230K - Factor Timing Project

Submitted by Team 07

Akshay Joshi
Alexandre Godard
Bingcheng Liu
Garik Vardanyan
Khalil Belghouate
Prajakta Chavan
Prakarsh Duhoon
Yanggang (Daniel) Fang
Yvonne Zhu

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1. Executive Summary

In this project, we attempt to solve the problem of managing a certain set out wealth of our asset owner Elon Musk, especially for retirement planning. In order to model the problem, we analyse the risk factors Elon Musk is exposed to. Given his wealth is highly concentrated in Tesla Inc. stock, we mostly focus on the risk factors for Tesla shares. These risk factors shape the choice of our portfolio constituents' assets which are centered around achieving returns while hedging the major risk factors faced by our client.

Our reference portfolio is Ray Dalio's "All Weather" portfolio: 30% Equities, 55% Treasury Bonds, 15% Hard Assets (7.5% Gold and 7.5% Commodities). We then experimented with two portfolio schemes: 1. Traditional Portfolio Optimization with Musk's Risk Factor Hedging Assets; 2. Valuation-Timing Portfolios with Tag-Based Allocation. The first set of hedging asset results suggested poorer performance than that of "All Weather" due to the overweights on risk-free like treasury bonds. **<u>Valuation-Timing Portfolios with "Premium" and "Discount" tags with momentum-based weights (buy premium sell discount) suggest an excellent risk-return profile for Musk's retirement portfolio.</u>**

2. Reference Portfolio

Our client is Elon Musk. Based on his risk profiles, particularly his familial responsibilities and exposure to Tesla equities, we intend to build his retirement portfolio that hedges his fundamental risks while exploring the plausibility of timing factor risk premia.

2.1 All Weather Portfolio

Reference portfolio provides a strategic portfolio framework based on the fund and personal account's liability structure. Based on Musk's liabilities and the main purpose here is to provide him and his family a stable stream of cash flows, we will use Ray Dalio's "All Weather" portfolio as the baseline. The "All Weather" portfolio is designed to protect investors from inflationary and deflationary environments and generates returns in both economic growth and stagnation periods.

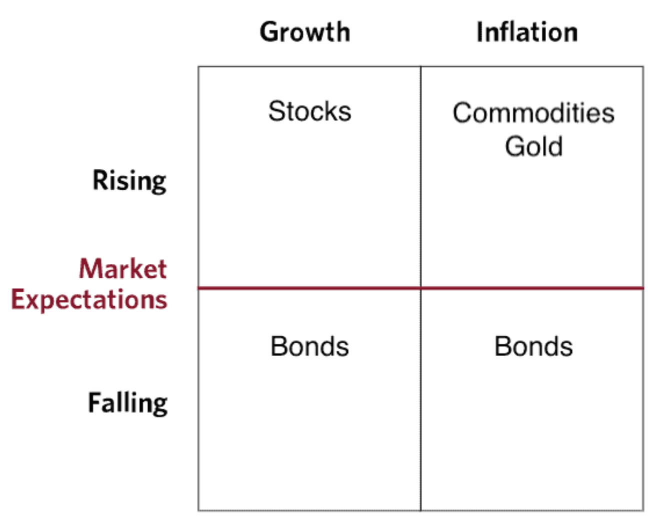


Figure 1: All Weather Components

The portfolio consists of 30% Equities, 55% Treasury Bonds, 15% Hard Assets (7.5% Gold and 7.5% Commodities). These three categories cover all four economic scenarios:

1. During growth periods, exposure to equities boosts retirement income while commodities and gold provide hedges to inflation
2. During economic downturns, bonds can shine in and buffer the portfolio against major tough turns.

2.2 Rebalancing Frequency Selection

We tested various rebalancing frequencies for the reference portfolio: monthly, quarterly, semi-annually, and annually with the intention of incorporating transaction and holding costs. Theoretically, since the All-Weather portfolio features a constant allocation to different asset classes, the only determinant that makes rebalancing frequency relevant is the momentum premium and cyclical exposure brought by a longer rebalancing period. For example, we observe that **Semi-Annual** and **Annual** rebalancing provide us with a slightly higher Sharpe Ratio while the skewness and kurtosis for all portfolios are close to normally distributed returns.

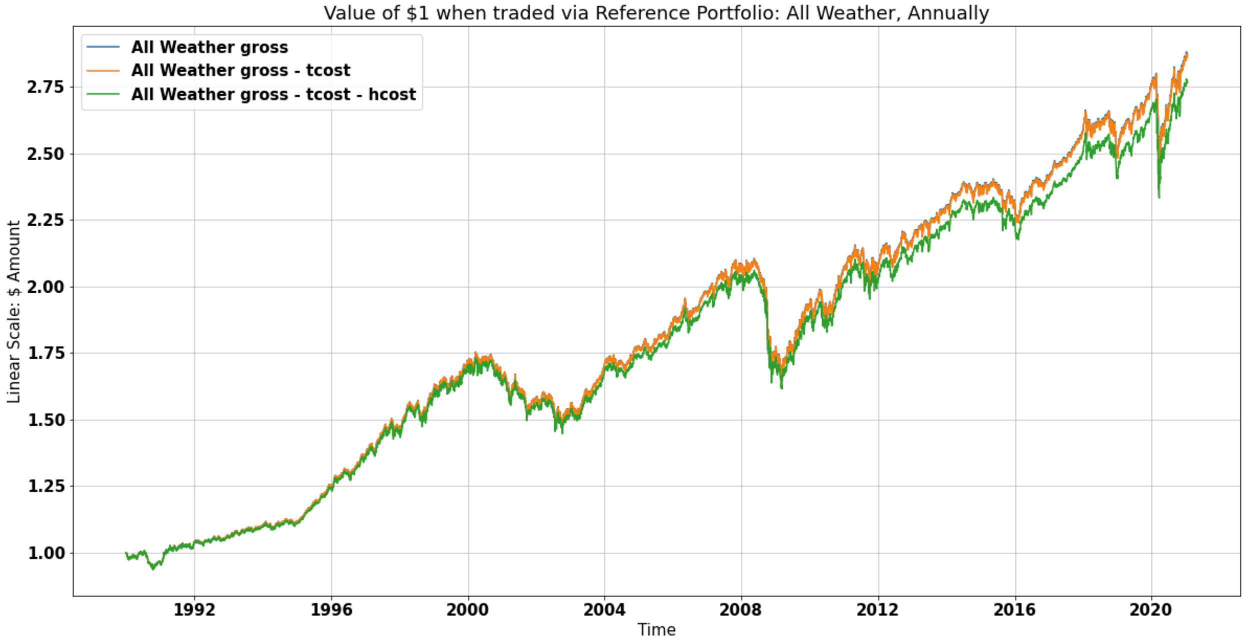


Figure 2: Annually Rebalanced Reference Portfolio, All Weather

	Sharpe	**Skewness**	**Kurtosis**	**Max Drawdown**
Weekly	0.35	-0.04	0.05	-24.95%
Monthly	0.35	-0.04	0.04	-24.68%
Quarterly	0.36	-0.04	0.04	-24.32%
Semi-Annually	0.38	-0.04	0.03	-23.32%
Annually	0.38	-0.05	0.04	-21.63%

Table 1: Net Returns (Gross - Trans. Cost - Holding Cost)

3. Factor Universe

3.1 General Musk Risk Factors Description

In this section, we cover the risk factors or "the bad times" for Elon Musk. Given Elon's wealth is highly concentrated in Tesla stocks (please see below figure). We consider the risk factors that are essentially Tesla's equity risk factors.

