$0.60 each is a repetitive calculation. You may quickly calculate 3 packs of gum costs \$1.80 while 4 packs of gum costs \$2.40. This may help you decide how many packs to purchase. The number of packs you buy may change but (at least for that visit) the price will not. An amount that may change can be represented by a variable. An amount that does not change, is a constant. In this example, the number of packs of gum purchased is a variable and the cost of a pack is a constant. Variables are typically represented by a letter. For instance, if x represents the number of packs of gum purchased then the total purchase price for the gum could be represented by $0.60x$.

In an algebraic expression such as $5x + 2$ we have both variables and constants. x is a variable while 5 and 2 are constants. This expression has 2 terms. Terms are separated by addition or subtraction. If a term does not have a variable in it we call that the constant term. The constant 5 is called the coefficient of x.

Examples: Terminology

EXAMPLE 1 *Answer the following questions for the expression $x - 3y + 9$*

1. *How many terms does this expression have?*
2. *What is the constant term?*
3. *What is the coefficient of y?*
4. *What is the coefficient of x?*

Solution:

1. There are 3 terms: x, $-3y$ and 9. Notice the subtraction belongs to the constant its right, since $x-3y+9 = x+(-3y)+9$.
2. 9 is the constant term since it is the term with no variable.
3. -3 is the coefficient of y since it is the constant in the term which contains the y, $-3y$.
4. 1 is understood to be the coefficient of x since $x = 1x$.

PRACTICE 2 *Answer the following questions for the expression $2x - y - 5$*

1. *How many terms does this expression have?*
2. *What is the constant term?*
3. *What is the coefficient of y?*
4. *What is the coefficient of x?*

Solution: Click here to check your answer.

Terms are considered "like" terms when the variable is exactly the same (including the exponents on the variable) in the terms. $7x - 5x^2 + 2x - 1$ presently has 4 terms but $7x$ and $2x$ are like terms since they have exactly the same variable. $-5x^2$ is not like them since the variable has a power of 2. When we have like terms we can add their coefficients and just keep the shared variable, $7x + 2x = 9x$. (If x represented apples, this could be thought of as 7 apples plus 2 apples equaling 9 apples.) Therefore, $7x - 5x^2 + 2x - 1 = -5x^2 + 9x - 1$. This process is called combining like terms or simplifying the expression.

Examples: Combining Like Terms

EXAMPLE 3 *Simplify:* $5x - 3x^2 + 7x^2 - 9x + 8 - 14$

Solution: $5x - 3x^2 + 7x^2 - 9x + 8 - 14 = 4x^2 - 4x - 6$ since $-3x^2, 7x^2$; $5x, -9x$; $8, -14$ are like terms we add the coefficients of each yielding $4x^2$; $-4x$; -6.

PRACTICE 4 *Simplify:* $7x + x^2 - 9 - 11x^2 - 3x - 10$

Solution: Click here to check your answer.

EXAMPLE 5 *Simplify:* $3(4m - 2) - 2(3m + 2n - 5)$

Solution:

$3(4m - 2) - 2(3m + 2n - 5)$	Change subtraction to addition
$3(4m - 2) + (-2)(3m + 2n - 5)$	Distribute 3 across first parentheses and -2 across the second
$3 \cdot 4m - 3 \cdot 2 + (-2) \cdot 3m + (-2) \cdot 2n - (-2) \cdot 5$	
$12m - 6 - 6m - 4n + 10$	Combine like terms
$6m - 4n + 4$	notice n has no term to combine

PRACTICE 6 *Simplify:* $5(2x + y + 1) - 3(x - 2y)$

Solution: Click here to check your answer.

EXAMPLE 7 *Simplify:* $-2(3a - 5) - (a + 10)$

Solution:

$-2(3a - 5) - 1(a + 10)$	Multiply 1 by $(a + 10)$ which does not change the problem
$(-2)(3a - 5) + (-1)(a + 10)$	Change subtraction to addition
$(-2)(3a) - (-2)(5) + (-1)(a) + (-1)(10)$	Distribute
$-6a + 10 + (-a) + (-10)$	
$-7a + 0$	Combine like terms
$-7a$	

PRACTICE 8 *Simplify:* $-4(2m - n) - (m + n)$

Solution: Click here to check your answer.

Solutions for Practice Problems:
Practice 2
Solution:

1. There are 3 terms: $2x$, $-y$ and -5.
2. -5 is the constant term since it is the term with no variable.
3. -1 is the constant in the term which contains the y, $-y = -1y$.
4. 2 is the coefficient of x.

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Practice 4
Solution: $7x + x^2 - 9 - 11x^2 - 3x - 10 = -10x^2 + 4x - 19$ since $-11x^2, x^2$; $7x, -3x$; $-9, -10$ are like terms. Back to Text

Practice 6
Solution: $5(2x + y + 1) - 3(x - 2y) = 5(2x + y + 1) + (-3)(x - 2y) = 10x + 5y + 5 - 3x + 6y = 7x + 11y + 5$ Back to Text

Practice 8
Solution: $-4(2m - n) - (m + n) = -4(2m - n) + (-1)(m + n) = -8m + 4n - m - n = -9m + 3n$ Back to Text

2.1.1 Exercises 2.1

List (in order) the number of terms, coefficient of x, coefficient of y, and constant term for each expression. Click here to see examples.

1. $x - 2y + 1$
2. $3y + 5z$
3. $w - x + 2z - 9$

Combine like terms. Click here to see examples.

4. $5x - 3 + 2x^2 - 3x + 1$
5. $(m + 3n - 4) - (4m - 3n + 1)$
6. $2(x + 3) - 4(2x - 7)$
7. $5(3m) + 6(2m^2 - m)$
8. $3(7x - 0.1y + 3.1) - (0.3y + 9.3)$
9. $0.7(2 - 0.1x) - 0.3(x + 1)$
10. $4(\frac{1}{2}x - \frac{3}{4}) - 12(\frac{2}{3}x + \frac{1}{6})$
11. $\frac{1}{2}(4x - 3) + \frac{2}{3}(9x - 6)$
12. $(\frac{2}{5}x + \frac{1}{2}) - (\frac{2}{5}x - \frac{1}{2})$

Click here to see the solutions.

2.2 Introduction to Solving Equations

Solving equations is something we naturally begin doing at a young age. If you know someone is 2 years older than you, you automatically add 2 years onto your age. This is solving an equation. It is just a simple equation where the variable is already isolated. Algebraically, if x represents your friend's age and you are 20, $x = 20 + 2$ so $x = 22$.

The purpose of solving equations is to find out the values that you could plug into x for which the equation is true. An effective way to get to obtain our goal is to isolate x on one side of the equation. For a simple example such as $x + 5 = 15$ we can try different values. With little effort we can identify that if $x = 10$ then $x + 5 = 10 + 5 = 15$ so we found the solution to be 10. The difficulty occurs when we have more complex problems such as $x + 3.143 = 2.1$ where a procedure would be useful to help us find the solution.

Examples: Determining if x is a Solution

Example 1 *Show that $x = 3$ is a solution for $4x - 3 = 12 - x$ while $x = 1$ is not.*
Solution: To show $x = 3$ is a solution, substitute 3 in for x into the equation to see if we end up with a true statement. The left side of the equation is $4(3) - 3 = 12 - 3 = 9$. The right side is $12 - (3) = 9$. Since the right side equals the left side $(9 = 9)$ when substituting 3, it is the solution.

For $x = 1$, substitute 1 in for x. On the left side $4(1) - 3 = 4 - 3 = 1$. On the right side $12 - (1) = 11$. Since the left side does not equal the right side, $1 = 11$ is not true, 1 is not a solution.

Practice 2 *Show that $x = 2$ is a solution for $5x + 3 = 15 - x$ while $x = -1$ is not.*
Solution: Click here to check your answer.

Example 3 *Determine if $\frac{1}{2}$ is a solution of $2 - (3x + 4) = 11x - 2(x + 4)$.*
Solution: Substitute $\frac{1}{2}$ in for x into the equation to see if we end up with a true statement. The left side of the equation is $2-(3(\frac{1}{2})+4) = 2-(\frac{3}{2}+\frac{8}{2}) = \frac{4}{2}-\frac{11}{2} = -\frac{7}{2}$. The right side is $11(\frac{1}{2})-2(\frac{1}{2}+4) = \frac{11}{2}-2(\frac{1}{2}+\frac{8}{2}) = \frac{11}{2}-2(\frac{9}{2}) = \frac{11}{2}-\frac{18}{2} = -\frac{7}{2}$. Since the right side equals the left side when substituting, $-\frac{7}{2} = -\frac{7}{2}$, it is the solution.

Practice 4 *Determine if $\frac{1}{3}$ is a solution of $x+4=3-(x+\frac{5}{3})$.*
Solution: Click here to check your answer.

