HW2-2 Report
Exponential Mechanism

- Utility u(D,O) = #(D,O) (for most frequent education)
- Probability = $\Pr[A(D) = r] \propto e^{\frac{(\varepsilon \, * \, u(D,r))}{2 * \Delta u}}$
- In this case Δu (Sensitivity of utility function) $= 1$

2. Task (a) for $\varepsilon = 0.5$
- For group 1 & $\varepsilon = 0.5$

```
Enter full path for the dataset file
adult.data.txt

For epsilon = 0.5 query = 1
 HS-grad

Utility of each education result value
 HS-grad        = 10501
 Some-college   = 7291
 Bachelors      = 5355
 Masters        = 1723
 Assoc-voc      = 1382
 11th           = 1175
 Assoc-acdm     = 1067
 10th           = 933
 7th-8th        = 646
 Prof-school    = 576
 9th            = 514
 12th           = 433
 Doctorate      = 413
 5th-6th        = 333
 1st-4th        = 168
 Preschool      = 51

Probability of each education result value
 HS-grad        = 37.036903
 Some-college   = 35.577583
 Bachelors      = 34.343143
 Masters        = 29.80729
 Assoc-voc      = 28.925148
 11th           = 28.276094
 Assoc-acdm     = 27.890425
 10th           = 27.35362
 7th-8th        = 25.883198
 Prof-school    = 25.42443
 9th            = 24.968893
 12th           = 24.282951
 Doctorate      = 24.09379
 5th-6th        = 23.23257
 1st-4th        = 20.495855
 Preschool      = 15.727303
```

- For group 2 & $\varepsilon = 0.5$

```
 Console    Problems    Debug Shell    History
<terminated> hw2_2_exponential [Java Application] /Library/Java/JavaVirtualMachine

For epsilon = 0.5 query = 2
 HS-grad

Utility of each education result value
 HS-grad        = 10500
 Some-college   = 7291
 Bachelors      = 5355
 Masters        = 1723
 Assoc-voc      = 1382
 11th           = 1175
 Assoc-acdm     = 1067
 10th           = 933
 7th-8th        = 646
 Prof-school    = 576
 9th            = 514
 12th           = 433
 Doctorate      = 413
 5th-6th        = 333
 1st-4th        = 168
 Preschool      = 51

Probability of each education result value
 HS-grad        = 37.036522
 Some-college   = 35.577583
 Bachelors      = 34.343143
 Masters        = 29.80729
 Assoc-voc      = 28.925148
 11th           = 28.276094
 Assoc-acdm     = 27.890425
 10th           = 27.35362
 7th-8th        = 25.883198
 Prof-school    = 25.42443
 9th            = 24.968893
 12th           = 24.282951
 Doctorate      = 24.09379
 5th-6th        = 23.23257
 1st-4th        = 20.495855
 Preschool      = 15.727303
```

- For group 3 & $\varepsilon = 0.5$


```
Console        Problems    Debug Shell    History
         nated> hw2_2_exponential [Java Application] /Library/Java/JavaVirtua
Restore
  For epsilon = 0.5 query = 3
   HS-grad

  Utility of each education result value
   HS-grad        = 10501
   Some-college   = 7290
   Bachelors      = 5355
   Masters        = 1723
   Assoc-voc      = 1382
   11th           = 1175
   Assoc-acdm     = 1067
   10th           = 933
   7th-8th        = 646
   Prof-school    = 576
   9th            = 514
   12th           = 433
   Doctorate      = 413
   5th-6th        = 333
   1st-4th        = 168
   Preschool      = 51

  Probability of each education result value
   HS-grad        = 37.036903
   Some-college   = 35.577034
   Bachelors      = 34.343143
   Masters        = 29.80729
   Assoc-voc      = 28.925148
   11th           = 28.276094
   Assoc-acdm     = 27.890425
   10th           = 27.35362
   7th-8th        = 25.883198
   Prof-school    = 25.42443
   9th            = 24.968893
   12th           = 24.282951
   Doctorate      = 24.09379
   5th-6th        = 23.23257
   1st-4th        = 20.495855
   Preschool      = 15.727303
```

- For group 4 & $\varepsilon = 0.5$


```
Console        Problems    Debug Shell    History
<terminated> hw2_2_exponential [Java Application] /Library/Ja
  For epsilon = 0.5 query = 4
   HS-grad

  Utility of each education result value
   HS-grad        = 10501
   Some-college   = 7291
   Bachelors      = 5355
   Masters        = 1723
   Assoc-voc      = 1382
   11th           = 1175
   Assoc-acdm     = 1067
   10th           = 933
   7th-8th        = 646
   Prof-school    = 576
   9th            = 514
   12th           = 433
   Doctorate      = 413
   5th-6th        = 333
   1st-4th        = 168
   Preschool      = 50

  Probability of each education result value
   HS-grad        = 37.036903
   Some-college   = 35.577583
   Bachelors      = 34.343143
   Masters        = 29.80729
   Assoc-voc      = 28.925148
   11th           = 28.276094
   Assoc-acdm     = 27.890425
   10th           = 27.35362
   7th-8th        = 25.883198
   Prof-school    = 25.42443
   9th            = 24.968893
   12th           = 24.282951
   Doctorate      = 24.09379
   5th-6th        = 23.23257
   1st-4th        = 20.495855
   Preschool      = 15.648092
```

2. Task(b) for $\varepsilon = 0.5$

The global sensitivity for the first 4 groups with $\Delta u = 1$
Comparing the 1st group with 2nd group:

   $Pr[A(D) = r]/Pr[A(D') = r] <= exp(\varepsilon) : 0.929 \leq 1.648$

   Thus, we can validate that the 2 sets are 0.5-distinguishable

- Comparing the 1st group with 3rd group:

   $Pr[A(D) = r]/Pr[A(D') = r] <= exp(\varepsilon) : 1.018 \leq 1.648$

   Thus, we can validate that the 2 sets are 0.5-distinguishable

- Comparing the 1st group with 4th group:

   $Pr[A(D) = r]/Pr[A(D') = r] <= exp(\varepsilon) : 0.999 \leq 1.648$

   Thus, we can validate that the 2 sets are 0.5-distinguishable

2. Task(c) for $\varepsilon = 1$
    - For group = 1 & $\varepsilon = 1$

- For group = 2 & $\varepsilon = 1$


```
Console          Problems     Debug Shell     History
<terminated> hw2_2_exponential [Java Application] /Library/Java/

For epsilon = 1.0 query = 2
Debug -grad

Utility of each education result value
 HS-grad         = 10500
 Some-college    = 7291
 Bachelors       = 5355
 Masters         = 1723
 Assoc-voc       = 1382
 11th            = 1175
 Assoc-acdm      = 1067
 10th            = 933
 7th-8th         = 646
 Prof-school     = 576
 9th             = 514
 12th            = 433
 Doctorate       = 413
 5th-6th         = 333
 1st-4th         = 168
 Preschool       = 51

Probability of each education result value
 HS-grad         = 18.518261
 Some-college    = 17.788792
 Bachelors       = 17.171572
 Masters         = 14.903645
 Assoc-voc       = 14.462574
 11th            = 14.138047
 Assoc-acdm      = 13.945212
 10th            = 13.67681
 7th-8th         = 12.941599
 Prof-school     = 12.712215
 9th             = 12.484447
 12th            = 12.141476
 Doctorate       = 12.046895
 5th-6th         = 11.616285
 1st-4th         = 10.247928
 Preschool       = 7.8636513
```

- For group = 3 & $\varepsilon = 1$


```
Console          Problems     Debug Shell     History
<terminated> hw2_2_exponential [Java Application] /Library/Java

For epsilon = 1.0 query = 3
 HS-grad

Utility of each education result value
 HS-grad         = 10501
 Some-college    = 7290
 Bachelors       = 5355
 Masters         = 1723
 Assoc-voc       = 1382
 11th            = 1175
 Assoc-acdm      = 1067
 10th            = 933
 7th-8th         = 646
 Prof-school     = 576
 9th             = 514
 12th            = 433
 Doctorate       = 413
 5th-6th         = 333
 1st-4th         = 168
 Preschool       = 51

Probability of each education result value
 HS-grad         = 18.518452
 Some-college    = 17.788517
 Bachelors       = 17.171572
 Masters         = 14.903645
 Assoc-voc       = 14.462574
 11th            = 14.138047
 Assoc-acdm      = 13.945212
 10th            = 13.67681
 7th-8th         = 12.941599
 Prof-school     = 12.712215
 9th             = 12.484447
 12th            = 12.141476
 Doctorate       = 12.046895
 5th-6th         = 11.616285
 1st-4th         = 10.247928
 Preschool       = 7.8636513
```

- For group = 4 & $\varepsilon = 1$

```
For epsilon = 1.0 query = 4
 HS-grad

Utility of each education result value
 HS-grad        = 10501
 Some-college   = 7291
 Bachelors      = 5355
 Masters        = 1723
 Assoc-voc      = 1382
 11th           = 1175
 Assoc-acdm     = 1067
 10th           = 933
 7th-8th        = 646
 Prof-school    = 576
 9th            = 514
 12th           = 433
 Doctorate      = 413
 5th-6th        = 333
 1st-4th        = 168
 Preschool      = 50

Probability of each education result value
 HS-grad        = 18.518452
 Some-college   = 17.788792
 Bachelors      = 17.171572
 Masters        = 14.903645
 Assoc-voc      = 14.462574
 11th           = 14.138047
 Assoc-acdm     = 13.945212
 10th           = 13.67681
 7th-8th        = 12.941599
 Prof-school    = 12.712215
 9th            = 12.484447
 12th           = 12.141476
 Doctorate      = 12.046895
 5th-6th        = 11.616285
 1st-4th        = 10.247928
 Preschool      = 7.824046
```

2. Task(c) for $\varepsilon = 1$

```
Validating that each of the last 3 groups of results and the first group are 1.0-indistinguishable.

0.92947143 <= 2.7182817

1.018243 <= 2.7182817

0.9998172 <= 2.7182817
```

The global sensitivity for the first 4 groups with $\Delta u = 1$
Comparing the 1st group with 2nd group:

$Pr[A(D) = r]/Pr[A(D') = r] <= exp(\varepsilon) : 0.929 \leq 2.718$

Thus, we can validate that the 2 sets are 0.5-distinguishable

- Comparing the 1st group with 3rd group:

$Pr[A(D) = r]/Pr[A(D') = r] <= exp(\varepsilon) : 1.018 \leq 2.718$

Thus, we can validate that the 2 sets are 0.5-distinguishable

- Comparing the 1st group with 4th group:

$Pr[A(D) = r]/Pr[A(D') = r] <= exp(\varepsilon) : 0.999 \leq 2.718$

Thus, we can validate that the 2 sets are 0.5-distinguishable

2. Task (d)

- The sensitivity for each group is 1.
- With increase in the ε value for 0.5 to 1 the privacy decreases and all the results are much closer to the actual result.
- By changing the number of tuples in the dataset and comparing the neighboring datasets, we can see that the results are ε-indistinguishable compared to the original dataset with all tuples.