Geometry via Gestures: Learning 3D geometry using gestures

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***Abstract*— Concepts of 3-dimensional (3D) Geometry are challenging to grasp for school students. The skill of manipulating 3D objects and interpreting their structure and properties are difficult. Traditionally to teach topics that have three dimensions, 3D artifacts have been used. However the opportunity of the learner to interact during the construction and manipulation of 3D objects is desirable. In this paper, we present an application - Geometry via Gestures (G-v-G), which enables learners to interact with 3D objects using their gestures.**

We report observation and analysis from an exploratory study that was performed to identify the different aspects of 3D geometry that students could learn in the process of using the application. We also examined the students' perception of learning with the application during the study. The analysis indicates that students learn about structure and property of 3D geometrical objects after using G-v-G. In addition, students participating in the study, expressed keen interest in learning additional topics of geometry using gestures.

***Index Terms*— 3D geometry; gestures; 3D structure; 3D property; volume**

I. INTRODUCTION

Understanding and interpreting structure and properties of 3-dimensional (3D) objects is important in several areas of science and engineering. Students begin learning about 3D Geometry at high school but find it difficult to visualize 3D structures and derive important properties from them. When de-constructing a 3D structure into its constituent 2D parts, or constructing 3D structure from 2D primitives, the student has to mentally visualize the intermediate structures. For instance, a student has to think of a right circular cylinder (3D) as being made of a rectangle (2D primitive) or imagine that a cylinder (3D) can be constructed from rectangles (2D primitive) or visualize formation of cylinder from rectangles to look like opening of a book. Traditionally to teach topics that have three dimensions, 3D artifacts or models and computer applications have been used. However, physical models of 3D structures, do not lend themselves to manipulations such as breaking open, changing shape, 2D - 3D translation to name a few. Although computer applications provide interactivity with 3D objects with I/O interfaces such as keyboard and mouse, these interactions are restricted by the affordances which keyboard and mouse provide.

3D geometry thinking is closely connected to students' ability to calculate properties of solids such as volume and surface area [1]. It is desirable that students be able to link the structure of the 3D object with the formulae of its properties and properties of underlying 2D primitives that combine to form the 3D object. Visualization and construction are two main thinking process in geometry [2]. Geometrical knowledge can be constructed in a meaningful way in contexts that serve as "fields of experience". Fields of experience encompasses the internal and external context of the learner as well as the teacher context. In case of traditional setup, the "field of experience"[2] is severely limited by blackboard and viewing of models. Also by physically providing models and technology tools with limited affordances there is no/minimal control over the learners internal context. Activities in class room are more oriented towards the abstract proofs and theorems rather than concrete mental construction.

In this paper, we present the design, implementation and preliminary evaluation of an application, "Geometry via Gestures" (G-v-G). G-v-G along with certain activities, enable learners to visualize and manipulate 3D structures using gestures. The application guides learners through a series of activities involving 2D and 3D structures. Learners use gestures to do the activities. The application senses gestures and modifies the object on the screen accordingly. For example, a swipe gesture will enable the user to construct a right circular cylinder from a rectangle. The user will be able to view on the screen the initial state, trajectory made by the rectangle and modified state due to effect of user gesture on the rectangle.

We performed an exploratory pilot study in order to answer two research questions viz. (1) What can students learn about the properties of 3D structure after interacting with Geometry via Gestures? (2) What is students' perception of learning about 3D geometry after using Geometry via Gestures?

We conducted the research study with nine school students who use G-v-G to solve three tasks. The researcher acted as a facilitator, who observed the student and asked guiding questions or prompts, the responses to which were later analyzed using thematic analysis [3]. Learning is also evaluated using a post-test that mainly looked into transfer of knowledge obtained during the use of G-v-G, to new problems. Analysis of data from verbal interaction during the intervention, post-test responses and learning perception indicate that G-v-G application in conjunction with reflection prompts from a facilitator, promises to be a valuable supplement to students' Geometry lessons.

978-1-5090-6115-0/16 $31.00 © 2016 IEEE
DOI 10.1109/T4E.2016.13

II. THEORETICAL BASIS

A. Existing related work

With the advent of technology tools the visualization and construction of 3D objects have been made more realistic. They offer the learner various ways to explore an object. GeoGebra is a popular freeware that has been used to teach mathematics from primary school to university level [4]. The software is predominantly used to interactively explore the mathematical representations of an object. Although GeoGebra is rich in connection between abstract and concrete aspects, it lacks the 3D object modelling and visualization which is made available from version 5.0 beta.