While the goal in solving equations is to get the variable by itself, the rule to help you achieve this is that you must do the same thing to both sides to keep the equation balanced (equal). That is, if you add 2 to the left side you must add 2 to the right side. If you subtract 3 you must do it to both sides. Similarly, multiplication and division work the same. The idea is that we are going to undo the equation. Read through this first example.

$$\begin{array}{rcrcrl} 3x & - & 4 & = & 11 & \text{Undo the subtraction of 4 on the same side as the variable by adding 4} \\ & & \underline{+4} & & \underline{+4} & \text{Line up the addition under the constant terms} \\ 3x & + & 0 & = & 15 & \text{adding vertically} \\ 3x & & & = & 15 & \\ \frac{3x}{3} & & & = & \frac{15}{3} & \text{Undo the multiplication of 3 by dividing by 3 on both sides} \\ x & & & = & 5 & \text{Simplify} \end{array}$$

In this problem, did you notice how we "undid" the equation in the reverse order of the order of operations? We performed the addition first, then we applied division. Below are the formalized properties we just used.

> Properties of Equality
>
> If $a=b$ then $a+c=b+c$
> If $a=b$ then $a-c=b-c$
> If $a=b$ then $a\cdot c=b\cdot c$ for $c\neq 0$
> If $a=b$ then $a\div c=b\div c$ for $c\neq 0$

We will use these properties to first "gather" the variable terms on one side of the equation and the constant terms on the other. Then we will isolate the variable by dividing by the coefficient of the variable. As a final step, check the solution.

Examples: Solving Equations

Example 5 *Solve for m.* $-5m+3=14$

$$\begin{array}{rcrcrl} -5m & + & 3 & = & 14 & \text{Undo the addition by subtracting 3} \\ & & \underline{-3} & & \underline{-3} & \text{Line the like terms up vertically} \\ -5m & + & 0 & = & 11 & \text{Add like terms} \\ -5m & & & = & 11 & \text{Undo the multiplication by dividing by } -5 \\ \frac{-5m}{-5} & & & = & \frac{11}{-5} & \text{Simplify} \\ m & & & = & -\frac{11}{5} & \text{Convert to mixed number} \\ m & & & = & -2\frac{1}{5} & \end{array}$$

Check the solution:
Substitute $-\frac{11}{5}$ into $-5m+3=14$ to verify that it is the solution.
LHS: $-5(-\frac{11}{5})+3=11+3=14$ (LHS stands for Left Hand Side)
RHS: 14 (RHS stands for Right Hand Side)
Since LHS=RHS, $-\frac{11}{5}$ is the solution.

Practice 6 *Solve for n.* $7n+9=-12$
Solution: Click here to check your answer.

Example 7 *Solve for x.* $2-9x=4x+8$

$$\begin{array}{rcrcrcrl} 2 & - & 9x & = & 4x & + & 8 & \text{Gather the } x\text{-terms on the LHS by subtracting } 4x \\ & & \underline{-4x} & & \underline{-4x} & & & \text{Subtract like terms} \\ 2 & - & 13x & = & 0 & + & 8 & \text{RHS: simplify} \\ 2 & - & 13x & = & 8 & & & \text{Gather the constant terms on RHS by subtracting 2} \\ \underline{-2} & & & & \underline{-2} & & & \text{Subtract like terms} \\ 0 & - & 13x & = & 6 & & & \text{LHS: simplify} \\ & & -13x & = & 6 & & & \text{Isolate } x \text{ by dividing by the coefficient of } x \\ & & \frac{-13x}{-13} & & \frac{6}{-13} & & & \text{Simplify} \\ & & x & = & -\frac{6}{13} & & & \end{array}$$

Check the solution:
LHS: $2-9(-\frac{6}{13})=2+\frac{54}{13}=\frac{26}{13}+\frac{54}{13}=\frac{80}{13}$
RHS: $4(-\frac{6}{13})+8=-\frac{24}{13}+8=-\frac{24}{13}+\frac{104}{13}=\frac{80}{13}$
Since LHS=RHS, $-\frac{6}{13}$ is the solution.

PRACTICE 8 *Solve for x.* $-12+3x=8x+1$
Solution: Click here to check your answer.

Equations are not always in such a nice form. Often, we must simplify the expressions on either side of the equality before we begin the process of isolating the variable. This requires you to use the techniques discussed in the previous section. The general procedures to follow when solving equations is now given.

> Solving Equations
>
> 1. Simplify the expression on either side of the equation.
> 2. Gather the variable term on the left-hand side (LHS) by adding to both sides. the opposite of the variable term on the right-hand side (RHS).
>   - Note: either side is fine but we will consistently place the variable on the LHS. when a variable is present on both sides of the equation
> 3. Gather the constant terms on the RHS by adding to both sides the opposite of the constant term on the LHS.
> 4. Divide both sides by the coefficient of the variable
> 5. Simplify if possible.
> 6. Check your solution.

Examples: Equations First Requiring Simplifying

EXAMPLE 9 *Solve for x.* $2(x+4)-7=3-9x-6$

$$\begin{array}{rcll}
2(x+4)-7 & = & 3-9x-6 & \text{Simplify both sides of the equation first} \\
2x+8-7 & = & -9x-3 & \text{LHS: distribute the 2. RHS: combine constant terms} \\
2x+1 & = & -9x-3 & \text{LHS: combine constant terms. RHS: copy} \\
\underline{+9x} & = & \underline{+9x} & \text{Get } x\text{-term on LHS by adding 9x} \\
11x+1 & = & -3 & \\
\underline{-1} & = & \underline{-1} & \text{Get constant terms on RHS by subtracting 1} \\
11x & = & -4 & \\
\frac{11x}{11} & = & \frac{-4}{11} & \text{Divide by coefficient of } x \\
x & = & \frac{-4}{11} & \text{Simplify}
\end{array}$$

Check the solution:
LHS: $2(\frac{-4}{11}+4)-7=2(\frac{-4}{11}+\frac{44}{11})-7=2(\frac{40}{11})-7=\frac{80}{11}-7=\frac{80}{11}-\frac{77}{11}=\frac{3}{11}$
RHS: $3-9(\frac{-4}{11})-6=3+\frac{36}{11}-6=\frac{36}{11}-3=\frac{36}{11}-\frac{33}{11}=\frac{3}{11}$
Since LHS=RHS, $\frac{-4}{11}$ is the solution.

PRACTICE 10 *Solve for t.* $3-2(2t+1)=3(t+5)$
Solution: Click here to check your answer.

EXAMPLE 11 *Solve for y.* $-1-(y+7)=5y+2(3y-4)$

$$\begin{array}{rcll}
-1-(y+7) & = & 5y+2(3y-4) & \text{LHS: change to subtraction to addition of } -1 \\
-1+(-1)(y+7) & = & 5y+2(3y-4) & \text{LHS \& RHS: Distribute} \\
-1+(-1y)-7 & = & 5y+6y-8 & \text{LHS \& RHS: Combine like terms} \\
-y-8 & = & 11y-8 & \text{Get } y\text{-terms on the LHS by } -11y \text{ to both sides} \\
\underline{-11y\quad} & = & \underline{-11y} & \text{Combine like terms} \\
-12y-8 & = & -8 & \text{Get the constant terms on RHS by } +8 \text{ to both sides} \\
\underline{+8} & = & \underline{+8} & \text{Combine like terms} \\
-12y & = & 0 & \text{Divide by the coefficient of the } y\text{-term} \\
\frac{-12y}{-12} & = & \frac{0}{12} & \text{Simplify} \\
y & = & 0 &
\end{array}$$

Check the solution:
LHS: $-1-(0+7) = -1-7 = -8$
RHS: $5(0)+2(3(0)-4) = 0+2(0-4) = 2(-4) = -8$
Since LHS=RHS, 0 is the solution.

Practice 12 *Solve for x.* $2x-2(x+3) = 3-(4x+1)$
Solution: Click here to check your answer.

There are two special cases for solution sets, both which happen when the variable zeros out. When the variables zero out, this leaves constants only on both sides of the equation. One possibility is that you end up with a true statement such as $2 = 2$. When you have a statement that is always true, the solution is the **set of all real numbers** (or all reals). The other possibility is that you end up with a false statement such as $2 = 0$. When you have a statement that is always false, the answer for the solution to the equation is **no solution**. Consider the examples below.

Examples: Equations - Special Cases

Example 13 *Solve for x.* $2(3x+5) = 7+3(2x+1)$

$$\begin{array}{rcll}
2(3x+5) & = & 7+3(2x+1) & \text{LHS \& RHS: Distribute} \\
6x+10 & = & 7+6x+3 & \text{LHS: copy, RHS: combine constant terms} \\
6x+10 & = & 6x+10 & \text{LHS: combine constant terms, RHS: copy} \\
\underline{-6x\quad} & = & \underline{-6x} & \text{Get } x\text{-term on LHS by subtracting } 6x. \\
10 & = & 10 & \text{Combine like terms}
\end{array}$$

Since $10 = 10$ is a true statement, the solution is all real numbers.

Practice 14 *Solve for x.* $4x-(2x+1) = 3+2(x-2)$
Solution: Click here to check your answer.

