1to1 GURU

Learn to Lead, Learn to Succeed

Date :-21/01/2022 **Time :-50 Minutes** **Exam Name :-IIT-JEE-1to1Guru-3** **Mark :- 84**

1. **a**

2. **c**

(c) The time period of simple pendulum in air

$T = t_0 = 2\pi\sqrt{\left(\frac{l}{g}\right)}$ (i)

l , being the length of simple pendulum.

In water, effective weight of bob

$w' =$ weight of bob in air $-$ upthrust

$\Rightarrow \quad \rho V g_{eff} = mg - m'g$

$= \rho V g - \rho' V g = (\rho - \rho')Vg$

where $\rho' =$ density of bob,

$\rho =$ density of water

$\therefore \quad g_{eff} = \left(\frac{\rho-\rho'}{\rho}\right)g = \left(1 - \frac{\rho'}{\rho}\right)g$

$\therefore \quad t = 2\pi\sqrt{\left[\frac{l}{\left(1-\frac{\rho'}{\rho}\right)g}\right]}$ (ii)

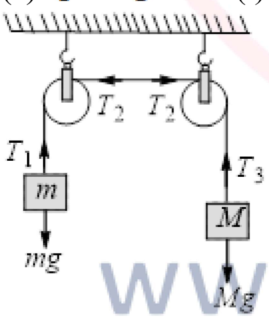
Thus, $\frac{t}{t_0} = \sqrt{\left[\frac{1}{\left(1-\frac{\rho'}{\rho}\right)}\right]}$

$= \sqrt{\left(\frac{1}{1-\frac{1000}{(4/3\times1000)}}\right)} = \sqrt{\left(\frac{4}{4-3}\right)} = 2$

$\Rightarrow \quad t = 2t_0$

3. **a**

(a) $T_1 - mg = ma$ (i)



$Mg - T_3 = Ma$ (iii)

$r(T_3 - T_2) = I\alpha$ (iv)

and $a = R\alpha$

From Eqs. (ii) and (iv), we get

$T_3 - T_1 = \frac{2la}{R^2}$

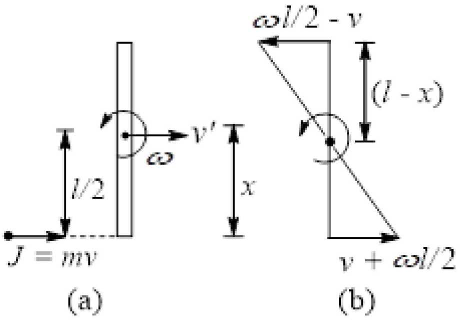
From Eqs. (i) and (iii), we get

$(M - m)g = (M + m)a + T_3 - T_1$

$(M - m)g = (M - m)a + \frac{2la}{r^2}$

$\Rightarrow a = \frac{(M-m)g}{\left(M+m+\frac{2l}{r^2}\right)}$

4. **d**

		(d) $J = mv = mv'$ $\Rightarrow$ Velocity of the CM of rod = v Applying impulse momentum equation about the CM of rod  $J\frac{l}{2} = I_{CM}\omega \Rightarrow \frac{mvl}{2} = \left(\frac{ml^2}{12}\right)\omega \Rightarrow \omega = \frac{6V}{\ell}$ About instantaneous axis of rotation the rod is considered to have pure rotation Let instantaneous axis of rotation be located at a distance x from the colliding end $\frac{\frac{\omega\ell}{2}-v}{\ell-x} = \frac{v+\frac{\omega\ell}{2}}{x}$ (i) Substituting the value of $\omega = 6V/l$ in Eq.(i), we get $x = \frac{2}{3}\ell$
5.	**b**	(b) The brass rod and the lead rod will suffer expansion when placed in steam bath ∴ Length of brass rod at 100 ° C $L_{brass} = L_{brass}(1 + \alpha_{brass}\Delta T)$ = 80[1+18 × 10^{-6} × 100] and the length of lead rod at 100 ° C $L_{lead} = L_{lead}(1 + \alpha_{lead}\Delta T) = 80[1 + 28 \times 10^{-6} \times 100]$ Separation of free ends of the rods after heating $= L_{lead} - L_{brass} = 80[28 - 18] \times 10^{-4} = 8 \times 10^{-2}$ cm =0.8 mm
6.	**8**	The energy of the electron in the n^{th} state of He^+ ion of atomic number Z is given by E_n = - (13.6)eV $\frac{Z^2}{n^2}$ for He^+ ion Z - 2. Therefore $E_n = -\frac{(13.6eV)\times(2)^2}{n^2} = -\frac{54.4}{n^2}eV$ The energies E_1 and E_2 of the two emitted photons in eV are $E_1 = \frac{12431}{1085}$ eV = 11.4 eV and $E_2 = \frac{12431}{304}$ eV = 40.9 eV Thus total energy E = $E_1 + E_2$ = 11.4 + 40.9 = 52.3 eV Let n be the principle quantum number of excited state. Now we have for the transition from n = n to n = 1 E = - (54.4) eV $\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$ But E = 52.3 eV. Therefore 52.3 eV = 54.4eV x $\left(1 - \frac{1}{n^2}\right)$ or 1 - $\frac{1}{n^2} = \frac{52.3}{54.4}$ = 0.96 which gives n^2 = 25 or n - 5 The energy of the incident electron = 100eV (given). The energy supplied to He^+ ion = 52.3eV. Therefore, the energy of the electrons left after the collision = 100 - 52.3 = 47.7eV
7.	**449**	