Apart from GeoGebra, there exists another category of software that fall in the category of Dynamic Geometry Software (DGS) [5]. CABRI-GOMTRE [6] and Geometer's Sketchpad [7] are two instances of DGS. DGS provides a wide range of activities that prompt the learners to delve deeper and reflect on the construction and realization of the geometrical object [6]. This would not have been possible in conventional paper pencil setup. This contributes towards the "fields of experience" but the learner interaction is still limited to keyboard and mouse. In the teaching-learning of Geometry, learners find transformation between 2D to 3D and vice-versa difficult [8]. One such attempt [8] to bridge the transformation gap is by using Blender 3D software. The study aimed at improving the Mental Rotation (MR) capability of the students by drill and practice. But this lacked the gesture based interactivity. In the other study, the authors used gestures to map between rigid 3D structures to 2D static images [9]. However, the learner has to imagine the 2D image to 3D object transformation via gesture. It would be useful if the transformation can be visualized by the learner as and when the gesture is made.

B. Theories of Geometry Learning

Most research studies involving children's thinking of 3D solids are done using Van Hieles model of 3D geometric thinking. The model consists of five levels of understanding. They are "visualization", "analysis", "informal deduction", "formal deduction", and "rigor" [10]. These levels are in the increasing order of understanding. With our application, we intend learners to progress from the "visualization" level, where they can recognize 3D structures, to the "informal deduction" level, where they can derive properties, like volume and surface area of 3D structures. Understanding the properties of a solid is equivalent to understanding the characteristic parts of 3D structure [11]. Using our application, we allow the learner to construct the 3D shape using the constituent 2D primitives which helps the learner understand that 3D shapes can be constructed using 2D shapes. The assumption is that the learner is already exposed to the properties of the 2D shape. This understanding can now be extended to derive the properties of the 3D shapes.

Three-dimensional geometry thinking is closely connected to students' ability to calculate the volume and surface area of a solid [1]. Research findings showed that students mainly focus on the formulas and the numerical operations required in calculating the volume or the surface area of a solid and completely ignore the structure of the unit measure [12]. Our application helps learners come up with their own explanation of the volume of the 3D shape. This explanation is then extended to the formula of the 3D shape, which improves the geometry thinking and prevents rote learning of the formula.

Battista [13] suggested that students should develop two necessary skills to calculate conceptually the volume and surface area of a solid: (a) linking the formulae with the structure of the solid (b) visualizing the internal structure of the solid. The relationship between the formula and the structure of the solid is not emphasized while learning about 3D solids.

C. On the role of gestures

There is enough empirical evidence that the actions performed by a learner in the "field of experience" positively influences ones internal representations [14]. Gestures are special kind of actions that people produce when talking [15]. Gestures are representational modes that convey spatial, relational and embodied concepts [16]. Gestures are particularly good at expressing spatial and motor information [17]. Gestures have information about problem solving process as well as problem representation [18]. This aspect of gestures is used by defining gestures closest to the action that needs to be performed in real world to bring about the desired change to the system. So in our application, a slide gesture akin a revolving door, is used to cue the students about how the door - a rectangle, moves about a central axis to give right circular cylinder and how circles of unit height when stacked up yields a cylinder.

Attentional anchors are real or imagined objects that aid in channeling user's attention and thereby reducing operational complexity of an otherwise complicated task [16]. In our application, the trace of rectangles around central axis or stacked discs are the attentional anchors that enable learners to associate 2D primitives and the final 3D object. It also motivates the learner to think about deriving volume of the 3D object in terms of 2D primitives. Sense Making is aided by reflection on the consequences of the interaction with objects [19]. Reflection generally follows activities which include planning of what gesture can help the learner complete a given task, execution of the gesture, and analysis of the output seen as a consequence of enacting the gesture. An iterative interaction of such sequence of activities can help learner form new ideas. In our system, we expect that the activities followed by reflection, will help students identify the underlying concepts that go into deriving volume of the cuboid and cylinder.