Example 15 *Solve for x.* $7x+2(3x-1) = 3x+5(2x+3)$

$$\begin{array}{rcll}
7x+2(3x-1) & = & 3x+5(2x+3) & \text{Distribute} \\
7x+6x-2 & = & 3x+10x+15 & \text{Combine like terms} \\
13x-2 & = & 13x+15 & \text{Get } x\text{-terms on the LHS by } -13x \text{ to both sides} \\
\underline{-13x\quad} & = & \underline{-13x} & \text{Combine like terms} \\
-2 & = & 15 &
\end{array}$$

Since $-2 = 15$ is a false statement, there is no solution.

Practice 16 *Solve for x.* $8x-2(x+4) = 3+6(x+2)$
Solution: Click here to check your answer.

Solutions to Practice Problems:

Practice 2

Solution: Substituting $x = 2$ into $5x+3 = 15-x$ we get $5(2)+3 = 10+3 = 13$ on the left side and $15-(2) = 13$ on the right side. Since the left side equals the right $x = 2$ is a solution. Likewise, substituting $x = -1$ into $5x+3 = 15-x$ we get $5(-1)+3 = -5+3 = -2$ on the left side and $15-(-1) = 16$ on the right side. Since these are not equal it is not a solution.

Back to Text

Practice 4

Solution: Substitute $x = \frac{1}{3}$ into $x + 4 = 3 - (x + \frac{5}{3})$. On the left side, $\frac{1}{3} + 4 = \frac{1}{3} + \frac{12}{3} = \frac{13}{3}$. On the right side, $3 - (\frac{1}{3} + \frac{5}{3}) = 3 - \frac{6}{3} = 3 - 2 = 1$. Since the left side does not equal the right side, $\frac{13}{3} \neq 1$, $\frac{1}{3}$ is not a solution. Back to Text

Practice 6

Solution:

$$\begin{array}{rcrcrl} 7n & + & 9 & = & -12 & \text{Undo addition by subtracting 9} \\ & & \underline{-9} & & \underline{-9} & \text{Line the like terms up vertically} \\ 7n & + & 0 & = & -21 & \text{Add like terms} \\ \frac{7n}{7} & & & = & \frac{-21}{7} & \text{Undo the multiplication by dividing by 7} \\ n & & & = & -\frac{21}{7} & \\ n & & & = & -3 & \end{array}$$

Check the solution:

LHS: $7(-3) + 9 = -21 + 9 = -12$

RHS: -12

Since LHS=RHS, -3 is the solution. Back to Text

Practice 8

Solution:

$$\begin{array}{rcrcrcrl} -12 & + & 3x & = & 8x & + & 1 & \text{Gather the } x\text{-terms on the LHS by subtracting } 8x \\ & & \underline{-8x} & & \underline{-8x} & & & \text{Subtract like terms} \\ -12 & + & (-5x) & = & 0 & + & 1 & \text{RHS: simplify} \\ -12 & + & (-5x) & = & 1 & & & \text{Gather the constant terms on RHS by adding 12} \\ \underline{12} & & & & \underline{12} & & & \text{add like terms} \\ 0 & + & (-5x) & = & 13 & & & \text{LHS: simplify} \\ & & -5x & = & 13 & & & \text{Isolate } x \text{ by dividing by the coefficient of } x \\ & & \frac{-5x}{-5} & & \frac{13}{-5} & & & \text{Simplify} \\ & & x & = & -\frac{13}{5} & & & \text{Convert to a mixed number} \\ & & x & = & -2\frac{3}{5} & & & \end{array}$$

Check the solution:

LHS: $-12 + 3(-\frac{13}{5}) = -12 - \frac{39}{5} = -\frac{60}{5} - \frac{39}{5} = -\frac{99}{5}$

RHS: $8(-\frac{13}{5}) + 1 = -\frac{104}{5} - 1 = -\frac{104}{5} + \frac{5}{5} = -\frac{99}{5}$

Since LHS=RHS, $-2\frac{3}{5}$ is the solution.

Back to Text

Practice 10

Solution:

$$\begin{array}{rcrl} 3 - 2(2t + 1) & = & 3(t + 5) & \text{LHS: distribute } -2\text{, RHS: distribute 3} \\ 3 - 4t - 2 & = & 3t + 15 & \text{LHS: combine constant terms, RHS: copy} \\ -4t + 1 & = & 3t + 15 & \text{Gather the } x\text{-terms on LHS by subtracting } 3t \text{ both sides} \\ \underline{-3t} & = & \underline{-3t} & \text{combine like terms} \\ -7t + 1 & = & 15 & \text{Gather the constant terms on RHS by subtracting 1} \\ \underline{-1} & = & \underline{-1} & \text{combine like terms} \\ -7t & = & 14 & \text{Isolate } t \text{ by dividing by the coefficient of } t \\ \frac{-7t}{-7} & = & \frac{14}{-7} & \text{Simplify} \\ t & = & -2 & \end{array}$$

Check the solution:

LHS: $3 - 2(2(-2) + 1) = 3 - 2(-4 + 1) = 3 - 2(-3) = 3 + 6 = 9$

RHS: $3(-2 + 5) = 3(3) = 9$

Since LHS=RHS, -2 is the solution.

Back to Text

Practice 12

Solution:

$$\begin{array}{rcll}
2x-2(x+3) &=& 3-(4x+1) & \text{RHS: Change the subtraction to addition of } -1\\
2x-2(x+3) &=& 3+(-1)(4x+1) & \text{LHS: distribute } -2\text{, RHS: distribute } -1\\
2x-2x-6 &=& 3-4x-1 & \text{LHS \& RHS: combine like terms}\\
-6 &=& 2-4x & \text{Gather the constant terms on LHS by subtracting 2}\\
\underline{-2} &=& \underline{-2} & \text{combine like terms}\\
-8 &=& 0-4x & \text{Isolate } x \text{ by dividing by the coefficient of } x\\
\frac{-8}{-4} &=& \frac{-4x}{-4} & \text{Simplify}\\
2 &=& x &
\end{array}$$

Check the solution:
LHS: $2(2)-2(2+3)=4-2(5)=4-10=-6$
RHS: $3-(4(2)+1)=3-(8+1)=3-9=-6$
Since LHS=RHS, 2 is the solution.
Note that this is a case when gathering the variables on the right hand side does not make sense since the variable had a coefficient of zero when we simplified the right hand side.

Back to Text

Practice 14
Solution:

$$\begin{array}{rcll}
4x-(2x+1) &=& 3+2(x-2) & \text{RHS: Change the subtraction to addition of } -1\\
4x+(-1)(2x+1) &=& 3+2(x-2) & \text{LHS: distribute } -1\text{, RHS: distribute 2}\\
4x-2x-1 &=& 3+2x-4 & \text{LHS \& RHS: combine like terms}\\
2x-1 &=& 2x-1 & \text{Gather the } x\text{-terms on LHS by subtracting } 2x\\
\underline{-2x} &=& \underline{-2x} & \text{Combine like terms}\\
-1 &=& -1 & \text{True}
\end{array}$$

Since this is true, the solution set is all real numbers. Back to Text

Practice 16
Solution:

$$\begin{array}{rcll}
8x-2(x+4) &=& 3+6(x+2) & \text{LHS: distribute } -2\text{, RHS: distribute 6}\\
8x-2x-8 &=& 3+6x+12 & \text{LHS \& RHS: combine like terms}\\
6x-8 &=& 6x+15 & \text{Gather the } x\text{-terms on LHS by subtracting } 6x\\
\underline{-6x} &=& \underline{-6x} & \text{Combine like terms}\\
-8 &=& 15 & \text{False}
\end{array}$$

Since this is false, there is no solution to this equation. Back to Text

2.2.1 Exercises 2.2

Determine if the value is a solution to the equation preceding it. Click here to see examples.

1. $3x-5=2(x+4);\ -1$
2. $-2(x+5)=1-(3x+11);\ 0$
3. $5x+4=8x-1;\ 1\frac{2}{3}$

Solve these equations. Click here to see examples.

4. $3x-4=11$
5. $11x+9=9x-21$
6. $8x-7=11x-2$

Solve these equations which require simplifying first. Click here to see examples.

7. $2(x-1)=3(2x+1)+7$
8. $4-(2x+7)=x+5(x+1)$
9. $3-4(x+2)=5x+2(3x-2)$

Solve these equations which are special cases. Click here to see examples.

10. $3x+1=3x$
11. $2(x-4)=2x-8$
12. $5x-(1-x)=3(2x-1)+4$

Solve the equations.

13. $4-(x+1)=7x+2x$
14. $5x+7=4x-(-x-7)$
15. $2x+3(x+2)=x-(4-x)$
16. $-2(5x-4)=2+(x-11)$
17. $2x+4=8x-1$
18. $3x-8=4x-3(2x-4)$

Click here to see the solutions.

2.3 Introduction to Problem Solving

Translating English into mathematics can be challenging especially if you do not know the mathematical meaning of words. Developing techniques to help in this translation is the focus of this section.

