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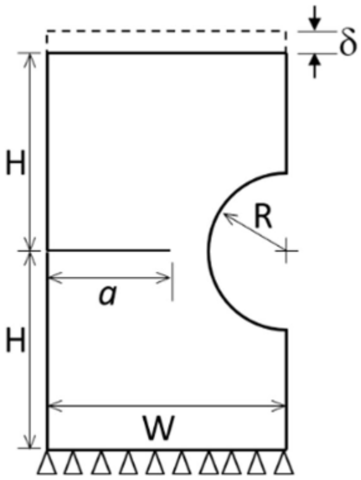
ME728- Fracture and Fatigue: Project

Case 1: Semi-Circular Boundary



1) Problem: The loading is a uniform displacement of the top edge by δ = 5 × 10-5 m, while the bottom edge is restricted from moving vertically. Having plane stress conditions with unit thickness.

    Dimensions:
    W = 60 mm, H = 84 mm, a= 20 mm, R = 25 mm.
    Material Properties:
    E, elastic modulus is 70 GPa and ν, Poisson's ratio is 0.3.

2) Software Used: ABAQUS/CAE2019
3) Details of the finite element mesh and boundary conditions:
    a) Part geometry:
        - W = 0.06 m, H = 0.084 m, a= 0.02 m, R = 0.025 m
    b) Meshed model:
        - Plate: Size of element: 5x10^{-4} m, Type of element: CPS4, No. of element: 11126 (excluded near crack tip)
        - Near the crack tip: Size of element: 6.28x10^{-5} m, Type of element: Triangular, No. of element: 32 in the diameter of 0.5x10^{-3} m, Mid side node parameter – 0.25 (Collapsed element side single node).

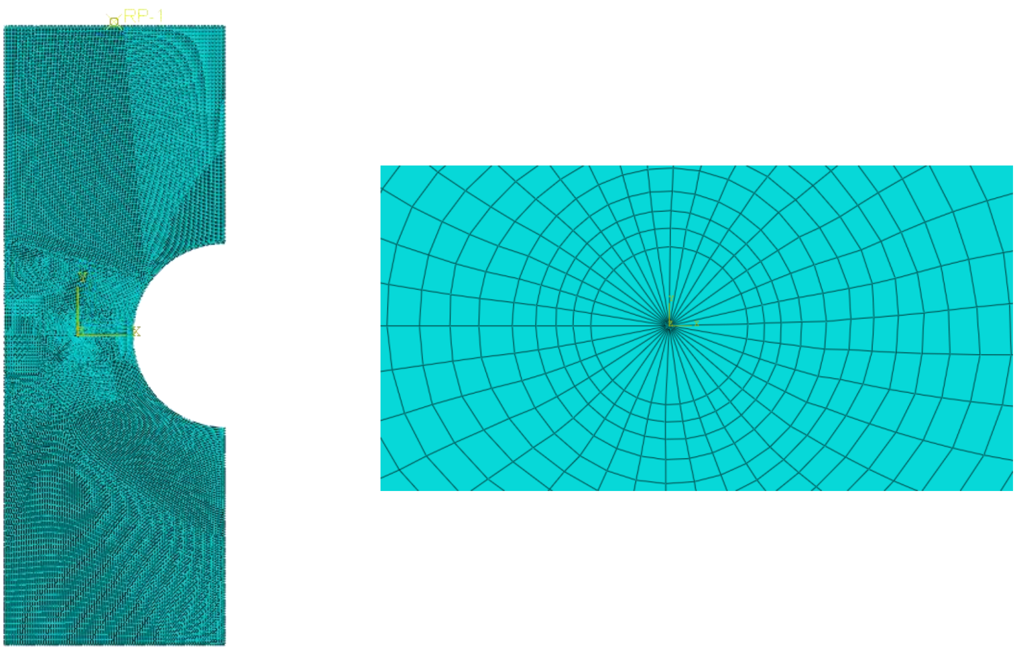


Figure 1: Meshed model.

    c) Boundary conditions:

    The loading is uniform displacement of the top edge by δ = 5 x 10-5 m, while the bottom edge is restricted from moving vertically.

d) Interaction:

- Line ‘ab’ is assigned as crack line and first circular region is assigned as crack front.
- Crack extension direction is given as X-axis direction.
- Collapsed element side; single node is given for singularity to move all the nodes together and given the behaviour of all triangular element.
- To induce the singularity behaviour $\left(\frac{1}{\sqrt{r}}\right)$, the mid-side node is shifted towards the crack tip. In Abaqus, an option is provided for specifying this singularity behaviour. The parameter for the mid-side node shift is set to 0.25, indicating that the mid-side node is shifted by 0.25Δ towards the crack tip, where Δ represents the length of the element near the crack tip. A triangular element is placed in the vicinity of the crack tip, as illustrated in Figure 1. The circular region encompassing the triangular element is designated as the crack front region. This designation informs the software to shift the mid-side node by the specified parameter in this region.
- Five no. of contours are given to check that J- integral value is not depend on the contour.

All the boundary conditions are given in Figure no. 2.