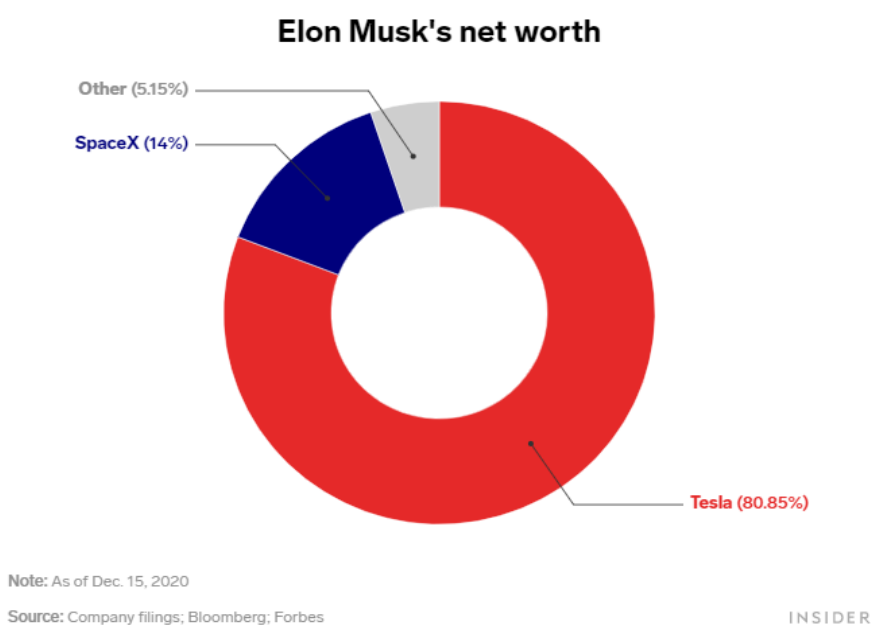


Figure 3: Elon Musk's Net Worth Components

We use the risk factor breakdown as explained in the snippet below from the lecture 4 slides on Leveraged Portfolios, Rebalancing, and Factor Theory where we notice that equities have strong positive exposure to growth factor; negative exposure to inflation, volatility, credit, and liquidity.

Factor Style	Factor	Asset Class Public Equities
Macro	Growth	++
	Inflation	-
	Volatility	-
Investment	Equity	+++
	Bond	?
	Credit	+
Dynamic	Value, Momentum, Carry, Etc.	x
Investor Specific	Leverage	x
Liquidity Preference	Liquidity	-

Figure 4: Risk factor exposures

Further, we think that Elon is also exposed to some other minor factors:

1) **Crude Oil**

   Price variations in crude as it is the fuel source for the internal combustion engine-powered vehicles; these vehicles form the competitor products of electric vehicles.

2) **Crypto-currency (Bitcoin)**

   Given Tesla has invested 1.5 billion USD in Bitcoin for transaction purposes, BTC price movements and volatility would pose to be a risk factor for Elon, though minor at the moment since Tesla has not started accepting payments in the said cryptocurrency.

3.2 Portfolio Constituents Assets:

While deciding our portfolio constituents, we not only consider the assets which provide diversification to the portfolio but also hedge Musk's risk factors. The following are described below:

1) **Nominal Bonds**

   Nominal bonds work as a good hedge for periods of declining economic growth. It is this period when the equities tend to underperform. We use 'iShares 1-3 Year Treasury Bond ETF' for capturing this asset class; the ETF tracks the investment results of an index composed of the U.S. Treasury bonds with remaining maturities between one and three years. For the lookback periods when the data is not available, we use Dow Jones Corporate Bond Price Index.

2) **Volatility**

   In order to hedge the equities downside with increased volatility, we use the volatility ETF: PRO VIX MT FUT; the ETF provides long exposure to the S&P 500 VIX Mid-Term Futures Index, which measures the returns of a portfolio of monthly VIX futures contracts with a weighted average of five months to expiration. For the lookback periods when the data is not available, we use the CBOE VIX Index.

3) **Commodities**

   Commodities provide an effective hedge against the increased inflation. Equities returns have historically shown negative exposure to the inflation factor; however, in the low-interest rate regimes (recent times), this relationship can get altered to a moderate positive. In order to capture the commodities exposure, we use the 'Invesco Optimum Yield Diversified Commodity Strategy No K-1 ETF'; the ETF invests in the commodity-linked futures and other financial instruments that provide economic exposure to a diverse group of the world's most heavily traded commodities across the energy, precious metals, industrial metals, and agriculture sectors. For the lookback periods when the data is not available, we use the Bloomberg Commodity Index.

4) **Corporate Bonds**

   We also consider the corporate bonds in our portfolio as it exposes us to a balanced risk somewhere between the nominal treasury bonds (essentially risk-free) and much riskier equities. We use the iShares iBoxx $ Investment Grade Corporate Bond ETF; the ETF seeks to track the investment results of an index composed of U.S. dollar-denominated investment-grade corporate bonds. For the lookback periods when the data is not available, we use the Dow Jones Corporate Bond Price Index.

5) **Broader US Equities (S&P 500)**

   S&P 500 is also included in our portfolio as it diversifies the wealth across the broader US equity instruments, as compared to Elon's concentrated wealth in Tesla's stock. Also, the index helps us to achieve higher returns over time in the portfolio context. We use the SPDR® S&P 500® ETF Trust for capturing the index; the ETF seeks to provide investment results that, before expenses, correspond generally to the price and yield performance of the S&P 500 index. For the lookback periods when the data is not available, we use the S&P 500 Total Return index.

4. Investment Strategies

In the project, we introduce two strategies, one based on asset classes and the other based on tilt on factor portfolio or investable ETFs. The flowchart below details how the asset universe and portfolio construction proceed:

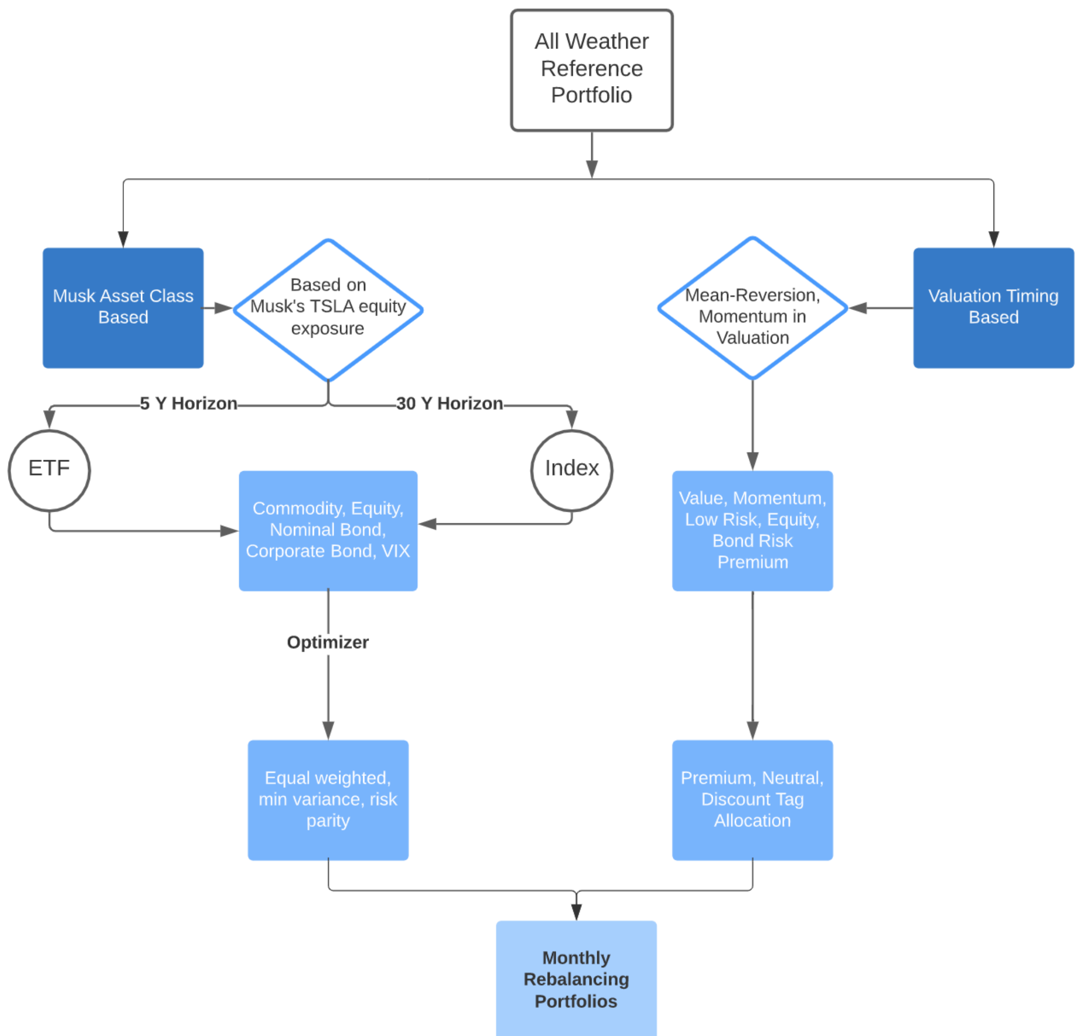


Figure 5: The Flowchart of the Asset Allocation Process

4.1 Traditional Portfolio Optimization

For traditional portfolio optimization methods, we tried the equal-weighted portfolio, minimum variance portfolio, and risk parity portfolio.

1) Equal Weighted

For the equal-weighted portfolio, we use equal weights for all assets we consider in our portfolio. This is our baseline model to see how the assets' unbiased allocation can perform in our given portfolio construction. Our investing instruments include equity, commodity, nominal bond, corporate bond, and VIX, therefore we allot a weight of ⅕ to each asset in our pool. This creates a portfolio with weights of ⅕ each for equity, nominal bond, corporate bond, commodity, and VIX. Equal weighted portfolios are anti-momentum in nature since we need to sell the winners and to buy the losers when rebalancing the portfolios, which is the reason why people are not in favor of this method these days. During a period of time when momentum is doing well, equal-weighted portfolios will suffer from this. However, during a time when mean-reversion occurs, equal-weighted portfolios will benefit from it.

2) Minimum Variance

For the minimum variance portfolio, we calculated the minimum variance weights by the following formula:

$$w_{minimum\ var} = \Sigma^{-1} \cdot e \,/\, e^{T} \cdot \Sigma^{-1} \cdot e$$

Where Σ is the covariance matrix of the 5 assets' returns and e is a column vector of ones.

We use a rolling window of 12 months to calculate the covariance matrix for asset returns. After obtaining the covariance matrix, we then find the inverse of the matrix to obtain the set of weights resulting in minimal portfolio variance. The caveat in the approach here is that our assets are a mix of extremely volatile assets such as VIX while other assets such as nominal bonds (treasury return index) are considered safe assets with very little volatility. Therefore we might end up taking huge positions in bonds as well as allocating only a tiny portion to volatile assets such as VIX.

3) Risk Parity

Risk Parity is a construction focusing on the allocation of risk as opposed to the allocation of capital. Given target volatility of 10%, we allowed equal risk contribution for the asset classes, such as corporate bonds, nominal bonds, commodities, and VIX. By calculating the assets' historical returns and volatility, we obtained rolling historical covariance matrix estimation as the input to the optimizer.

The optimizer will then find the optimal set of portfolio weights with the following objective and constraints:

$$argmin \sum_{j=1}^{n} w_j - \frac{w\,V\,w}{(V\,w)_j N}$$

$$\text{s.t. } \sum_{j=1}^{n} w_j = 1$$

The three optimizers and the defined asset universe are run in a 30-year horizon to match Elon Musk's retirement plan scope and in a shorter, 5-year horizon with ETFs. Ideally, we shall use a total return product for the respective asset classes for the most realistic returns, but since our backtest would require data from 30 years ago (1990 onwards), using the index data is the sub-optimal yet functional choice for us to test the hypotheses. With the data available for the past five years, we use the ETFs as the proxy for the asset classes to display the feasibility for Musk to invest in our retirement portfolio going forward.

4.2 Valuation Timing

Ilmanen detailed case studies on return sources in his book "Expected Returns." In particular, he coined the constant expected returns on various risk premiums, so we imply that asset, dynamic, and underlying risk premia all exhibit long-term mean-reversion patterns. We can exploit the short-term mispricing to capture valuation alpha. In this project, we will present a monthly-rebalancing strategy based on the idea of buying the discounted and selling the premium factors. This method represents the ideology of mean-reversion in risk premium.

However, in the lectures, we recall the discussion on the momentum effects in the risk premium, so we will also proceed with a second scheme on buying the premium and selling the discounted factors. To be coherent with the previous discussion on Musk's specific risk factors, we will focus on equity, bond, value, volatility, and tail risk premiums. After consulting Ilmanen's book and consideration for monthly-frequency data availability, we will use the CAPE ratio, Kim-Wright term-structure model, Fama-French's Value (HML) Factor, Momentum Factor, and AQR's Betting Against Betas portfolio for corresponding factor representations.

At each month, we compute the 10-year moving average of the risk premia as the baseline of our historical mean. By setting up factor-specific thresholds to determine a confidence interval band of premia above or below the historical mean, we label each month, each factor with a discount, neutral, or premium tag to construct long-only, and long-short portfolios. For example, if we calculated the Value factor's 10-year average to be 4.72%, if the factor exceeds 5.03% (which is one standard deviation away from the historical average) in May 2004, we will assign the premium flag for that month.

The portfolio allocation is directly related to the factor premium tag. During the project, we considered multiple weighting schemes to find the risk-adjusted, optimal allocation.
In the long-only case, we assign the following scheme:

- Premium: 10% or 40%
- Neutral: 25%
- Discount: 40% or 10%

In the long-short case, the scheme is changed as:

- Premium: -10% or 30%
- Neutral: 10%
- Discount: 30% or -10%

Note that the results shown in this project can variate significantly depending on how the neutral tag confidence interval is set up. We are keen to attempt either an equal distribution of tags for the factors or a more restrictive distribution of tags which will underweight the volume of neutral flags.