Gesture usage in G-v-G, is expected to aid understanding of construction of 3D object from known 2D primitives. This understanding is expected to aid in discovery of parameters affecting volume of the 3D object. Learning 3D geometry by building over knowledge of 2D primitives using gestures has not been reported, to the best of our knowledge.

III. Design and Implementation of G-v-G

Geometry via Gestures (G-v-G) is a web based application, in which learners use gestures to construct 3D structures from their constituent 2D shapes. G-v-G uses the Leap Motion Controller device to detect gestures from the learner. There is an initial set of demo activities to familiarize the learner with the application and the Leap Motion Controller. After this, the learner proceeds to do three activities using G-v-G.

The first version of G-v-G consisted of a Java application, which we evaluated in a pilot study with four students. Based on feedback on usability and usefulness of the application, we refined our application to its current form. The main changes made included a shift from Java based desktop application to Javascript based web application, initial demo and scaffolding in the form of hints on type of gesture to be used.

A. Design of G-v-G

The activity screen of G-v-G application, is divided into the following areas as seen in Fig. 1.

1) Question Area - This is the space on the screen where the activity question is provided to the learner.
2) Activity Area - This is the space on the screen where the learner initially sees a 2D shape. Based on the learner's gestures, transformations occur in this area and finally a 3D structure is formed. A progress bar is also present, which indicates when the activity has been successfully completed.
3) Hint Area - This is the space on the screen where appropriate hints are provided to the learner. During the activities, learners might not be able to identify the right gesture to transform the 2D shape into a 3D structure. To help the learner proceed with the activity, an animation of the correct gesture is shown when the learner selects the hint button. The learner can use the hint at any time during the course of the activity.

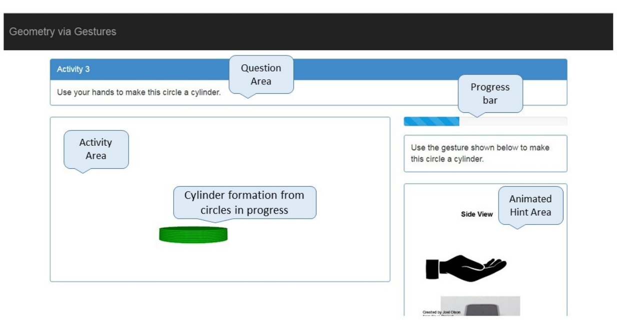

Fig. 1. G-v-G application

The learner interacts with the application by placing his/her hand above the Leap Motion Controller at a particular height. A circular cursor shows the position of the hand on the screen. The controller is capable of detecting hand movements and gestures of the learner. Based on these gestures, appropriate transformations occur on the screen.

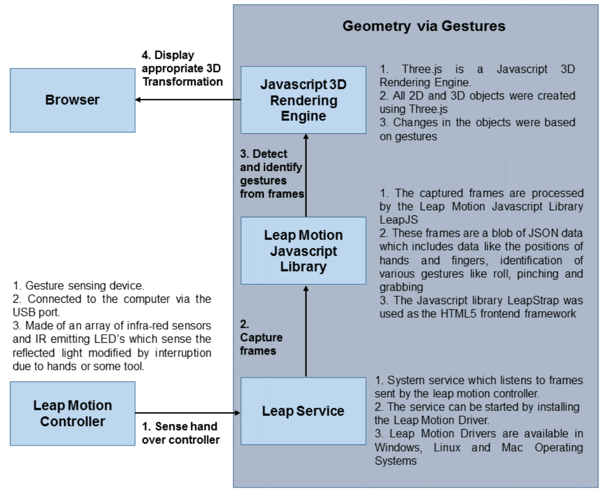

Fig. 2. Functional blocks of G-v-G

The different functional blocks that interact in the G-v-G application are as shown in Fig. 2.

B. Activities in G-v-G

Once a student gets familiar with manipulating on-screen objects with gestures, and viewing a 3D object from different perspectives, the actual intervention begins. This involves 3 activities that the student must complete using only hand gestures. The gestures in the activities that we designed, were aligned as closely as possible to natural intuition about how a 3D structure can be formed using 2D primitives. The activities in G-v-G span the first three levels of Van Hiele model [20], namely Visualization, Analysis and Abstraction. Formal deduction and Rigor are not the objectives of G-v-G and hence have not been addressed.