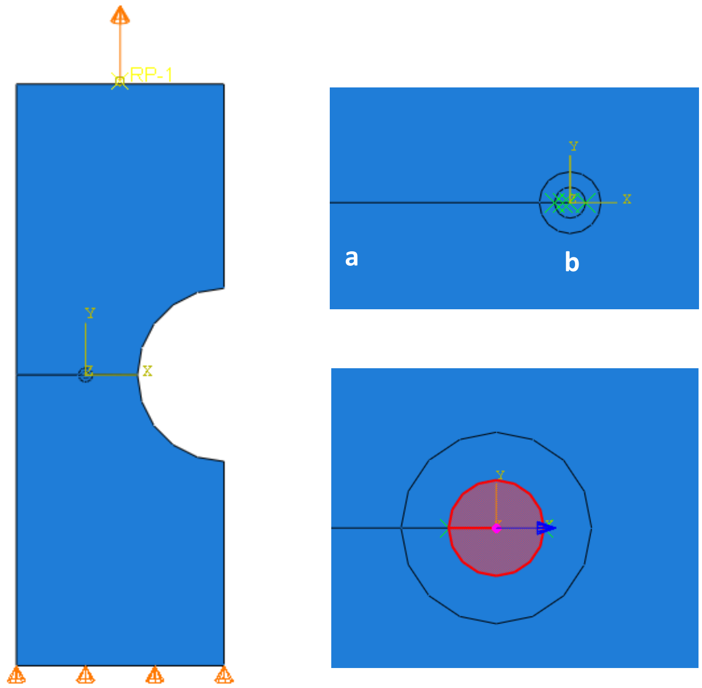


Figure 2: Boundary conditions.

e) Material properties:

- E, elastic modulus is $70x10^9$Pa
- ν, Poisson’s ratio is 0.3.

4) Results and discussion:

a) Estimation of SIF using stress extrapolation method:

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi x}} + A_1\sqrt{x},$$

$$\sigma_{yy}\sqrt{2\pi x} = K_I + \sqrt{2\pi}A_1 x,$$

$$K_{app} = K_I + \sqrt{2\pi}A_1 x, \qquad (1)$$

Where, σ_{yy} = stresses in y-direction, which is indicated as S22 in FEM, x = true distance along path at θ = 0°. Stresses in Y-direction near the crack tip are shown in Figure 3 and path taken for opening stress is shown in Figure 4. From the figure no. 3, it is visible that stresses in Y-direction having symmetry through the crack line, it means crack will be open along the crack line direction.

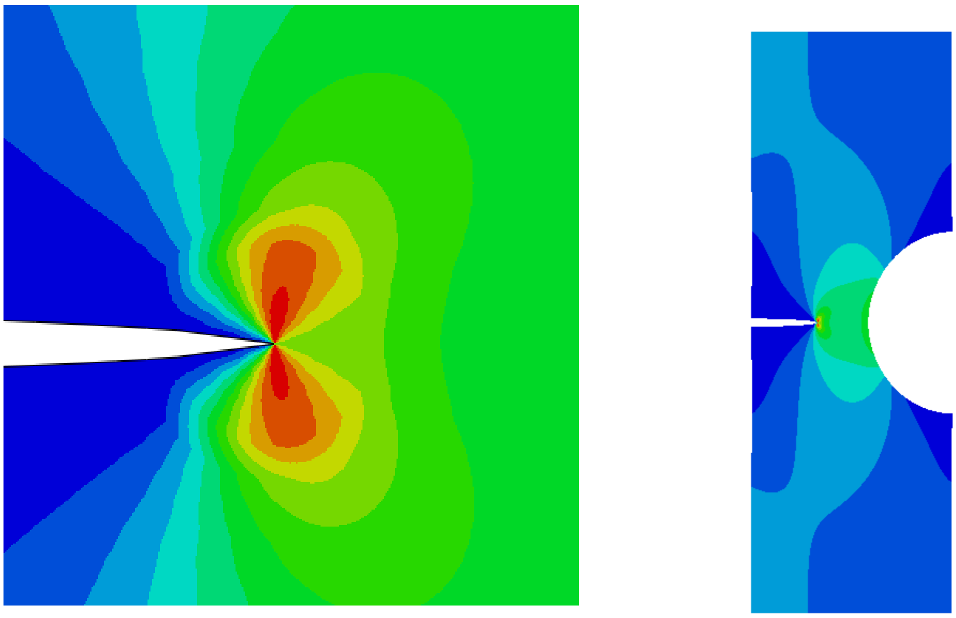


Figure 3: Stress in the y direction (S22)

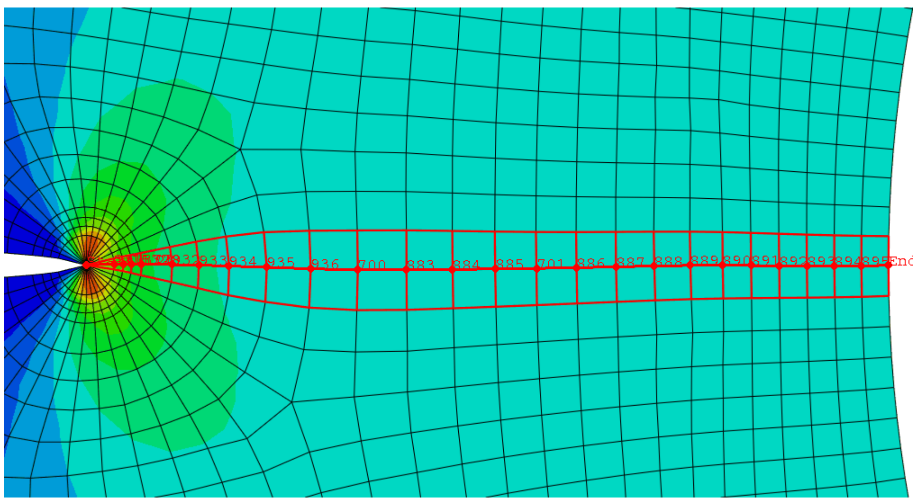


Figure 4: Path for opening stress

The stress data along the opening stress path are also extracted from the software and plotted, as depicted in Figure 5. The data presented in Figure 5 undergoes post-processing using the stress extrapolation method and Singular term of the stress using the formula $\frac{K_1}{\sqrt{2\pi x}}$.

