COP290

DESIGN PRACTICES IN CS

Project 1-Engineering Drawing Software Package

MATHEMATICAL MODELLING

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ABSTRACT

This is a mathematical model for implementation of a software package for **Engineering Drawing**. In this document we will look at certain aspects of Engineering Drawing like:-

- How many views are necessary to uniquely define a 3D object?
- How many views are sufficient?
- How can one compute projections given the 3D description?
- How can one compute the 3D description given one or more projections?
- What interactions are necessary?

INTRODUCTION

An engineering drawing, a type of technical drawing, is used to fully and clearly define requirements for engineered items. Engineering drawing(the activity) produces engineering drawings(the documents). More than merely the drawing of pictures, it is also a graphical language that communicates ideas and information from one mind to another. Most especially, it communicates all needed information from the engineer who designed a part to the workers who will make it.

Two out of many goals of Engineering Drawing are:-

- Given the 3D model description we should be able to generate projections on to any planes, cross section or cutting plane.
- Given two or more projections we should be able to recover the 3D description and produce an isometric drawing from any view direction.

So, as we have to implement a software which could perform these actions we need to know into how could we convert this problem into a mathematical problem and do mathematical calculations in computer to convert either a 3D object to its 2D projections or 2D projections to its actual 3D object.

HOW TO COMPUTE 2-D PROJECTIONS GIVEN A 3-D DESCRIPTION

Every 3D solid is just an aggregate of points, edges, planes and curved surfaces. A 3-D object can be analyzed with respect to its 0-D, 1-D, and 2-D components. A mathematical point is an example of a geometric object that has zero dimensions, because a mathematical point (unlike a point made by a pencil) has no length, no width, and no height/depth. So the 3 components which can describe the 3-D solid are:-

- **Vertices**- These are the 0-D component and described as each corner where three faces of a solid meet.
- **Edges**- These are the 1-D component and described as the lines where two faces of a solid meet.
- **Faces**- These are the 2-D component and described as each flat part of a solid.

Hence to draw the 2D projections of a 3D solid our job is to transfer all these details to its 2D projections. This job is done in steps where the first step is to transfer the details of corner points of a solid called as vertices to 2D projections.

1-Mapping vertices to their corresponding 2D projection

Firstly we will look at how we can find the projection of a 3D point on an any arbitrary plane.

Let us assume we have a 3D point $A(x_1, y_1, z_1)$ and a plane P given by the equation $ax + by + cz + d = 0$ on which we want to take the projection of point A as given in Figure 1

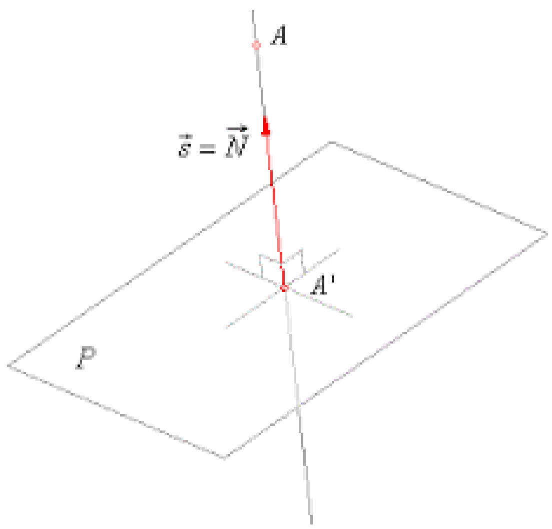

Figure 1: Projection of a point on a Plane

So to find the projection of the point A on plane P we need to find the point A'. Let this point A' be (α, β, γ) to find the point A' first we will find the equation of line given by AA' this line is parallel to the normal vector of Plane P and passes through point A. Thus equation of line AA' is given by

$$\frac{x - x_1}{a} = \frac{x - x_2}{b} = \frac{x - x3}{c} = t$$

where t is a varying parameter, thus any arbitrary point on line AA' is of the form $(at + x_1, bt + x_2, ct + x_3)$ which implies point A' is also of this form and it will also satisfy the equation of the plane P hence putting this parametric form of point A' in equation of Plane P we get

$$a(at + x_1) + b(bt + x_2) + c(ct + x_3) + d = 0$$

from here we can solve for t and find our point A', which is projection of point A on plane P. So this method could give the projection of a point given any arbitrary plane and as this method uses linear equations to solve for the projection point we will try to make a matrix based approach to this problem.