1) In the first activity, the student learns about formation of a 3D right circular cylinder by the rotation movement of a 2D rectangle. This enables the student to construct as well as look into the process of formation of 3D objects from the 2D primitive.
2) In the second activity, student learns about formation of cuboid by the translational movement of a rectangle. This enables the student to construct a 3D object from known 2D primitive and at the same time reflect on how area of rectangle can affect volume of the cuboid. The theoretical basis for this activity was Battista's suggestions [13] that it is necessary to develop skill of visualizing internal structure of solid and connecting formula of properties of solid with its structure.
3) In the final activity, the student learns about formation of 3D right circular cylinder by the translational movement of a disc. This enables the student to construct a 3D object from known 2D primitive and also think of volume of cylinder in terms of area of a circle.

As the student is interacting with the application, the facilitator poses questions which have been scripted. Questions posed by the facilitator are expected to make students reflect on what action they did, why they did that particular action/gesture, what happened to the object on screen, what insight did the student gain from this activity and finally can the student conceptually describe the volume property of the 3D object in terms of the 2D objects.

C. An instance of student interaction with G-v-G

Student X is quite studious and interested in Maths, Science and Technology. Before proceeding to the activities, the researcher explained the aim of G-v-G and allowed the student to try out the demo. The student took some time to figure out the position of the hand above the leap motion controller, but was then able to proceed through the demo.

The first activity involved changing the rectangle given on the screen to a cuboid. The horizontal swipe gesture had to be used. This gesture was not intuitive to the student. The student also asked if he could bend the rectangle. The researcher then prompted the student to use the hint on the screen. Using the hint, the student was able to complete the activity successfully. The second and the third activity involved using the vertical swipe gesture. In both these activities, the gesture to be used was quite intuitive, and the student was able to complete the activities in the first try, without the use of the hints provided.

IV. RESEARCH METHODOLOGY

The research questions that we are addressing in this study are:

1) What can students learn about the properties of 3D structure of objects after interacting with Geometry via Gestures?
2) What is students' perception of learning about 3D geometry after using Geometry via Gestures?

A. Participants

Our participants were students who had completed their 8th grade and begun with their 9th grade. Geometry as a subject is taught over several grades beginning from primary (4th grade) to high school. Students learn about 2D geometric structures, their area, and other properties during their 8th grade. At the beginning of their 9th grade, they are not yet exposed to properties of 3D structures such as volume and surface area. Thus, students who have just begun their 9th grade, would make for appropriate participants for learning properties of 3D structure using Geometry via Gestures.

We used convenience sampling and selected a diverse set of 9 students who were from different schools, education boards and achievement levels, from schools near our Institute. While students 1,2 and 3 were from SSC board, the rest were from ICSE board. In terms of achievement levels, there were 2 high achieving, 3 medium achieving and 4 low achieving students. Our study aimed at exploring what students can learn from our intervention, regardless of their achievement levels and background.

B. Procedure of study

Our study was exploratory in nature and intended to examine the effectiveness of the G-v-G intervention on learning about 3D geometry. Before beginning the intervention, an informed consent was obtained from the student as well as the guardian since the students were minors. A facilitator was present along with the student during the study. The role of the facilitator included prompting, scaffolding and guiding the student through activities designed in the application. A list of reflection questions was prepared, that the facilitator (one of the authors of this paper) would ask the student during the intervention. The questions were semi-structured but sufficiently open-ended to allow for detailing, digressions, and exploration in order to elicit rich response from the student.

The study was conducted in a room with the facilitator and the student present. A laptop with the Geometry via Gestures application loaded on it was placed before the student along with the leap motion controller. The activities of the student was captured using screen-capture software called 'Kazam'[1] and the verbal interaction of the student with the facilitator was audio recorded. The facilitator began the study with an overview of the application, and a demo activity built into the application. The objective of the demo was to familiarize the student with using the leap motion controller to interact with the Geometry via Gestures application.

The students then proceeded to the activities in the application. Three activities were designed in the application and each activity was followed by discussion between the student and the facilitator, and mainly comprised of reflection on the activity performed. On completion of the activities, students were given a post-test that tested them for transfer. A perception questionnaire was also included so as to obtain student perception about the application and learning.