An important part of problem solving is identifying the variable. The variable is always an unknown quantity. Often, the variable is the item we are asked to find. In the cases below the unknown quantity is the number we are asked to find and so we let the variable that number.

There are words we will consider "math" words when it comes to translation: sum (+), difference (-), product (·) and quotient (÷). These words not only indicate a specific operation, but they also indicate grouping. They keep statements in order. Consider the translation of these statements below.

- The sum of 2 and a number: $(2 + x)$.
- The difference of a number and 3: $(x - 3)$.
- The product of 2 and a number: $(2 \cdot x)$.
- The quotient of a number and 7: $(x \div 7)$.

There are other words we use in every day language that translate into mathematical expressions. These words take statements out of order when we express them in a mathematical sentence and they do not group.

- Three more than a number: $x + 3$
- Four less than a number: $x - 4$

Some other mathematically translated words are below.

- Double a number: $2 \cdot x$
- Twice a number: $2 \cdot x$
- Triple a number: $3 \cdot x$
- Subtract 5 from a number: $x - 5$

Examples: Translation of Phrases

EXAMPLE 1 *twice the sum of 4 and a number*

Solution: Let x represent the number.

$$\underbrace{\text{twice}}_{2\cdot}\text{ the sum of }\left(\underbrace{4}_{4}\underbrace{\text{and}}_{+}\underbrace{\text{a number}}_{x}\right)$$

Therefore, it translates to $2(4 + x)$.

PRACTICE 2 *triple the difference of a number and 7*

Solution: Click here to check your answer.

EXAMPLE 3 *the difference of twice a number and 8*

Solution: Let x represent the number.

$$\text{the difference of }\left(\underbrace{\text{twice}}_{2\cdot}\underbrace{\text{a number}}_{x}\underbrace{\text{and}}_{-}\underbrace{8}_{8}\right)$$

Therefore, it translates to $(2x - 8) = 2x - 8$. In this problem the word "and" translated into $-$ since we had the operator word "difference". Notice in this case the parentheses did not change the meaning of the translation but it is best to determine that after the translation.

PRACTICE 4 *the sum of triple a number and* 1
Solution: Click here to check your answer.

EXAMPLE 5 9 *less than five times the number*
Solution: This phrase does not get translated in the order that we say it. We can remember how this translation works by imagining the "than" to be the pivot word that flips the sentence. Let x represent the number.

$$\underbrace{\text{five times the number}}_{5x}\ \text{than}\ \underbrace{\text{less } 9}_{-9}$$

Therefore, it translates to $5x-9$

PRACTICE 6 11 *more than three times the number*
Solution: Click here to check your answer.

The last thing to build into an introduction of applications is the verb, that is, when we are no longer dealing with just phrases. The verb (often times "is") indicates an equality. With the incorporation of an equation we can get a specific number which will answer the problem.

Examples: Number Applications

EXAMPLE 7 *Triple a number is the same as double the sum of a number and* 3*. Find the number.*
Solution: Let x represent the number.

$$\underbrace{\text{triple a number}}_{3x}\ \underbrace{\text{is the same}}_{=}\ \underbrace{\text{double}}_{2\cdot}\ \text{the sum of}\ \left(\underbrace{\text{a number}}_{x}\ \underbrace{\text{and}}_{+}\ \underbrace{3}_{3}\right)$$

This translates to $3x=2(x+3)$. Next, find x.

$$\begin{array}{rcll} 3x &=& 2(x+3) & \text{RHS: distribute} \\ 3x &=& 2x+6 & \text{subtract } 2x \text{ from LHS \& RHS} \\ \underline{-2x} &=& \underline{-2x} & \text{combine like terms} \\ x &=& 6 & \end{array}$$

PRACTICE 8 *Seven less than a number is the same as triple the difference of* 2 *and the number. Find the number.*
Solution: Click here to check your answer.

EXAMPLE 9 *The product of a number and* 5 *is the same as* 8 *more than* 3 *times the number. Find the number.*
Solution: Let x represent the number.

$$\text{The product of}\ \left(\underbrace{\text{a number}}_{x}\ \underbrace{\text{and}}_{\cdot}\ \underbrace{5}_{5}\right)\ \underbrace{\text{is the same}}_{=}\ \underbrace{\text{3 times the number}}_{3\cdot x}\ \underbrace{\text{8 more}}_{+8}$$

This translates to $5x=3x+8$. Next, find x.

$$\begin{array}{rcll} 5x &=& 3x+8 & \text{subtract } 3x \text{ from LHS \& RHS} \\ \underline{-3x} &=& \underline{-3x} & \text{combine like terms} \\ 2x &=& 8 & \text{divide by the coefficient of } x \\ \frac{2x}{2} &=& \frac{8}{2} & \text{simplify} \\ x &=& 4 & \end{array}$$

PRACTICE 10 *Twice the sum of a number and* 4 *is equal to the sum of triple the number and* 8*.*
Solution: Click here to check your answer.

We also use these translations in what many refer to as story problems. Consider the examples below.

Examples: Story Applications

Example 11 *A 20 inch board is cut into three pieces. The first piece is 2 inches shorter than the middle piece and the last piece is twice the sum of the middle piece and 1. Find the length of the middle piece.*

Solution: Labeling is important in a story problem with multiple pieces. Often the variable will be what you are asked to find. In this case the length of the middle piece. The length of the first and last pieces is also unknown but there are descriptions of them. Look at each phrase as an individual problem of translation.

Let $x =$ length of the middle piece.

"The first piece is 2 inches shorter than the middle piece" translates as seen below when you flip the sentence around the word "than".

$$\underbrace{\text{The first piece}}_{\text{length of first piece}} \underbrace{\text{is}}_{=} \underbrace{\text{than the middle piece}}_{x} \underbrace{\text{2 inches shorter}}_{-2}$$

"The last piece is twice the sum of the middle piece and 1" is translated below.

$$\underbrace{\text{the last piece}}_{\text{length of last piece}} \underbrace{\text{is}}_{=} \underbrace{\text{twice}}_{2\cdot} \underbrace{\text{the sum of the middle piece and 1}}_{(x+1)}$$

Summing this up:

$$\begin{array}{lcl} x-2 & = & \text{the length of the first piece} \\ x & = & \text{the length of the middle piece} \\ 2(x+1) & = & \text{the length of the last piece} \end{array}$$

Now the length of the pieces when added together must equal the length of the board. That is,

$$\underbrace{\text{the length of the first piece}}_{x-2} + \underbrace{\text{the length of the middle piece}}_{x} + \underbrace{\text{the length of the last piece}}_{2(x+1)} = \underbrace{\text{the length of the whole board}}_{20}$$

Next, solve the equation.

$$\begin{array}{rcll} x-2+x+2(x+1) & = & 20 & \text{LHS: distribute} \\ x-2+x+2x+2 & = & 20 & \text{LHS: combine like terms} \\ 4x & = & 20 & \text{divide by coefficient of x} \\ \frac{4x}{4} & = & \frac{20}{4} & \text{simplify} \\ x & = & 5 & \end{array}$$

Therefore the length of the middle piece is 5 inches.

Practice 12 *There is a 15 foot board cut into three pieces. The middle piece is 1 foot longer than the first piece. The last piece is twice the difference of the first piece and 1. Find the length of the first piece.*

Solution: Click here to check your answer.

Example 13 *The boss makes \$40,000 more than the secretary. Together they make \$130,000. How much does the boss earn?*

Solution: This is an example where it is easier to choose the variable to be something other than the question. When we read this problem we notice that there is very little information about the secretary's salary. This is a good indication that you should let the variable be the secretaries salary.

$$\begin{array}{rcl} x & = & \text{secretary's salary} \\ x+40000 & = & \text{boss's salary} \end{array}$$

The equation comes from

$$\underbrace{\text{secretary's salary}}_{x} + \underbrace{\text{boss's salary}}_{x+40000} = \underbrace{\text{salaries together}}_{130000}$$

Next, solve $x + x + 40000 = 130000$ for x.

$$\begin{array}{rcll} x+x+40000 & = & 130000 & \text{LHS: combine like terms} \\ 2x+40000 & = & 130000 & \text{Gather constant terms to RHS by subtracting 40000} \\ \underline{-40000} & = & \underline{-40000} & \text{combine like terms} \\ 2x & = & 90000 & \text{divide by the coefficient of } x \\ \frac{2x}{2} & = & \frac{90000}{2} & \text{simplify} \\ x & = & 45000 & \end{array}$$

So the secretaries salary is \$45,000. But, we were asked to find the boss's salary which is $x + 40000 = 45000 + 40000 = 85000$ when we substitute in for x. Therefore, the boss makes \$85,000.

Practice 14 *The janitor makes \$10,000 less than the secretary. Together they make \$70,000. How much does the janitor earn?*
Solution: Click here to check your answer.