The most common way of representing 2D projections of a 3D solid is by taking its projections on three mutually perpendicular coordinate planes that are XY, YZ, ZX. The axes are fixed to the 3D object and projections is taken on these planes.

Now suppose we have a point P (x, y, z) in XYZ coordinate system as shown in Figure 2.

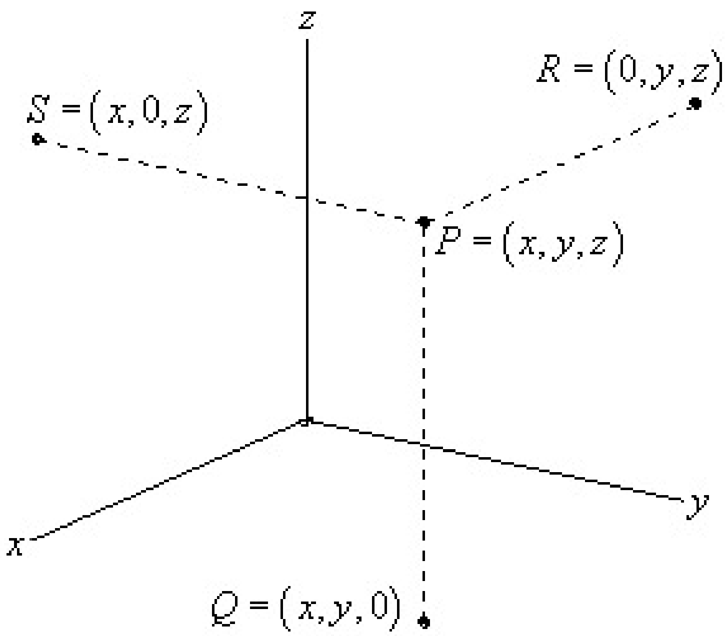

Figure 2: Point P in XYZ coordinate system

If we apply the method described above to find the projections if this point P on XY, YZ, ZX planes we get projection on XY plane = $Q = (x, y, 0)$ projection on YZ plane = $R = (0, y, z)$ and similary on ZX plane =$S = (x, 0, z)$. Hence for every point P of the from (X, Y, Z) the projection of this point on XY plane = $(X, Y, 0)$. So if we represent our point in form of a column matrix like

$$P(X, Y, Z) \Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Then we can define a $2 * 3$ **Conversion Matrix** let say

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

which when multiplied by point P maps it to its projection on XY plane, i.e,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

Similarly we can find the projections on another two planes using the conversion matrix accordingly. But these 3 projections are on 3 different planes and our main aim is to make these 3 projections on a single plane let say XY. For which we have to look into a concept called **Rotation of Coordinate Axes.**

Suppose we have a point P (x, y) marked in XY plane and now this plane is rotated about the Z axis by an angle α as shown in Figure 3.

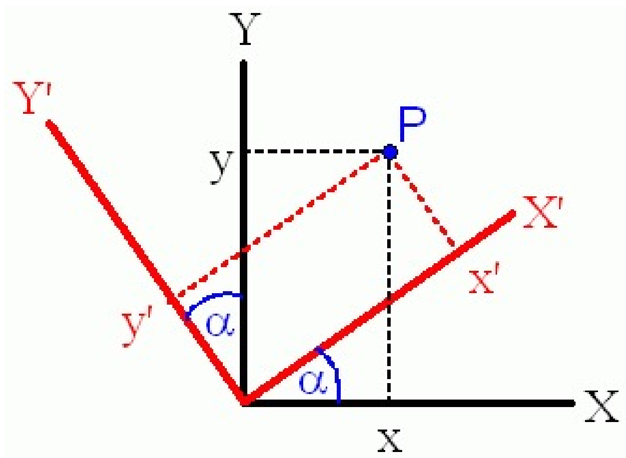

Figure 3: Rotation of XY plane about Z axis

In such case the coordinates of Point P w.r.t to new coordinate axes $X'Y'$ will be different from previous values and we can easily find out these values by multiplying the original point respective rotation matrix.