C. Data sources and Instruments

The instruments used in the study included

1) Post-test - The post-test comprised mainly of questions that evaluated the ability of the students to transfer the knowledge they gained from the activities in G-v-G, to a new scenario. The answers were expected to validate the insights on learning obtained from the semi-structured interview. Questions included identification of 2D shapes that contribute to formation of a 3D object, identification of likely 3D objects that could be constructed from a given 2D object and reasoning about the volume of a given 3D object. Rubrics with three levels of measurement - Target, needs improvement and poor, were developed to evaluate responses to the post-test questions. Table I shows the rubrics for the post-test questions.
2) Semi structured interview questions - A set of questions were prepared in order to determine level of prior knowledge, identify misconceptions if any, delve deeper into student thought process during activity,

[1] https://apps.ubuntu.com/cat/applications/kazam/

TABLE I
RUBRICS USED TO EVALUATE STUDENTS' POST-TEST RESPONSE

Question	Target	Needs improvement	Poor
What do you think are the 2D shapes that can combine to form the given structure (3D Hexagon)	Hexagon or Hexagon and rectangle or Triangle and rectangle	Rectangle only or Triangle only	No 2D shape mentioned
What 3D structure do you think you can build out of the 2D figure (Triangle)	Prism or Pyramid or Picture of Prism	Cone or 3D triangle	No 3D structure mentioned
What do you think is the volume of the following 3D structure (3D Pentagon)	Identification of volume concept as product of area of base (pentagon) and height or Identification of area of pentagon as a contributor to volume of 3D object, and identification of height of 3D object being contributed by stacking of n pentagons and identification of n pentagons forming the height of 3D object	Either identification of the 2D structure's area as contributor of volume Or identification of height as contributor of volume or Reasoning about contribution of height in the form of stacking of number of base structure	Writing volume of other 3D figures such as cuboid

and elicit knowledge constructed if any after a given activity. The content of the questions were validated by fellow researchers. Responses to these questions helped to identify things that students could learn from the intervention thereby answering the first research question. The questions were broadly categorized as a) pertaining to interaction with the system, which dealt with types of gesture used, the reason for using the gesture and effect of gesture on the on-screen object; b) prior knowledge questions on area and volume and c) specific questions on structure and volume of cuboid and cylinder. In addition students perception about the application was also obtained using interview questions at the end of the activity.

3) Perception questionnaire - Students perception about their learning after using G-v-G was obtained using a questionnaire that had one open-ended question on what students learnt using G-v-G application and one 5-point Likert scale (Strongly agree, agree, neutral, disagree and strongly disagree) question on their willingness to learn other topics of Geometry using G-v-G. The perception questionnaire was adapted from the System Usability Scale literature [21].

D. Data analysis technique

In this study, we collected and used data from multiple sources to answer our research questions. To answer RQ1, rubrics were used to score the post-test responses, and thematic analysis was used to analyze the verbal interaction between the student and the facilitator. Thematic analysis is a rich and detailed qualitative analysis method, that involves identifying, analyzing and reporting patterns or themes, within the given data [3]. To do thematic analysis, we transcribed the interview data and coded them by identifying themes which were related to the process of learning using G-v-G. Post-test and interview responses were initially independently analyzed and subsequently they were analyzed together to draw inferences regarding common patterns observed between interview and post-test responses. To answer RQ2, responses to the questions on the perception questionnaire, along with responses to the relevant interview questions at the end of the activity, were analyzed.

V. RESULTS

A. Results related to students' learning from G-v-G (RQ1)

1) Results from post-test: The post-tests were scored using the rubric shown in Table 1 to identify which performance level the student had reached (target, needs improvement or poor performance) for each question on the post test. Fig. 3 summarizes student post-test performance in terms of how many students reached each level of performance on each post-test item.