The stress intensity factor, K_I, is determined to be $K_I = 5.2X10^6\ Pa\sqrt{m}$, which corresponds to the intercept of the trend line shown in Figure 6, in comparison to equation no. 1.

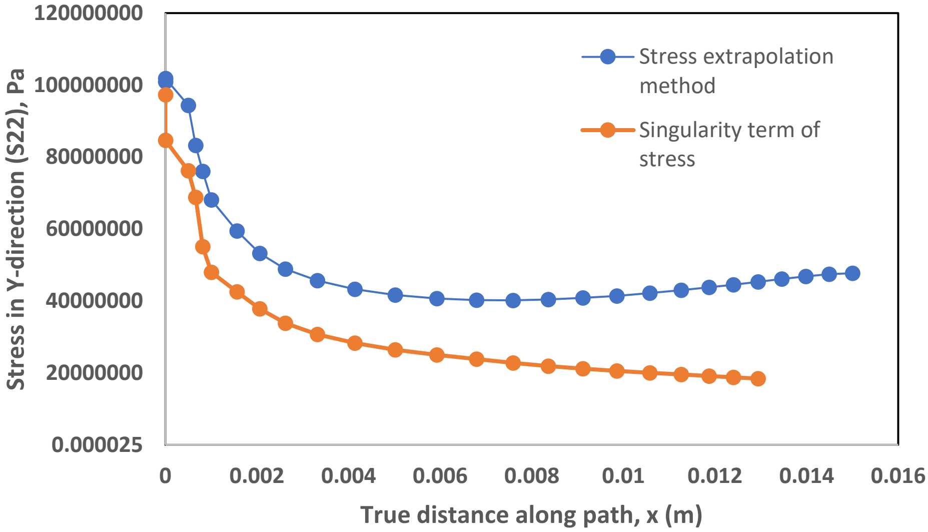


Figure 5: Opening stress along the path (FEM) at θ = 0°

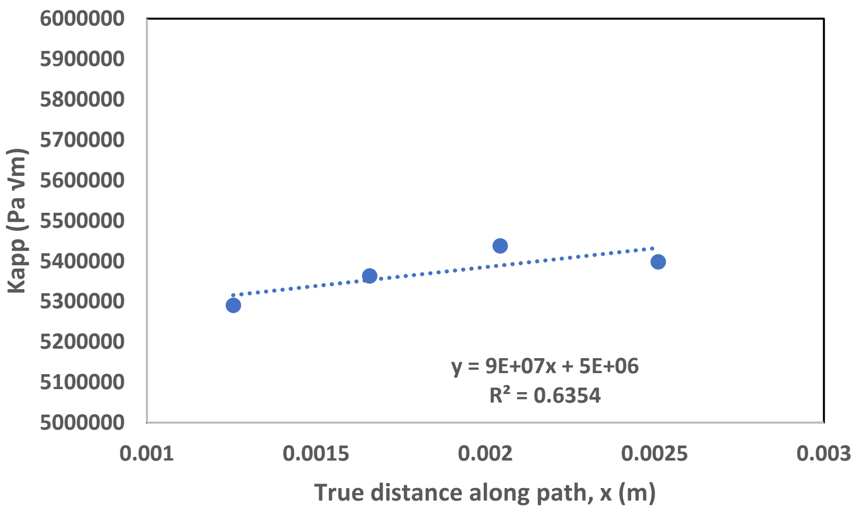


Figure 6: K_{app} along the path (FEM) at θ = 0°

The value of 'stress intensity factor' is obtained from the 'history output' from the Abaqus software, which is given in Table 1.

Contour No.	1	2	3	4	5
$K_I(Mpa\sqrt{m})$	5.44	5.42	5.43	5.42	5.42
J- integral (J/m)	427.1	427.5	427.1	421.7	423.1

As we know, the J-integral values do not depend on the path. It is observed that the J-integral and stress intensity factor, K_I, values for all contours are consistent and identical as indicated in Table 1. It is observed that the values of stress intensity factor obtained from the software shown in Table 1 and the stress extrapolation method are coming closer with 4.7% error.

b) Estimation of SIF using the displacement extrapolation method:

$$E\ v = \frac{2}{\pi} K_I \sqrt{2\pi x} - \frac{4}{3}\ A_1 x^{\left(\frac{3}{2}\right)},$$

$$\frac{\frac{\pi}{2} E v}{\sqrt{2\pi x}} = K_I - \left(\frac{\sqrt{2\pi}}{3}\right) A_1 x,$$

$$K_{app} = K_I - \left(\frac{\sqrt{2\pi}}{3}\right) A_1 x, \quad (2)$$

Where v is the vertical displacement, E is the modulus of elasticity for a plane stress condition, and x is the path for the opening displacement starting from the crack tip. K_I is stress intensity factor, A_1 is constant.

Figure 7: Displacement in the y direction (U2)

Displacement in the Y-direction is shown in Figure 7, and the path taken for opening displacement at θ =π is shown in Figure 8. The displacement data along the opening displacement path at θ = π are also extracted from the software and plotted, as depicted in Figure 9. The data presented in Figure 9 undergoes post-processing using the displacement extrapolation method and the Singular term of the displacement by using the formula $\frac{8K_1}{E}\sqrt{\frac{x}{2\pi}}$.