Solutions to Practice Problems:

Practice 2

Solution: Let x represent the number.

$$\underbrace{\text{triple}}_{3\cdot}\text{ the difference of }\left(\underbrace{\text{a number}}_{x}\underbrace{\text{and}}_{-}\underbrace{7}_{7}\right)$$

Therefore, it translates to $3(x-7)$. Back to Text

Practice 4

Solution: Let x represent the number.

$$\text{the sum of }\left(\underbrace{\text{triple}}_{3\cdot}\ \underbrace{\text{a number}}_{x}\underbrace{\text{and}}_{+}\underbrace{1}_{1}\right)$$

Therefore, it translates to $(3x+1)=3x+1$. Back to Text

Practice 6

Solution: Let x represent the number.

$$\underbrace{\text{three times the number}}_{3x}\text{ than }\underbrace{\text{more 11}}_{+11}$$

Therefore, it translates to $3x+11$. Back to Text

Practice 8

Solution: Let x represent the number.

$$\underbrace{\text{than a number}}_{x}\underbrace{\text{Seven less}}_{-7}\underbrace{\text{is the same}}_{=}\underbrace{\text{triple}}_{3\cdot}\text{ the difference of }\left(\underbrace{2}_{2}\underbrace{\text{and}}_{-}\underbrace{\text{the number}}_{x}\right)$$

This translates to $x-7=3(2-x)$. Next, find x.

$$\begin{array}{rcll}
x-7 & = & 3(2-x) & \text{RHS: distribute} \\
x-7 & = & 6-3x & \text{add } 3x \text{ to LHS \& RHS} \\
\underline{+3x} & = & \underline{+3x} & \text{combine like terms} \\
4x-7 & = & 6 & \text{add 7 to LHS \& RHS} \\
\underline{+7} & = & \underline{+7} & \text{combine like terms} \\
4x & = & 13 & \text{divide by 4 to LHS \& RHS} \\
\frac{4x}{4} & = & \frac{13}{4} & \text{simplify} \\
x & = & \frac{13}{4} & \text{convert to mixed number} \\
x & = & 3\frac{1}{4} &
\end{array}$$

Back to Text

Practice 10

Solution: Let x represent the number.

$$\underbrace{\text{Twice}}_{2\cdot}\text{ the sum of }\left(\underbrace{\text{a number}}_{x}\underbrace{\text{and}}_{+}\underbrace{4}_{4}\right)\underbrace{\text{is equal to}}_{=}\text{ the sum of }\left(\underbrace{\text{triple the number}}_{3\cdot x}\underbrace{\text{and}}_{+}\underbrace{8}_{8}\right)$$

This translates to $2(x+4)=(3x+8)$. Next, find x.

$$\begin{array}{rcll}
2(x+4) & = & 3x+8 & \text{LHS: distribute} \\
2x+8 & = & 3x+8 & \text{subtract } 3x \text{ from LHS \& RHS} \\
\underline{-3x} & = & \underline{-3x} & \text{combine like terms} \\
-x+8 & = & 8 & \text{subtract 8 from LHS \& RHS} \\
\underline{-8} & = & \underline{-8} & \text{combine like terms} \\
-x & = & 0 & \text{divide by the coefficient of } x \\
\frac{-x}{-1} & = & \frac{0}{-1} & \text{simplify} \\
x & = & 0 &
\end{array}$$

Back to Text

Practice 12

Solution: Let $x =$ length of the first piece.

"The middle piece is 1 foot longer than the first piece." translates as seen below when you flip the sentence around the word "than".

$$\underbrace{\text{The middle piece}}_{\text{length of middle piece}} \quad \underbrace{\text{is}}_{=} \underbrace{\text{(than the first piece)}}_{x} \underbrace{\text{1 feet longer}}_{+1}$$

" The last piece is twice the difference of the first piece and 1." is translated below.

$$\underbrace{\text{the last piece}}_{\text{length of last piece}} \quad \underbrace{\text{is}}_{=} \underbrace{\text{twice}}_{2\cdot} \underbrace{\text{the difference of the first piece and 1}}_{(x-1)}$$

Summing this up:

$$\begin{array}{lcl} x & = & \text{the length of the first piece} \\ x+1 & = & \text{the length of the middle piece} \\ 2(x-1) & = & \text{the length of the last piece} \end{array}$$

Now the length of the pieces when added together must equal the length of the board. That is,

$$\underbrace{\text{the length of the first piece}}_{x} + \underbrace{\text{the length of the middle piece}}_{x+1} + \underbrace{\text{the length of the last piece}}_{2(x-1)} = \underbrace{\text{the length of the whole board}}_{15}$$

Next, solve the equation.

$$\begin{array}{rcll} x+x+1+2(x-1) & = & 15 & \text{LHS: distribute} \\ x+x+1+2x-2 & = & 15 & \text{LHS: combine like terms} \\ 4x-1 & = & 15 & \text{LHS \& RHS: } +1 \\ \underline{+1} & = & \underline{+1} & \text{LHS \& RHS: combine like terms} \\ 4x & = & 16 & \text{divide by coefficient of x} \\ \frac{4x}{4} & = & \frac{16}{4} & \text{simplify} \\ x & = & 4 & \end{array}$$

Therefore the length of the first piece is 4 feet. Back to Text

Practice 14

Solution:

$$\begin{array}{rcl} x & = & \text{secretary's salary} \\ x-10000 & = & \text{janitor's salary} \end{array}$$

The equation comes from

$$\underbrace{\text{secretary's salary}}_{x} + \underbrace{\text{janitor's salary}}_{x-10000} = \underbrace{\text{salaries together}}_{70000}$$

Next, solve $x + x - 10000 = 70000$ for x.

$$\begin{array}{rcll} x+x-10000 & = & 70000 & \text{LHS: combine like terms} \\ 2x-10000 & = & 70000 & \text{Gather constant terms to RHS by adding 10000} \\ \underline{+10000} & = & \underline{+10000} & \text{combine like terms} \\ 2x & = & 80000 & \text{divide by the coefficient of } x \\ \frac{2x}{2} & = & \frac{80000}{2} & \text{simplify} \\ x & = & 40000 & \end{array}$$

So the secretaries salary is \$40,000. But, we were asked to find the janitor's salary which is $x - 10000 = 40000 - 10000 = 30000$ when we substitute in for x. Therefore, the janitor makes \$30,000. Back to Text

2.3.1 Exercises 2.3

Translate the phrase into a mathematical expression. Click here to see examples.

1. triple the difference of 6 and double a number
2. the sum of twice a number and 4
3. five times the difference of 3 and double a number
4. 7 more than twice a number
5. 8 less than triple a number
6. subtract double a number from 9

Write an equation for the following problems and solve. Click here to see examples.

7. Triple the difference of a number and 4 is 6.

8. Three less than twice the sum of a number and 5 is 8 more than the number.

9. Subtracting 7 from triple a number yields three times the number.

10. The sum of twice a number and 4 is the same as 6 less than the number.

11. The difference of 3 and the sum of a number and 4 is 15.

12. Twice the sum of a number and 4 is equal to 6 less than triple the number.

Write an equation for the following problems and solve. Click here to see examples.

13. A 23 foot board is cut into two pieces. The second piece is 1 inch shorter than triple the length of the first piece. Find the length of the larger piece.

14. A 52 inch string is cut into three pieces. The second piece is 3 inches shorter than twice the length of the first piece and the third piece is 2 inches shorter than the second piece. Find the length of the smallest piece.

15. A 37 centimeter board is cut into four pieces. The first piece and the second piece are the same length. Similarly, the third and the fourth piece are the same length. If the fourth piece is quadruple the difference of the first piece and 1, find the length of the second piece.

16. The cook makes $7,000 more than the waitress. Together they make $57,000. How much does the cook earn?

17. The owner of a business earns twice as much as the secretary. Together they make $79,200. How much does the owner earn?

18. The principal's salary is twice the sum of the salary of the librarian and $5000. The teacher makes twice as much as the librarian. If all together they make $145,000, how much does the principal earn?

Click here to see the solutions.

2.4 Computation with Formulas

Formulas are used constantly in our modern world. Buying carpeting requires a formula for area, filling a pool with water requires a formula for calculating volume, even calculating taxes on your purchase requires a formula. These are just a few that happen behind the scenes of our every day world. In this section our focus will not be on the development of the formulas but on calculating given the formulas.

Examples: Evaluate a given Formula

EXAMPLE 1 *Given the formula, $V = lwh$, if $V = 108$, $l = 4$, $h = 9$, find w.*

Solution: Substitute into the equation and solve.

V	$=$	lwh	Substitute the values of the known variables
108	$=$	$(4)w(9)$	Use the commutative property and multiply
108	$=$	$36w$	Divide by the coefficient of w
$\frac{108}{36}$	$=$	$\frac{36w}{36}$	Simplify
3	$=$	w	

Practice 2 *Given the formula, $V = lwh$, if $V = 210$, $h = 7$, $w = 3$, find l.*
Solution: Click here to check your answer.