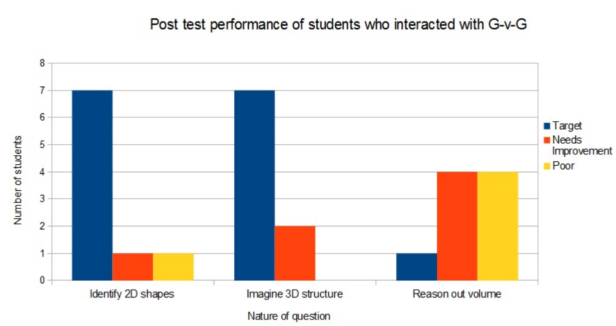

Fig. 3. Post-test performance of students who interacted with G-v-G

In Fig. 3 we show a bar chart of the number of students who have scored Target, Needs improvement and Poor on each rubrics item. It was observed that 7 out of 9 students could identify 2D shapes that combine to form the given 3D object, and imagine 3D structure that could be formed from given 2D shape, successfully. However, while only 1 student could reason about the volume of the given 3D object successfully, 4 students correctly identified at least one factor that would affect the volume of the 3D object. The remaining 4 students wrongly identified the volume of the given 3D object as the volume of a cuboid.

2) Observations from interview responses: The transcripts of the interviews were coded and thematic analysis was done. To ensure validity of the codes, two authors of the paper independently coded the transcripts and there was about 75% agreement to the codes. Thematic analysis of responses to the interview questions, gave valuable insights on the process of learning when using the G-v-G application. A number of themes emerged from students statements regarding the different ways in which performing activities in G-v-G helped them in their knowledge construction. Table II identifies themes from the interview transcripts of the students, that indicates some of the ways in which students explained their observations while they performed different activities in G-v-G. This is supported by key quotes made by students while interacting with G-v-G.

TABLE II
THEMATIC ANALYSIS TO IDENTIFY PROCESS OF LEARNING FROM G-V-G

Theme	Indicative instances from student quotes during interview
Intuitive usage of gesture for constructing cylinder from circle	I did the gesture because I wanted to - 1. “Pile more shapes.”, 2. “Make it big.”, 3. “Stretch the shape.”, 4. “Increase the volume.”
Connect commonly encountered structure and dynamics to explain 3D in terms of 2D.	1. “A number of rectangles were formed in a circular way.” 2. “The rectangles ended up at the same place that they started from.” 3. “Rectangles are arranged in a round manner. One end of the rectangles intersects at the middle.” 4. “Combined rectangles in a round form and gave shape in the form of a circle.” 5. “It looks like a book.” (during formation process) 6. “Its like a paper when you fold it and open it you get a cylinder.” 7. “We can spin it.” (reasoning before formation process)
“Give height” to flat 2D object to create 3D object	1. “Many rectangles come one on top of other and they rise to form cuboid.” 2. “Gesture caused layers to increase double, multiply and become a cuboid.” 3. “Cuboid has same length and breadth as rectangle. By adding rectangles, we are stretching it. When we stretch rectangle, we get height.” 4. “I gave height by adding more circles to form a cylinder.”
Reasoning on why 3D objects have volume while 2D objects do not.	1. “Basically volume of rectangle is zero as height is zero. But in cuboid, there is height and so it has volume.”
Multiple ways of forming cylinder from rectangle	1. “You can join a square to form a cylinder or by stacking circles you can make a cylinder.”
Viewing cylinder from all directions	1. “This is good as we can see it from all directions 360 degrees. It looks like circle from one side and cylinder from other side.”

During the study it was observed that 7 out of 9 students got at least one of the two gestures right. This was seen especially in the formation of cuboid from rectangle and cylinder from circle.

3) Results from post-test and interview responses: Post-test responses and relevant interview sections during the intervention, were compared to gain insight into the correlation between the G-v-G activity and the corresponding post-test performance. Specifically, we examined the extent to which students learnt a given concept during the G-v-G activity and how much prompting (and of what nature) was required from the facilitator for that. The correlation was observed for two broad categories of post-test questions viz. 3D structure formation (identification of 2D shapes that contribute to formation of a 3D object, identification of likely 3D objects that could be constructed from a given 2D object) and reasoning about the volume of a given 3D object. We present in Fig. 4 and Fig. 5, the different variations in performances of students in post-test when they sought different amounts of prompts during intervention. The variations seen can be classified into four groups viz. students who required much prompting and went on to do well in the post-test, students who needed more prompting but failed to perform well in the post-test, students who did not need more prompting during intervention but still performed well in the post-test and finally students who did not require much prompting during the intervention but did not do well in post-test. We refer to the nine students using numerals 1-9. We qualify less prompting as straight-forward questions that conveyed an idea which was immediately grasped by the student. On the other hand, more prompting required facilitator to convey ideas by multiple reflection questions or focusing prompts.