4.2.1 Premia Universe

1) Equity risk premium - CAPE

We used Schiller (2005) approach to calculate equity risk premium. This model captures not only the dividends but also the potential buyback operation that eventually arises in the long term. The equity risk premium is defined as:

$$ERP = 1/CAPE - r_{f10Y} = E/P - r_{f10Y}$$

2) Bond risk premium

The bond risk premium (BRP) is the expected return advantage of long-duration government bonds over the short-term (one-year) period.

BRP ≡ E(excess bond return over the riskless rate for the next year)

$$BRP \cong Y_{10} - Y_1 - Duration_{10} * E(\Delta Y_{10})$$

where long term bond is represented by a 10-year maturity and short term bond by 1-year maturity

3) Yield Curve Steepness:

The yield curve reflects both the BRP and the market's interest rate expectations. However, the yield curve (YC) is a poor BRP proxy because mean-reverting rate expectations dominate curve steepness when short-term rates are exceptionally high or low.

4) Kim-Wright BRP model:

By extracting the term structure dynamics and rate expectations from the yield curves and utilizing the no-arbitrage condition on bonds with different maturities, Kim-Wright's approach blends both the macroeconomic theory and yield data to obtain a real-time series of bond risk premium.

5) **Value - HML**

We use the Fama-French (1992) HML factor as a proxy for value here. It would capture the value premium by relying on investing in assets with a high book-to-market ratio.

6) **Momentum - MOM**

We use Carhart (1997) momentum factor generated as the difference between equal-weighted average high performing firms and low performing firms, lagged by a month. This helps in capturing the momentum premium present in the market.

7) **Tail Risk Premium (BAB) - AQR portfolio**

Since leveraging a lot of mutual funds and asset managers is not an extensive option, they tend to invest heavily in highly volatile risky assets and in turn on high beta stocks suggesting a crowding move. AQR constructed a portfolio that is long on low beta assets and short on high beta assets to answer some questions that they posit to understand how betting against beta works. We use the dataset from their data library for a monthly rebalancing frequency as one of the risk factors.

5. Evaluation & Results

Here we will present the returns of our different strategies: the allocation-based portfolio and the factor timing strategy.

5.1 Asset-based Portfolios

Although we tried to put the VIX in our portfolio, we decided not to include it since it cost too much during normal time. Given that Elon has high exposure to equities and growth stock, we want to avoid taking even more risk on highly risky assets such as small stocks and Nasdaq-like indices. Hence, our portfolio does not give enough returns to offset the cost of rolling down the VIX during normal time, even if it can pay during bad-times. Therefore, we chose to only use these assets: equity, commodities, corporate bonds, and nominal bonds.

As we wanted to study the performance of our asset allocation on a long time horizon (last 30 years) but also assess the practical performance of our asset-based portfolio we have proceed with two approaches:

- We will compute the performance of our asset-based portfolio on the last 30 years using indices
- We will do the same in the last 5 years, a time horizon where we can find ETFs for all the components of our portfolio.

5.1.1 Indices based - 30 Year Horizon

In Appendix A, we showed the portfolio performance in the last 30 years with different allocation: Equal Weighted, Minimum Variance, and Risk Parity.

Below the summary of the three different allocation strategies:

	Sharpe	Skewness	Kurtosis	Max Drawdown
Equal Weighted	0.14	-0.05	0.04	-32.79
Min-Variance	-18.69	-0.02	-0.00	-0.00
Risk Parity	0.18	-0.05	0.72	-18.67

Table 2: Net Results on last 30 years (Gross - Trans. Cost - Holding Cost)

Even with a very low Sharpe Ratio of 0.18, the best allocation strategy is the risk parity one. It reduces the risk and permits to increase the Sharpe ratio. On the other hand, we also observe that the Min-Variance parity tilts toward 100% low-risk bond allocation which gives a very poor performance while adjusted by the risk-free rate (negative Sharpe ratio). To finish, the Equal Weighted suffers from large drawdowns which impact too much its returns in the long-run and also its Sharpe Ratio.

5.1.2 ETF based - 5 Year Horizon

In Appendix B, we showed the portfolio performance in the last 5 years using ETFs with different allocation strategies: Equal Weighted, Minimum Variance, and Risk Parity.

Below the summary of the three different allocation strategies:

	Sharpe	Skewness	Kurtosis	Max Drawdown
Equal Weighted	0.40	-0.11	0.07	-20.37
Min-Variance	-0.21	0.13	0.10	-1.83
Risk Parity	0.77	0.02	0.13	-13.05

Table 3: Net Results on last 5 years (Gross - Trans. Cost - Holding Cost)

Same conclusion than the simulation of 30 years with the indices: the best performer is the Risk Parity allocation thanks to its capacity to reduce the drawdown and hence, improve its Sharpe Ratio. The Min-Variance is the worst performance while the Equal Weighted provides good returns but suffers from large volatility in its returns.

In conclusion, the asset-based strategies provide good returns only if we apply the risk parity allocation which prevents the drawdowns and improves the Sharpe ratio.

5.2 Valuation Timing - Based Portfolios

For valuation timing models, we have two competing theories, 1) mean-reversion in risk premium, 2) momentum in risk premium. To test the hypotheses, we will run through four schemes as detailed in the previous section: long-only by weighting more on discount over premium (mean-reversion), long-only by weighting more on premium over discount (momentum), and repeat the long-only analysis with a long-short framework. The investment vehicles here are the broad S&P equities, Treasury bonds, Fama-French's Value, Momentum portfolios, AQR Betting against Beta portfolio. These instruments are chosen on the basis of investability and tradability.

To establish a benchmark portfolio for our factor-based portfolios, we created an equal-weighted portfolio first. The 30-year horizon backtest shows a promising start:

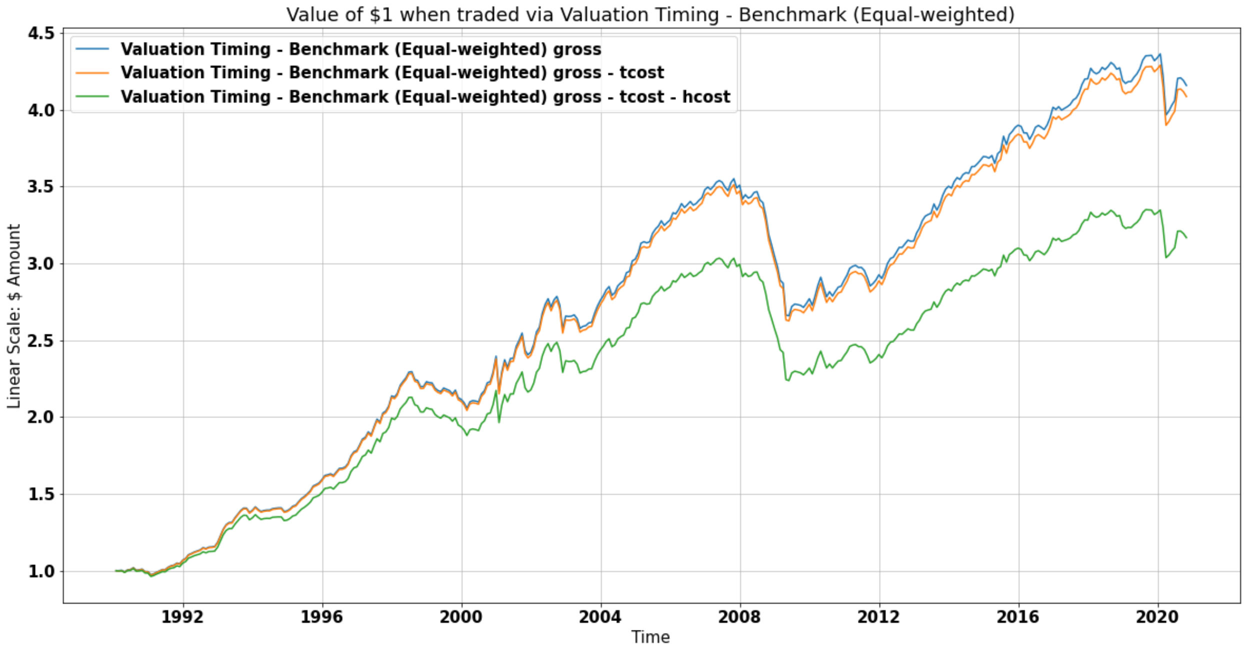


Figure 6: Equal-weighted Benchmark Portfolio of the Valuation Timing

We then applied our fixed weight, tag-based scheme with the long-only, mean-reversion on factor premium portfolio by taking premium factors with 10%, neutral with 25% and discounted with 40% (the difference between allocation weight numbers is designed to be symmetric). Note that we normalize the overall portfolio weights to 100% net-exposure to obtain a fully invested portfolio in different time periods. Results are rather dismal and are hintting a rejection on the mean-reversion hypothesis:

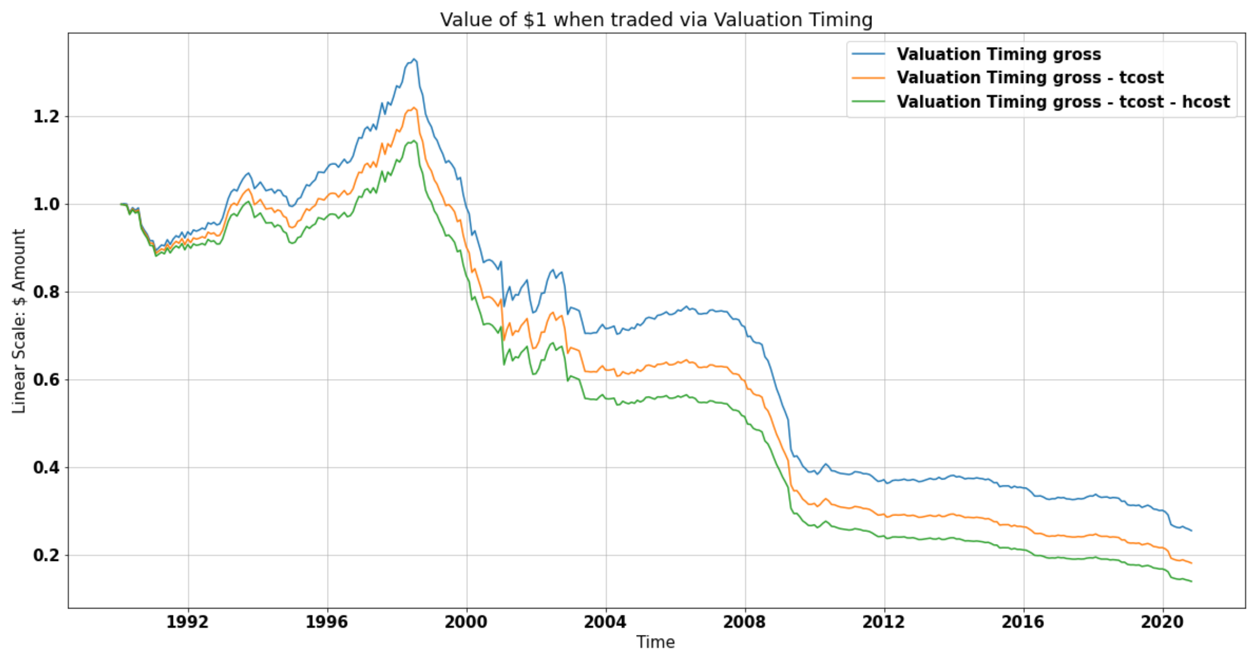


Figure 7: Long-only, Mean-reversion factor premium portfolio of the Valuation Timing with 0 Std.

We then tested the momentum-based valuation timing structure by taking premium factors with 40%, neutral still with 25%, discounted with 10%. The results this time suggest the effectiveness of valuation-timing if done properly with the investment vehicles:

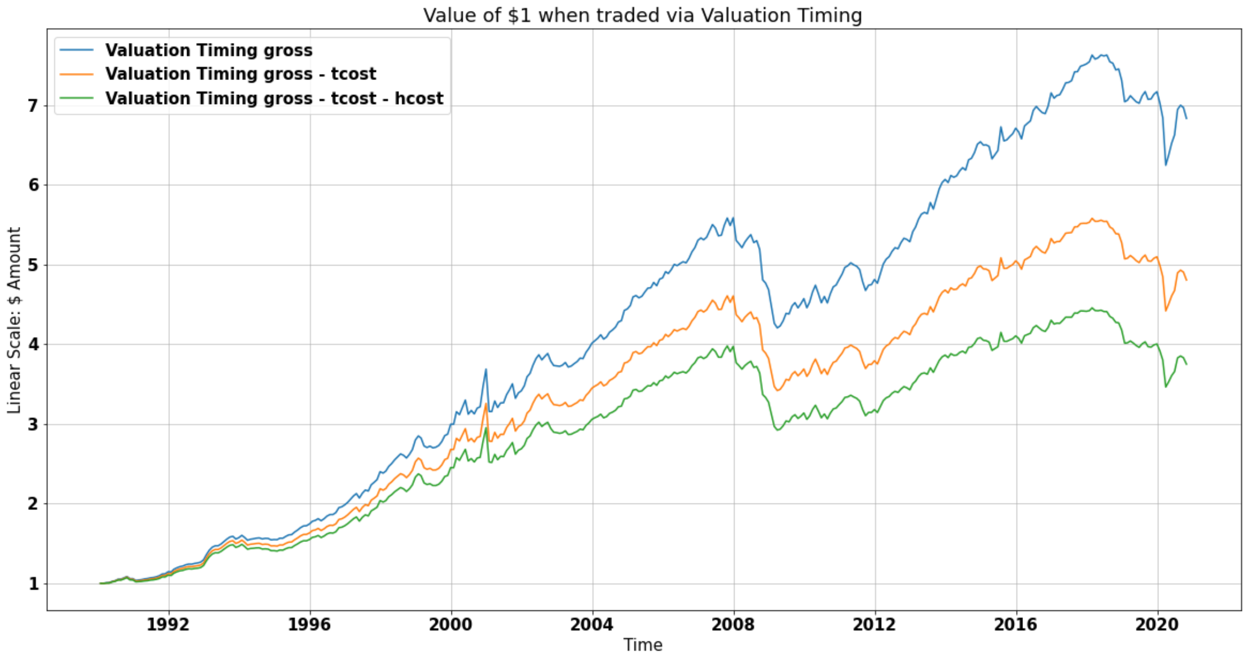


Figure 8: Long-only, Momentum factor premium portfolio of the Valuation Timing with 0 Std.