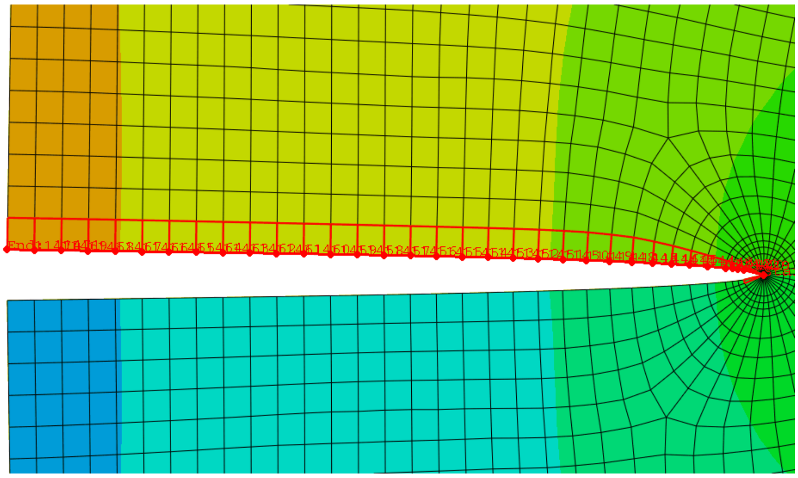


Figure 8: Path for opening displacement

The stress intensity factor, K_I, is determined to be $K_I = 5.64X10^6\ Pa\sqrt{m}$, which corresponds to the intercept of the trend line shown in Figure 10, in comparison to equation no. 2.

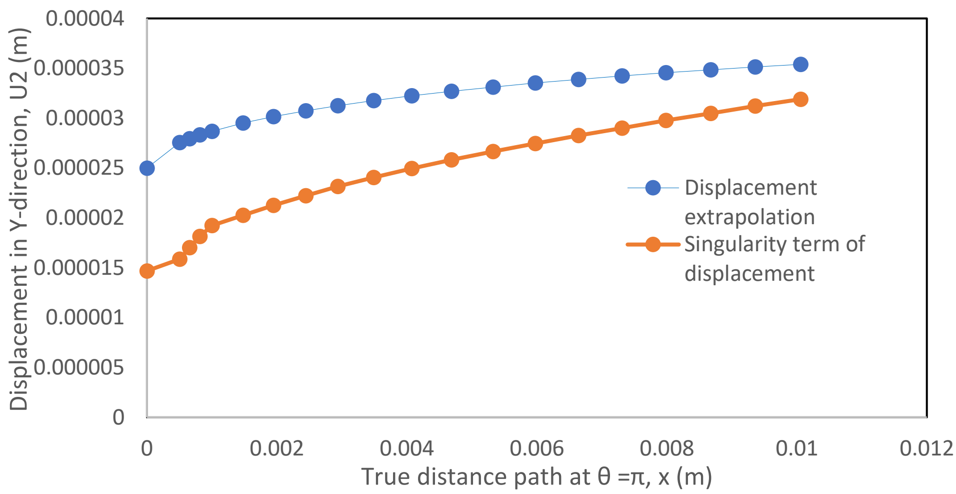


Figure 9: Opening displacement along the path (FEM) at θ = π

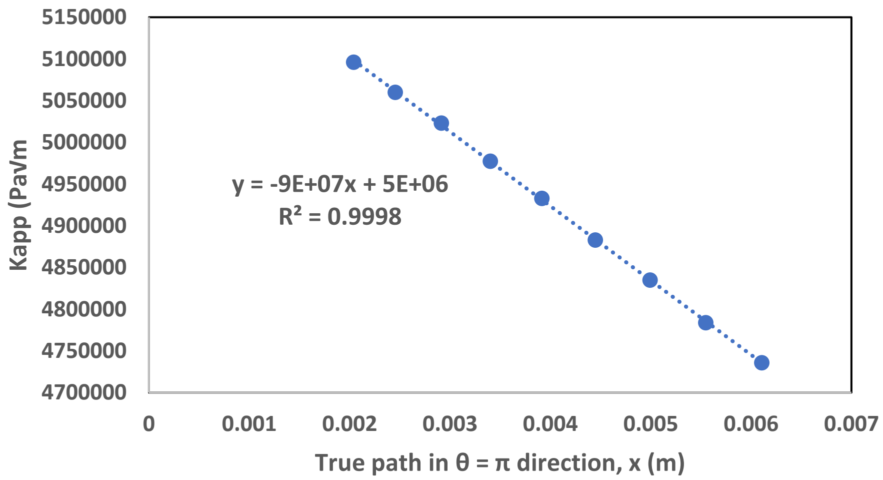


Figure 10: K_{app} along the path (FEM) at θ = π

It is observed that the values of stress intensity factor obtained from the software shown in Table 1 and stress extrapolation method are coming closer with 3.2% error.

C) Estimation of SIF using Modified virtual crack closure method (MVCCM):

$$W = \frac{1}{2B}\{F_{yi}(v_k - v_{k'})\},$$

$$G_I = \lim_{\Delta \to 0} \frac{W}{\Delta} \sim \frac{1}{2\Delta B}\{F_{yi}(v_k - v_{k'})\} \quad (3)$$

$$J = G_I = \frac{K_I^2}{E} \; for\ linear\ elastic\ and\ plane\ stress.$$

F_{yi} = Load at the crack tip act through the element above the crack tip from θ = 0° to θ =π.

v_k and $v_{k'}$ are the vertical displacements of crack opening of elements attached to the crack tip. Length of element closet to the crack tip, Δ = $0.5x10^{-3}$m.

Thickness of plate, B is assumed as unit thickness.