The rotation matrix for rotation around X, Y, Z axes (also called as elemental rotation) in case of 3D coordinate system are given respectively:-

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\theta) & -sin(\theta) \\ 0 & sin(\theta) & cos(\theta) \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} cos(\theta) & 0 & sin(\theta) \\ 0 & 1 & 0 \\ -sin(\theta) & 0 & cos(\theta) \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

These three rotation matrices rotate axes by an angle θ about $X, Y \text{or} Z$ axis, in three dimensions, using the right-hand rule. That is the direction of rotation according to right-hand rule is taken to be positive(θ is positive). So if we have a point P (x, y, z) in original XYZ coordinate system and want to find out it's coordinate w.r.t to the rotated coordinate(let say $(x', y'z')$) axes $X'Y'Z'$ we just need to multiply the coordinate vector with appropriate rotation matrix.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = R_x(\theta_x)R_y(\theta_y)R_z(\theta_z) \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where θ_x,θ_y and θ_z are the angles by which axes are rotated about X, Y and Z axes respectively. One can note that if either of θ_x,θ_y and θ_z are 0 then the corresponding rotation matrix turns to identity matrix, denoted by I.

Now we will see that how we can use these rotation vectors to get all the projections of a 3D solid on a single plane. Suppose we have a solid kept in a 3D space with X, Y, Z coordinate axes as shown in Figure 4 and we want to draw orthographic projections of this solid all on XY plane.

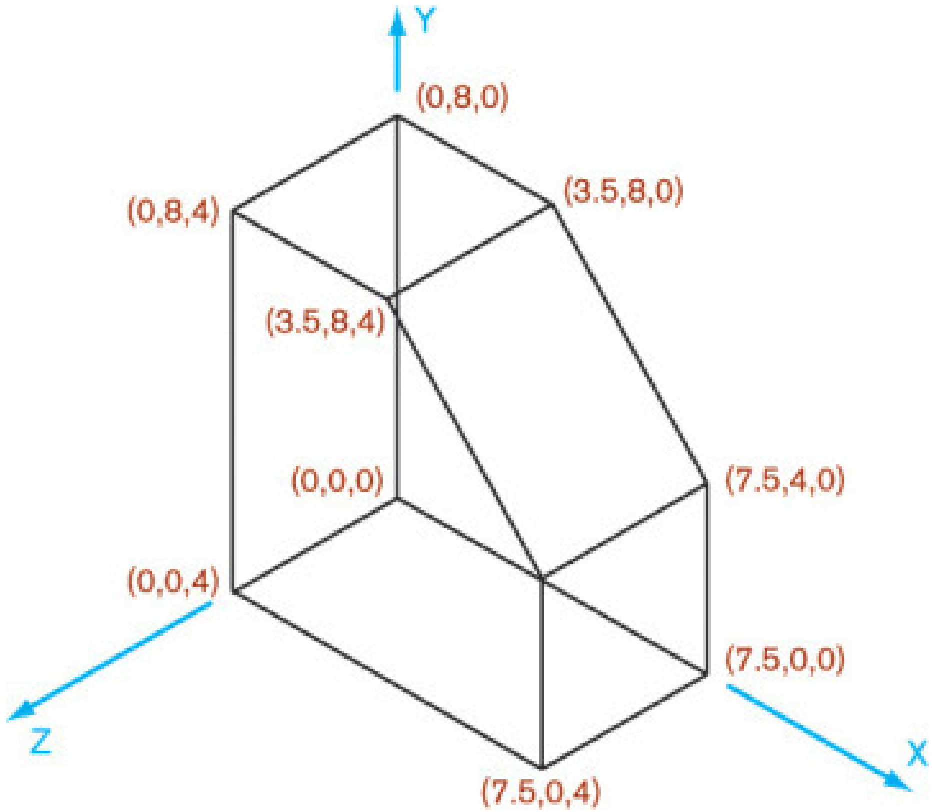

Figure 4: Solid kept in a 3D space

To draw the **Front View** of this object that is when viewed along the Z axis we multiply each of its vertex point with Conversion Matrix for XY plane projection. That is each vertex of the 3D solid of the form (α, β, γ) the corresponding point on XY plane let say $(\alpha_{front}, \beta_{front})$ is given by

$$\begin{bmatrix} \alpha_{front} \\ \beta_{front} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