Example 3 *Given the formula, $A = \frac{1}{2}h(B + b)$, if $A = 48$, $h = 8$, $b = 3$ find B.*
Solution: Substitute into the equation and solve.

$$\begin{array}{rcll}
A &=& \frac{1}{2}h(B+b) & \text{Substitute the values of the known variables} \\
48 &=& \frac{1}{2}(8)(B+3) & \text{RHS: Multiply} \\
48 &=& 4(B+3) & \text{RHS: distribute} \\
48 &=& 4B+12 & \text{LHS \& RHS: } -12 \\
\underline{-12} && \underline{\quad -12} & \text{Combine like terms} \\
36 &=& 4B & \text{Divide by the coefficient of } B \\
\frac{36}{4} &=& \frac{4B}{4} & \text{Simplify} \\
9 &=& B &
\end{array}$$

Practice 4 *Given the formula, $y - y_1 = m(x - x_1)$, if $y = 1$, $y_1 = -2$, $x = -3$, $x_1 = -1$, find m.*
Solution: Click here to check your answer.

A perimeter is the length around an object. Perimeter is used to calculate such things as fencing around a yard, trimming a piece of material, and the amount of baseboard needed for a room. While for a perimeter it is not necessary to have a formula since it is always just calculated by adding the lengths of all sides, sometimes a formula can be useful. If we have a rectangle with width w and length l, the formula for the perimeter is $P = 2w + 2l$. Consider the following perimeter examples.

Examples: Perimeter

Example 5 *Calculate how much fencing is needed for a rectangular garden with a width of 10 feet and a length of 15 feet.*
Solution: The formula for the perimeter of a rectangle is $P = 2w + 2l$. In this example $w = 10$ and $l = 15$. Now, substitute these numbers in for the variables. $P = 2w + 2l = 2(10) + 2(15) = 20 + 30 = 50$. That is, 50 feet of fencing is needed.

Practice 6 *Calculate how much fencing is needed for a rectangular garden with a width of 8 feet and a length of 11 feet.*
Solution: Click here to check your answer.

Example 7 *Calculate how long a garden can be if the width is 6 feet and you have 26 feet of fencing to enclose the garden.*
Solution: The formula for the perimeter of a rectangle is $P = 2w + 2l$. In this example $w = 6$ and $P = 26$. Now, substitute these numbers in for the variables. $26 = 2(6) + 2l$. Solve for l.

$$\begin{array}{rcll}
26 &=& 2(6)+2l & \text{RHS: simplify} \\
26 &=& 12+2l & \text{Constant terms to the LHS: subtract 12} \\
\underline{-12} &=& \underline{-12} & \text{Simplify} \\
14 &=& 2l & \text{Divide by coefficient of } l \\
\frac{14}{2} &=& \frac{2l}{2} & \text{Simplify} \\
7 &=& l &
\end{array}$$

Therefore, the length is 7 feet.

Practice 8 *Calculate how long a garden can be if the width is 8 feet and you have 36 feet of fencing to enclose the garden.*
Solution: Click here to check your answer.

Example 9 *Calculate (to the nearest tenth) how much fencing is needed to enclose a round pool if the diameter of the pool is 28 ft.*
Solution: The formula for the perimeter of a rectangle is $C = 2r\pi$ where π is estimated using 3.14. In this example, the diameter is $28ft$ therefore the radius is half the diameter so $r = \frac{28}{2} = 14$. Substituting we get the equation $C = 2(14)\pi = 2(14)(3.14) = 87.92$. Rounding to the nearest tenth, this pool requires $87.9ft$ of fencing.

Practice 10 *Calculate how much lace would be needed to trim a round tablecloth whose diameter is 5 feet. (Use 3.14 as an estimate for π.)*
Solution: Click here to check your answer.

Example 11 *A triangular shaped pool is known to have the length of the shortest side 2 ft less than the length of the next to shortest side and the longest side is 1 ft more than the length of the next to the shortest side. If it took 71 ft of fencing to enclose it, calculate the length of the shortest side.*
Solution: For this problem we use the translations learned in the last section and then sum up the sides to get the perimeter.

$$\begin{aligned} x-2 &= \text{length of the shortest side} \\ x &= \text{length of the next to shortest side} \\ x+1 &= \text{length of the longest side} \end{aligned}$$

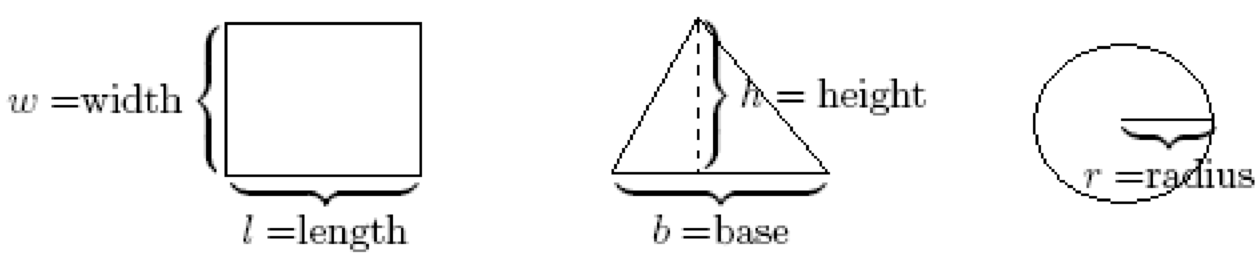
Adding all these up we get the perimeter is $x-2+x+x+1=3x-1$. Therefore the equation to solve is $3x-1=71$.

$$\begin{array}{rcll} 3x-1 &=& 71 & \text{LHS \& RHS: } +1 \\ \underline{+1} & & \underline{+1} & \text{combine like terms} \\ 3x &=& 72 & \text{divide by coefficient of } x \\ \frac{3x}{3} &=& \frac{72}{3} & \text{simplify} \\ x &=& 24 & \end{array}$$

Therefore, the length of the shortest side is $x-2=24-2=22$ ft.

Practice 12 *A horse pen has five sides and it takes 93 ft of fencing to enclose it. It is known that three sides are all the same length and the remaining two sides also have the same length which is 4 ft more than the other sides. What are the dimensions of the pen?*
Solution: Click here to check your answer.

Area is used when discussing carpeting a room, painting a wall or covering any kind of surface. The formulas for area that we will use are area of a rectangle $A=lw$, triangle $A=\frac{1}{2}bh$ and circle $A=\pi r^2$ where A represents area in each set and is always expressed in square units. The pictures below show what the other variables represent in the formulas.



Examples: Area

Example 13 *Calculate the width of a floor in a rectangular room, if the area of the carpet covering the floor is known to be 105 ft^2 (square feet) and the length is 15 ft.*
Solution: The formula for the area of a rectangle is $A=lw$. In this example $A=105$ ft^2 and $l=15$ ft. Substituting, we get the equation $105=15w$. Solve for w.

$$\begin{array}{rcll} 15w &=& 105 & \text{divide by coefficient of } w \\ \frac{15w}{15} &=& \frac{105}{15} & \text{simplify} \\ w &=& 7 & \end{array}$$

Therefore, the width of the room is 7 ft.

Practice 14 *Calculate the length of a rectangular pool, if the solar cover is 221 ft^2 (square feet) and the width is 13 ft.*
Solution: Click here to check your answer.

Example 15 *Calculate the base of a triangle, if the area of the triangle is 40 cm^2 and the height is 10 cm.*
Solution: The formula for the area of a triangle is $A = \frac{1}{2}bh$. In this example $A = 40$ cm^2 and $h = 10$ cm. Substituting, we get the equation $40 = \frac{1}{2}b(10)$. Solve for b.

$$\begin{array}{rcll} \frac{1}{2}b(10) & = & 40 & \frac{1}{2}\cdot 10 \text{ using the commutative property} \\ 5b & = & 40 & \text{divide by coefficient of } b \\ \frac{5b}{5} & = & \frac{40}{5} & \text{simplify} \\ b & = & 8 & \end{array}$$

Therefore, the base of the triangle is 8 cm.

Practice 16 *Calculate the height of a triangle, if the area of the triangle is 88 in^2 and the base is 16 in.*
Solution: Click here to check your answer.

Example 17 *Calculate the area needed for a cover on a round pool if the diameter is 27 ft.*
Solution: The formula for the area of a circle is $A = \pi r^2$ where π is estimated using 3.14. Since the diameter is 27 ft the radius is half the diameter $\frac{27}{2} = 13.5$. In this example $r = 13.5$. Substituting we get the equation $A = (3.14)(13.5)^2$. Solve for A.

$$\begin{array}{rcll} A & = & (3.14)(13.5)^2 & \text{Order of operations requires exponents first} \\ A & = & (3.14)(182.25) & \text{multiply} \\ A & = & 572.265 & \end{array}$$

Therefore, the area is 572.265 ft^2.

Practice 18 *Calculate how much material would be needed to make a round tablecloth whose diameter is 5 feet. (Use 3.14 as an estimate for π.)*
Solution: Click here to check your answer.