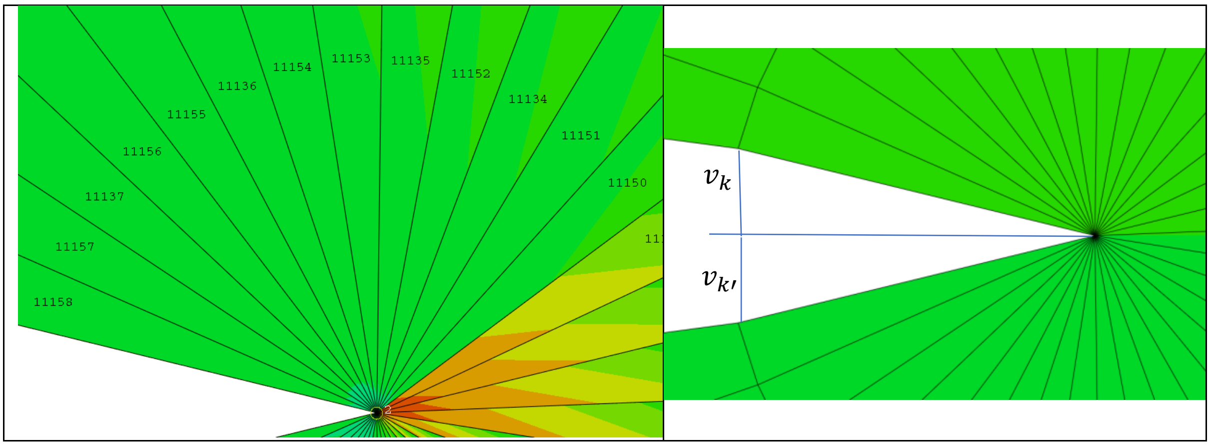


Figure 11: Elements attached to the crack tip from θ = 0° to θ =π

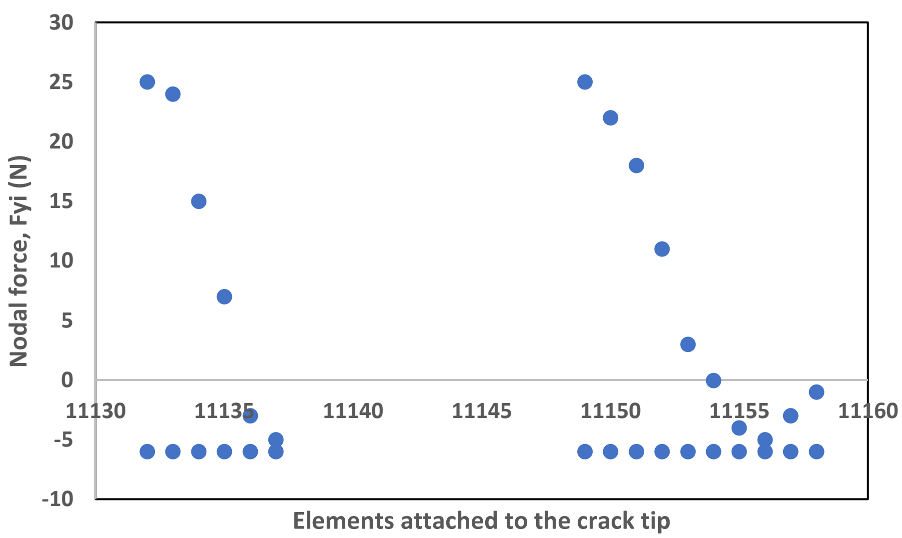


Figure 12: Nodal force on elements attached to the crack tip from θ = 0° to θ =π

From the Figure 12, $F_{yi} = 33N$ by using equation no. 3, K_I is calculated.

$K_I = 4.85MPa\sqrt{m}$ with 11% error.

Case 2: Straight Boundary

For case applied same material properties and boundary conditions.

1) Results and discussion:
    a. Estimation of SIF using stress extrapolation method:

The stress data along the opening stress path are also extracted from the software and plotted, as depicted in Figure 15. The data presented in Figure 15 undergoes post-processing using the stress extrapolation method and Singular term of the stress using the formula $\frac{K_1}{\sqrt{2\pi x}}$.

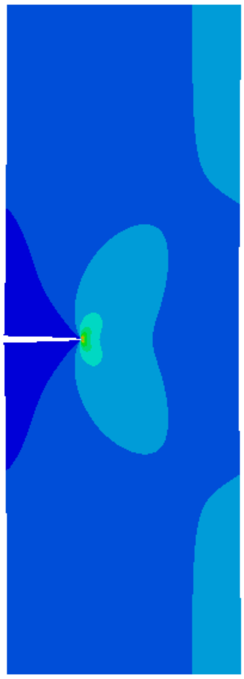


Figure 13: Stress in the y direction (S22)