To draw the **Top View** on XY plane we first rotate the coordinate axes by -90° degree about X axis according to the right-hand rule so that top view of the object becomes parallel to XY plane. Now to find the corresponding projections of each vertex (α, β, γ) in top view let say $(\alpha_{top}, \beta_{top})$ we first multiply each coordinate vector by corresponding Rotation Matrix and then by Conversion Matrix.

$$\begin{bmatrix} \alpha_{top} \\ \beta_{top} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\theta) & -sin(\theta) \\ 0 & sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \qquad where\ \theta = -90°$$

Similarly, to draw the **Side View** on XY plane we first rotate the coordinate axes by +90° degree about Y axis according to the right-hand rule so that side view of the object becomes parallel to XY plane. Now to find the corresponding projections of each vertex (α, β, γ) in side view let say $(\alpha_{side}, \beta_{side})$ we first multiply each coordinate vector corresponding by Rotation Matrix and then by Conversion Matrix.

$$\begin{bmatrix} \alpha_{side} \\ \beta_{side} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} cos(\theta) & 0 & sin(\theta) \\ 0 & 1 & 0 \\ -sin(\theta) & 0 & cos(\theta) \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \qquad where\ \theta = 90^\circ$$

So, in this fashion we could map each vertex of our 3D solid to its corresponding projection in Front, Top and Side view. After this our next step is to to transfer the details of **Edges** to these 2D projections. This is done in following way:-

Transferring Edges-We iterate over all edges of our 3D solid and suppose if there exists an edge E between 2 vertices AB then we add the corresponding edge between A and B in all the projections.

After this step we have almost completed our task of projecting 3D solid to its 3 orthographic projections.

Next step is storing the details of faces present in our 3D solid. These details as for storing as details of faces are not shown in 2D projections. These details will help us making our 2D projections more accurate when hidden lines are encountered.

Dealing with hidden lines

According to the standard rules of Engineering Drawing all the edges of a solid are not shown in each view. Only the lines which are directly visible from that view direction are shown as solid lines and any other edges which is present but not directly visible from that view direction i.e, hidden when seen from that view is shown as dashed line and called as hidden line. This gives our 2D projections more details about 3D solid and is helpful is regenerating 3D solid from 2D projections.

To draw complete 2D projection of a solid we will first draw all the lines as solid lines and after that find the hidden lines and represent them as dashed lines. To find which lines are dashed in a particular view we will start viewing the object from that direction from some distance more than the boundary of our 3D solid. We will keep traversing in this direction and whenever we find a Face we will store all lines behind that Face and mark those part of those lines as hidden which directly fall behind that Face. We will keep on moving in this fashion unless we cross the opposite boundary of our 3D solid.

How to compute the 3-D description given one or more 2-D projections

By convention three orthographic views(i.e top, face side) have been widely used for the re-construction of a solid. These views are produced by making parallel projections of objects onto three mutually perpendicular planes. We'll explain it later why we need these three views. An object projected in the three orthographic views can be denoted by a set of points P and a set of lines L. The set of lines represents the connectivity between the points in the set P. The drawing consists of two types of line first is in which features that are directly visible which are represented by solid lines, secondly the features that are not directly visible which are shown as dashed lines. We assume that these information are given to us. And, in each views the vertices are labelled for us to identify. We have used B-rep(Boundary representation) oriented method for the generation of 3-D polyhedral solids from these projections. B-rep is a method for representing shapes using the limits.