	Sharpe	**Skewness**	**Kurtosis**	**Max Drawdown**
Equal Weighted	0.54	-0.41	0.61	-26.26
Mean-Reversion	0.03	-0.43	0.98	-34.07
Momentum	0.51	-0.51	1.02	-26.55

Table 4: Net Results on last 5 years (Gross - Trans. Cost - Holding Cost) with 0 Std. Tolerance

Since most factors are of long-only and long-short construction, our additional long-short framework based on factors' relative premium on historical means did not work. Results are hence omitted.

We are also aware that due to the different behaviors in risk premium distribution, the threshold for our confidence interval band to tag "Neutral" for the risk premia could differ significantly, we attempted to find an optimal width by controlling the standard deviation of the tag differences:

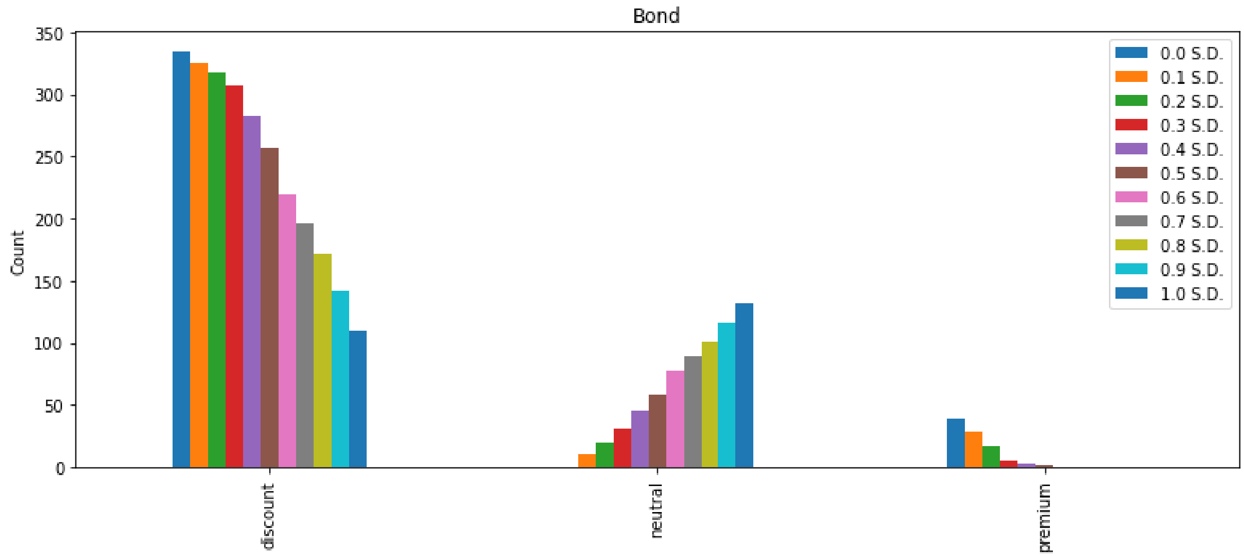


Figure 9: Bond Risk Premium Partitioned by Different Tolerance Ranges

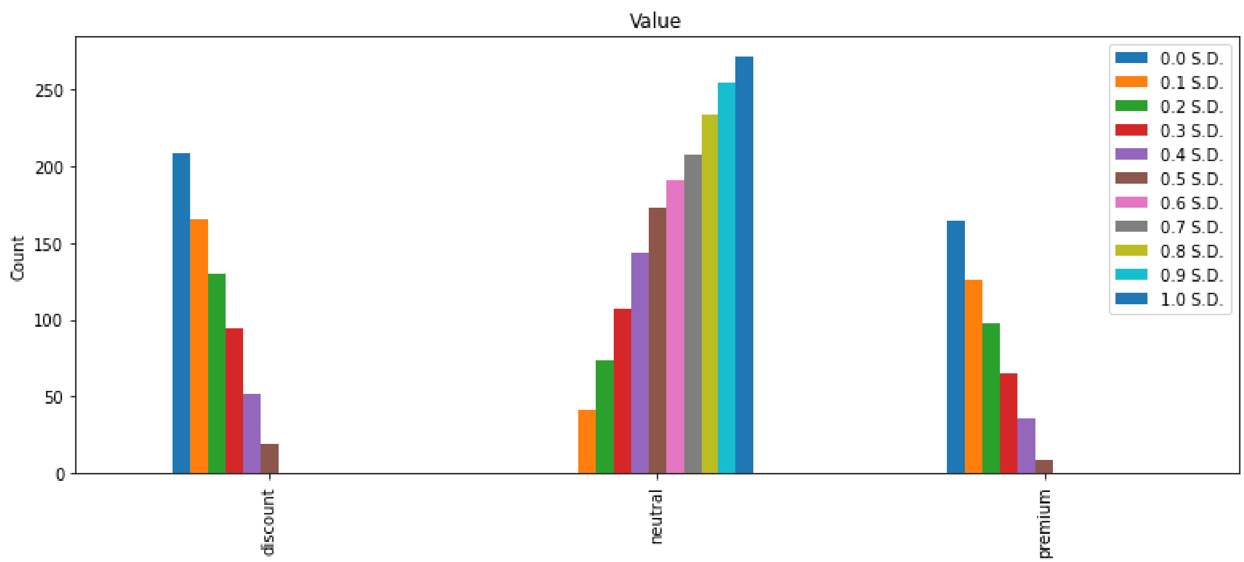


Figure 10: Value Risk Premium Partitioned by Different Tolerance Ranges

We observe that for bond risk premium, the distribution is tighter and resulting in less volume of “premium” cases. For a more uniform distribution of tag volume, we settled with a standard deviation of **0.6**. Below is Momentum-based valuation timing strategy with 0.6 standard deviation tag difference band:

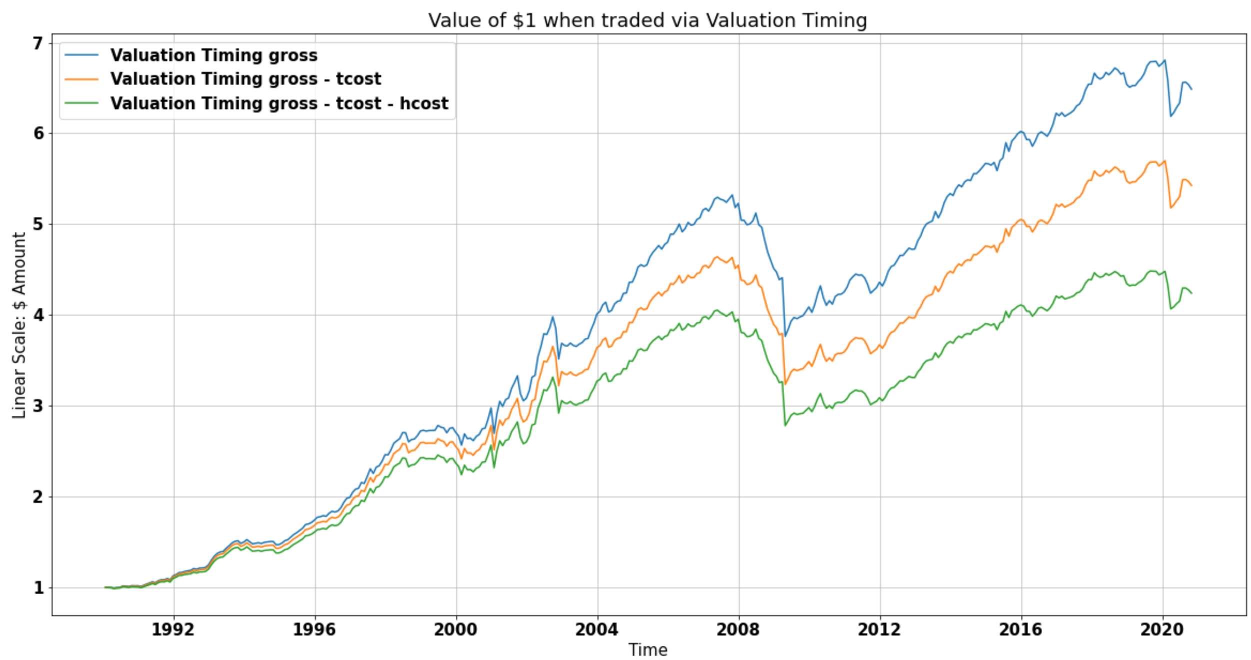


Figure 8: Long-only, Momentum factor premium portfolio of the Valuation Timing with Distribution-based Std.

