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CAMBRIDGE INSTITUTE OF TECHNOLOGY

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Department of Basic Sciences

Preparatory Examination - Odd Semester 2018-19

Sub. Name: Calculus and Linear Algebra **Sub. Code**: 18MAT11 **Semester**: I

Date: 07-01-2019 **Time:** 9:00 AM **Duration**: 3 Hours **Max. Marks**: 100

NOTE:

Answer five full questions, choosing one from each module, each full question carries maximum of 15 Marks.

Sl. No.	QUESTIONS	COs	RBT Levels	Marks
1	**Module - I**			
	a) Find the Pedal equation of the curve $\frac{2a}{r} = 1 + \cos\theta$.	CO1	L1	06M
	b) Determine the center of curvature of the parabola $y^2 = 4ax$ at (x, y). Also find the equation of the evolute of the given parabola.	CO1	L2	07M
	c) Determine the radius of curvature of the curve $y^2 = \frac{a^2(a-x)}{x}$ $at\,(a,0)$.	CO1	L2	07M
	OR			
2	a) Find the Pedal equation of the curve $r = a\,\mathrm{cosec}^2\frac{\theta}{2}$.	CO1	L1	06M
	b) Determine the angle of intersection between the curves $r^n = a^n(\sin n\theta + \cos n\theta)$ and $r^n = a^n \sin n\theta$.	CO1	L2	07M
	c) Show that the square of radius of curvature of the curve $r(1-cos\theta) = 2a$ varies as r^3.	CO1	L2	07M
3	**Module - II**			
	a) Find the value of the indeterminate form $\lim_{x\to\frac{\pi}{2}} (\tan x)^{\cos x}$.	CO2	L1	06M
	b) If $u = f(2x-3y, 3y-4z, 4z-2x)$ prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$.	CO2	L2	07M
	c) Determine the stationary values of $f(x,y,z) = x^2y^3z^4$ subject to the condition $x+y+z=5$ using Lagrange's method of undetermined multipliers.	CO2	L2	07M

	OR			
4	a) Express $\tan^{-1}x$ in Maclaurin's series up to the terms containing fifth degree.	CO2	L1	06M
	b) Determine the total derivative of $u=\tan^{-1}\left(\frac{x}{y}\right)$ where $x=2t, y=1-t^2$.	CO2	L2	07M
	c) Determine the maximum and minimum values of $\sin x \sin y+\sin(x+y)$.	CO2	L2	07M
5	**Module - III**			
	a) Determine the value of $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dxdy$ by changing into polar coordinates.	CO3	L2	06M
	b) Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.	CO3	L2	07M
	c) Use double integration to find the volume of a tetrahedron bounded by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and the coordinate planes.	CO3	L3	07M
	OR			
6	a) Determine the value of $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$.	CO3	L2	06M
	b) Determine the value of $\int_0^1\int_x^{\sqrt{x}} xy\,dydx$ by changing the order of integration.	CO3	L2	07M
	c) Use double integration to find the area between the parabolas $y^2=4ax$ and $x^2=4ay$.	CO3	L3	07M
7	**Module - IV**			
	a) Find the general solution and singular solution of $xp^2+px-py+1-y=0$.	CO4	L1	06M
	b) A series circuit with resistance R, inductance L and electromotive force E is governed by the differential equation $L\frac{di}{dt}+Ri=E$, where L and R are constants and initially the current' i ' is zero. Determine the current at any time t.	CO4	L2	07M
	c) Solve $\frac{dy}{dx}-\frac{dx}{dy}=\frac{x}{y}-\frac{y}{x}$.	CO4	L3	07M

	OR			
8	a) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$, where '$\lambda$' being the parameter.	CO4	L1	06M
	b) If the temperature of air is $30^\circ C$ and a metal ball cools from $100^\circ C$ to $70^\circ C$ in 15 minutes, determine the time taken by the metal ball to reach a temperature of $40^\circ C$?	CO4	L2	07M
	c) Determine the general and singular solution of $(px-y)(x-py)=2p$, by using the substitution $x^2=X$ and $y^2=Y$.	CO4	L3	07M
9	**Module - V**			
	a) Test for consistency and hence find the solution for the system of linear equations $x+2y+3z=14;\ 4x+5y+7z=35;\ 3x+3y+4z=8$.	CO5	L1	06M
	b) Determine the largest eigen value and the corresponding Eigen vector for the given matrix by Rayleigh power method, considering $[1\quad 0\quad 0]'$ as an initial eigen vector(carry out 6 iterations) : $\begin{bmatrix}2&0&1\\0&2&0\\1&0&2\end{bmatrix}$.	CO5	L2	07M
	c) Solve the following system of equations using Gauss-Jordan method $x+4y-z=-5;\ x+y-6z=-12;\ 3x-y-z=4$.	CO5	L3	07M
	OR			
10	a) Find the values of λ and μ for which the following system of equations has (i) no solution (ii) a unique solution and (ii) infinite number of solutions $x+y+z=6;\ x+2y+3z=10;\ x+2y+\lambda z=\mu$.	CO5	L1	06M
	b) Reduce the matrix A=$\begin{bmatrix}-19&7\\-42&16\end{bmatrix}$ into diagonal form.	CO5	L2	07M
	c) Find the solution for the given system of equations using Gauss-Seidel iterative method $2x-3y+20z=25;\ 20x+y-2z=17;\ 3x+20y-z=-18$.	CO5	L3	07M

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