USN 1CD18CS106

CAMBRIDGE INSTITUTE OF TECHNOLOGY

K.R. PURAM, BENGALURU-560036

CAMBRIDGE

Department of Basic Sciences

Program: B.E. ■ M.Tech. □ Specialization:

Preparatory Examination - Even Semester 2018-19

Sub. Name: Advanced Calculus & Numerical Methods **Sub. Code**: 18MAT21 **Semester**: II

Date: 13-06-2019 **Time:** 1:15 PM **Duration**: 3 Hours **Max. Marks**: 100

[**Instructions**: Answer any five full questions, choosing one from each module, each full question carries maximum 20 marks]

Sl. No.		QUESTIONS	COs	RBT Levels	Marks
		Module 1			
1.	a)	Find the directional derivative of $\varphi = 4xz^3 - 3x^2y^2z$ at $(2,-1,2)$ along $2\hat{i} - 3\hat{j} + 6\hat{k}$.	CO1	L1	04M
	b)	Prove that the vector $\bar{F} = \dfrac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational.	CO1	L2	08M
	c)	Determine the total work done by the force $\bar{F} = (3x^2 + 6y)\hat{i} - 14yz\,\hat{j} + 20zx^2\hat{k}$ along the curve given by $x = t, y = t^2, z = t^3$ and $0 \le t \le 1$.	CO1	L3	08M
		OR			
2.	a)	Find the constants a, b, c so that the vector function $\bar{F} = (x + 2y + az)\hat{i} + (bx - 3y + z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational. Hence find the scalar function φ such that $\bar{F} = \nabla\varphi$.	CO1	L1	04M
	b)	Determine the value of the integral $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ using Green's theorem in a plane, where C is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$.	CO1	L2	08M
	c)	Use divergence theorem to evaluate $\iint_S \bar{F}.\hat{n}\,ds$ over the surface of the region above xy plane, bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 4$ where $\bar{F} = 4xz\,\hat{i} + xyz^2\hat{j} + 3z\,\hat{k}$.	CO1	L3	08M

		Module II			
3.	a)	Find the solution of $\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+11\frac{dy}{dx}-6y=3e^x$.	CO2	L1	04M
	b)	Determine the solution of $\frac{d^2y}{dx^2}+y=\frac{1}{1+\sin x}$ using the method of variation of parameters.	CO2	L2	08M
	c)	Solve: $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+9y=3x^2+\sin(3\log x)$.	CO2	L3	08M
		OR			
4.	a)	Find the solution of $(D^3+8)y=x^4+2x+1$ where $D=\frac{d}{dx}$.	CO2	L1	04M
	b)	Determine the solution for the equation : $(1+x)^2y''+(1+x)y'+y=2\sin[\log(1+x)]$.	CO2	L2	08M
	c)	In an L-C-R circuit, the charge q on a plate of a condenser is given by $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=E\sin pt$. The circuit is tuned to resonance so that $p^2=\frac{1}{LC}$. If initially the current i and the charge q be zero, prove that for the small values of R/L, the current in the circuit at time t is given by (Et/2L) Sinpt.	CO2	L3	08M
		Module III			
5.	a)	Find the partial differential equation by eliminating the arbitrary function from $z=yf(x)+x\varphi(y)$.	CO3	L1	04M
	b)	Determine the solution of $\frac{\partial^2u}{\partial x\partial t}=e^{-t}\cos x$, given that $u=0$ *when* $t=0$ and $\frac{\partial u}{\partial t}=0$ at $x=0$.	CO3	L2	08M
	c)	Derive one dimensional wave equation in the standard form.	CO3	L3	08M
		OR			
6.	a)	Find the partial differential equation by eliminating the arbitrary constants from $(x-a)^2+(y-b)^2+z^2=c^2$.	CO3	L1	04M
	b)	Determine the solution of $\frac{\partial^2z}{\partial x^2}=\sin x\sin y$ given $\frac{\partial z}{\partial y}=-2\sin y$ when $x=0$ and $z=0$ when y is an odd multiple of $\frac{\pi}{2}$.	CO3	L2	08M
	c)	Solve one dimensional heat equation using the method of separation of variables.	CO3	L3	08M

		Module IV			
7.	a)	Find the nature of the series: $\sum_{n=1}^{\infty} \frac{n!}{(n^n)^2}$.	CO4	L1	04M
	b)	Express $f(x) = x^4 + 3x^3 + 2x^2 - x - 3$ in terms of Legendre polynomial.	CO4	L2	08M
	c)	Determine the solution of Bessel's differential equation leading to $J_n(x)$.	CO4	L3	08M
		OR			
8.	a)	Find the nature of the series : $\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$.	CO4	L1	04M
	b)	Determine the solution of Legendre's equation leading to $P_n(x)$.	CO4	L2	08M
	c)	Use Rodrigue's formula to show that $P_3(\cos\theta) = \frac{1}{8}(3\cos\theta + 5\cos 3\theta)$.	CO4	L3	08M
		Module V			
9.	a)	Find the real root of the equation $xe^x - \cos x = 0$ correct to 3 decimal places which lies between 0 and 1 by using Regula Falsi method.	CO5	L1	04M
	b)	Determine the value of the integral $\int_0^{\pi/2} \sqrt{\sin x}\, dx$ by using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule and dividing the interval into ten equal parts.	CO5	L2	08M
	c)	Use an appropriate interpolation formula to compute $f(98)$ for the following data:	CO5	L3	08M

x	80	85	90	95	100
f(x)	5026	5674	6362	7088	7854

		OR			
10.	a)	Find a real root of the equation $x \log_{10} x - 1.2 = 0$, near x=2.5 using Newton-Raphson method correct to 4 decimal places.	CO5	L1	04M
	b)	Determine the interpolating polynomial using divided difference formula for the following data:	CO5	L2	08M

x	0	1	2	3	4	5
f(x)	3	2	7	24	59	118

	c)	Use Weddle's rule to evaluate $\int_0^1 \frac{1}{1+x^2}\, dx$ by taking seven ordinates and hence find an approximate value of π.	CO5	L3	08M