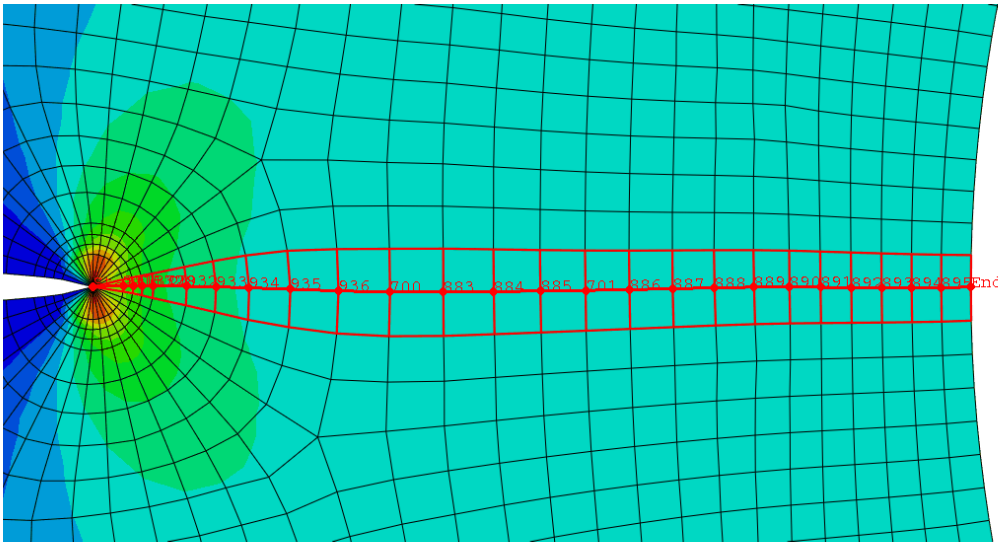


Figure14: Path for opening stress

The stress data along the opening stress path are also extracted from the software and plotted, as depicted in Figure 15. The data presented in Figure 15 undergoes post-processing using the stress extrapolation method. The stress intensity factor, K_I, is determined to be $K_I = 5.72X10^6\ Pa\sqrt{m}$, which corresponds to the intercept of the trend line shown in Figure 16, in comparison to equation no. 1.

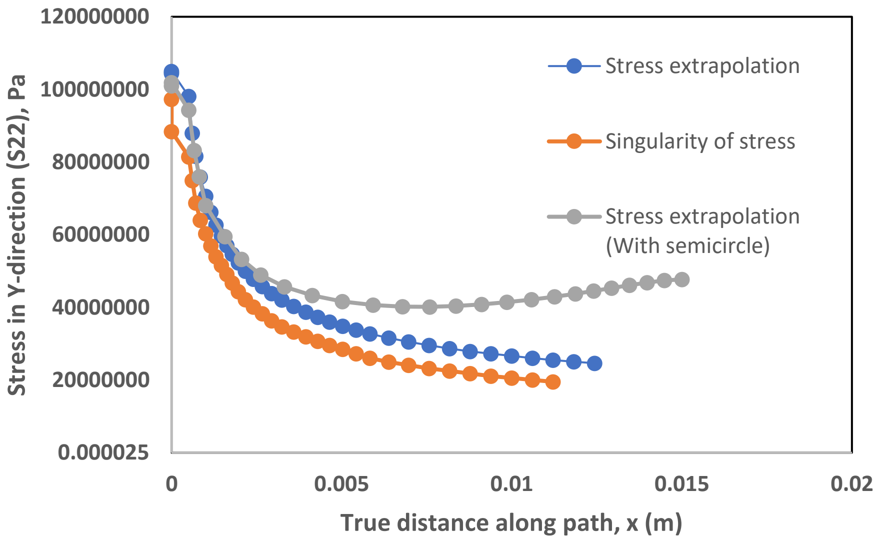


Figure 15: Opening stress along the path (FEM) at θ = 0°

It is observed from Figure 15 that stresses exhibit a downward trend followed by an upward trend, attributable to geometric effects. This behaviour is contingent upon the distance between the crack tip and the initiation of the semicircular region. When this distance is substantial, stress does not exhibit a continuous increase over distance. However, when the crack tip and the start of the semicircular region are in close proximity, the slope of stress increase becomes steeper.

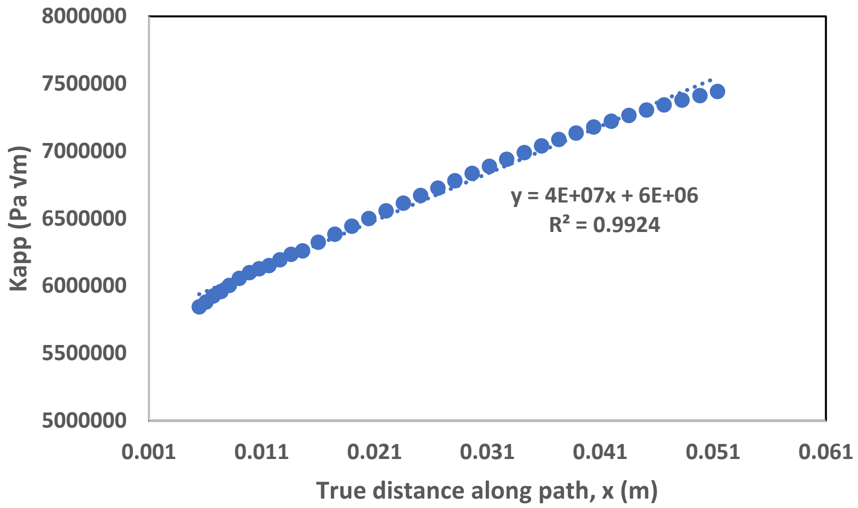


Figure 16: K_{app} along the path (FEM) at θ = 0°

The value of 'stress intensity factor' is obtained from the 'history output' from the Abaqus software, which is given in Table 2.

Contour No.	1	2	3	4	5
$K_I(Mpa\sqrt{m})$	5.77	5.77	5.77	5.76	5.76
J- integral (J/m)	427.1	427.5	427.1	421.7	423.1

As we know, the J-integral values do not depend on the path. It is observed that the J-integral and stress intensity factor, K_I, values for all contours are consistent and identical as indicated in Table 2. It is observed that the values of stress intensity factor obtained from the software shown in Table 2 and stress extrapolation method are coming closer with 1.2% error.

c) Estimation of SIF using displacement extrapolation method:

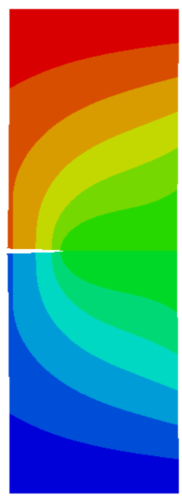


Figure 17: Displacement in the y direction (U2)

Displacement in Y-direction shown in Figure 17 and path taken for opening displacement at θ =π, shown in Figure 18. The displacement data along the opening displacement path at θ = π are also extracted from the software and plotted, as depicted in Figure 19. The data presented in Figure 19 undergoes post-processing using the displacement extrapolation method and Singular term of the displacement by using the formula $\frac{8K_1}{E}\sqrt{\frac{x}{2\pi}}$.