Solutions to Practice Problems:

Practice 2

Solution: Substitute into the equation and solve.

$$\begin{array}{rcll} V & = & lwh & \text{Substitute the values of the known variables} \\ 210 & = & l(3)(7) & \text{Multiply} \\ 210 & = & 21l & \text{Divide by the coefficient of } l \\ \frac{210}{21} & = & \frac{21l}{21} & \text{Simplify} \\ 10 & = & l & \end{array}$$

Back to Text

Practice 4

Solution: Substitute into the equation and solve.

$$\begin{array}{rcll} y - y_1 & = & m(x - x_1) & \text{Substitute the values of the known variables} \\ 1 - (-2) & = & m(-3 - (-1)) & \text{LHS \& RHS: Combine like terms} \\ 3 & = & -2m & \text{Divide by the coefficient of } m \\ \frac{3}{-2} & = & \frac{-2m}{-2} & \text{Simplify} \\ -\frac{3}{2} & = & m & \text{Convert to a mixed number} \\ -1\frac{1}{2} & = & m & \end{array}$$

Back to Text

Practice 6

Solution: The formula for the perimeter of a rectangle is $P = 2w + 2l$. In this example $w = 8$ and $l = 11$. Now, substitute these numbers in for the variables. $P = 2w + 2l = 2(8) + 2(11) = 16 + 22 = 38$. That is, 38 feet of fencing is needed. Back to Text

Practice 8

Solution: Solution: The formula for the perimeter of a rectangle is $P = 2w + 2l$. In this example $w = 8$ and $P = 36$. Now, substitute these numbers in for the variables. $26 = 2(6) + 2l$. Solve for l.

$$\begin{array}{rcll} 36 & = & 2(8) + 2l & \text{RHS: simplify} \\ 36 & = & 16 + 2l & \text{Constant terms to the LHS: subtract 12} \\ \underline{-16} & = & \underline{-16} & \text{Simplify} \\ 20 & = & 2l & \text{Divide by coefficient of } l \\ \frac{20}{2} & = & \frac{2l}{2} & \text{Simplify} \\ 10 & = & l & \end{array}$$

Therefore, the length is 10 feet. Back to Text

Practice 10

Solution: The formula for the perimeter of a rectangle is $C = 2r\pi$ where π is estimated using 3.14. In this example, the diameter is 5 ft therefore the radius is half the diameter so $r = \frac{5}{2}$. Substituting we get the equation $C = 2(\frac{5}{2})\pi = 5(3.14) = 15.7$ ft. Rounding to the nearest tenth, the requires 15.7 ft.

Back to Text

Practice 12

Solution:

$$\begin{aligned} x &= \text{length of first side} \\ x &= \text{length of second side} \\ x &= \text{length of third side} \\ x+4 &= \text{length of fourth side} \\ x+4 &= \text{length of fifth side} \end{aligned}$$

Adding all these up we get the perimeter is $x+x+x+x+4+x+4 = 5x+8$. Therefore the equation to solve is $5x+8 = 93$.

$$\begin{array}{rcll} 5x+8 & = & 93 & \text{LHS \& RHS: } +1 \\ \underline{-8} & & \underline{-8} & \text{combine like terms} \\ 5x & = & 85 & \text{divide by coefficient of } x \\ \frac{5x}{5} & = & \frac{85}{5} & \text{simplify} \\ x & = & 17 & \end{array}$$

Therefore, the length of the third and fourth sides is $x+4 = 17+4 = 21$ ft. So the dimensions of the pen is 17 ft $\times$ 17 ft $\times$ 17 ft $\times$21 ft $\times$21 ft. Back to Text

Practice 14

Solution: The formula for the area of a rectangle is $A = lw$. In this example $A = 221$ ft^2 and $w = 13$ ft. Substituting, we get the equation $221 = l(13)$. Solve for w.

$$\begin{array}{rcll} 13l & = & 221 & \text{divide by coefficient of } l \\ \frac{13l}{13} & = & \frac{221}{13} & \text{simplify} \\ l & = & 17 & \end{array}$$

Therefore, the length of the pool is 17 ft. Back to Text

Practice 16

Solution: The formula for the area of a triangle is $A = \frac{1}{2}bh$. In this example $A = 88$ in^2 and $b = 16$ in. Substituting, we get the equation $88 = \frac{1}{2}(16)h$. Solve for h.

$$\begin{array}{rcll} \frac{1}{2}(16)h & = & 88 & \text{multiply} \\ 8h & = & 88 & \text{divide by coefficient of } b \\ \frac{8h}{8} & = & \frac{88}{8} & \text{simplify} \\ h & = & 11 & \end{array}$$

Therefore, the height of the triangle is 11 in.

Back to Text

Practice 18

Solution: The formula for the area of a circle is $A = \pi r^2$ where π is estimated using 3.14. Since the diameter is 5 ft the radius is half the diameter $\frac{5}{2} = 2.5$. In this example $r = 2.5$. Substituting we get the equation $A = (3.14)(2.5)^2$. Solve for A.

$$\begin{array}{rcll} A & = & (3.14)(2.5)^2 & \text{Order of operations requires exponents first} \\ A & = & (3.14)(6.25) & \text{Multiply} \\ A & = & 19.625 & \end{array}$$

Therefore, the needed material is 19.625 ft^2. Back to Text

2.4.1 Exercises 2.4

Solve for the indicated variable. Click here to see examples.

1. Given the formula, $I = Prt$, and $P = \$7000$, $r = 0.05$, $t = 3$, find I.

2. Given the formula, $d = rt$, and $d = 400$, $r = 55$, find t.

3. Given the formula, $d = rt$, and $d = 125$, $t = 2$, find r.

4. Given the formula, $y = mx + b$, and $y = 12$, $x = 3$, $b = -5$, find m.

5. Given the formula, $y = mx + b$, and $y = 15$, $m = -2$, $x = 4$, find b.

Solve the problems. Click here to see examples.

6. A rectangular garden is 5 feet long and 4 feet wide. How much fencing is needed to completely enclose it?

7. If a rectangular room requires 66 feet of baseboard and the length is 18 feet, how wide is the room?

8. A square garden has length 7 feet. How much fencing is needed to completely enclose it?

9. Approximately (to the nearest tenth) how much lace is needed to trim a circular table cloth whose diameter is 10 feet? (Use 3.14 as an estimate for π.)

10. Approximately (to the nearest tenth) how much lace is needed to trim a circular table cloth whose diameter is 12 feet? (Use 3.14 as an estimate for π.)

11. A triangular shaped pig pen has 2 sides that are the same length the third side is 2 feet less than the sum of the lengths of the other two sides. If 61 feet of fencing is needed how long is the longest side?

Solve the problems. Click here to see examples.

12. Determine how much carpeting would be needed for a 10 ft by 12 ft room.

13. What is the length of a room whose width is 5 ft and the amount of carpeting needed to cover the floor is 55 ft^2?

14. Find the area of a triangle whose base is 7 in and height is 9 in.

15. A triangular shaped wall is used for partitioning a room. Determine the length of the base given that its height is known to be 6 ft and 42 ft^2 of wallpaper was used to cover it.

16. Find the area of a circle whose diameter is known to be 8 m. (Use $\pi = 3.14$ and round to the nearest tenth.)

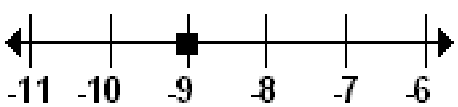
17. Find the area of a circle whose radius is known to be 2.1 cm. (Use $\pi = 3.14$ and round to the nearest tenth.)

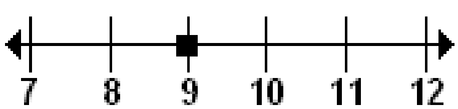
Click here to see the solutions.

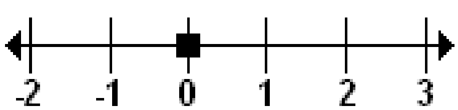
Chapter 3

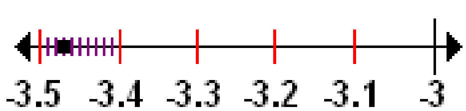
Solutions to Exercises

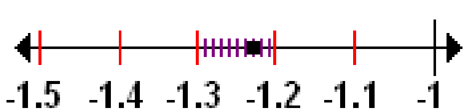
Solutions to Exercises 1.1

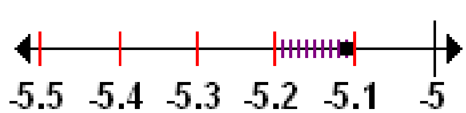
1. rational, integers
2. rational, integers, whole numbers, positive integers
3. rational
4. rational
5. irrational
6. rational
7. 

8. 

9. 

10. 

11. 

12. 