What we observe is that there is less turnover with the introduction of more "neutral" tags and hence smaller transaction costs. The trade-off, however, is that the more neutral and discounted tags introduced, the less aggressive the allocation, hence the less performance from the results.

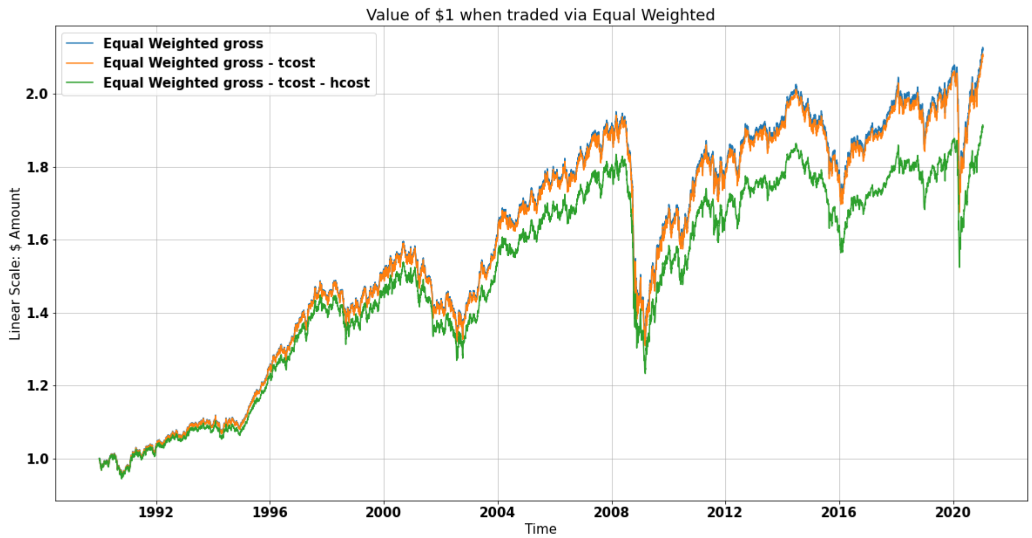
6. Conclusion & Future Work

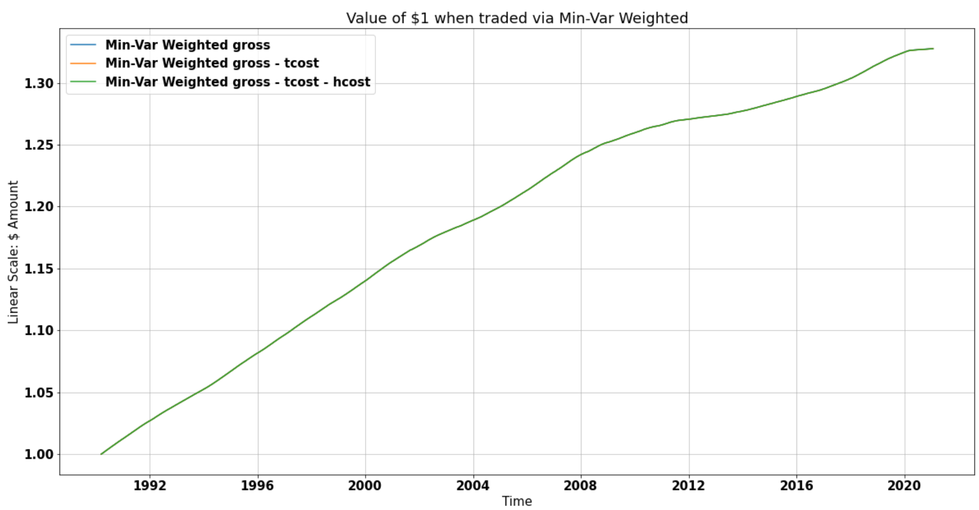
In this project, we designed various portfolios for our client, Elon Musk, to retire while taking into consideration his significant exposure to Tesla equities and personal, familial responsibilities. Starting with the "All-Weather" portfolio, we experimented with Ray Dalio's asset allocation with a fixed set of weights of equities, stocks, and hard assets. Then, based on Musk's specific risk factors, we designed a set of portfolios with emphasis on inflation, equity, and volatility hedges with assets such as broad equities, commodity, investment grade corporate bonds, nominal bonds, and VIX. The portfolios are then constructed using the equal-weighted, min-variance, and risk-parity (with 10% target volatility) optimizers. Results from Musk-specific asset-based portfolios suggest superior performance on the All-Weather portfolio. We theorized that introduction of risk-budget optimizers along with treasury bonds dampen the net returns.

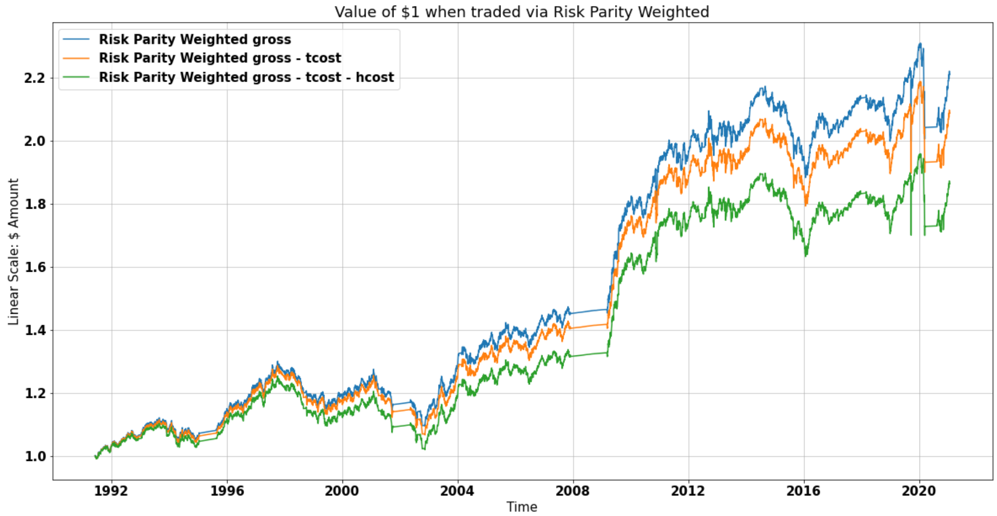
The highlight of the project is our Valuation-Timing Based portfolios, in which we estimated the monthly tag on whether specific factors are trading with premium or discounts. After comparing the two theories on factor premium's mean reversion and momentum behaviors, we conclude that a momentum-driven, Valuation-Timing based portfolio, with small confidence interval on neutral tag assignment, would yield the optimal set of portfolio for Musk to have as his retirement portfolio. The results suggest that factor timing can be promising once we define the appropriate factor universe and trading rules.