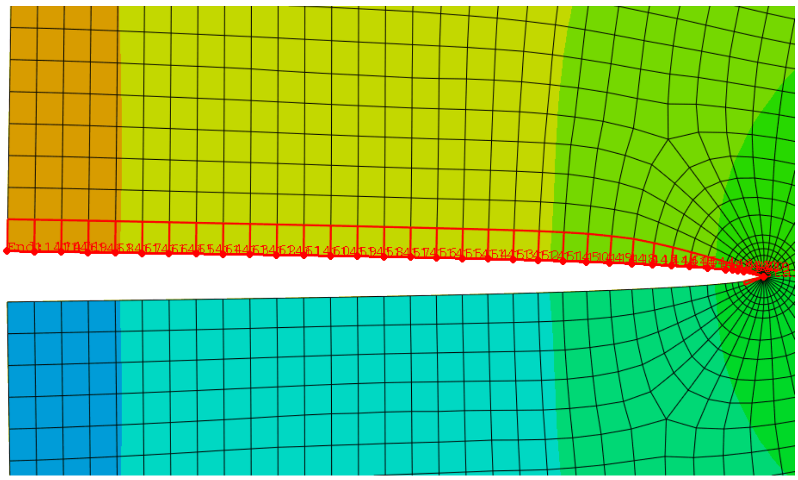


Figure 18: Path for opening displacement

The stress intensity factor, K_I, is determined to be $K_I = 5.64X10^6\ Pa\sqrt{m}$, which corresponds to the intercept of the trend line shown in Figure 20, in comparison to equation no. 2.

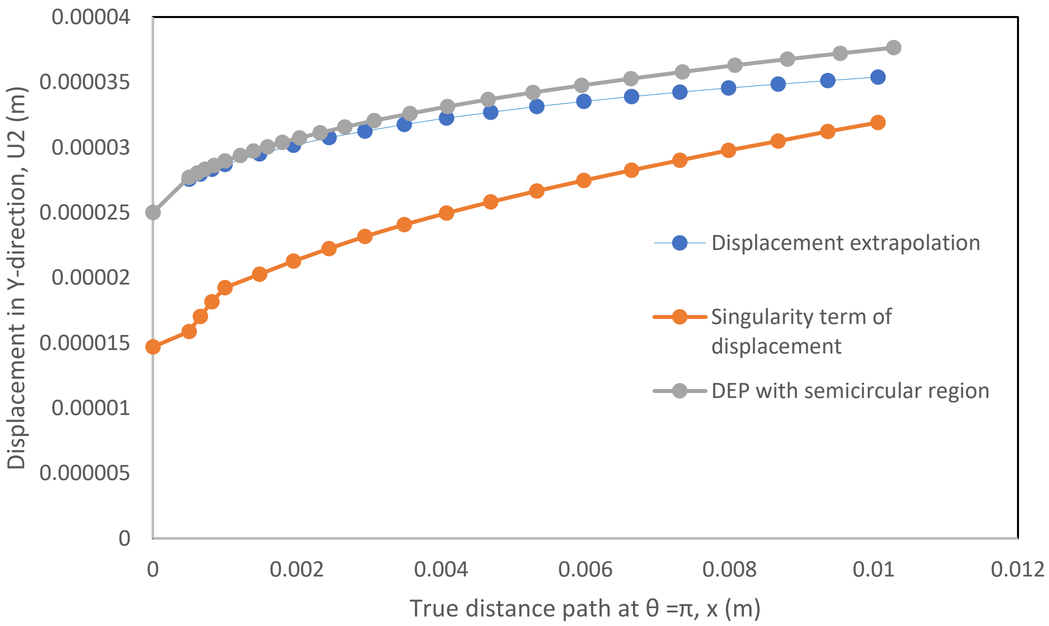


Figure 19: Opening displacement along the path (FEM) at θ = π

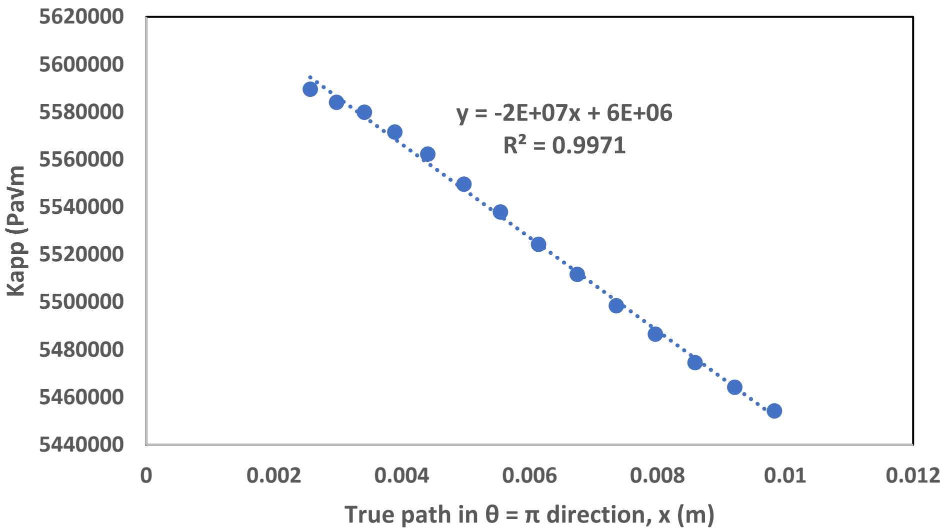


Figure 20: K_{app} along the path (FEM) at θ = π

It is observed that the values of stress intensity factor obtained from the software shown in Table 2 and stress extrapolation method are coming closer with 2.3% error.