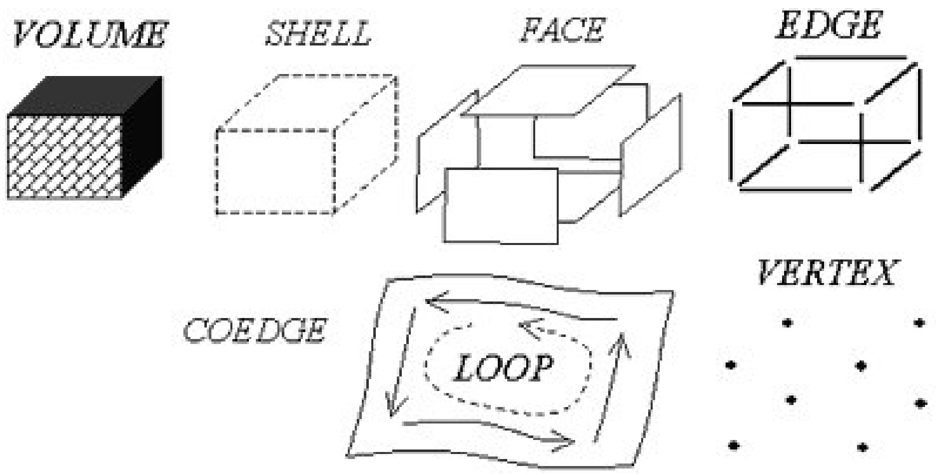

Figure 5: B-rep technique

The steps involved are :

- Generate 3D candidate vertices from 2D vertices in each view.
- Generate 3D candidate edges from 3D candidate vertices.
- Construct 3D candidate faces from 3D candidate edges on the same surface.

■ Construct 3D objects from candidate faces.

For constructing our wire frame model, we assume that all the data regarding the vertices, edges and faces are provided to us through the 2-D projection views. For examples we create a set of vertices say P = $\{p_1, p_2, p_3, ...\}$ from the given three projected views. And also we create a set of lines L = $\{l_2, l_2, ...\}$ also storing the fact that which of those points forms a line. Similarly a set of area A = $\{a_1, a_2, ...\}$ and also the fact about which set of lines constitutes the area like in the figure shown:

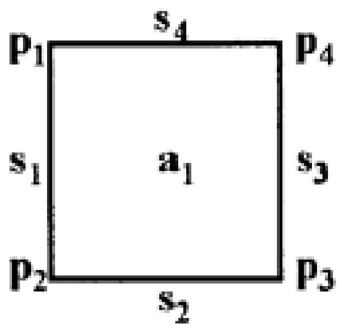

Figure 6: Projected view

Firstly, we construct all the possible 3-D vertices that constitutes the wire frame by establishing correspondences among the orthographic views(i.e front, top, side). A candidate vertex is created from the 2-D vertices given in the three views. Let

$$P_f = (P_x, P_z), P_t = (P_x, P_Y), P_s = (P_y, P_z)$$

be the vertices in the front, top and side view respectively and P_x , P_y , P_z is equal for all the three case. Or in our case,

$$|P_x(f) - P_x(t)| < \epsilon$$

$$|P_z(f) - P_z(s)| < \epsilon$$

$$|P_y(s) - P_y(t)| < \epsilon$$

where ϵ is the amount of error we allow to overcome inexact matching problem. Then, P_x , P_y , P_z is the corresponding vertex for the 2-D vertices in the projection plane like in the figure shown:

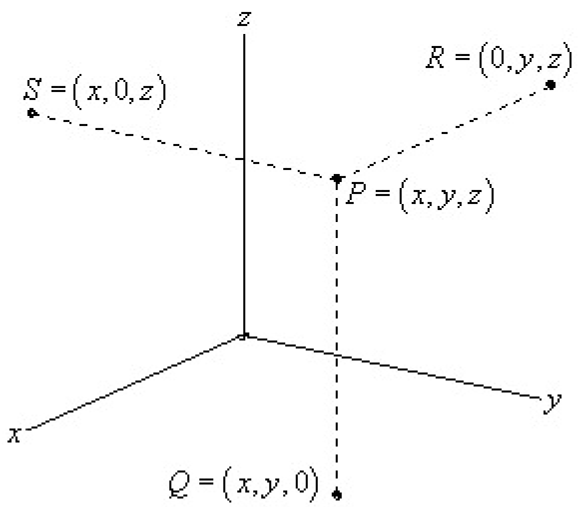

Figure 7: Locating a vertex through its projection

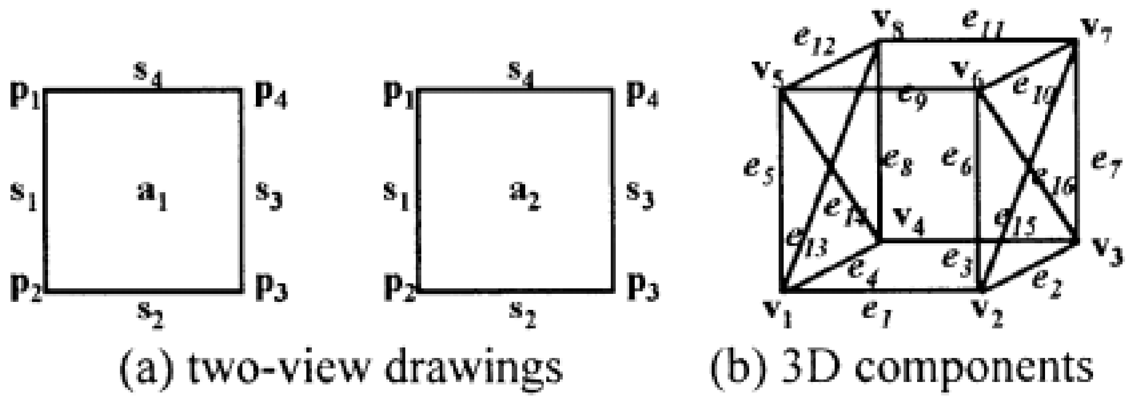

Figure 8: Two-view drawings and corresponding 3D components

These give us the sets of Vertices V = $\{v_1, v_2, ...\}$ for our wire-frame model. Now to construct edges we first choose a set of vertices and construct an edge between the two vertices and check if the edge is valid by its projection on the three perpendicular planes and the original set of edges extracted from our 2-d projection planes given to us(including both Hidden and solid lines). If the set is valid we add the given pair of vertices To our edge set E=$\{e_1, e_2, ...\}$. Now repeat the same process with the remaining set of pair of vertices.

Now By finding out the set of vertices V and set of edges E we have create a pseudo wire-frame model like in Fig - (though these set may contain some extra edges). The pseudo wire frame model will now be used to generate faces and surfaces.

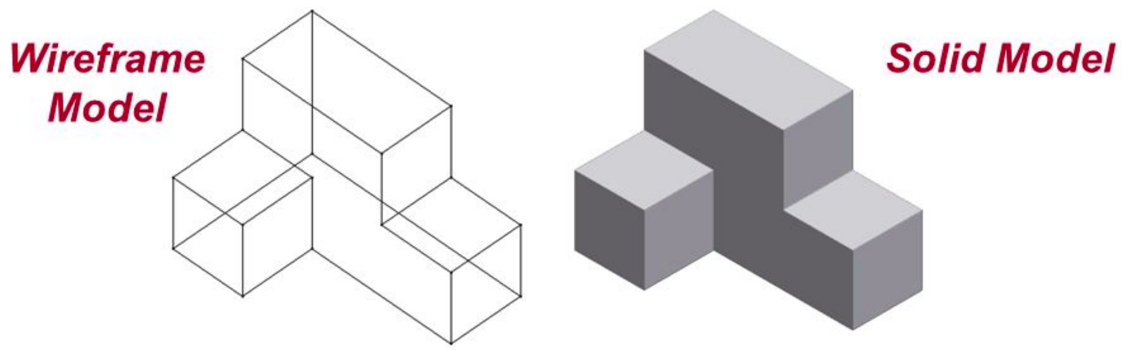

Figure 9: Wire-frame model

We now want to generate surfaces(Unbounded face) and faces(bounded by an edge loop). We can identify a surface by the fact about two edges sharing a common vertex. (i.e two co-planar edge sharing an edge will form a planar surface). After the surface is identified we'll look to find the corresponding face by searching through the list of edges we have to form a loop of edges making the face. This process is used for making the set of faces for all possible surfaces. Now After making the set of faces we remove the redundant edges that doesn't constitute at least two non-planar faces. So set of edges are reduced by number. Again we repeat the process of making the faces using the new set of edges and vertices. After the set of faces have been made, We find all set of solids that can be made using these faces and compare these with actual projection (i.e the set of points, line, area given to us through projection).