		$\frac{1}{T} = \frac{1}{T_\alpha} + \frac{1}{T_\beta}$ $T = \frac{T_\alpha T_\beta}{T_\alpha + T_\beta} = 324$ years $\frac{N}{N_0} = e^{-1t}$ $t = \frac{1}{\lambda} \ln \frac{N_0}{N} = T \ln \frac{N_0}{N}$ t = 324 x 2 ln 2 t = 449 .06 years t ≈ 449 years
8.	**d**	(d) In s -block elements, electron enter into the ns -orbitals. For atomic number 3= $1s^2, 2s^1$ Atomic number 12= $1s^2, 2s^2 2p^6, 3s^2$
9.	**c**	(c) $PV = RT$ at temperature T for one mole $P(V + \Delta V) = R(T + 1)$ at temperature $(T + 1)$ for one mol $\therefore\ P\,\Delta V = R$
10.	**d**	
11.	**c**	
12.	**a**	(1) F_2 is highly reactive gas.
13.	**217**	
14.	**4**	$t = \frac{2.303}{K_2} \log \frac{100}{6}$ and $t = \frac{2.303}{K_1} \log \frac{100}{6}$ $\frac{K_1}{K_2} = \frac{\log 100 - \log 6}{\log 2} = 4$
15.	**d**	(d) Given, $x^2 + y^2 + 2gx + 2fy + c = 0$ $\therefore$ Radius of circle $= \sqrt{g^2 + f^2 - c}$ $= \sqrt{c - c} = 0 \quad [\text{given} g^2 + f^2 = c]$
16.	**b**	(b) We have, $f(x) = 1 + \frac{\sin x}{1 - \sin^2 x} = 1 + \frac{\sin x}{\cos^2 x} = 1 + \tan x \sec x$ $\therefore f'(x) = \sec^3 x + \sec x \tan^2 x > 0$ for all $x \in (-\pi/2, \pi/2)$ $\Rightarrow f(x)$ is an increasing function on $(-\pi/2, \pi/2)$ Now, $\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} \left(1 + \frac{\sin x}{1 - \sin^2 x}\right) = \infty$ and, $\lim_{x \to -\pi/2} f(x) = \lim_{x \to -\pi/2} \left(1 + \frac{\sin x}{1 - \sin^2 x}\right) = -\infty$ Hence, range $(f) = (f(-\pi/2), f(\pi/2)) = (-\infty, \infty) = R$
17.	**b**	(b) We have, $\frac{C_k}{C_{k-1}} = \frac{{}^nC_k}{{}^nC_{k-1}} = \frac{n - k + 1}{k}$ $\therefore \sum_{k=1}^{n} k^3 \left(\frac{C_k}{C_{k-1}}\right)^2$

		$=\sum_{k=1}^{n} k^3 \frac{(n-k+1)^2}{k^2} = \sum_{k=1}^{n} k(n-k+1)^2$ $=(n+1)^2\left(\sum_{k=1}^{n} k\right) - 2(n+1)\left(\sum_{k=1}^{n} k^2\right) + \left(\sum_{k=1}^{n} k^3\right)$ $=(n+1)^2 \frac{n(n+1)}{2} - \frac{2(n+1)n(n+1)(2n+1)}{6} + \left\{\frac{n(n+1)}{2}\right\}^2$ $=\frac{n(n+1)^2}{12}\{6(n+1)-4(2n+1)+3n\}$ $=\frac{n(n+1)^2(n+2)}{12}$
18.	**b**	(b) Let the roots be α and 2α. Then, $3\alpha = -\frac{b}{a}$ and $2\alpha^2 = \frac{c}{a}$ $\Rightarrow \alpha = -\frac{b}{3a}$ and $\alpha^2 = \frac{c}{2a} \Rightarrow \left(-\frac{b}{3a}\right)^2 = \frac{c}{2a} \Rightarrow 2b^2 = 9ac$
19.	**a**	(a) Radius $= \sqrt{(a-\pi)^2 + (b-e)^2}$ =irrational $= k$ $\therefore$ Circle $(x-\pi)^2 + (y-e)^2 = k^2$
20.	**3**	$4\left(\frac{\sqrt{5}+1}{4}\right) - 4\left(\frac{\sqrt{5}-1}{4}\right) + 4\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right)$ = 3
21.	**8**	(b) $3^{2n}+7$ is divisible by 8. This can be checked by putting $n = 1, 2, 3$ etc.