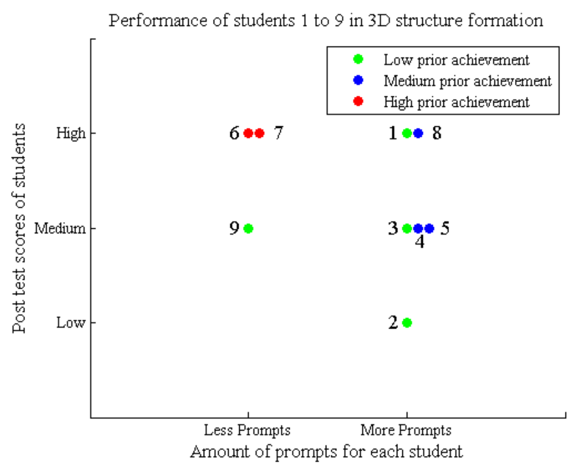

Fig. 4. Post-test score on 3D structure formation versus amount of prompts

From the graphs in Fig. 4 and Fig. 5, we observe that most students need some prompting from facilitator along with the application to learn. By 'learn', here we imply gradual improvement in the performance of student from intervention to post-test. From the graphs, we can also see that with adequate prompting, the performance of low achievement level students (Student 1, 3, 9) was comparable to medium achievement level students (Student 4, 5, 8) in

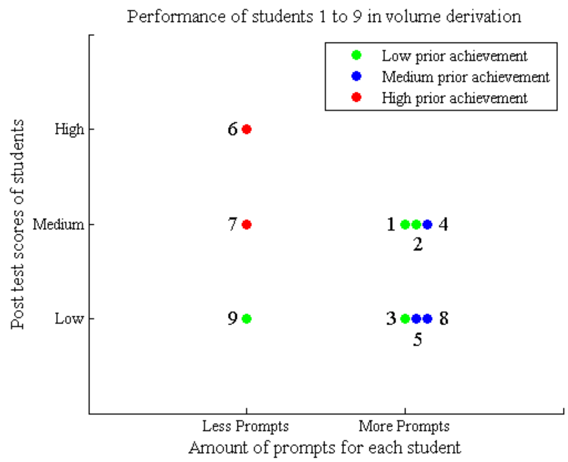

Fig. 5. Post-test score on volume derivation versus amount of prompts

question category of structural formation of 3D objects. Here, 5 out of 9 students benefited from prompting. However, both low and medium achievement level students had difficulty in question category of deducing volume of 3D object. Only 3 out of 9 students benefited from prompting and 3 students did not benefit even after extensive prompting from the facilitator. The high achieving students (Student 6, 7) needed less prompting as compared to others and learnt the concept. Since prompts from the facilitator had some effect on learning among low and medium achievement students, we looked into the types of prompts and classified them into 5 main categories. Table III lists the categories of prompts and example instances from the interview.

TABLE III
CLASSIFICATION OF PROMPTS AND SAMPLE INSTANCES

Prompt type	Example instances from interview
Concept related	"Area of circle is pi r squared. So volume of cylinder is?"
Process related	"What happened on the screen when you did the gesture?", "How are the rectangles arranged?"
G-v-G usage related	"Try moving your hands faster"
Recall	"What is the area of the rectangle?"
Reflection	"What do you think is the difference between a 2D and 3D"

From the transcripts, it was also observed that recall, reflect and process related prompts were the most useful and required by the students, most of the time.

B. Results related to students' perception (RQ2)

Student perception of learning after using G-v-G, was obtained using a questionnaire. Out of 9 students, 7 strongly agreed and 2 agreed that G-v-G would be suitable application for learning other topics in Geometry. Some of the things that students perceived they learnt after using G-v-G were:

- "2D structures have area and 3D structures have volume."
- "We can find volume with the help of given area and height."
- "I learnt about 3D shapes practically."
- "I learnt how 3D shapes can be made out of 2D shapes."
- "I learnt how volume is dependent on area."

Some of the statements regarding students perception of their own learning that emerged during interview are tabulated in Table IV.