In the future works, we would like to focus on testing more variants on the factors for valuation-timing framework and exploring the trade-off between more neutral tag assignment (through higher standard deviation threshold to set the "neutral" tag) and more aggressive allocations with different factors. The current stages laid a promising foundation for valuation-timing portfolios.

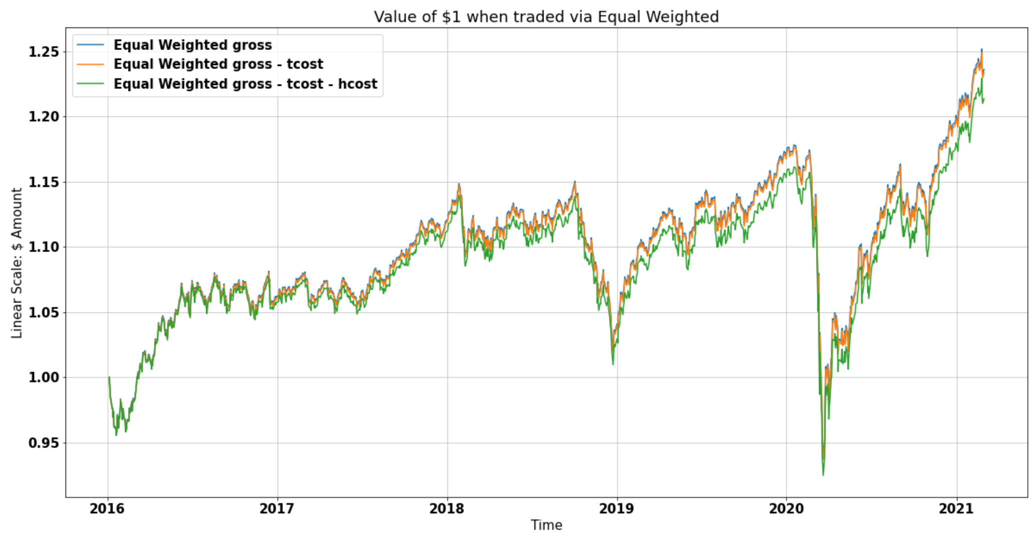
Appendix A - 30 Year Performance

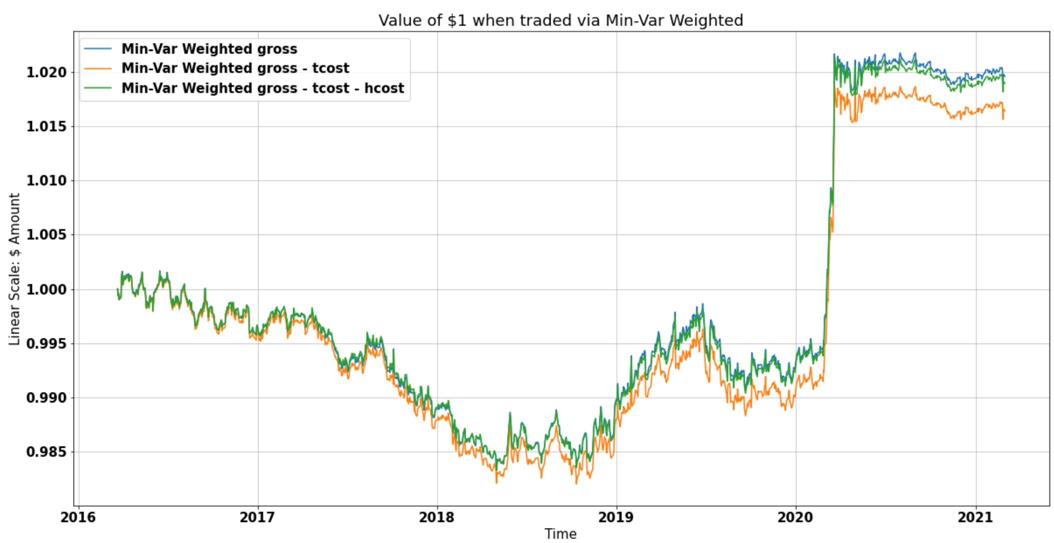


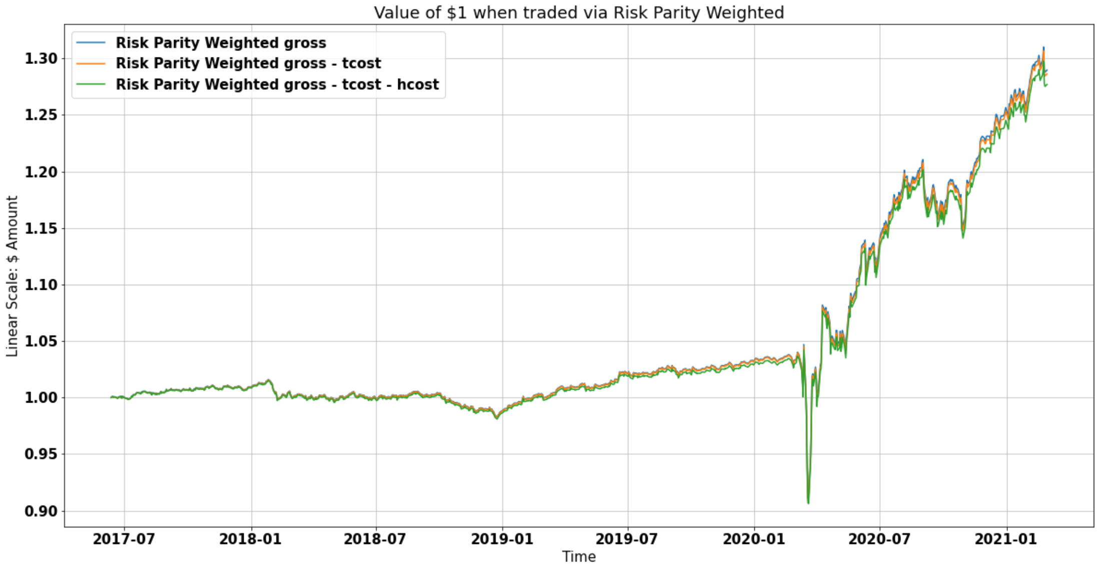



Value of $1 when traded via Risk Parity Weighted
Risk Parity Weighted gross
Risk Parity Weighted gross - tcost
Risk Parity Weighted gross - tcost - hcost
Linear Scale: $ Amount
2.2
2.0
1.8
1.6
1.4
1.2
1.0
1992
1996
2000
2004
2008
2012
2016
2020
Time

Appendix B - 5 Year Performance






Value of $1 when traded via Risk Parity Weighted
Risk Parity Weighted gross
Risk Parity Weighted gross - tcost
Risk Parity Weighted gross - tcost - hcost
Linear Scale: $ Amount
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0.90
2017-07
2018-01
2018-07
2019-01
2019-07
2020-01
2020-07
2021-01
Time