Multiview drawings usually require several orthographic projections to define the shape of a three-dimensional object. Each orthographic view is a two-dimensional drawing showing only two of the three dimensions of the three-dimensional object. Consequently, no individual view contains sufficient information to completely define the shape of the three-dimensional object. All orthographic views must be looked at together to comprehend the shape of the three-dimensional object. The arrangement and relationship between the views are therefore very important in multiview drawings A photograph shows an object as it appears to the observer, but not necessarily as it is. It cannot describe the object accurately. No matter what distance or which direction it is taken from , because it does not show the exact shapes and sizes of the parts. It would be impossible to create an accurate three-dimensional model of an object using only a photograph for reference because it shows only one view. It is a 2D representation of a 3D object.

A sketch or drawing should only contain the views needed to clearly and completely describe the object. These minimally required views are referred to as the **necessary views.**

Some symmetric objects can be completely described by two projection views(eg. simple cuboid). But for a general 3D object we need a minimum of **three projections.** The most commonly used views in Engineering Drawing are front view, top view and side view.

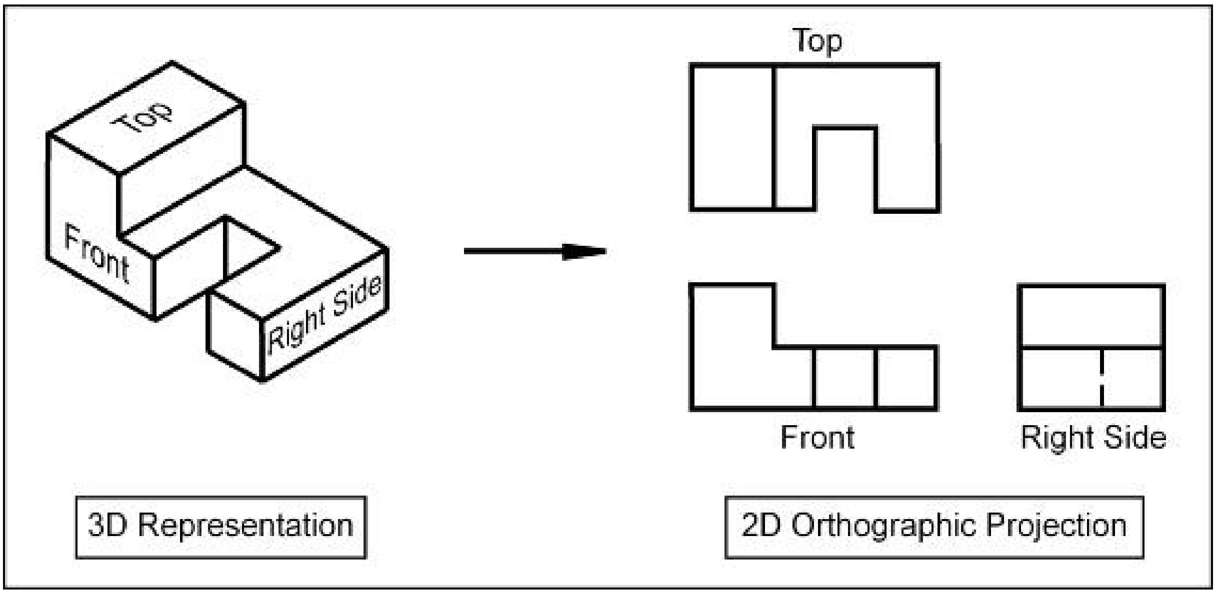

Figure 10: Projections of a 3D solid

But now let us look into the fact that what made us say that 3 projections of a 3D solid are sufficient to uniquely define an 3D object.

In an abstract way, to find the location of a point in 3-dimension we just need to know projection of the point in 2 views(i.e two non parallel planes). If we draw two lines perpendicular to these projection planes from the respective projection point, their intersection will give us our point. let the equations of projection planes be :

$$a_1 x + b_1 y + c_1 z + d_1 = 0$$

$$a_2 x + b_2 y + c_2 z + d_2 = 0$$

and the two projection points be:

$$(x_1, y_1, z_1), \ (x_2, y_2, z_2)$$

This gives the equation of line as :

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

$$\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$$

Solving these two lines will give us our point.

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- 2.Reconstruction of quadric surface solids from three-view engineering drawings M H Kuo