TABLE IV
THEMATIC ANALYSIS TO IDENTIFY PERCEPTION OF LEARNING AMONG STUDENTS AFTER USING G-V-G

Theme	Student statements
About gesture	"By doing with hand, we can understand faster." "I think concepts remains in mind for longer." "It was good but hands pain." "This is more interesting. Because we are doing it ourselves whereas there [with mouse] the process is automatic."
About 3D visualization	"We could move it in different directions. I could understand, I could visualize the image."
About G-v-G application as a whole	"Not bookish, it was practical learning. When we learn actual Maths we solve problems. This was a practical way of learning Maths. You showed 2D figure and then a 3D figure and then how to make 3D from 2D. The differences were very clear." "You can do science subjects. Physics and Chemistry"

VI. DISCUSSION

The first research question of this study was - What can students learn about the properties of 3D structure after interacting with Geometry via Gestures? From the results, we can see that students are able to learn about construction of 3D structure from 2D shape and also identify the possible 3D structures that could be constructed from a given 2D shape. It is our conjecture that the act of doing task appropriate gesture and visualizing the intermediate structures during the formation of 3D object from 2D primitive, helped students in structural understanding of 3D objects by adding to their "fields of experience". G-v-G was also partially successful in helping them reason about the volume of the 3D object from the structural information. The results also show the pivotal role that a facilitator plays while the student is performing different activities in G-v-G. We also identified the nature of the prompts necessary for sense-making and reflection.

The G-v-G application provides the affordance of visualization and manipulation of 3D structure from 2D primitives. Learning to reason about the volume of 3D structure however needs reflection. From the results, we observe that without the facilitator, students rarely reflect on the activity that they performed. Therefore, reflection prompts are vital for the sense-making process of a student during which the student links prior knowledge to new knowledge thereby creating new knowledge [19]. In the absence of appropriate reflection prompts, students tend to undermine the importance of the role of an event or a process thereby focusing more on the big picture rather than the fine grained details. The student consequently loses out on knowledge construction or knowledge integration. In our intervention, reflection prompts posed by facilitator encourage students to reflect on the process and reason about the properties. The prompts help students tie their prior knowledge to what they do and see during the activity.

The implication is that G-v-G application alone is not sufficient for promoting the learning process. This can be seen by the fact that until the facilitator prompted the students with questions, they did not reflect deeply on their activities. Moreover, facilitator alone is not sufficient for the learning process. Given appropriate prompts, most of the students learnt the concepts after only one round of prompting by the facilitator. It was observed from the transcripts that once the students grasped the idea of linking area of the 2D shape with the volume of 3D object for cylinder, they tried to apply the same principle for deriving volume of cylinder constructed from circles. In the future, appropriate facilitator prompts can be built into the G-v-G application.

According to Markopoulos et al [11], once students understand the characteristic parts of 3D shapes, they can understand the properties of a solid. The post-test results confirm that using G-v-G, students were able to identify characteristic parts of 3D shapes. Correlation between post-test and interview responses indicate that 5 students out of 9 were able to identify at least one characteristic part of the given 3D object that contribute to volume of the 3D object.

Gestures hold information on both problem representation as well as the problem solving process [18]. The intuitive use of task appropriate gestures by 7 out of 9 students based on some problem solving approach for the activity, indicates the value of gestures in the learning process. This is corroborated by student responses to interview questions. The second RQ was - What is students' perception of learning about 3D geometry after using Geometry via Gestures? From the results about student perception on learning, it is evident that most of the students find G-v-G an interesting and useful application to learn about 3D geometry.

One of the main logistics related limitation of G-v-G intervention is the need to train teachers so that they can facilitate effectively. In terms of the nature of the study, the small sample size implies non-representativeness among the targeted population leading to lack of statistical generalizability to the results. Though our study aimed at exploring what students could learn from their interactions with G-v-G, regardless of student background and achievement levels, a more rigorous study with larger N and random sampling, would validate the claims and implications that we have drawn from this study.

Despite small N (N=9), we have analyzed data from multiple measures, triangulated post-test scores with interview data and student perception. Thus given the richness of data and multiple methods of analysis, we claim that G-v-G is an effective TEL environment for students to learn the nature of 3D objects and fundamental concepts regarding volume of 3D objects. Hence we recommend that teachers can use it as an additional resource in Mathematics classrooms.

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