13. True
14. False
15. True
16. True
17. False
18. False

Back to Exercises

Solutions to Exercises 1.2

1. 50
2. 33
3. $\frac{3}{7}$
4. 3.5
5. 21
6. 55
7. -8
8. 7
9. 147
10. 125
11. 2103
12. 2906
13. -922
14. -1057
15. 2
16. 10
17. 7
18. 5
19. 8
20. 4
21. 91.38
22. 262.47
23. 3706
24. 8411
25. false
26. true
27. false
28. true

29. true
30. false

[Back to Exercises]

Solutions to Exercises 1.3

1. 32
2. 21
3. 36
4. 263
5. 1779
6. 784
7. -5
8. -21
9. -99
10. -198
11. 720
12. 1050
13. -66
14. -164
15. 0
16. -30
17. -21
18. -31
19. -144
20. -355
21. -3876
22. -8238.1
23. 539.67
24. 2.43
25. true
26. true
27. false
28. false
29. true
30. false
31. -5
32. -123
33. -11
34. -7
35. 12
36. -24

[Back to Exercises]

Solutions to Exercises 1.4

1. $161,668$
2. $108,664$
3. -63
4. -48
5. 24
6. -45
7. 140
8. -740
9. $-1,260$
10. $-4,602$
11. $13,176$
12. $133,518$
13. 0.000072
14. 0.00002008
15. 0.056714
16. -25.9
17. -0.14416
18. 370.5576
19. $false$
20. $true$
21. $false$

[Back to Exercises]

Solutions to Exercises 1.5

1. 6
2. 7
3. -6
4. -9
5. 8
6. 6
7. -13
8. -6
9. 27

10. -24
11. 141
12. 58
13. -524
14. -643
15. 811
16. 0.08
17. 3024.41
18. -4000.1
19. $50,700,000$
20. -61.4
21. -35.005
22. $25\frac{5}{7}$
23. $37\frac{6}{11}$
24. $141\frac{4}{15}$

Back to Exercises

Solutions to Exercises 1.6

1. 3200
2. $54,300,000$
3. $9,021,000$
4. 100
5. $400,000$
6. $8,120,000,000$
7. 4.05×10^5
8. 3.2×10^3
9. 1.23×10^7
10. 1.589×10^6
11. 2.1×10^8
12. 1×10^3
13. 64
14. 216
15. 64
16. 9
17. 1.21
18. 1.44
19. -1.21
20. -1.44
21. -27
22. -125
23. 1
24. -5
25. 1
26. -1
27. -1
28. $-1,000,000$
29. $10,000,000$
30. $1,000,000$
31. -0.064
32. .01
33. 1
34. undefined
35. 0
36. 1

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Solutions to Exercises 1.7

1. 19
2. 32
3. -68
4. -4
5. -4
6. -33
7. -3
8. 3
9. 35
10. 46
11. undefined
12. 0
13. 1.09
14. 1.2
15. -5.4

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Solutions to Exercises 1.8

1. composite
2. prime

3. prime
4. composite
5. composite
6. composite
7. $\{1, 2, 3, 5, 6, 10\}$
8. $\{1, 2, 3, 6, 11\}$
9. $\{1, 2\}$
10. $\{1, 2, 3, 4, 6, 8, 12\}$
11. $\{1, 2, 3, 6, 9\}$
12. $\{1, 5\}$
13. $3 \cdot 5$
14. $2 \cdot 3^2 \cdot 5$
15. $2 \cdot 5$
16. 2^4
17. $2^2 \cdot 3$
18. $2^3 \cdot 3$
19. 15
20. 6
21. 2
22. 8
23. 1
24. 2
25. 90
26. 60
27. 60
28. 48
29. 33
30. 16

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Solutions to Exercises 1.9

1. $\frac{.5}{100} = \frac{1}{200}, 0.005$
2. $\frac{27}{100}, 0.27$
3. $5\%, \frac{5}{100} = \frac{1}{20}$
4. $112\%, \frac{112}{100} = \frac{28}{25}$
5. $8\%, 0.08$
6. $2.5\%, 0.025$
7. $30\%, 0.3$
8. $0.003, \frac{3}{1000}$
9. $0.0035, \frac{35}{10000} = \frac{7}{2000}$
10. 0.04
11. 1.6
12. 1.11
13. $\frac{20}{48}$
14. $-\frac{8}{48}$
15. $\frac{32}{48}$
16. $\frac{3}{100}$
17. $\frac{12}{1000} = \frac{3}{250}$
18. $-\frac{11}{10000}$
19. $\frac{23}{50}$
20. $-\frac{200}{167} = -1\frac{33}{167}$
21. $\frac{132}{50} = \frac{66}{25} = 2\frac{16}{25}$
22. $\frac{2}{5}$
23. $-\frac{3}{21} = -\frac{1}{7}$
24. $-\frac{5}{9}$
25. $\frac{3}{11}$
26. $\frac{5}{3} = 1\frac{2}{3}$
27. $-\frac{9}{4} = -2\frac{1}{4}$
28. $-3\frac{6}{7}$
29. $5\frac{5}{6}$
30. $-30\frac{9}{10}$
31. $\frac{21}{8}$
32. $-\frac{25}{7}$
33. $-\frac{63}{5}$
34. $\frac{1}{2}$
35. $-\frac{9}{2} = -4\frac{1}{2}$
36. -12
37. $\frac{1}{28}$
38. $\frac{25}{21} = 1\frac{4}{21}$
39. $\frac{1}{3}$
40. $-\frac{19}{12} = -1\frac{7}{12}$
41. $\frac{103}{60} = 1\frac{43}{60}$
42. $\frac{37}{11} = 3\frac{4}{11}$
43. $\frac{10}{7} = 1\frac{3}{7}$
44. $-\frac{3}{2} = -1\frac{1}{2}$

45. -4

46. 11

47. $-\frac{1}{3}$

48. 8

49. 4

50. $-\frac{5}{4} = -1\frac{1}{4}$

51. $\frac{18}{23}$

52. $-\frac{9}{5} = -1\frac{4}{5}$

53. $\frac{11}{4} = 2\frac{3}{4}$

54. $\frac{3}{20}$

55. $\frac{7}{12}$

56. $-\frac{1}{3}$

57. $-\frac{5}{16}$

58. $\frac{1}{15}$

59. $-\frac{27}{5} = -5\frac{2}{5}$

60. $\frac{13}{6} = 2\frac{1}{6}$

Back to Exercises

Solutions to Exercises 1.10

1. 9
2. 3
3. 2
4. -2
5. not real
6. not real
7. 12
8. 63
9. not real
10. -6
11. 5
12. -6
13. $10 < \sqrt{104} < 11$
14. $7 < \sqrt{55} < 8$
15. $-4 < \sqrt[3]{-33} < -3$

Back to Exercises

Solutions to Exercises 1.11

1. commutative property of addition
2. commutative property of multiplication
3. distributive property
4. associative property of addition
5. commutative property of addition
6. associative property of multiplication
7. -23
8. 0
9. $5\frac{3}{4}$
10. $\frac{1}{12}$
11. 1
12. $-\frac{7}{4}$
13. multiplicative identity
14. multiplicative inverse
15. additive inverse
16. additive identity
17. distributive property
18. multiplicative identity
19. commutative property of multiplication
20. commutative property of multiplication
21. associative property of addition

Back to Exercises

Solutions to Exercises 2.1

1. $3; 1; -2; 1$
2. $2; 0; 3; 0$
3. $4; -1; 0; -9$
4. $2x^2 + 2x - 2$
5. $-3m + 6n - 5$
6. $-6x + 34$
7. $12m^2 + 9m$
8. $21x - 0.6y$
9. $-0.37x + 1.1$
10. $-6x - 5$
11. $8x - \frac{11}{2}$
12. 1

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Solutions to Exercises 2.2

1. not a solution
2. solution

3. solution
4. 5
5. -15
6. $-\frac{5}{3} = -1\frac{2}{3}$
7. -3
8. -1
9. $-\frac{1}{15}$
10. no solution
11. all real numbers
12. no solution
13. $\frac{3}{10}$
14. all real numbers
15. $-\frac{10}{3} = -3\frac{1}{3}$
16. $\frac{17}{11} = 1\frac{6}{11}$
17. $\frac{5}{6}$
18. 4

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Solutions to Exercises 2.3

1. $3(6-2x)$
2. $(2x+4) = 2x+4$
3. $5(3-2x)$
4. $2x+7$
5. $3x-8$
6. $9-2x$
7. 6
8. 1
9. There is no such number
10. -10
11. -16
12. 14
13. 17 feet
14. 12 inches
15. 4.5 centimeters
16. $\$32,000$
17. $\$52,800$
18. $\$64,000$ Back to Exercises

Solutions to Exercises 2.4

1. \$1050
2. $7\frac{3}{11}$
3. $62\frac{1}{2}$
4. $\frac{17}{3} = 5\frac{2}{3}$
5. 23
6. 18 ft
7. 15 ft
8. 28 ft
9. 31.4 ft
10. 37.7 ft
11. 29.5 ft
12. 120 ft^2
13. 11 ft
14. $31.5 = 31\frac{1}{2}$ in^2
15. 14 ft
16. 50.2 m^2
17. 13.8 cm^2

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