D) Estimation of SIF using Modified virtual crack closure method (MVCCM):

F_{yi} = Load at the crack tip act through the element above the crack tip from θ = 0° to θ =π.

v_k and $v_{k'}$ are the vertical displacements of crack opening of elements attached to the crack tip. Length of element closet to the crack tip, Δ = $0.5x10^{-3}$m.

Thickness of plate, B is assumed as unit thickness.

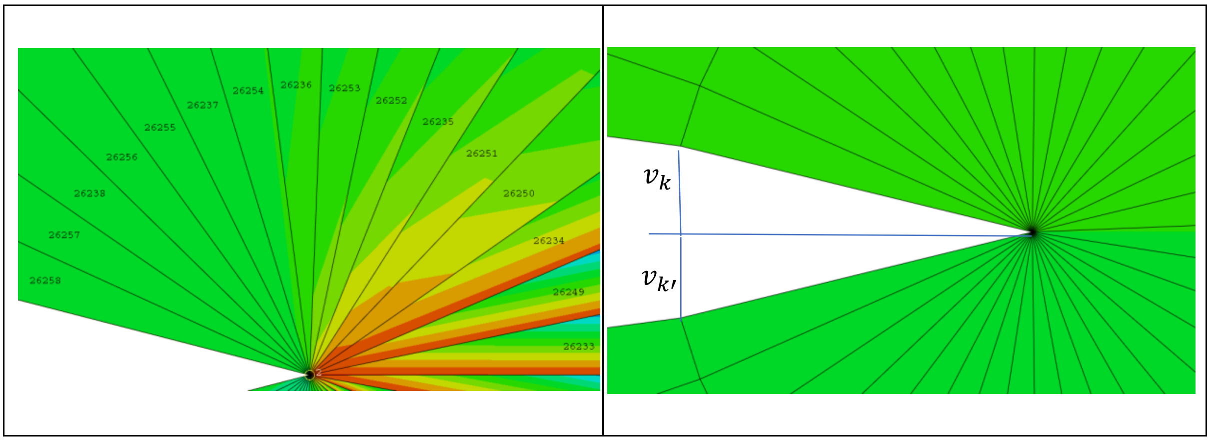


Figure 11: Elements attached to the crack tip from θ = 0° to θ =π

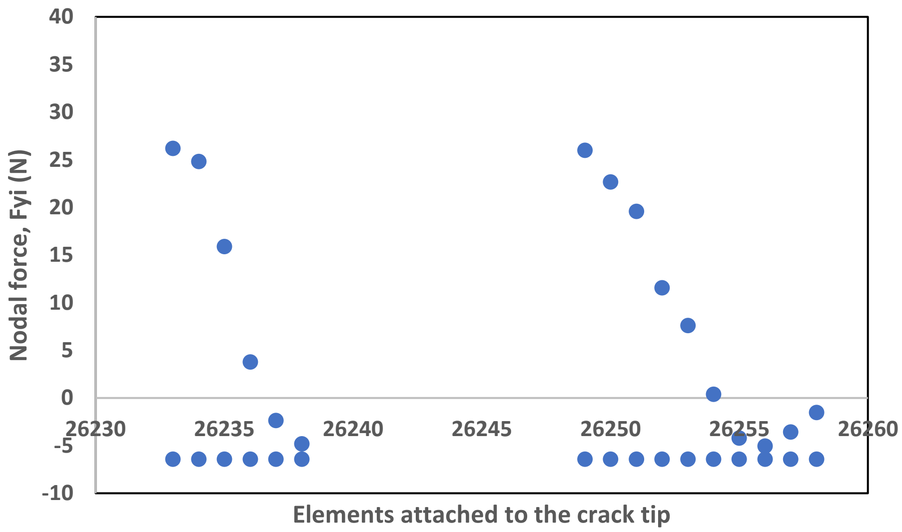


Figure 12: Nodal force on elements attached to the crack tip from θ = 0° to θ =π

From the Figure 12, $F_{yi} = 34.3N$ by using equation no. 3, K_I is calculated.

$K_I = 5.1MPa\sqrt{m}$ with 11.6% error.

Conclusion: The comparison of stress intensity factor found by using different method with and without semicircular boundary shown in Table3.

Table3: Comparison of Stress Intensity Factor

With semicircular boundary, $K_I(MPa\sqrt{m})$				Without semicircular boundary, $K_I(MPa\sqrt{m})$			
Abaqus	SEM	DEM	MVCCM	Abaqus	SEM	DEM	MVCCM
5.46	5.2	5.64	4.85	5.77	5.7	5.64	5.1
Error (%)	4.7%	3.2%	11.2%	Error (%)	1.2%	2.25%	11.6%

The comparison of stress intensity factor (K_I), values obtained with and without the semicircular boundary provides insight into the influence of the singularity domain zone on structural behaviour. With the inclusion of the semicircular boundary, the stress intensity factors exhibit variations across different simulation methods (Abaqus (FEM software name), SEM (Stress extrapolation method, DEM (Displacement extrapolation method), MVCCM (Modified virtual crack closure method). This suggests that the presence of the boundary alters the stress distribution near the crack tip, affecting the size and behaviour of the singularity domain zone. The observed differences in KIKI values, along with the associated error percentages, indicate the importance of considering geometric features, such as boundaries, in accurately predicting the stress state near crack tips. Further investigation into the interaction between boundaries and crack tips could provide valuable insights into structural integrity and failure mechanisms.