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Q> $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

→ Preprocessing

$\rightarrow F(x,y) \times (-1)^{x+y}$

$$\begin{bmatrix} (-1)^{0+0} & (-1)^{0+1} & (-1)^{0+2} & (-1)^{0+3} \\ (-1)^{1+0} & (-1)^{1+1} & (-1)^{1+2} & (-1)^{1+3} \\ (-1)^{2+0} & (-1)^{2+1} & (-1)^{2+2} & (-1)^{2+3} \\ (-1)^{3+0} & (-1)^{3+1} & (-1)^{3+2} & (-1)^{3+3} \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

{ Butterworth Highpass
Gaussian Highpass }

$$\begin{bmatrix} -1 & 0 & -1 & 0 \\ -1 & 0 & -1 & 0 \\ -1 & 0 & -1 & 0 \\ -1 & 0 & -1 & 0 \end{bmatrix}$$

(2) DFT

Kernal $\times$ $F(x,y)$ $\times$ Kernal^T

$$\text{Kernal} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & j \end{bmatrix}$$

$$F(u,v) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \leq D_0 \\ 1 & D(u,v) > D_0 \end{cases}$$

$$D(u,v) = \sqrt{\left(u - M/2\right)^2 + \left(v - M/2\right)^2}$$

$$M = 4, \quad N = 4$$

$$D(u,v) = \sqrt{(u-2)^2 + (v-2)^2}$$

$$D(0,0) = \sqrt{(0-2)^2 + (0-2)^2} = \sqrt{8} = 4.24$$

$$D(0,1) = \sqrt{(0-2)^2 + (1-2)^2} = \sqrt{5} = 2.23$$

$$D(0,2) = \sqrt{(0-2)^2 + 0^2} = 2$$

$$D(1,0) = \sqrt{(1-2)^2 + (-2)^2} = \sqrt{5} = 2.23$$

$$D(1,1) = \sqrt{(1-2)^2 + (1-2)^2} = \sqrt{2} = 1.414$$

$$D(1,2) = \sqrt{(1-2)^2 + (0)^2} = 1$$

$$D(1,3) = \sqrt{(1-2)^2 + (3-2)^2} = \sqrt{2}$$

$$D(2,0) = \sqrt{(0)^2 + (-2)^2} = 2$$

$$D(2,1) = \sqrt{0 + 1^2} = 1$$

$$D(2,2) = \sqrt{0} = 0$$

$$D(2,3) = \sqrt{0+1} = 1$$

$$D(3,0) = \sqrt{1 + (-2)^2} = \sqrt{5} = 2.23$$

$D(3,1) = \sqrt{1+1} = \sqrt{2}$

$D(3,2) = \sqrt{1} = 1$

$D(3,3) = \sqrt{2} = 1.414$

$$D(u, v) = \begin{bmatrix} 2.82 & 2.23 & 2 & 2.23 \\ 2.23 & 1.41 & 1 & 1.41 \\ 2 & 1 & 0 & 1 \\ 2.23 & 1.41 & 1 & 1.41 \end{bmatrix}$$

$D_0 = 0.5$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$G(u, v) = F(u, v) * H(u, v)$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

$$\boxed{G(u, v) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

IDFT

$$g(x,y) = \frac{1}{4}\,\text{Kernal} \times G(U,V) \times \frac{1}{4}\,\text{Kernal}^{T}$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\times \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$g(x,y) = \frac{8}{16}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \end{bmatrix}$$

$$= g(x,y) = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

Butter Worth Soln. (High Pass)

$$H(u,v) = \frac{1}{1+\left[D_0 / D(u,v)\right]^{2n}} \qquad n=2$$

$$H(u,v) = 1 - e^{-D^2(-u,v)^{2n}} \qquad n=2$$

So from last question we have

$$D(-u,v) = \begin{bmatrix} 2.82 & 2.32 & 2 & 2.32 \\ 2.32 & 1.41 & 1 & 1.41 \\ 2 & 1 & 0 & 1 \\ 2.32 & 1.41 & 1 & 1.41 \end{bmatrix}$$

$$H(u,v) = \frac{1}{1+\left(\frac{D_0}{D(u,v)}\right)^{2n}}$$

$\because n=2$ & $D_0 = 0.5$

$$\text{So } H(0,0) = \frac{1}{1+\left(\frac{0.5}{2.82}\right)^4} = 0.999$$

$$H(0,1) = \frac{1}{1+\left(\frac{0.5}{2.32}\right)^4} = 0.997$$

$H(0,3) = 0.997$

$H(1,0) = 0.997$

$H(1,1) = 0.984$

$H(1,2) = 0.941$

$H(1,3) = 0.984$

$H(2,0) = 0.996$

$H(2,1) = 0.941$

$H(2,2) = 0$

$H(2,3) = 0.941$

$H(3,0) = 0.997$

$H(3,1) = 0.984$

$H(3,2) = 0.941$

$H(3,3) = 0.989$

So
$$H(u,v) = \begin{bmatrix} 0.999 & 0.997 & 0.996 & 0.99 \\ 0.997 & 0.984 & 0.941 & 0.984 \\ 0.996 & 0.941 & 0 & 0.941 \\ 0.997 & 0.989 & 0.941 & 0.989 \end{bmatrix}$$

Now

$$G(u,v) = F(x,y) * H(u,V)$$

$$F(u,v) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

&

$$G(u,v) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 7.96 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now IDFT

$$g(x,y) = \frac{1}{4} \times \text{Kernel} \times G(u,V) \times \frac{1}{4} \text{Kernel}^T$$

$H(0,3) = 0.997$

$H(1,0) = 0.997$

$H(1,1) = 0.984$

$H(1,2) = 0.941$

$H(1,3) = 0.984$

$H(2,0) = 0.996$

$H(2,1) = 0.941$

$H(2,2) = 0$

$H(2,3) = 0.941$

$H(3,0) = 0.997$

$H(3,1) = 0.984$

$H(3,2) = 0.941$

$H(3,3) = 0.989$

So $H(u,v) = \begin{bmatrix} 0.999 & 0.997 & 0.996 & 0 \\ 0.997 & 0.984 & 0.941 & 0.984 \\ 0.996 & 0.941 & 0 & 0.941 \\ 0.997 & 0.989 & 0.941 & 0. \end{bmatrix}$

Now

$G(u,v) = F(x,y) * H(u,v)$

$F(u,v) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

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& $G(u,v) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 7.96 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Now IDFT

$g(x,y) = \frac{1}{4} \times \text{Kernel} \times G(u,v) \times \frac{1}{4}\, \text{Kernel}^T$

So

$$\frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 7.96 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \frac{1}{16}\begin{bmatrix} 7.96 & 0 & 0 & 0 \\ -7.96 & 0 & 0 & 0 \\ 7.96 & 0 & 0 & 0 \\ -7.96 & 0 & 0 & 0 \end{bmatrix}$$

$$= \frac{1}{16}\begin{bmatrix} 7.96 & 7.96 & 7.96 & 7.96 \\ -7.96 & -7.96 & -7.96 & -7.96 \\ 7.96 & 7.96 & 7.96 & 7.96 \\ -7.96 & -7.96 & -7.96 & -7.96 \end{bmatrix}$$

$$= \frac{7.96}{16}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \end{bmatrix}$$

Post Processing $\Rightarrow$

$$(-1)^{x+y} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

$$\rightarrow g(x,y) * (-1)^{x+y}$$

$$= \frac{7.96}{16} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

$$= \frac{7.96}{16} \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \text{ Ans}$$